Principles of Mathematical Analysis Test

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May 11, 2017

1 The Real and Complex Number Systems

1.1 Theorem *If a and b are real, then $(a, b) = a + bi$.*

Proof

$$\begin{aligned} a + bi &= (a, 0) + (b, 0)(0, 1) \\ &= (a, 0) + (0, b) = (a, b). \end{aligned}$$

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1.2 Notation If $x_1, \dots, x_n$ are complex numbers, we write

$$x_1 + x_2 + \cdots + \sum_{j=1}^{n} x_j.$$

$$i^2 = -1 \tag{1}$$

1.3 Theorem *Closed subsets of compact metric spaces are compact.*

Proof Let E be a closed subset of the compact metric space X. For any open cover $\{V_\alpha\}$ of E we may append the open set E^C to obtain an open cover of X. This open cover of X may be reduced to a finite subcover of X, and thus of E. If E^C is not present in this finite open cover of E, then the open cover is a subcover of $\{V_\alpha\}$. If E^C is present, then removing it leaves us a finite open cover of E that is a subcover of $\{V_\alpha\}$. It follows that E is compact. ■