R Notebook

```
knitr::opts_chunk$set(fig.show = "all", results = "hide")
```

Instructions: You may discuss the homework problems in small groups, but you must write up the .nal solutions and code yourself. Please turn in your code for the problems that involve coding. However, code without written answers will receive no credit. To receive credit, you must explain your answers and show your work. All plots should be appropriately labeled and legible, with axis labels, legends, etc., as needed.

1. In this problem, you will generate data with p = 2 features and a qualitative response with K = 3 classes, and n = 50 observations per class. You will then apply linear discriminant analysis to the data.

(a) Generate data such that the distribution of an observation in the kth class follows a N(?k, S) distribution, for k =1,. . . ,K. That is, the data follow a bivariate normal distribution with a mean vector ?k that is speci.c to the kth class, and a covariance matrix S that is shared across the K classes. Choose S and ?1,. . . ,?K such that there is some overlap between the K classes, i.e. no linear decision boundary is able to perfectly separate the training data. Specify your choices for S and ?1,. . . ,?K .

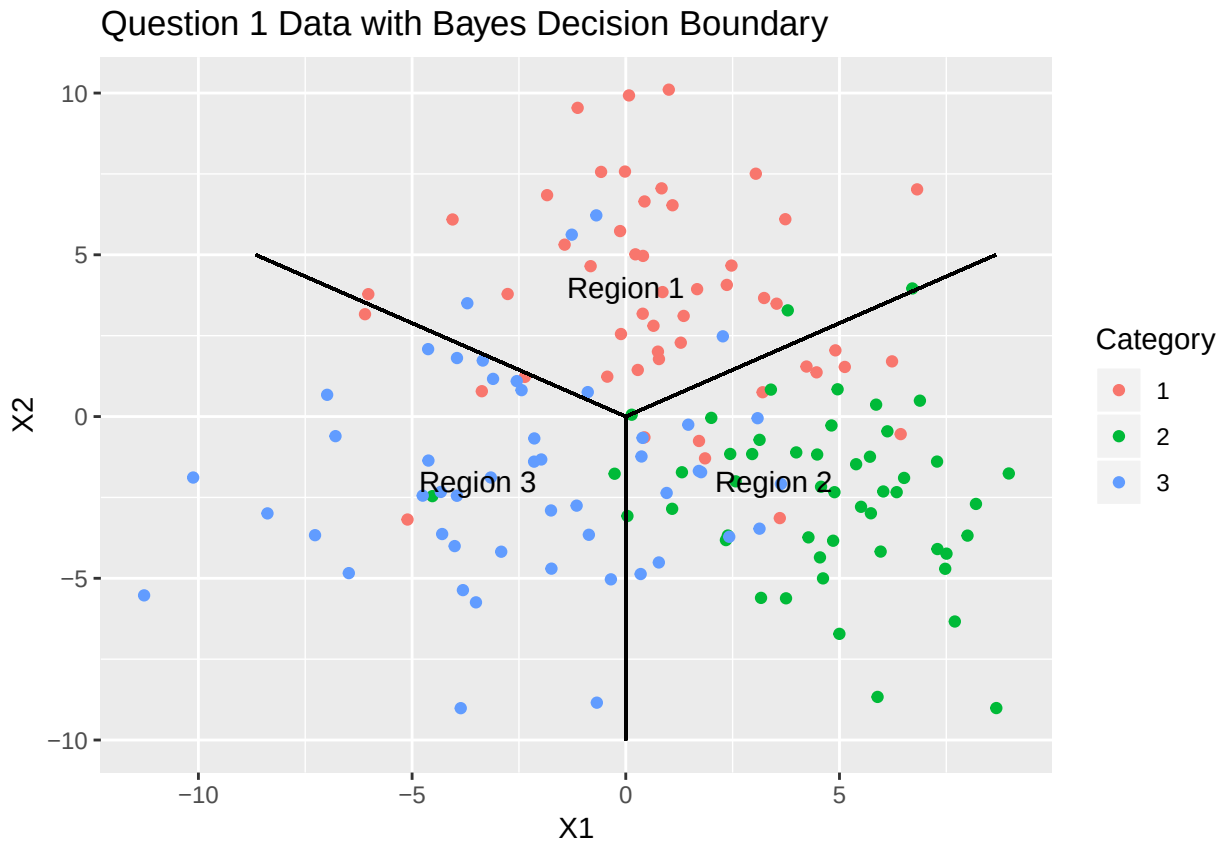
```
set.seed(990)
DOTDISTANCE <- 4
STANDARDDEV <- 3
findit <- function(length,angle,coord) {
  #not sure what I'm doing wrong, but x and y work when reverse, so here goes
  #Error, at 180 degrees, a very small number is reported, instead of 0
  if(coord == "x") {
    toReturn <- length*sin(angle*(pi/180))
  }
  if(coord == "y") {
    toReturn <- length*cos(angle*(pi/180))
  }
  return(toReturn)
}
findit(5,45,"x")


DF <- data.frame(X1 = c(rnorm(50,0,STANDARDDEV),
                        rnorm(50,findit(DOTDISTANCE,120,"x"),STANDARDDEV),
                        rnorm(50,findit(DOTDISTANCE,240,"x"),STANDARDDEV)) ,
```

```
                X2=  c(rnorm(50,DOTDISTANCE,STANDARDDEV),
                       rnorm(50,findit(DOTDISTANCE,120,"y"),STANDARDDEV),
                       rnorm(50,findit(DOTDISTANCE,240,"y"),STANDARDDEV)) ,
                Type = (c(rep(1,50),rep(2,50),rep(3,50) )))
```

3 sets of 50 points were created with mean locations equidistat from the origin (0,0) and at even angles (0, 120, and 240 degrees). Each set of 50 points exists 3 standard deviations from their central mean.

(b) Plot the data, with the observations in each class displayed in a di.erent color. Compute and display the Bayes decision boundary (or Bayes de?cision boundaries) on this plot. This plot should look something like the right-hand panel of Figure 4.6 in the textbook (although no need to worry about shading the background, and also you don?t need to display the LDA decision boundary for this sub-problem ? you will do that in the next sub-problem). Be sure to label which region(s) of the plot correspond to each class.

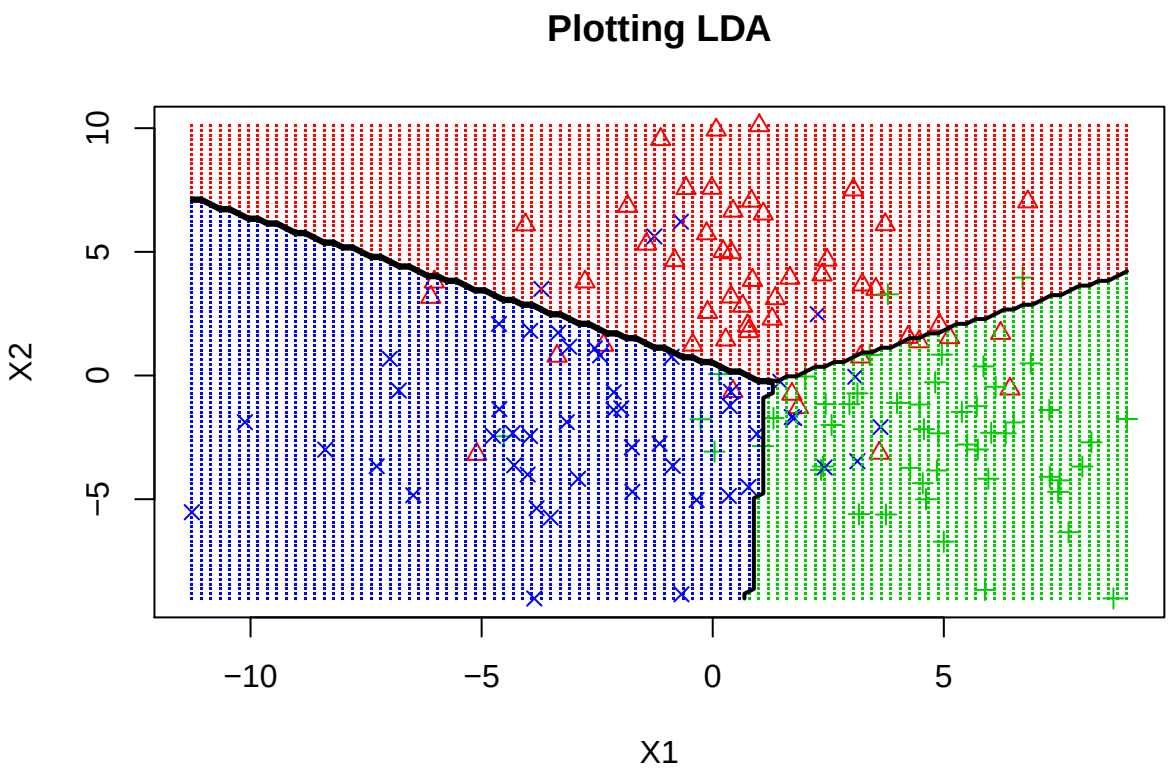
```
library(ggplot2)
linelength <- 10
ggplot(DF, aes(x = X1,y=X2, color=as.factor(Type))) +
  geom_point() +
  ggtitle("Question 1 Data with Bayes Decision Boundary") +
  labs(colour= "Category")+
  geom_segment(aes(x=0, y=0, xend = 0, yend = findit(linelength,180,"y")), color = "black") +
  geom_segment(aes(x=0, y=0, xend = findit(linelength,300,"x"), yend=findit(linelength,300,"y")), color
  geom_segment(aes(x=0, y=0, xend = findit(linelength,60,"x"), yend=findit(linelength,60,"y")), color =
  annotate(geom = "text", x= 0, y = DOTDISTANCE, label = "Region 1") +
  annotate(geom = "text",x = findit(DOTDISTANCE,120,"x"), y=findit(DOTDISTANCE,120,"y"),  label= "Regio
  annotate(geom = "text",x = findit(DOTDISTANCE,240,"x"), y=findit(DOTDISTANCE,240,"y"),  label= "Regio
```



(c) Fit a linear discriminant analysis model to the data, and make a plot that displays the observations as well as the decision boundary (or boundaries) corresponding to this .tted model. How does the LDA decision boundary (which can be viewed as an estimate of the Bayes decision boundary) compare to the Bayes decision boundary that you computed and plotted in (b)?

```
library(MASS)

#this function adopted from Michael Hahsler
#http://michael.hahsler.net/SMU/EMIS7332/R/viz_classifier.html
decisionplot <- function(model, data, class = NULL, predict_type = "class",
  resolution = 100, showgrid = TRUE, ...) {

  if(!is.null(class)) cl <- data[,class] else cl <- 1
  data <- data[,1:2]
  k <- length(unique(cl))

  plot(data, col = as.integer(cl)+1L, pch = as.integer(cl)+1L, ...)

  # make grid
  r <- sapply(data, range, na.rm = TRUE)
  xs <- seq(r[1,1], r[2,1], length.out = resolution)
  ys <- seq(r[1,2], r[2,2], length.out = resolution)
  g <- cbind(rep(xs, each=resolution), rep(ys, time = resolution))
  colnames(g) <- colnames(r)
```

```
  g <- as.data.frame(g)

  ### guess how to get class labels from predict
  ### (unfortunately not very consistent between models)
  p <- predict(model, g, type = predict_type)
  if(is.list(p)) p <- p$class
  p <- as.factor(p)

  if(showgrid) points(g, col = as.integer(p)+1L, pch = ".")

  z <- matrix(as.integer(p), nrow = resolution, byrow = TRUE)
  contour(xs, ys, z, add = TRUE, drawlabels = FALSE,
    lwd = 2, levels = (1:(k-1))+.5)

  invisible(z)
}

test <- lda(Type~., data = DF)

decisionplot(test, DF, class = "Type",main = "Plotting LDA")
```



The LDA decision boundaries are similar, but not geometric. The boundaries on the positive Y axis are less acute reltaive to each other, and the boundary on the negative y axis is no longer perfectly vertical.

(d) Report the K ? K confusion matrix for the LDA model on the training data. The rows of this confusion matrix represent the predicted class labels, and the columns represent the true class labels. (See Table 4.4 in the textbook for an example of a confusion matrix.) Also, report the training error (i.e. the proportion of observations that are mis-classi.ed.)

```
library(stargazer)
```

```
##
## Please cite as:
##  Hlavac, Marek (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables.
##  R package version 5.2.2. https://CRAN.R-project.org/package=stargazer
```

```
DFConfusionMatrix <- table(predict(test, type="class")$class, DF$Type)

DFConfusionMatrix
stargazer(DFConfusionMatrix, no.space = TRUE, header = FALSE, float = FALSE, ci = TRUE, table.placement
```

(e) Generate n = 50 test observations in each of the K classes, using the bivariate normal distributions from (a). Report the K ? K confusion matrix, as well as the test error, that results from applying the model .t to the training data in (c) to your test data.

(f) Compare your results from (d) and (e), and comment on your .ndings.

2. In this problem, you will apply quadratic discriminant analysis to the data from Q1.

(a) Fit a quadratic discriminant analysis model to the training data from Q1, and make a plot that displays the observations as well as the QDA decision boundary (or boundaries) corresponding to this .tted model. Be sure to label which region(s) of the plot correspond to each class. How does the QDA decision boundary compare to the Bayes decision boundary that you computed in Q1(b)?

(b) Report the K ? K confusion matrix for the QDA model on the training data, as well as the training error.

(c) Repeat (b), but this time using the test data generated in Q1. (That is, apply the model .t to the training data in (a) in order to calculate the test error.)

(d) Compare your results in (b) and (c), and comment on your .ndings.

(e) Which method had smaller training error in this example: LDA or QDA? Comment on your .ndings.

(f) Which method had smaller test error in this example: LDA or QDA? Comment on your .ndings.

3. In this example, you will generate data in such a way that QDA will have a smaller test error than LDA. Once again, take K = 3, p = 2, and n = 50 observations per class.

(a) Generate data such that the observations in the kth class follow a N(?k, Sk) distribution, for k =1,. . . ,K. This is similar to the data generated in Q1, except that now each class has its own covariance matrix. Choose ?1,. . . ,?K and S1,. . . , SK so that the Bayes decision boundary (or bound?aries) is highly non-linear, and also so that there is some overlap between the K classes. Specify your choices for S1,. . . , SK and ?1,. . . ,?K .

(b) Plot the data, with the observations in each class displayed in a di.erent color. Compute and display the Bayes decision boundary (or Bayes deci?sion boundaries) on this plot. Be sure to label which region(s) of the plot correspond to each class.

(c) Fit an LDA model to the data, and also .t a QDA model to the data. Make a plot that displays the observations in each class in a di.erent color, as well as the LDA decision boundary and the QDA decision boundary. Be sure to label which region(s) of the plot correspond to each class.

(d) Report the confusion matrix for the training data, as well as the training error, for each of the models .t in (c).

(e) Now generate n = 50 test observations per class, using the bivariate nor?mal distributions from (a). Report the K ? K confusion matrices, as well as the test errors, that result from applying the models trained in (c) on the training data to your test data.

(f) Which method had smaller training error in this example: LDA or QDA? Comment on your .ndings.

(g) Which method had smaller test error in this example: LDA or QDA? Comment on your .ndings.

4. In this problem, you will perform binary classi.cation on a real (not simulated) data set of your choice (from the book website, from the internet, or from another source). In order to perform binary classi.cation, you will of course need a qualitative response variable with values in K = 2 classes. Let nk denote the number of observations in the kth class, for k =1, 2. You will want to have nk ? p for k =1, 2 in this data set, so if you have chosen a data set with p = nk or p ? nk for some k, then you should just work with a subset of the features. Also, please choose a data set with p = 4.

(a) Describe the data. Where did you get it from? What is the value of p? What is the sample size in each class? What are the response and features measuring? Are the features qualitative or quantitative?

(b) Make some scatterplots displaying Y on the y-axis and Xj on the x-axis, for j =1,. . . ,p. Based on these scatterplots, do you think it will be possible to accurately predict Y using X1,. . . ,Xp? Explain your answer.

(c) Fit a logistic regression model to predict Y using X1,. . . ,Xp, and make a table like Table 4.3 in the textbook displaying the coe.cient estimates, standard errors, and p-values. If any of your predictors are qualitative, then be sure to clearly indicate how to interpret the corresponding coe.?cients in the table.

(d) For which predictors (if any) can you reject the null hypothesis H0 : ?j = 0, where ?j is the coe.cient in the logistic regression model from (c)? Comment on your results, and relate them to your answer in (b).

(e) Write a sentence providing an interpretation for the coe.cient associated with X1. For instance, you could say something along the lines of ?A one-unit increase in X1 is associated with . . . ? (complete the sentence!).

5. This problem has to do with logistic regression.

(a) Suppose you .t a logistic regression to some data and .nd that for a given observation x =(x1,. . . ,xp)T , the estimated log-odds equals 0.7. What is P (Y =1 | X = x)?

(b) In the same setting as (a), suppose you are now interested in the obser?vation x * =(x1 +1,x2 - 1, 2x3,x4,. . . ,xp)T . In other words, x1 * = x1 + 1,

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x = x2 - 1, x3 =2x3, and x = xj for j =4,. . . ,p. Write out a simple 2 j expression for P (Y =1 | X = x *). Your answer will involve the estimated coe.cients in the logistic regression model, as well as the number 0.7. 6. Suppose we wish to perform classi.cation of a binary response in a setting with p = 1: that is, X . R, and Y .{1, 2}. We assume that the observations in Class 1 are drawn from a N(?, s2) distribution, and that the observations in Class 2 are drawn from an Uniform[-2, 2] distribution. (a) Derive an expression for the Bayes decision boundary: that is, for the set of x such that P (Y =1 | X = x)= P (Y =2 | X = x). Write it out as simply as you can.

(b) Suppose (for this sub-problem only) that ? = 0, s = 1, p1 =0.45 (here, p1 is the prior probability that an observation belongs to Class 1). Describe the Bayes classi.er in this case: what range of x values will get assigned to Class 1, and what range of x values will get assigned to Class 2? Write out your answer as simply as you can. Draw a picture showing the set of x values assigned to Class 1 and the set of x values assigned to Class 2.

(c) Now suppose we observe n training observations (x1,y1),. . . , (xn,yn). Ex?plain how you could use these observations to estimate ?, s, and p1 (in?stead of using the values that were given in part (b)).

(d) Given a test observation X = x0, provide an estimate of

P (Y =1 | X = x0). Your answer should involve only the training observations (x1,y1),. . . , (xn,yn) and the test observation x0, and no unknown parameters.