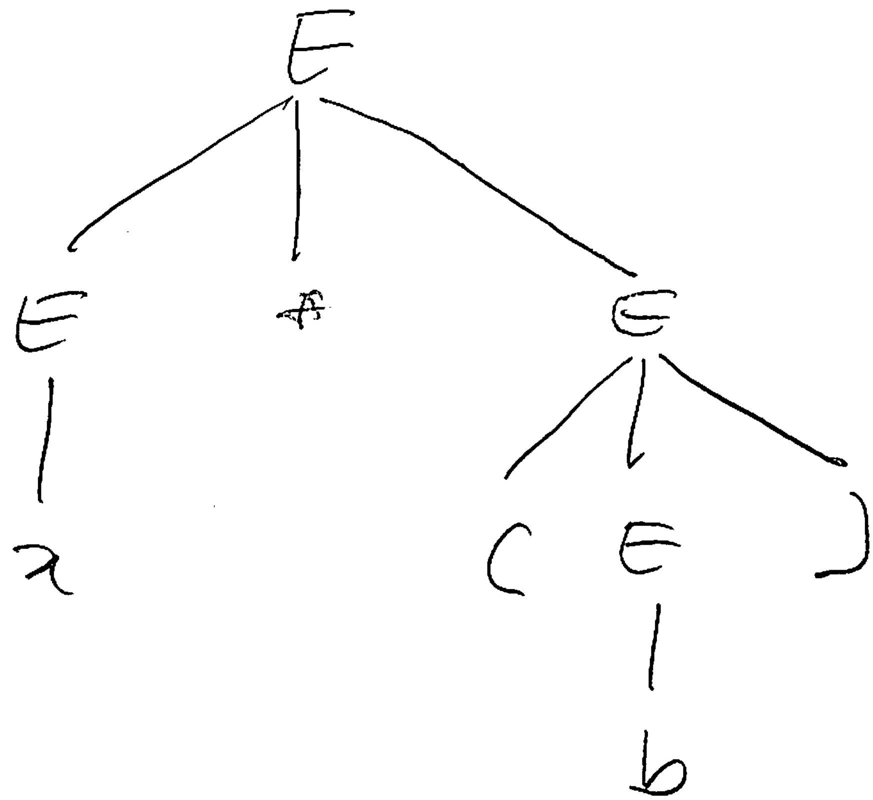
PARSE TREES

$$E \Rightarrow I \mid E+E \mid E*E \mid (E)$$

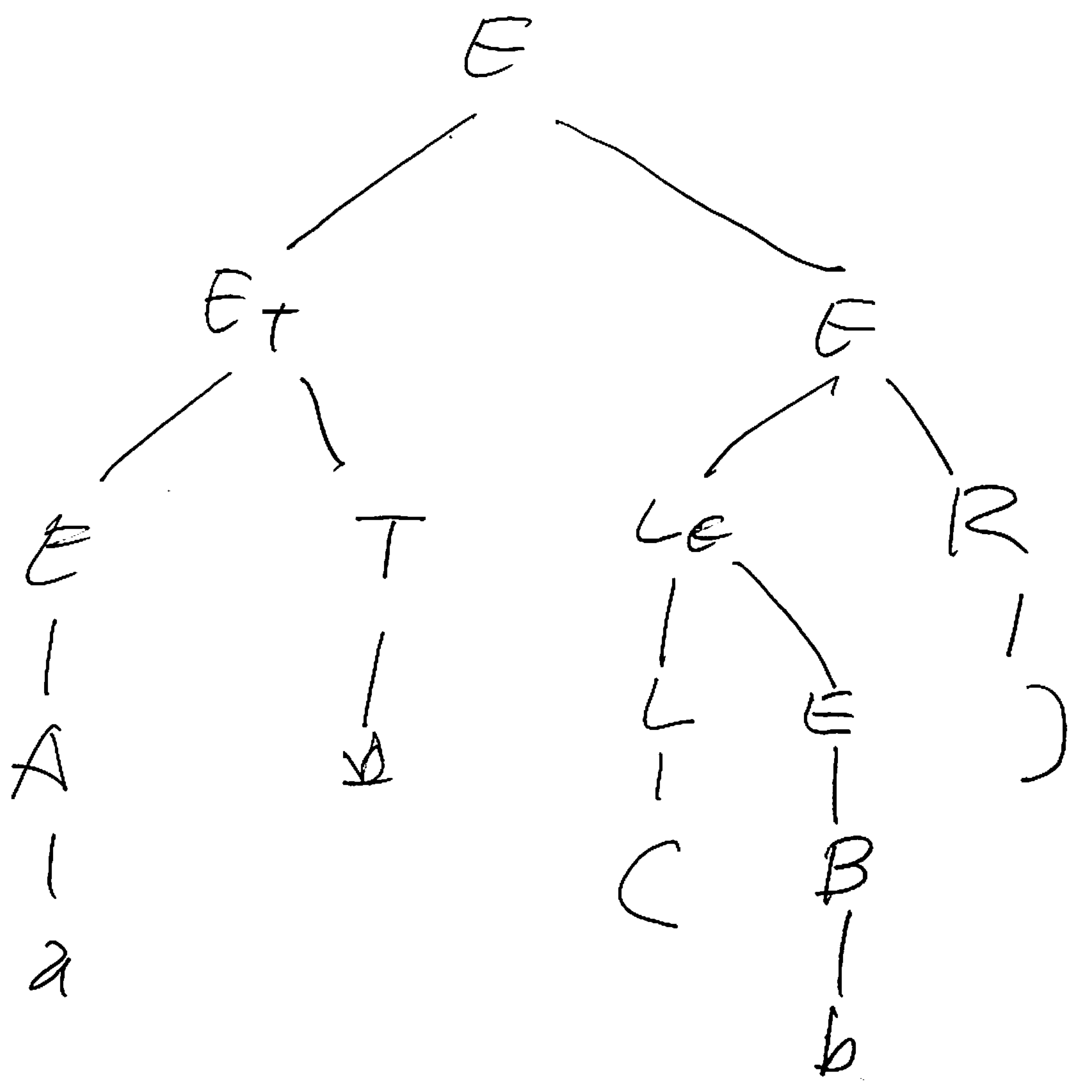
$$I \Rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$$

$$E \Rightarrow E*E \Rightarrow E*(E) \Rightarrow a*(E) \Rightarrow a*(b)$$



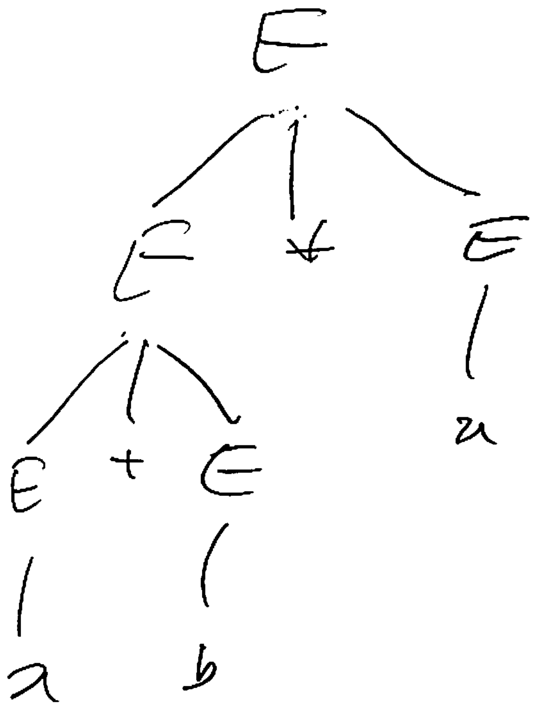
CNF - Chomsky Normal Form

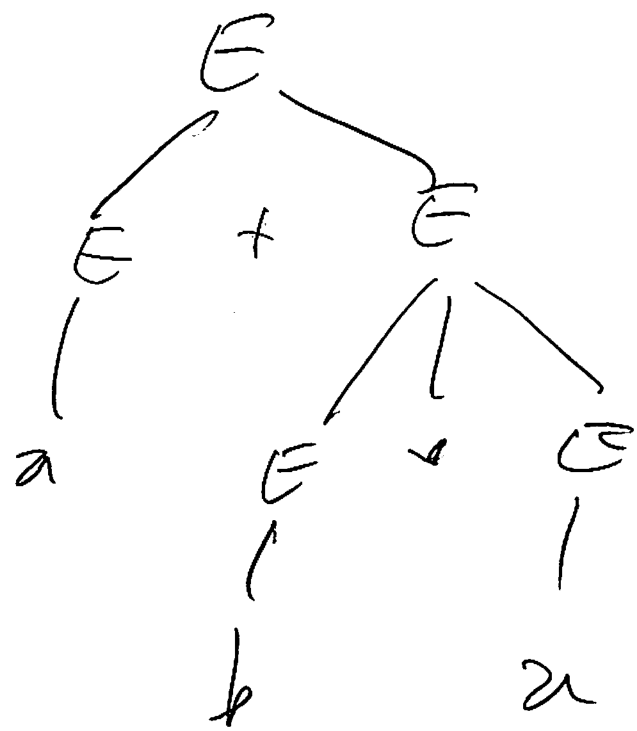
x # C b)



Ambiguity

$a + b * a$





$E \Rightarrow E * E \Rightarrow E + E * E \Rightarrow a + E * E \Rightarrow a + b * E \Rightarrow a + b * a$

$E \Rightarrow E + E \Rightarrow a + E \Rightarrow a + E * E \Rightarrow a + b * E \Rightarrow a + b * a$

FORMALLY

A STRING ω IS derived ambiguously in CFG G if it has TWO OR MORE different leftmost derivations.

Grammar G is ambiguous if it generates some string ambiguously

LEFTMOST DERIVATIONS

At any step replace the leftmost variable by one of it's productions

$a+b*a$

$E \Rightarrow E*E \Rightarrow E+E*E$

$\Rightarrow a+E*E$

$\Rightarrow a+b*E$

$\Rightarrow a+b*a$

$E \Rightarrow E+E \Rightarrow a+E$

$\Rightarrow a+E*E$

$\Rightarrow a+b*E$

$\Rightarrow a+b*a$

Disambiguation

$E \Rightarrow T \mid E + T$

$T \Rightarrow F \mid T * F$

$F \Rightarrow I \mid (E)$

$I \Rightarrow$ same

$a + b * a$

$E \Rightarrow E + T$

$\Rightarrow T + T \Rightarrow F + T \Rightarrow I + T$

$\Rightarrow a + T$

$\Rightarrow a + T * F \Rightarrow a + F * F \Rightarrow$

$\Rightarrow a + I * F \Rightarrow a + b * F \Rightarrow$

$\Rightarrow a + b * I \Rightarrow a + b * a$

CFG $\Rightarrow$ PDA

INPUT $G = (V, \Sigma, R, S)$

OUTPUT $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

1). CONVERT INTO GNF

$A \Rightarrow b$

$A \Rightarrow b\alpha$

$A \Rightarrow \varepsilon$

$\alpha \Rightarrow D_1 \ldots D_n$

2). SINGLE STATE $\{q\}$

3). $q_0 = q_0$

4). $\Gamma = V \cup \Sigma$

5). FOR $A \Rightarrow aX$ $\quad \delta(q, a, A)$

$G = (\{S, X\}, \{a, b\}, P, S)$

$S \Rightarrow XS \mid \varepsilon$

$X \Rightarrow aXb \mid Xb \mid ab$

PDA

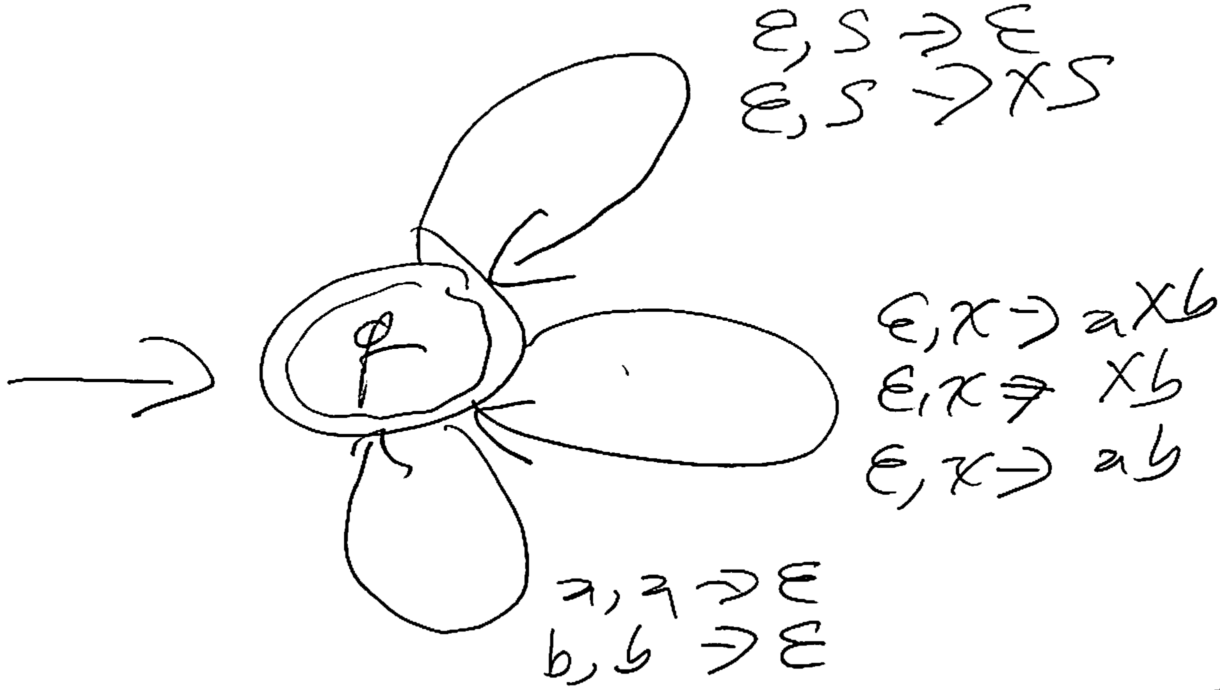
$$P = (\{q\}, \{a, b\}, \{S, X, a, b\}, \delta, q, \varepsilon, \emptyset)$$

$$\delta(q, \varepsilon, S) = \{(q, XS), (q, \varepsilon)\}$$

$$\delta(q, \varepsilon, X) = \{(q, aXb), (q, Xb), (q, ab)\}$$

$$\delta(q, a, a) = \{(q, \varepsilon)\}$$

$$\delta(q, b, b) = \{(q, \varepsilon)\}$$



Section 2.3
Pumping Lemma for CFL's

If A is a CFL, then there is a number p (pumping length) where, if s is any string in A of length at least p, then s may be divided into 5 pieces

$$s = uvwxy$$

satisfying:

1). for each $i \geq 0$ $uv^iwx^iy \in A$
2). $|vx| > 0$
3). $|vwx| \leq p$

$L = \{0^n 1^n 2^n \mid n \geq 1\}$

$z = 0^n 1^n 2^n \quad = u v^i w x^i y$

n is the pumping length

vwx cannot contain both 0's & 2's

- since $|vwx| \leq n$
  - n of those for example $|vwx| > n$

- Case 1
  vwx has no 2's
  - vx is only 0's & 1's
    - $|vx| > 0$
      at least on of them
    - uwy must have n 2's
      only possible if fewer than n 0's or n 1's.

- Case 2
  vwx has no 0's
  - uwy has n 0's
    so fewer than n 1's or 2's.