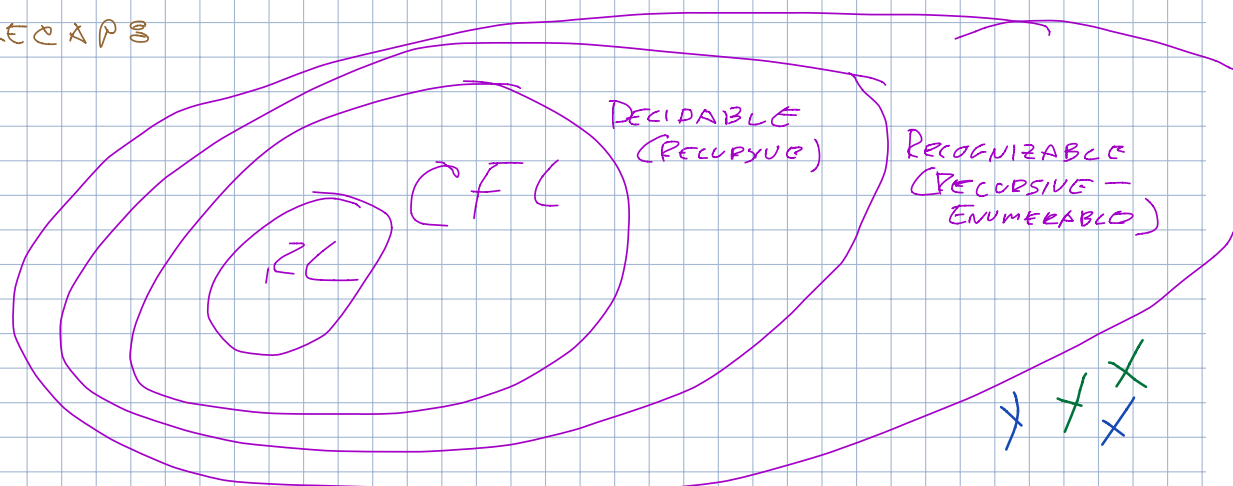


# Recaps



$A_{DFA} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts } w \}$

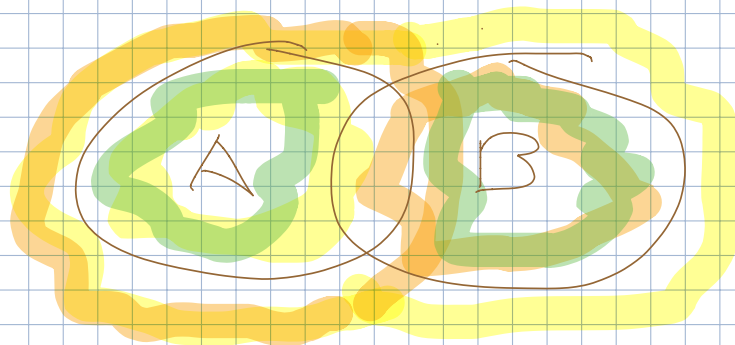
$A_{NFA} = \{ \dots \} \rightarrow \text{ALWAYS CONVERT AN NFA TO A DFA}$

$A_{RE} = \{ \dots \} \rightarrow \text{ALWAYS CONVERT AN RE TO A DFA}$

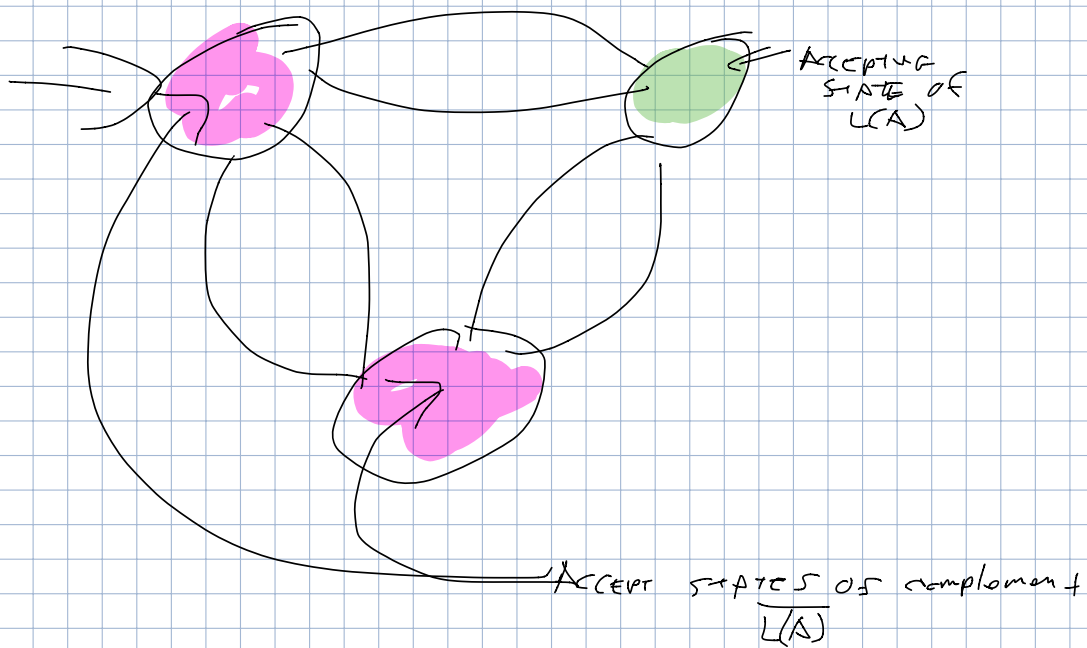
$E_{DFA} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) = \emptyset \}$

$E_{DFA} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFA's and } L(A) = L(B) \}$

$L(A) = (L(A) \cap \overline{L(B)}) \cup (L(B) \cap \overline{L(A)}) \rightarrow \text{Empty}$   
 $\hookrightarrow L(A) = L(B)$



Complement of a DFA  
 $L(A)$



INTERSECTION OF LANGUAGES  
 DE MORGAN'S LAW  
 $A \cap B = \overline{A \cup B}$

## CONTEXT-FREE LANGUAGES

$A_{CFG} = \{ \langle G, w \rangle \mid G \text{ is a CFG that generates string } w \}$

$E_{CFL} = \{ \langle G \rangle \mid G \text{ is a CFG and } L(G) = \emptyset \}$

$EQ_{CFG} = \{ \langle G, H \rangle \mid G \text{ and } H \text{ are CFG's and } L(G) = L(H) \}$

**EVERY CFG IS DECIDABLE**

## UNDECIDABILITY

$$A_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM and } w \text{ is a string in the language of } M \}$$

- $U \rightarrow$
- 1) SIMULATE  $M$  ON  $w$
  - 2) IF  $M$  EVER ENTERS AN ACCEPT STATE  $\rightarrow$  ACCEPT  
IF  $M$  EVER ENTERS A REJECT STATE  $\rightarrow$  REJECT

### Diagonalization

- ONE TO ONE  $f(a) \neq f(b)$  whenever  $a \neq b$
- ONTO, FOR EVERY  $b \in B$  there is a  $a \in A$  such that  $f(a) = b$
- IF  $A$  &  $B$  BOTH ONE TO ONE & ONTO THEN THERE IS A CORRESPONDENCE BETWEEN  $A$  &  $B$ 
  - $A$  &  $B$  ARE THE SAME SIZE
- $|A| > |B|$  IS NOT 1:1 BUT ONTO
- $|A| < |B|$  IF 1:1 BUT NOT ONTO

$\#$  ARE NOT COUNTABLE

SOME LANGUAGES ARE NOT RECOGNIZABLE

$$|TM| \stackrel{?}{=} |L|$$

- $|TM|$  IS COUNTABLE
- $|L|$  IS NOT COUNTABLE  
 $\{ \{0,1\}^* \}$   
 $0, 1, 00, 10, 11, \dots$

$$|TM| < |L|$$

$$A_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM and } w \text{ is a string in the language of } M \}$$

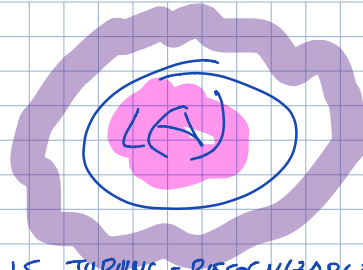
- THERE ARE STRINGS THAT ARE NOT ACCEPTED BY ANY TM
- CANNOT DECIDE  $A_{TM}$

$\Delta$  NON RECOGNIZABLE LANGUAGE

IF  $L(\Delta)$  IS TURING RECOGNIZABLE

IF  $\overline{L(\Delta)}$  IS ALSO TURING RECOGNIZABLE

$\Delta$  IS DECIDABLE



$\Delta$  LANGUAGE IS TURING DECIDABLE IFF IT IS TURING-RECOGNIZABLE & IT'S COMPLEMENT IS TURING-RECOGNIZABLE

$\Delta_{TM}$  IS TURING-RECOGNIZABLE

$\Delta_{TM}$  IS NOT TURING DECIDABLE

$\overline{\Delta_{TM}}$   $\rightarrow$  TURING RECOGNIZABLE  
 $\rightarrow$  CANNOT BE TURING RECOGNIZABLE