RELATIONSHIPS

- EVERY $O(n)$ MULTI-TAPE TM HAS A $O(n^2)$ SINGLE TAPE TM

$O(n)$ M-T TM
$\times\ O(n)$ REDUCTION TO SINGLE TAPE
$O(n^2)$ SINGLE TAPE TM

- EVERY $O(n)$ NON-DETERMINISTIC TM HAS A $O(2^n)$ DETERMINISTIC TM
$O(2^n)$ POSSIBLE N.D TREES

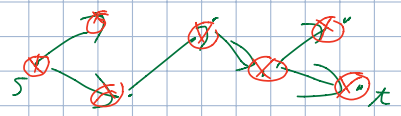
$\times\ O(n)$ A SINGLE PATH

$\times 2^n \Rightarrow O(2^n)$

P → POLYNOMIAL TIME

P IS THE CLASS OF LANGUAGES THAT ARE DECIDABLE IN POLYNOMIAL TIME ON A DETERMINISTIC, SINGLE TAPE TM

PATH Problem



IS THERE A PATH FROM s TO t?

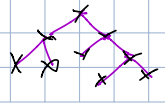
- Try every path $\Rightarrow O(m^m)$ → Not P
m nodes
- Breadth-first search: $O(n) \rightarrow P$

$P \neq NP$

IS A CFL DECIDABLE?

Brute force: $2n-1$ possible steps in CNF
$\Rightarrow \Omega(2^{(n-1)}) \Rightarrow O(2^n)$

- Dynamic programming
  - Try small pieces & record the results
    - Try 1 CNF production & save the results
      - Try another production using the results already obtained
      - Repeat



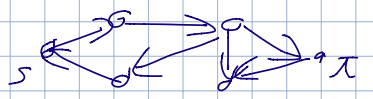
$O(n) \rightarrow P$

NON-DETERMINISTIC POLYNOMIAL TIME

- NP IS THE CLASS OF LANGUAGES THAT ARE DECIDABLE IN POLYNOMIAL TIME ON A NON-DETERMINISTIC TM
- NP IS THE CLASS OF LANGUAGES THAT CAN BE VERIFIED IN POLYNOMIAL TIME

HAMILTONIAN PATH PROBLEM

- DIRECTED GRAPH



IS THERE A PATH FROM s TO t THAT GOES THROUGH EACH VERTEX ONCE AND ONLY ONCE

BRUTE FORCE $O(m^m)$

$\Rightarrow$ NP

GIVEN SOME PATH $p = \{v_i, v_{i+1}, v_{i+2}, \ldots, v_{i+m}\}$

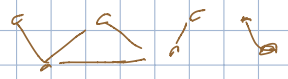
- WALK THROUGH THE GRAPH IN $O(m)$ & ACCEPT OR REJECT
  $\rightarrow$ VERIFICATION IS P

Problems

3-SAT - Boolean satisfiability
$(\neg a \vee b) \wedge (a \vee c) \wedge (b \vee c)$
- Is proof of a set of values that guarantees the expression is true

3-DM - 3 Dimensional Matching
- Given some set of vertices M divided into 3 sets W, X, Y
- Does M contain a matching, M', subset of M, such that $|M'| = q$ & no 2 elements of M' are the same

VC - Vertex Cover
- Graph problem
  - Subset of size K such that all vertices in K are connected



Ack NP problems

CLIQUE
HAMILTONIAN CIRCUIT
PARTITION

∃ GRAPH
- Set Theory

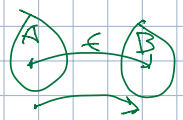
NP-Completeness

- Core NP problems
  - If any of these problems can be proven to be P-time then all NP problems are P. $\Rightarrow P = NP$

- A language B is NP-Complete if
  1. B is in NP
  2. Every problem A in NP is polynomial time reducible to B

Polynomial Time Reducability

- Language A is PTR to B, written $A \leq_p B$, if a polynomial time function $f: \Sigma^* \to \Sigma^*$ exists where for every w,
  $w \in A \iff f(w) \in B$

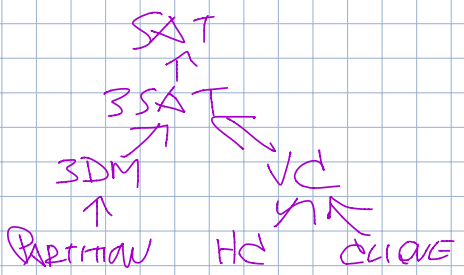


IF $A \leq_p B$ AND $B \in P$ THEN $A \in P$

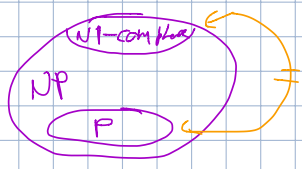
Cooks Theorem

Boolean Satisfiability
- Given any Boolean expression, is there a setting of var such that the expression is true

Cook proved SAT is in NP



Clique $\leq_p$ SAT



$\neq$ P = NP NOT KNOWN

NP-HARD

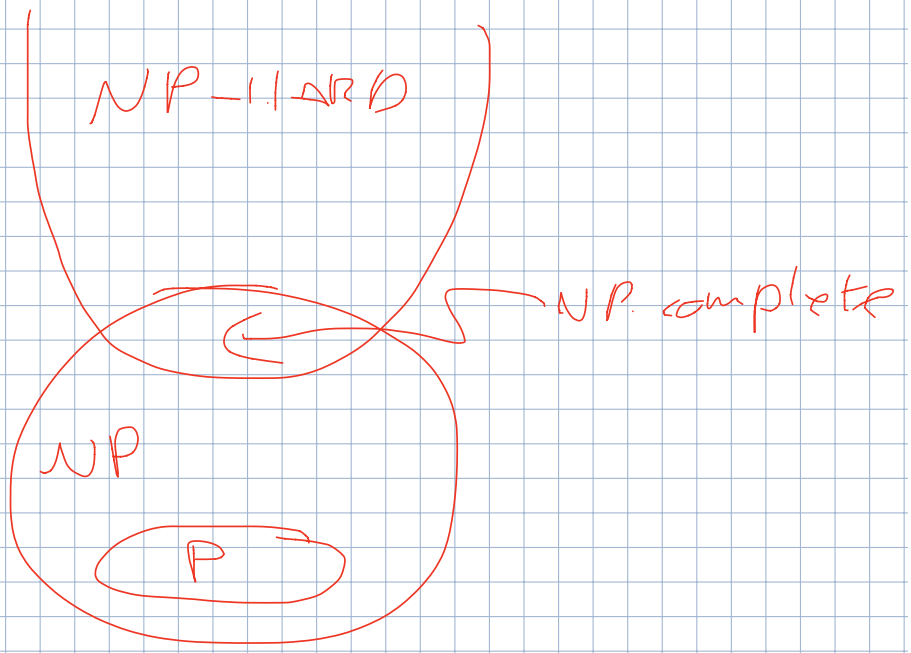
- INFORMALLY; AT LEAST AS HARD AS THE HARDEST problems IN NP
- A problem H IS NP-Hard when EVERY problem L IN NP CAN BE REDUCED IN polynomial TIME TO H.

SUBSET SUM PROBLEM

- SET OF INTEGERS S
  - IS THERE A SUBSET OF S THAT SUMS TO ZERO?

$\{-7, -3, -2, 5, 8\} \Rightarrow \{-3, -2, 5\}$

- Provable to be NP-Complete
- Provable that ANY NP problem can be polynomial time REDUCED TO Subset Sum.



$P \neq NP$

Core Concepts

Chomsky's Hierarchy
- 4 classes
- Conversions
  - NFA → DFA
  - CFL → CNF → PDA
  - Express why simpler languages use a more complex one
    DFA ⇒ CFL

Implementations
- Push down automata
- Turing machine
  - Universal representation of computation
  - Universal Emulators

Decidability
↓
Reducability
↓
Computational Complexity

Rust

- What problems is it attempting to solve
  - Memory Safety w/o overhead of garbage collection
- Restrictive Model
  - Mutable vs. Immutable References