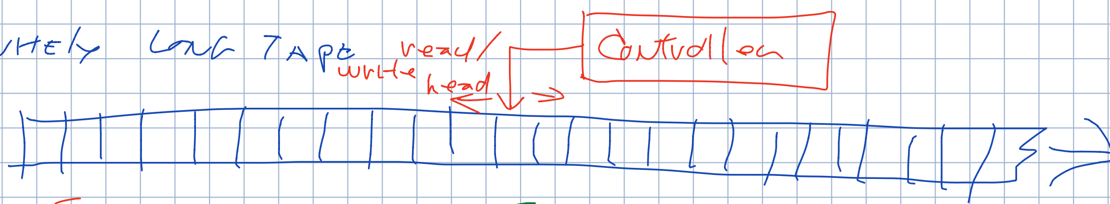
Regular Languages
- No memory

Context-Free Language
- Push Down Stack

Turing Machine
- Generalized memory

Infinitely Long Tape



Inputs
- Current State
- Character under head

Outputs
- New State
- Write to Tape (current position)
- Move head Left or Right

Accepting State
- Immediate

Rejecting State
- Rejecting State
- No transition available

DECIDABILITY

- CAN WE ANSWER (compute) SOMETHING?

FERMAT'S LAST THEOREM

$x^n + y^n = z^n$ for $n > 2$

7-Tuple

$\{Q, \Sigma, \Gamma, \delta, q_0, \{q_{accept}\}, \{q_{reject}\}\}$

$Q \rightarrow$ Valid States

abab_ _ _ _

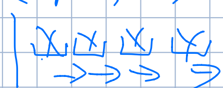
$\Sigma \rightarrow$ Input Symbols $\sqcup \notin \Sigma$

$\Gamma \rightarrow$ Tape Symbols $= \Sigma \cup \sqcup \cup \{\text{control symbols}\}$

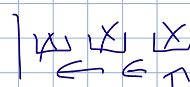
$\delta \rightarrow Q_n \times \Gamma_n \rightarrow Q_{n+1} \times \Gamma_{n+1} \times \{L, R\}$

↳ if at left edge, don't move

$(q_m, \sqcup) \rightarrow (q_m, X, R)$



$(q_m, \sqcup) \rightarrow (q_m, X, R)$



$q_0 \rightarrow$ Start State

$q_{accept} \rightarrow$ Set of accepting states

$q_{reject} \rightarrow$ Set of rejecting states

CONFIGURATION OF A MACHINE

→ Contents of tape
→ Current state
→ Position of head

0 1 1 0 0 0 0
$\uparrow q_3$

0110000
q_3
pos 2

$0 q_3 110000$

$u q v$

Symbols left of head — curr state — Symbol right of head

$101010 q_0 101010$ $\quad \delta(q_0, 1) \to (q_1, x, L)$

$10101 q_1 0x01010$ $\quad \delta(q_1, 0) \to (q_2, y, L)$

$1010 q_2 1yx01010$ $\quad \delta(q_2, 1) \to (q_3, 1, R)$

$10101 q_3 yx01010$ $\quad \delta(q_3, y) \to (q_4, y, R)$

$10101y q_4 x01010$

STANDARD CONFIGURATIONS

- START CONFIGURATION
  - START STATE
  - HEAD POSITION
- ACCEPT CONFIGURATION
  - ACCEPT STATE
- REJECT CONFIGURATION
  - REJECT STATE OR
    No transitions available

A TURING MACHINE M ACCEPTS INPUT w IS A SEQUENCE of configurations exists where:

1). C_1 IS the START configuration of M on w
2). Each C_i YIELDS C_{i+1}
3) C_k IS AN ACCEPTING CONFIGURATION

Palindromes of 0 and 1

0110
10101

$M = (\{q_0 \ldots q_6\}, \{0,1\}, \Sigma \cup \{\sqcup, X, Y\}, \delta, \{q_6\}, \{\text{no tr}\})$

Ex 1 0110

X → → X

Y → Y

$q_0 1001 \Rightarrow Y q_2 001 \Rightarrow Y0 q_2 01 \Rightarrow Y00 q_2 1 \Rightarrow$
$Y001 q_2 \sqcup \Rightarrow Y00 q_4 1 \Rightarrow Y0 q_5 0Y \Rightarrow Y q_5 00Y \Rightarrow$
$q_5 Y00Y \Rightarrow Y q_0 00Y \Rightarrow YX q_1 0Y \Rightarrow YX0 q_1 Y \Rightarrow$
$YX q_3 0Y \Rightarrow Y q_5 XXY \Rightarrow YX q_0 XY \Rightarrow YXX q_6 Y$

ACCEPT

Ex 2 10101

$q_0 10101 \Rightarrow Y q_2 0101 \Rightarrow Y0 q_2 101 \Rightarrow Y01 q_2 01 \Rightarrow Y010 q_2 1$
$Y0101 q_2 \Rightarrow Y010 q_4 1 \Rightarrow Y01 q_5 0Y \Rightarrow Y0 q_5 10Y \Rightarrow Y q_5 010Y$
$q_5 Y010Y \Rightarrow Y q_0 010Y \Rightarrow YX q_1 10Y \Rightarrow YX1 q_1 0Y \Rightarrow YX10 q_1 Y$
$YX1 q_3 0Y \Rightarrow YX q_5 1XY \Rightarrow Y q_5 X1XY \Rightarrow YX q_0 1XY \Rightarrow$
$YXY q_2 XY \Rightarrow YX q_4 YXY \Rightarrow YXY q_6 XY$

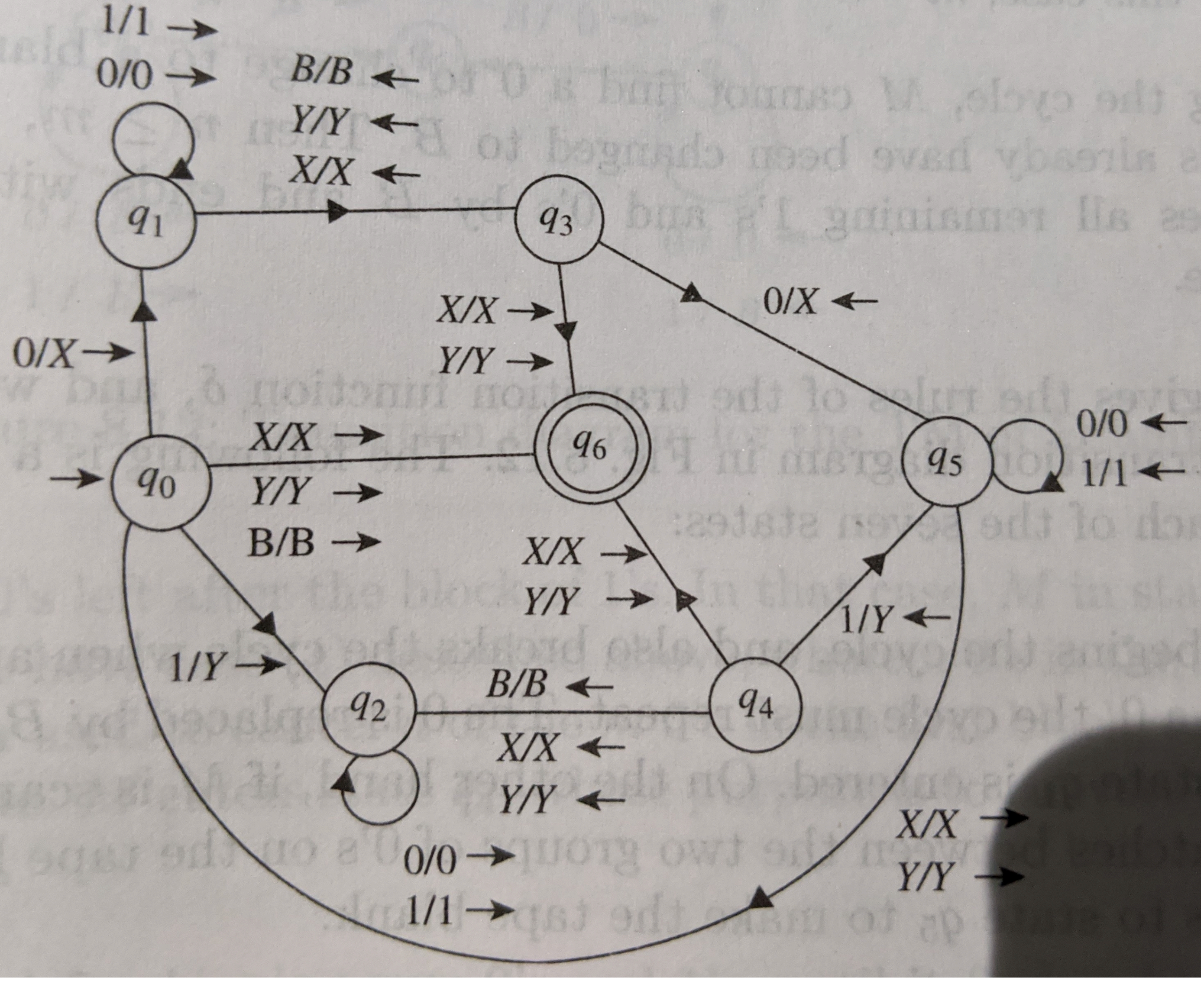
accept

Ex 3 1011

$q_0 1011 \Rightarrow Y q_2 011 \rightarrow Y0 q_2 11 \rightarrow Y01 q_2 1 \rightarrow Y011 q_2 \rightarrow$
$Y01 q_4 1 \rightarrow Y0 q_5 1Y \rightarrow Y q_5 01Y \Rightarrow q_5 Y01Y \rightarrow Y q_0 01Y$
$YX q_1 1Y \Rightarrow YX1 q_1 Y \rightarrow YX q_3 1Y$

No transition defined $\Rightarrow$ REJECT!

State	0	1	X	Y	B
q_0	(q_1, X, R)	(q_2, Y, R)	(q_6, X, R)	(q_6, Y, R)	(q_6, B, R)
q_1	$(q_1, 0, R)$	$(q_1, 1, R)$	(q_3, X, L)	(q_3, Y, L)	(q_3, B, L)
q_2	$(q_2, 0, R)$	$(q_2, 1, R)$	(q_4, X, L)	(q_4, Y, L)	(q_4, B, L)
q_3	(q_5, X, L)	—	(q_6, X, R)	(q_6, Y, R)	—
q_4	—	(q_5, Y, L)	(q_6, X, R)	(q_6, Y, R)	—
q_5	$(q_5, 0, L)$	$(q_5, 1, L)$	(q_0, X, R)	(q_0, Y, R)	—


1/1 →
0/0 →
B/B ←
Y/Y ←
X/X ←
q1
q3
X/X →
Y/Y →
0/X ←
0/X →
X/X →
q6
Y/Y →
B/B →
q0
q5
0/0 ←
1/1 ←
X/X →
Y/Y →
1/Y ←
1/Y →
B/B ←
q2
q4
X/X ←
Y/Y ←
X/X →
Y/Y →
0/0 →
1/1 →

Possibilities
- ACCEPT
- REJECT
- NEVER HALT

ACCEPT, REJECT } → DECIDABLE

TURING RECOGNIZABLE LANGUAGE
if some turing machine ACCEPTS IT
- RECURSIVELY ENUMERABLE

TURING DECIDABLE
A TURING MACHINE THAT HALTS ON ALL INPUTS is called a decider. A language accepted by a decider is Turing-decidable
- RECURSIVE

$M_1 = \{w\#w \mid w \in \{0,1\}^*\}$

1011#1011
1011 #1011
x011 #1011
x011 #1011
x011 #1011
x011 #1011
x011 #1011
x011 #x011

x011 #x011
x011 #
x011 #
x011 #
x011 #x011
xy11#x011
xy11#
xy11#x011
xy11#x011
xy11#xy11
xy11#xy11

x x
x x
#
␣ Accept

State Diagram
pg 173
Sipser

Sipser 3.11 pg 174

$M_2 = \{a^i b^j c^k \mid i \times j = k, \text{ and } i,j,k > 0\}$

aabbbcccccc

x

y z

y z

y z

xayyyzzzccc

xabbbzzzccc

x

y z

y z

y z

xxyyyzzzzzz

bbb

→ ⊔ ← accept