

# Biostats 546 HW 3

*Ronald Buie*

*February 10, 2019*

Due Via Online Submission to Canvas: Sunday, February 24 at 12 PM (Noon)

1. In this problem, you will make use of a (real) dataset of your choice in order to predict a binary response  $Y$  using predictors  $X_1; \dots; X_p$ . You should have p 5.

```
library(ISLR)

AutoData <- ISLR::Auto

AutoData$yearBinary <- "older"
AutoData[AutoData$year > 76,]$yearBinary <- "newer"
AutoData$yearBinary <- as.factor(AutoData$yearBinary)
AutoData$origin <- as.factor(AutoData$origin)
AutoData$cylinders <- as.factor(AutoData$cylinders)
AutoData$year <- NULL
AutoData$name <- NULL
```

(a) Describe the dataset. What is the response, and what are the predictors?

```
head(AutoData)
```

##	mpg	cylinders	displacement	horsepower	weight	acceleration	origin
## 1	18	8	307	130	3504	12.0	1
## 2	15	8	350	165	3693	11.5	1
## 3	18	8	318	150	3436	11.0	1
## 4	16	8	304	150	3433	12.0	1
## 5	17	8	302	140	3449	10.5	1
## 6	15	8	429	198	4341	10.0	1

```
## yearBinary
## 1 older
## 2 older
## 3 older
## 4 older
## 5 older
## 6 older
```

```
summary(AutoData)
```

##	mpg	cylinders	displacement	horsepower	weight
## Min.	: 9.00	3: 4	Min. : 68.0	Min. : 46.0	Min. :1613
## 1st Qu.	:17.00	4:199	1st Qu.:105.0	1st Qu.: 75.0	1st Qu.:2225
## Median	:22.75	5: 3	Median :151.0	Median : 93.5	Median :2804
## Mean	:23.45	6: 83	Mean :194.4	Mean :104.5	Mean :2978
## 3rd Qu.	:29.00	8:103	3rd Qu.:275.8	3rd Qu.:126.0	3rd Qu.:3615

```
## Max.      :46.60           Max.      :455.0   Max.      :230.0   Max.      :5140
## acceleration  origin  yearBinary
## Min.       : 8.00    1:245   newer:178
## 1st Qu.:13.78    2: 68   older:214
## Median :15.50    3: 79
## Mean      :15.54
## 3rd Qu.:17.02
## Max.      :24.80
```

The Auto data set contains information about the performance, origin and model of 392 vehicles. We have created a binary indicator variable for the year where “older” is 70 to 76, and “newer” is > 76 and will use this as our outcome. our predictors are mpg, cylinders, displacement, horsepower, weight, acceleration, and origin.

(b) Fit a logistic regression model to predict Y using  $X_1; \dots; X_p$ . What is the classification error (i.e. the fraction of incorrectly classied observations) on the training set?

```
LogFitAutoData <- glm(yearBinary ~ ., data = AutoData, family="binomial")
LogFitAutoDataPredictions <- predict(LogFitAutoData, type="response")
LogPredictions <- rep("newer",392)
LogPredictions[LogFitAutoDataPredictions >.5]<- "older"
LogPredictionsErrorRate <- 1-mean(LogPredictions==AutoData$yearBinary)
```

The training error rate is 21%

(c) Use the validation set approach in order to estimate the test classification error. Report the error you obtain.

```
set.seed(1000)
trainSample <- sample(392, 196)
LogFitAutoDataVS <- glm(yearBinary ~ ., data = AutoData, family="binomial", subset = trainSample)
LogFitAutoDataVSPredictions <- predict(LogFitAutoDataVS, newdata = AutoData, type="response")
LogPredictionsVS <- rep("newer",392)
LogPredictionsVS[LogFitAutoDataVSPredictions >.5]<- "older"
LogPredictionsVSErrorRate <- 1-mean(LogPredictionsVS==AutoData$yearBinary)[-trainSample]
```

The training error rate for the Validation set approach is 21%

(d) Use the leave-one-out cross-validation approach in order to estimate the test classification error. Report the error you obtain.

```
library(boot)
LogFitAutoDataCVMError <- cv.glm(AutoData, LogFitAutoData)
```

```
LogFitAutoDataCVMError$delta
```

```
## [1] 0.1382103 0.1381981
```

The cross validation approach yields a MSE of 0.1382103 and bias corrected MSE of 0.1381981

(e) Use the 5-fold cross-validation approach in order to estimate the test classification error. Report the error you obtain.

```
library(boot)
set.seed(1000)
attach(AutoData)
```

```
LogFitAutoDataCVK5Error <- cv.glm(AutoData, LogFitAutoData, K=5)
```

```
LogFitAutoDataCVK5Error$delta[1]
```

```
## [1] 0.1393391
```

The cross validation approach yields an MSE of 0.1382103 and bias corrected MSE of 0.1381981

(f) Comment on your findings in (b)-(e).

The error rates of our logistic regression and validation set approaches were similar, 21%. Also similar, at 14% were the error rates of our cross validation and 5-fold cross-validation approaches. The cross validation approaches identified models with superior fit than our

**2. In this problem, you will make use of a (real) dataset of your choice in order to predict a continuous response  $Y$  using predictors  $X_1; \dots; X_6$ . You need to choose a dataset with at least 6 predictors. If your dataset has more than 6 predictors, then please choose 6 of them now. In other words, you should have  $p = 6$ .**

```
AutoData6 <- AutoData[,!(names(AutoData) %in% "origin")]
AutoData6$cylinders <- as.numeric(as.character(AutoData6$cylinders))
```

(a) Describe the dataset. What is the response, and what are the predictors? 1

```
head(AutoData6)
```

```
##   mpg cylinders displacement horsepower weight acceleration yearBinary
## 1   18         8          307         130   3504          12.0      older
## 2   15         8          350         165   3693          11.5      older
## 3   18         8          318         150   3436          11.0      older
## 4   16         8          304         150   3433          12.0      older
## 5   17         8          302         140   3449          10.5      older
## 6   15         8          429         198   4341          10.0      older
```

```
summary(AutoData6)
```

```
##      mpg      cylinders      displacement      horsepower
## Min.   : 9.00    Min.   :3.000    Min.   : 68.0    Min.   : 46.0
## 1st Qu.:17.00    1st Qu.:4.000    1st Qu.:105.0    1st Qu.: 75.0
## Median :22.75    Median :4.000    Median :151.0    Median : 93.5
## Mean   :23.45    Mean   :5.472    Mean   :194.4    Mean   :104.5
## 3rd Qu.:29.00    3rd Qu.:8.000    3rd Qu.:275.8    3rd Qu.:126.0
## Max.   :46.60    Max.   :8.000    Max.   :455.0    Max.   :230.0
##      weight      acceleration      yearBinary
## Min.   :1613    Min.   : 8.00    newer:178
## 1st Qu.:2225    1st Qu.:13.78    older:214
## Median :2804    Median :15.50
## Mean   :2978    Mean   :15.54
## 3rd Qu.:3615    3rd Qu.:17.02
## Max.   :5140    Max.   :24.80
```

For this question we chose the same data set as above, but only included mpg, cylinders, displacement, horsepower, weight, and acceleration as predictors, and our binary year as our outcome. Cylinders, previously a dummy variable, has been converted to a continuous integer to meet the  $p=6$  requirement.

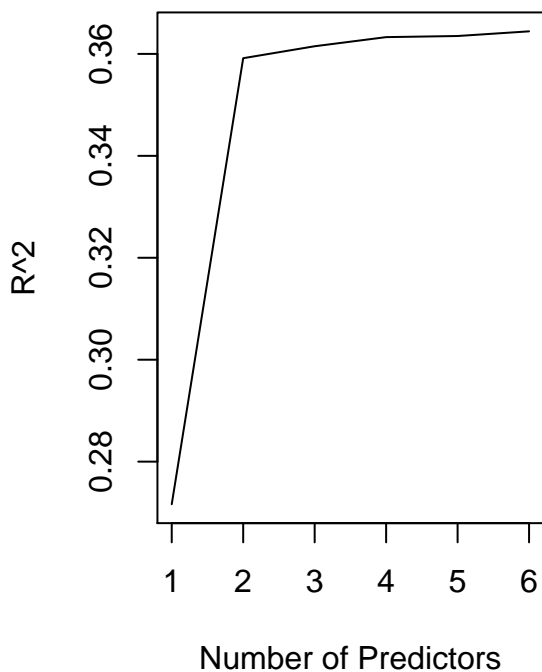
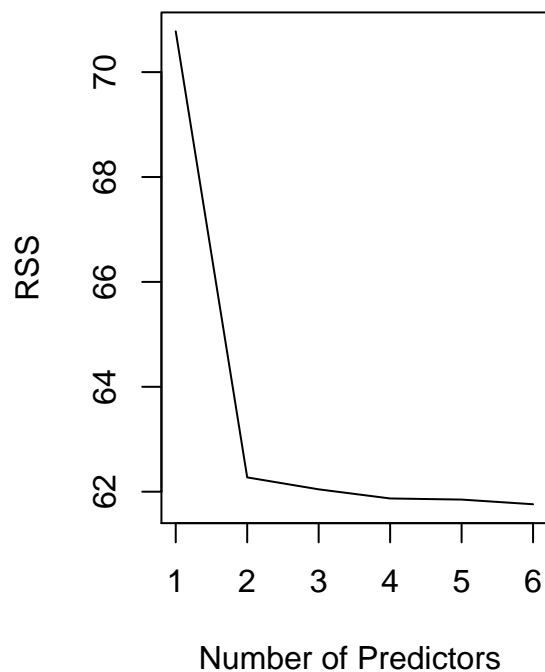
(b) Fit a least squares linear model using every possible subset of the features. How many models did you get?

```
library(leaps)
set.seed(1000)
a6Subsets <- regsubsets(yearBinary~., AutoData6)
a6SubsetsSummary <- summary(a6Subsets)
```

63 models are possible.

(c) Re-create Figure 6.1 in the textbook using your data. The left-hand panel should display (training set) RSS on the y-axis, and the right-hand panel should display the  $R^2$  on the y-axis. Both panels should display the number of predictors on the x-axis.

```
par(mfrow=c(1,2))
plot(a6SubsetsSummary$RSS, xlab="Number of Predictors", ylab="RSS", type="l")
plot(a6SubsetsSummary$rsq, xlab="Number of Predictors", ylab="R^2", type="l")
```



(d) Report the predictors in each of the models  $M_0; M_1; \dots; M_p$ .

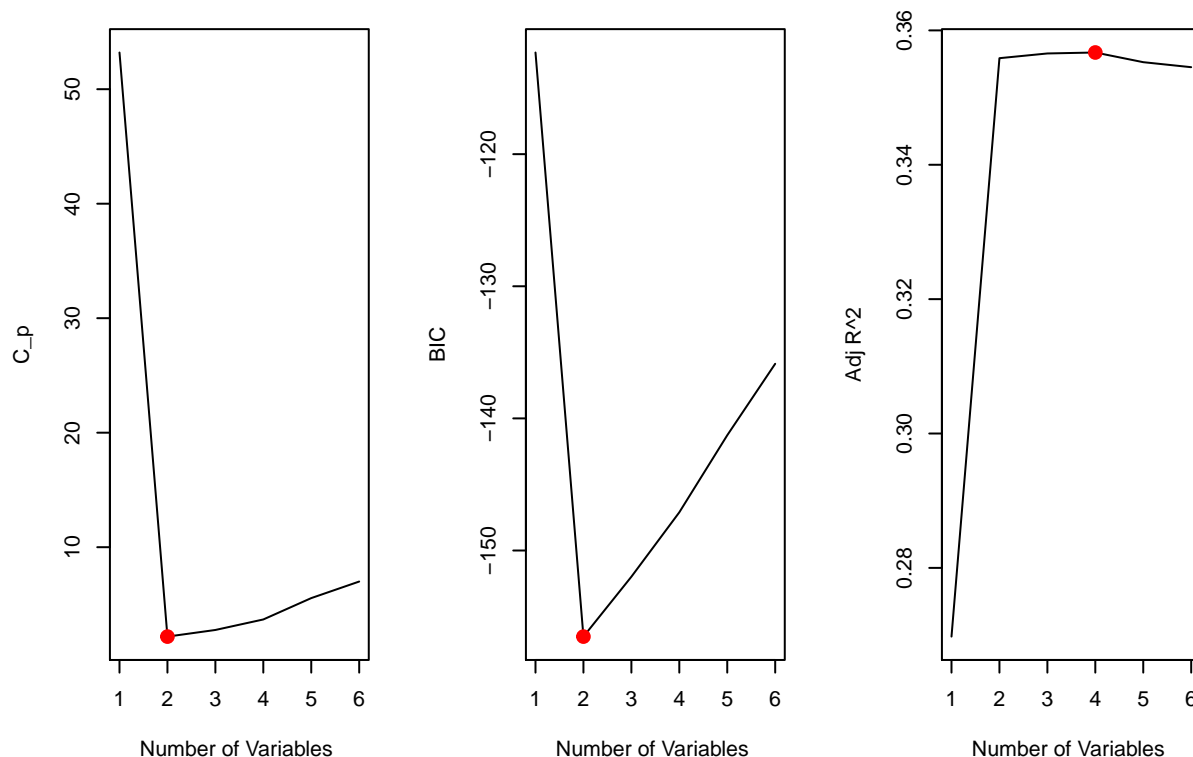
a6SubsetsSummary

```
## Subset selection object
## Call: regsubsets.formula(yearBinary ~ ., AutoData6)
## 6 Variables (and intercept)
##           Forced in Forced out
## mpg           FALSE      FALSE
## cylinders      FALSE      FALSE
## displacement   FALSE      FALSE
## horsepower     FALSE      FALSE
## weight         FALSE      FALSE
## acceleration   FALSE      FALSE
## 1 subsets of each size up to 6
## Selection Algorithm: exhaustive
##           mpg cylinders displacement horsepower weight acceleration
## 1 ( 1 ) "*" " "      " "      " "      " "      " "
## 2 ( 1 ) "*" " "      " "      " "      "*"      " "
## 3 ( 1 ) "*" " "      "*"      " "      "*"      " "
## 4 ( 1 ) "*" "*"      "*"      " "      "*"      " "
## 5 ( 1 ) "*" "*"      "*"      "*"      "*"      " "
## 6 ( 1 ) "*" "*"      "*"      "*"      "*"      "*"

```

(e) Re-create Figure 6.2 in the textbook using your data. The y-axes for the three panels should be  $C_p$ , BIC, and adjusted  $R^2$ ; all x-axes should display the number of predictors.

```
par(mfrow=c(1,3))
plot(a6SubsetsSummary$cp ,xlab="Number of Variables ",ylab="C_p", type='l')
points(which.min(a6SubsetsSummary$cp),min(a6SubsetsSummary$cp),col="red",cex=2,pch=20)
plot(a6SubsetsSummary$bic ,xlab="Number of Variables ",ylab="BIC", type='l')
points(which.min(a6SubsetsSummary$bic),min(a6SubsetsSummary$bic),col="red",cex=2,pch=20)
plot(a6SubsetsSummary$adjr2 ,xlab="Number of Variables ",ylab="Adj R^2", type='l')
points(which.max(a6SubsetsSummary$adjr2),max(a6SubsetsSummary$adjr2),col="red",cex=2,pch=20)
```



(f) Based on your results, what is the best" least squares linear model on this dataset? (Your answer should include not only the predictors, but also the coecient estimates.) Explain your answer.

Our best model based on two of our measures,  $C_p$  and BIC, contains 2 variables, mpg and weight.

```
library(stargazer)
```

```
##
```

```
## Please cite as:
```

```
## Hlavac, Marek (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables.
```

```
## R package version 5.2.2. https://CRAN.R-project.org/package=stargazer
```

```
stargazer(round(coef(a6Subsets, 2), 6), no.space=TRUE, header=FALSE, float= FALSE, ci = TRUE, table.pla
```

(Intercept)	mpg	weight
3.924	-0.062	-0.0003

However,  $R^2$  suggests the 4 variable model using mpg, cylinders, displacement, and weight.

```
library(stargazer)
stargazer(round(coef(a6Subsets, 4), 6), no.space=TRUE, header=FALSE, float= FALSE, ci = TRUE, table.pla
```

(Intercept)	mpg	cylinders	displacement	weight
4.078	-0.061	-0.040	0.001	-0.0004

### 3. Consider using the Auto data set to predict mpg using polynomial functions of horsepower in a least squares linear regression.

```
AutoDataPoly <- ISLR::Auto
AutoDataPoly <- AutoDataPoly[,-9]
```

(a) Perform the validation set approach, and produce a plot like the one in the right-hand panel of Figure 5.2 of the textbook. Your answer won't look exactly the same as the results in Figure 5.2, since you'll be starting with a different random seed. Discuss your findings. What degree polynomial is best, and why?

```
library(ggplot2)

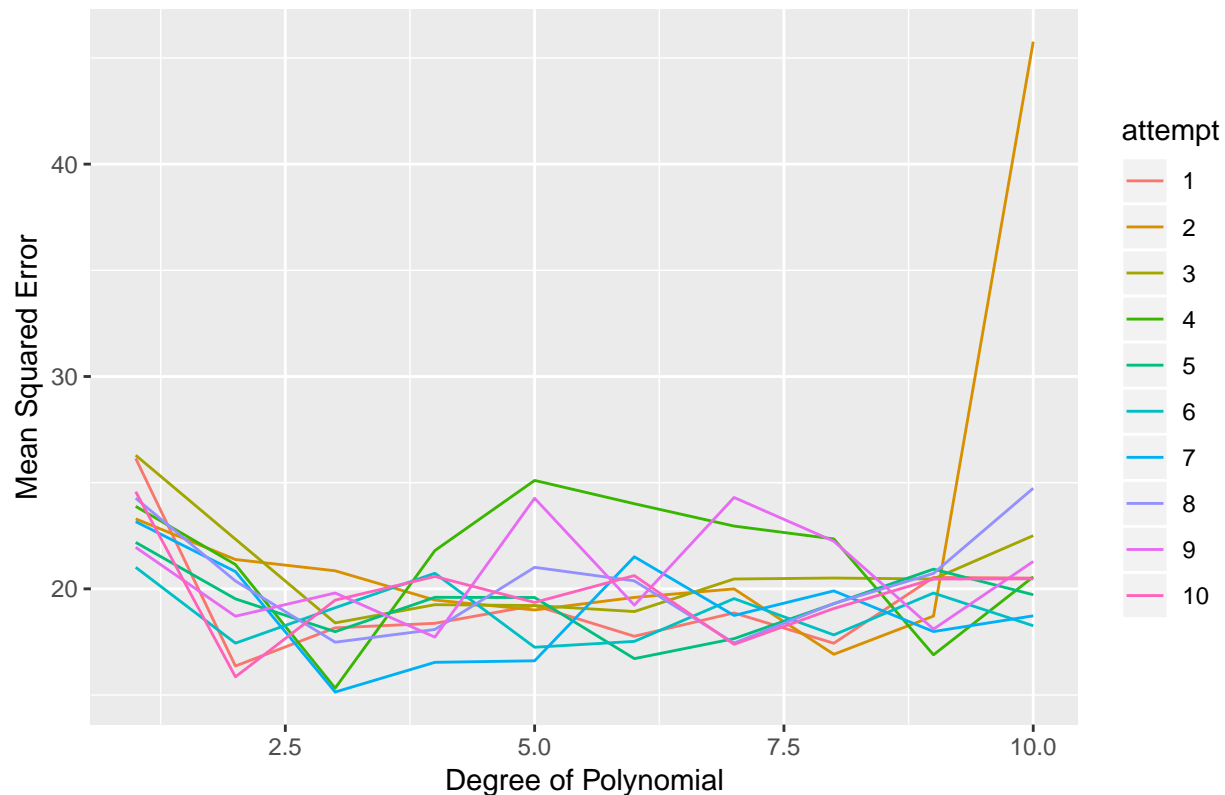
##
## Attaching package: 'ggplot2'

## The following object is masked from 'AutoData':
##
##      mpg

MSY <- data.frame(msy = numeric(), poly = numeric(), attempt = numeric())
counter <- 0
for(i in 1:10){
  set.seed(i)
  for(j in 1:10){
    counter <- counter+1
    train <- sample(nrow(AutoDataPoly), nrow(AutoDataPoly)/2)
    AutoDataPolyLM <- lm(mpg~poly(horsepower,j), data = AutoDataPoly, subset = train)
    MSY[counter,] <- c(mean((AutoDataPoly$mpg -predict(AutoDataPolyLM,AutoDataPoly))[-train]^2),j,i)
  }
}

MSY$attempt <- as.factor(MSY$attempt)
ggplot(MSY, aes(y=msy, x = poly, color = attempt)) +
  geom_line() +
  ggtitle("MSY of Multiple Polynomials Of Validation Set Approach") +
  xlab("Degree of Polynomial")+
  ylab("Mean Squared Error")
```

## MSY of Multiple Polynomials Of Validation Set Approach



```
MeanMSY <- aggregate(msy ~ poly, MSY, mean)[2]
which.min(MeanMSY$msy)
```

```
## [1] 3
```

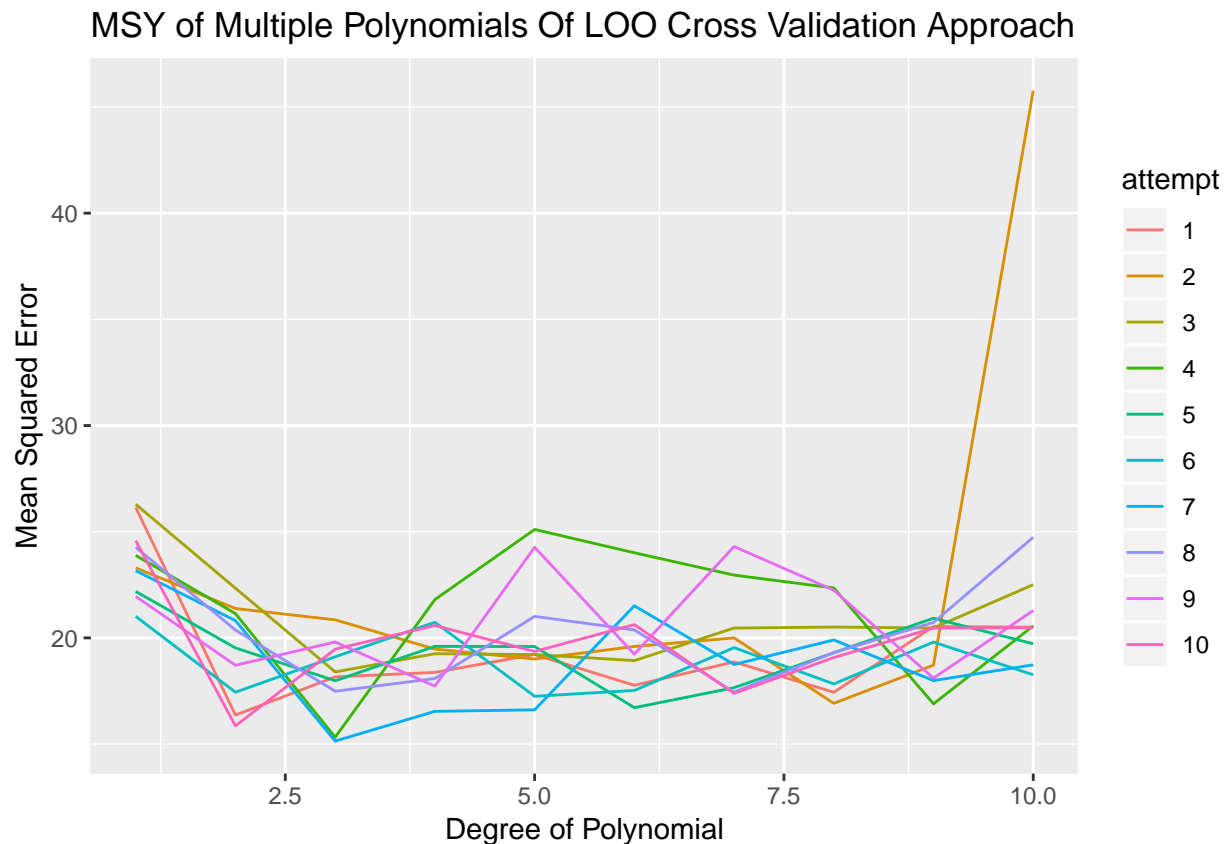
The resulting chart shows the performance of increasing polynomials for different samples of our data. This provides a range of guesses for possible results when sampling from a larger data set. Polynomial 3 has the lowest mean MSY across all samples, and so I'd take that to be the best polynomial to use.

(b) Perform leave-one-out cross-validation, and produce a plot like the one in the left-hand panel of Figure 5.4 of the textbook. Discuss your findings. What degree polynomial is best, and why?

```
library(ggplot2)
MSY <- data.frame(msy = numeric(), poly = numeric(), attempt = numeric())
counter <- 0
for(i in 1:10){
  set.seed(i)
  for(j in 1:10){
    counter <- counter+1
    train <- sample(nrow(AutoDataPoly), nrow(AutoDataPoly)/2)
    AutoDataPolyLM <- lm(mpg~poly(horsepower,j), data = AutoDataPoly, subset = train)
    MSY[counter,] <- c(mean((AutoDataPoly$mpg -predict(AutoDataPolyLM,AutoDataPoly))[-train]^2),j,i)
  }
}
MSY$attempt <- as.factor(MSY$attempt)
```



```
ggplot(MSY, aes(y=msy, x = poly, color = attempt)) +
  geom_line() +
  ggtitle("MSY of Multiple Polynomials Of LOO Cross Validation Approach") +
  xlab("Degree of Polynomial")+
  ylab("Mean Squared Error")
```



```
MeanMSY <- aggregate(msy ~ poly, MSY, mean)[2]
which.min(MeanMSY$msy)
```

```
## [1] 3
```

The mean MSY of our estimates using the third polynomial is lower than the others and so considered our best performing.

(c) Perform 10-fold cross-validation, and produce a plot like the one in the right-hand panel of Figure 5.4 of the textbook. Discuss your findings. What degree polynomial is best, and why?

```
library(ggplot2)
library(boot)
MSY <- data.frame(msy = numeric(), poly = numeric(), attempt = numeric())
counter <- 0
for(i in 1:9){
  set.seed(i)

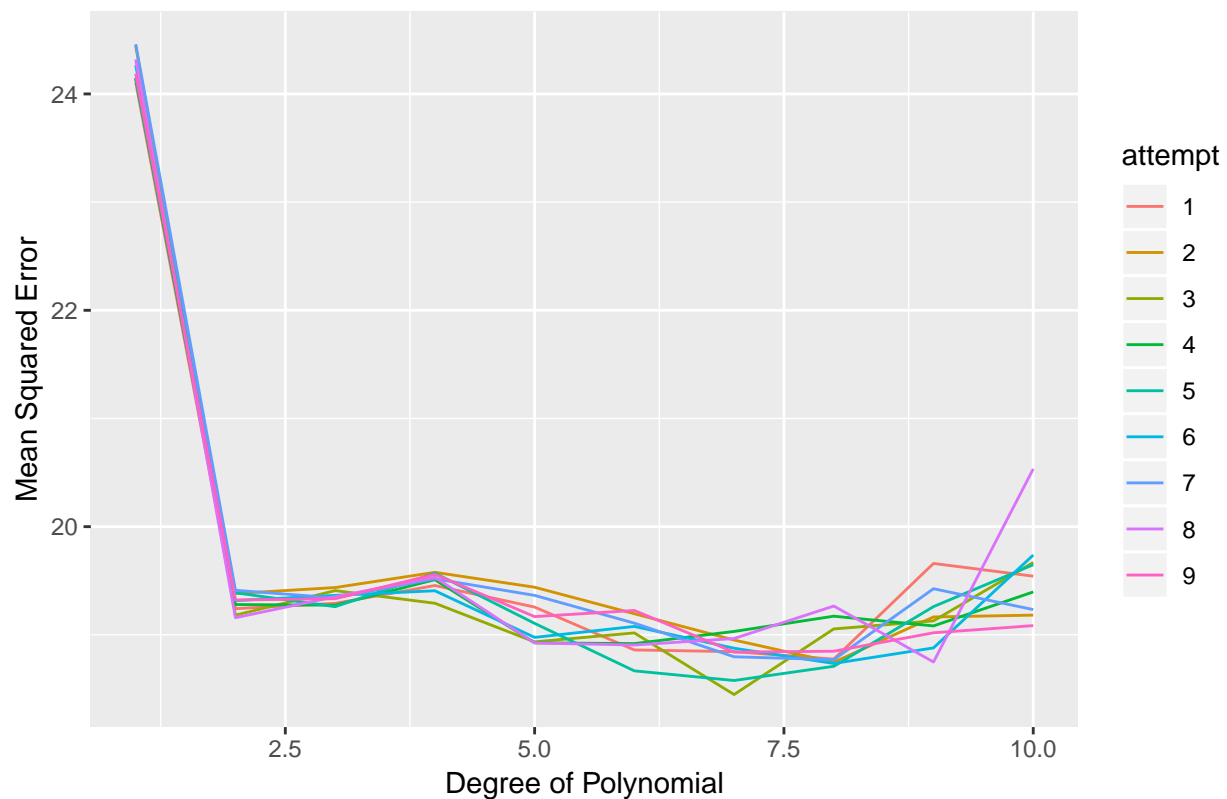
  for(j in 1:10){
```

```

    counter <- counter+1
    #train <- sample(nrow(AutoDataPoly), nrow(AutoDataPoly)/2)
    AutoDataPolyLM <- glm(mpg~poly(horsepower,j), data = AutoDataPoly)
    MSY[counter,] <- c(cv.glm(AutoDataPoly, AutoDataPolyLM, K=10)$delta[1],j,i)
    #MSY[counter,] <- c(mean((AutoDataPoly$mpg -predict(AutoDataPolyLM,AutoDataPoly))[-train]^2),j,i)
  }
}
MSY$attempt <- as.factor(MSY$attempt)
ggplot(MSY, aes(y=msy, x = poly, color = attempt)) +
  geom_line() +
  ggtitle("MSY of Multiple Polynomials Of K-Fold Cross-Validation Approach") +
  xlab("Degree of Polynomial")+
  ylab("Mean Squared Error")

```

MSY of Multiple Polynomials Of K-Fold Cross-Validation Approach



```

MeanMSY <- aggregate(msy ~ poly, MSY, mean)[2]
which.min(MeanMSY$msy)

```

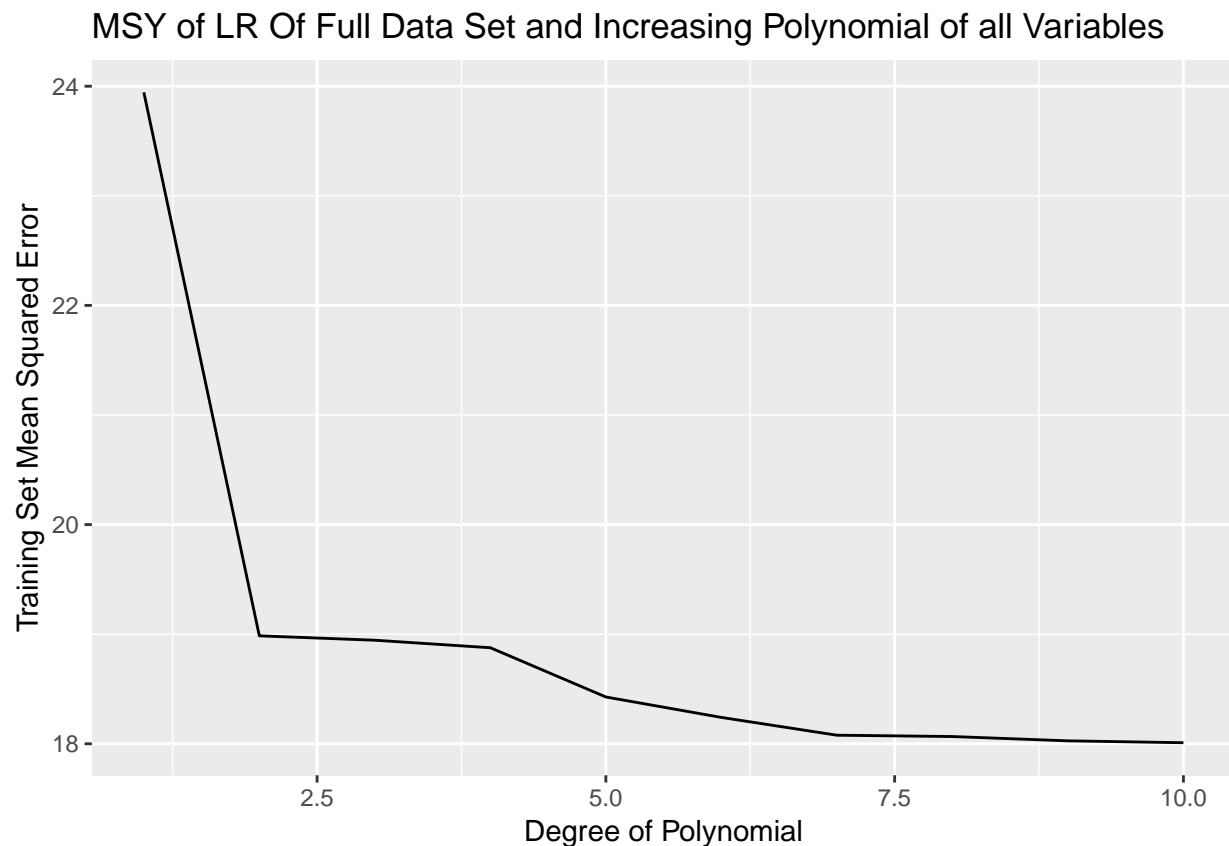
```
## [1] 7
```

The mean MSE of the 7th polynomial performed the best of our 9 attempts.

(d) Fit a least squares linear model to predict mpg using polynomials of degrees from 1 to 10, using all available observations. Make a plot showing "Degree of Polynomial" on the x-axis, and "Training Set Mean Squared Error" on the y-axis. Discuss your findings.

```
library(ggplot2)
MSY <- data.frame(msy = numeric(), poly = numeric())
for(i in 1:10){
  AutoDataPolyLM <- glm(mpg~poly(horsepower,i), data = AutoDataPoly)
  MSY[i,] <- c(mean((AutoDataPoly$mpg -predict(AutoDataPolyLM,AutoDataPoly))^2),i)
}

ggplot(MSY, aes(y=msy, x = poly)) +
  geom_line() +
  ggtitle("MSY of LR Of Full Data Set and Increasing Polynomial of all Variables") +
  xlab("Degree of Polynomial")+
  ylab("Training Set Mean Squared Error")
```



```
MeanMSY <- aggregate(msy ~ poly, MSY, mean)[2]
which.min(MeanMSY$msy)
```

```
## [1] 10
```

As the polynomial increases, so does the fit. So the MSE on the data decreases. Without the use of a validation approach, the result steadily improves MSE as the fit becomes tighter.

(e) Fit a least squares linear model to predict mpg using a degree-10 polynomial, using all available observations. Using the summary command in R, examine the output. Comment on the output, and discuss how this relates to your findings in (a)(d).

```
i<- 10
AutoDataPolyLM <- glm(mpg~poly(horsepower,i), data = AutoDataPoly)
summary(AutoDataPolyLM)

##
## Call:
## glm(formula = mpg ~ poly(horsepower, i), data = AutoDataPoly)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -15.7081  -2.5904  -0.1922   2.2859  14.8338
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      23.4459    0.2174 107.840  <2e-16 ***
## poly(horsepower, i)1 -120.1377    4.3046 -27.909  <2e-16 ***
## poly(horsepower, i)2  44.0895    4.3046  10.242  <2e-16 ***
## poly(horsepower, i)3  -3.9488    4.3046  -0.917  0.3595
## poly(horsepower, i)4  -5.1878    4.3046  -1.205  0.2289
## poly(horsepower, i)5  13.2722    4.3046   3.083  0.0022 **
## poly(horsepower, i)6  -8.5462    4.3046  -1.985  0.0478 *
## poly(horsepower, i)7   7.9806    4.3046   1.854  0.0645 .
## poly(horsepower, i)8   2.1727    4.3046   0.505  0.6140
## poly(horsepower, i)9  -3.9182    4.3046  -0.910  0.3633
## poly(horsepower, i)10 -2.6146    4.3046  -0.607  0.5440
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 18.52949)
##
##      Null deviance: 23819.0  on 391  degrees of freedom
## Residual deviance:  7059.7  on 381  degrees of freedom
## AIC: 2269.7
##
## Number of Fisher Scoring iterations: 2
```

The output from the polynomial regression shows the probability that the chance of the polynomial of a given level ( $\leq 10$ ) is a better fit to the data. Our output suggest that 1st through 3rd order polynomials, 5th, 6th, and possibly 7th order polynomials of horsepower are a good fit for our data. These observations align with our observations in parts a-c. In A and B, our suggested best model uses the 3rd order polynomial, an order strongly suggested in the above summary results. In C, the 7th order is suggested, which not strongly supported in the above summary data, save that a higher order polynomial, such as 5 and possibly 6, is. The results of d are not trully comparable as no validation method was used. So the highest order polynomial would always be the best fit (of the training data only).

4. We will now continue with the Auto data set. Note that the R package class contains the knn function, which can be used to perform k-nearest neighbors classification.

(a) Create a binary variable, HighMPG, that equals 1 if a car's gas mileage is above the median in the Auto data set, and equals 0 if the car's gas mileage is below the median.

```
library(ISLR)

FourthAutoData <- ISLR::Auto

FourthAutoData$HighMPG <- 0
FourthAutoData[FourthAutoData$mpg > median(FourthAutoData$mpg),]$HighMPG <- 1

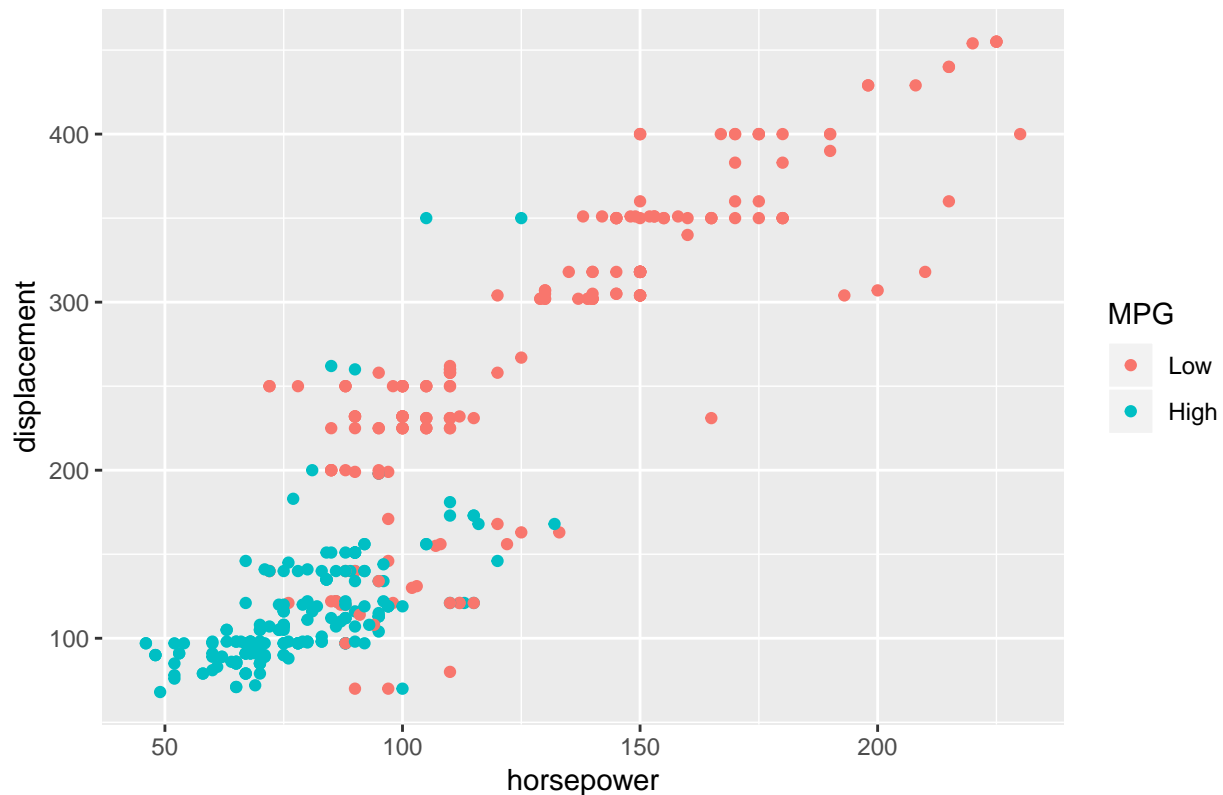
AutoData$yearBinary <- "older"
AutoData[AutoData$year > 76,]$yearBinary <- "newer"
AutoData$yearBinary <- as.factor(AutoData$yearBinary)
AutoData$origin <- as.factor(AutoData$origin)
AutoData$cylinders <- as.factor(AutoData$cylinders)
AutoData$year <- NULL
AutoData$name <- NULL
```

(b) Make a plot with horsepower on the x-axis, displacement on the y-axis, and with each of the cars in the Auto data set displayed as a point. The cars with gas mileage above the median should be displayed in one color, and the cars with gas mileage below the median should be displayed in another color. Be sure to create a legend and to label the axes appropriately.

```
library(ggplot2)

ggplot(FourthAutoData, aes(x = horsepower, y = displacement, color = as.factor(HighMPG))) +
  geom_point() +
  labs(color = "MPG") +
  scale_color_hue(labels = c("Low", "High")) +
  ggtitle("Distribution of High and Low MPG Vehicles")
```

Distribution of High and Low MPG Vehicles

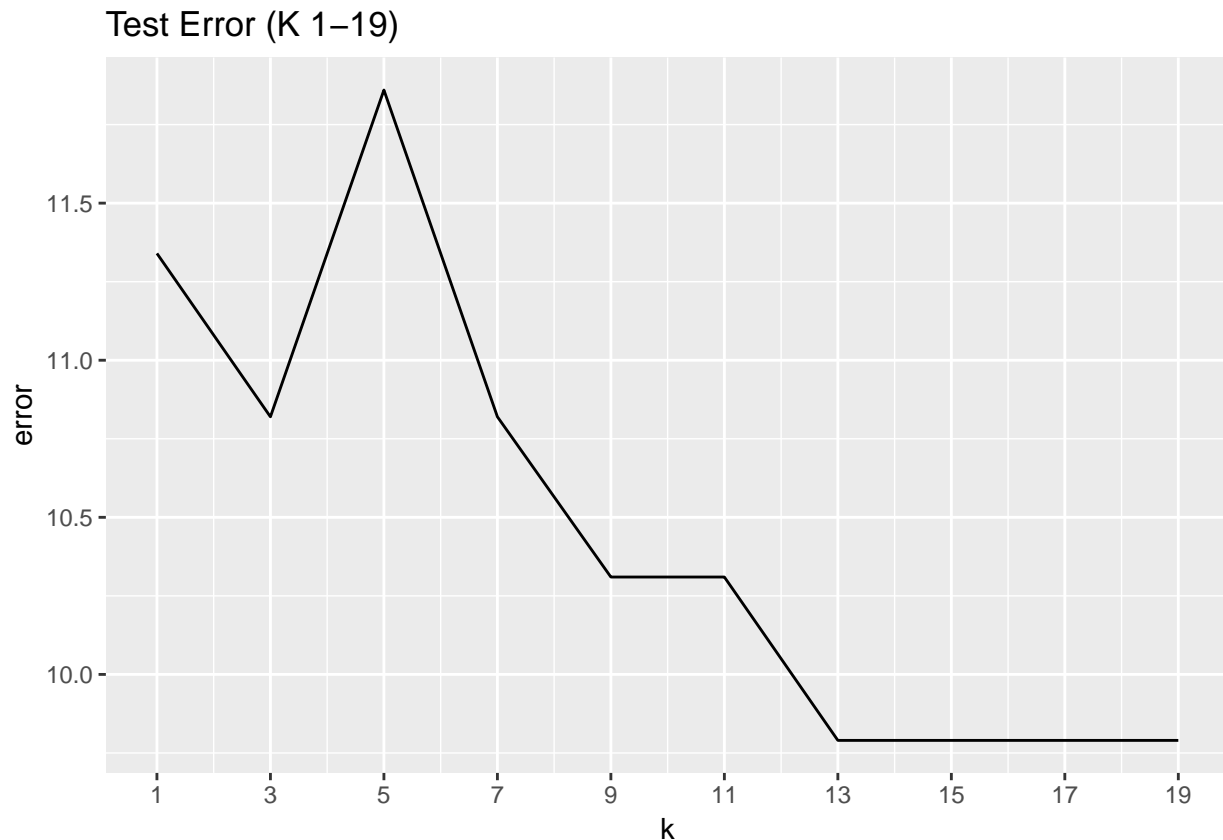


(c) Use the validation set approach in order to estimate the test error of knearest neighbors classification, when using horsepower and displacement to predict HighMPG. Since this is a classification problem, you can define test error as the fraction of test set observations that are incorrectly classified. Make a plot of the estimated test error, as a function of  $k$ . What value of  $k$  gives you the smallest estimated test error? Comment on your results.

```
library(class)
set.seed(100)
FourthAutoData$train <- sample(0:1, nrow(FourthAutoData), replace = TRUE)
calculatedError <- data.frame(error = rep(0,10), k = 1:10)
kcounter <- 1
for( i in 0:9) {
  kvalue <- kcounter + i*2
  KNNprediction <- knn(FourthAutoData[FourthAutoData$train == 1, c("horsepower", "displacement")],
    FourthAutoData[FourthAutoData$train == 0, c("horsepower", "displacement")],
    FourthAutoData[FourthAutoData$train == 1, "HighMPG"],
    k = kvalue)
  calculatedError[i+1,] <- c(round(1 - mean(KNNprediction == FourthAutoData[FourthAutoData$train == 0, "HighMPG"]), 2))
}

ggplot(calculatedError, aes(x = k, y = error)) +
  geom_line() +
  scale_x_continuous(breaks = seq(1, 19, 2)) +
```

```
ggtitle("Test Error (K 1-19)")
```



K of 7 to 19 (the highest tested) all performed the same, with an error rate of 9.79. These were our best performing models.

Each progressive K level increases the number of observations grouped together. Displacement has a slightly larger range, and so more effect on the outcome of KNN. No improvement in predictive error was found from K=13 and up. The models with the lowest fits performed best against the test data.

(d) Now perform k-nearest neighbors regression on the full data set, for various values of k. Make a plot displaying the training error rate obtained, as a function of k, for the same values of k considered in (c). Comment on your results, and discuss how they relate to your findings in (c). Hint: In (c), make sure to consider an appropriate range of values for k! I'd like to see values of k that are too small" (in terms of estimated test error) and also values of k that are too large".

```
library(class)

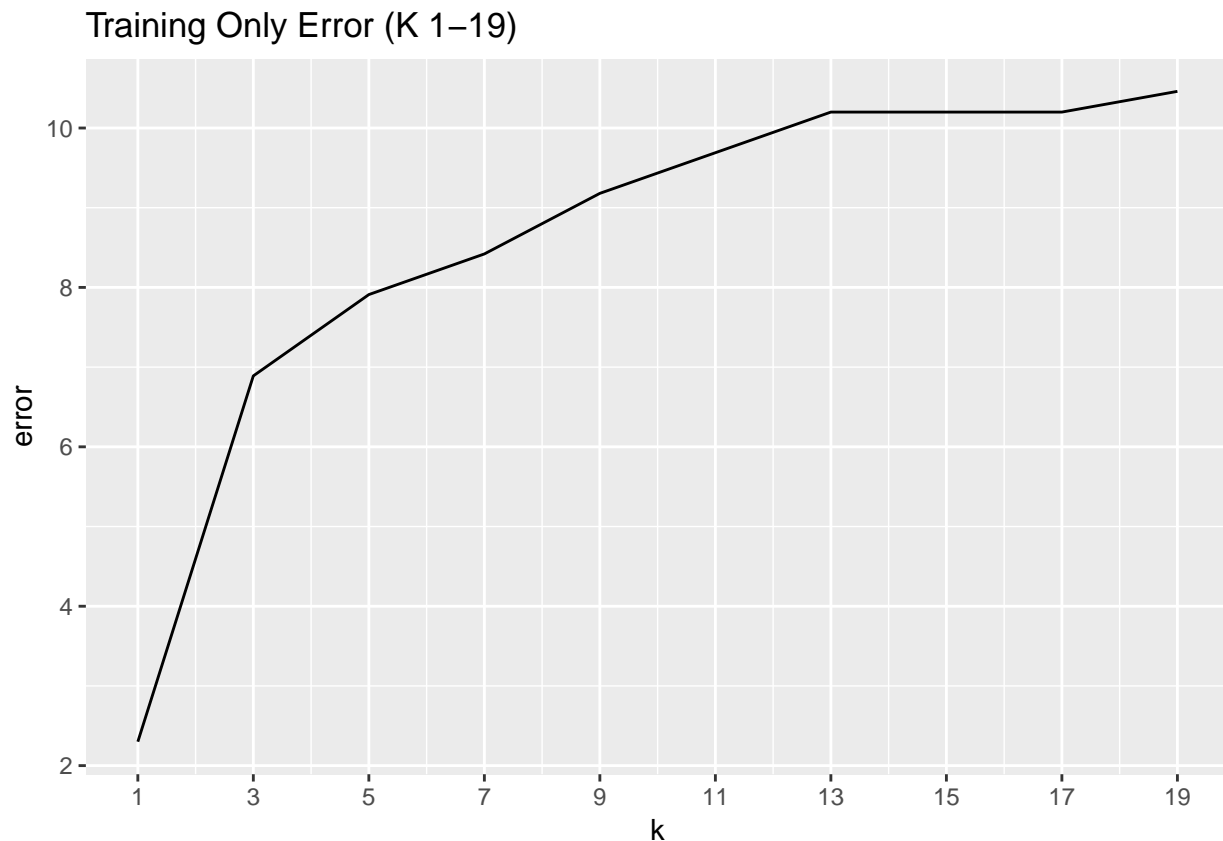
calculatedError <- data.frame(error = rep(0,10), k = 1:10)
kcounter <- 1
for( i in 0:9) {
  kvalue <- kcounter + i*2
  KNNprediction <- knn(FourthAutoData[,c("horsepower", "displacement")],
    FourthAutoData[,c("horsepower", "displacement")],
    FourthAutoData[, "HighMPG"],
```

```

    k = kvalue)
calculatedError[i+1,] <- c(round(1 - mean(KNNprediction ==FourthAutoData[, "HighMPG"]),4) *100, kvalue)
}

ggplot(calculatedError, aes(x = k, y = error)) +
  geom_line() +
  scale_x_continuous(breaks = seq(1, 19, 2)) +
  ggtitle("Training Only Error (K 1-19)")

```



Our KNN using all data to predict all data results in a higher and higher error rate as KNN becomes larger. This is because, using the same data, we are not measuring bias, but instead the increase in our variance. Larger K's results in less accurate predictions as the predictions become more generalized from the actual data. By comparison, with a training and test set, the generalization away from the training set allows for some of the difference between the training and test data to improve accuracy. Notably, the error of the worst performing models in C are similar to the middling and best performing models in B, but the best performing models in B perform slightly better ( $K \geq 13$ ) than the worst performing models in C. Unsurprisingly, the highest fitting model in C ( $K=1$ ) and several other models (up to  $K=9$ ) outperform all models in B. This would be a result of overfitting to our training data and drawing predictions on that same data.

**5. Prove the following claim: The (training) RSS of the model  $y = 0 + 1X + : 3$  is greater than or equal to the (training) RSS of the model  $y = 0 + 1X + : 3$**

The formula for RSS  $y = \beta_0 + \beta_1 X_1 + \epsilon$



$$RSS = \sum_{i=1}^N (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2$$

and for  $y = \beta_0 + \epsilon$  is:

$$RSS = \sum_{i=1}^N (Y_i - \hat{\beta}_0)^2$$

The former is a subset of the later. However, the uncorrelated  $\hat{\beta}_1$  of the former will tend to 0 if uncorrelated, and if correlated, will tend to reduce  $Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i$  or to move towards the less correlated, so larger RSS of  $Y_i - \hat{\beta}_0 + 0$  or  $Y_i - \hat{\beta}_0$