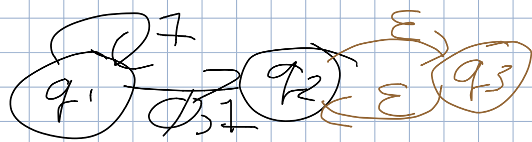
Nondeterministic Finite Automata



$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$

$P(Q)$ → POWER SET OF Q

ALL POSSIBLE SUBSETS OF Q

A NFA IS A FIVE TUPLE → $(Q, \Sigma_\varepsilon, \delta, q_0, F \subseteq Q)$

Q → FINITE SET OF STATES

Σ_ε → ALPHABET $\cup \{\varepsilon\}$

$\delta \rightarrow Q \times \Sigma_\varepsilon \rightarrow P(Q)$

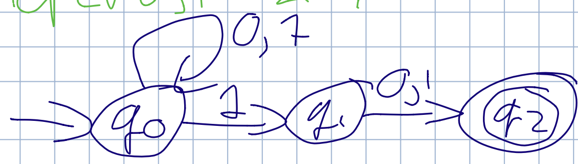
$q_0 \in Q$ → THE START STATE

$F \subseteq Q$ → THE ACCEPT STATES

NFA to DFA Conversion

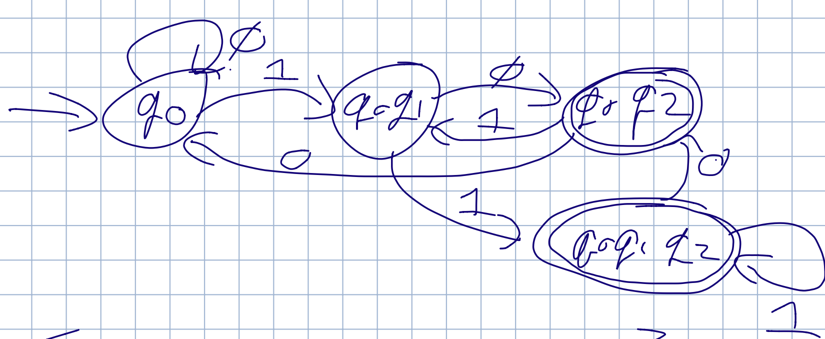
Hopcroft 2.9

W



$q_0 = q_0$

		0	1	
→	q_0	q_0	$q_0 q_1$	
	q_1	q_2	q_2	X
*	q_2	$\emptyset$	$\emptyset$	X
	$q_0 q_1$	$q_0 q_2$	$q_0 q_1 q_2$	
*	$q_0 q_2$	q_0	$q_0 q_1$	
*	$q_1 q_2$	q_2	q_2	X
*	$q_0 q_1 q_2$	$q_0 q_2$	$q_0 q_1 q_2$	



$Q = \{q_0, q_0 q_1, q_0 q_2, q_0 q_1 q_2\}$

$\Sigma_\varepsilon \rightarrow 0, 1, \varepsilon$

$\delta \rightarrow$ see above

$q_0 \rightarrow q_0$

$F \rightarrow \{q_0 q_2, q_0 q_1 q_2\}$

Regular Operations

UNION: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$A = \{0, 01\}$ $\quad A \cup B =$

$B = \{1, 101\}$ $\quad \{0, 01, 1, 101\}$

CONCAT: $A \circ B = \{xy \mid x \in A \text{ AND } y \in B\}$

$A \circ B = \{01, 0101, 011, 01101\}$

STAR: $A^* = \{x_1 x_2 x_3 \ldots x_k \mid k \geq 0 \text{ AND EACH } x_i \in A\}$

$A = \{0\}$

$A^* = \{\emptyset, 0, 00, 000, \ldots\ldots\}$

$L(R)$ IS THE LANGUAGE OF R

* $R \cup \emptyset = R$

$R \circ \varepsilon = R$

$R \circ \emptyset = \emptyset$

Regular Expressions

- SET OF SYMBOLS CONNECTED BY REGULAR OPERATORS

FORMALLY

R IS A REGULAR EXPRESSION IF R IS:

a) a for some $a \in \Sigma$
b) ε THE LANGUAGE $\{\varepsilon\}$
c) $\emptyset$ THE EMPTY LANGUAGE
d) $(R_1 \cup R_2)$ where R_1 AND R_2 ARE REG EXP'S
e) $(R_1 \circ R_2)$ " " " " " " "
f) (R_1^*) where R_1 IS A REG EXP

SHORT HANDS

$R^+ \Rightarrow RR^* \Rightarrow R \circ R^*$

- $\Rightarrow \{a \mid a \in \Sigma\}$

e.g. $\Sigma = \{a, b, c\}$ then . a OR b OR c

$[abc] \Rightarrow \{q \mid q \in \{a,b,c\} \in \Sigma\}$

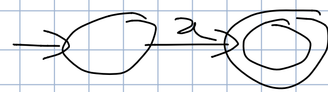
e.g. $\Sigma = \{a, b, c, d\}$ $[bc]$ b OR C

EQUIVALENCE W/ FA

A LANGUAGE IS REGULAR IF AND ONLY IF SOME REGEX DESCRIBES IT. SIPSER THEOREM 1.54

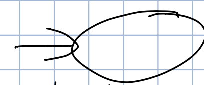
Regular expression to DFA

a). $R = a, \; a \in \Sigma$

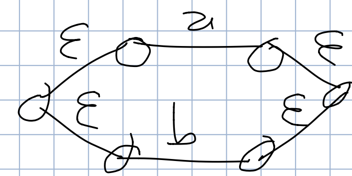


b). $R = \varepsilon$

c). $R = \emptyset$

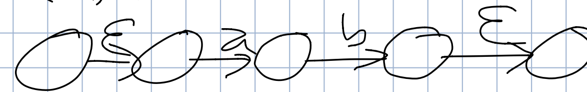


d). $R_1 \cup R_2$ R_1, R_2 symbols a, b

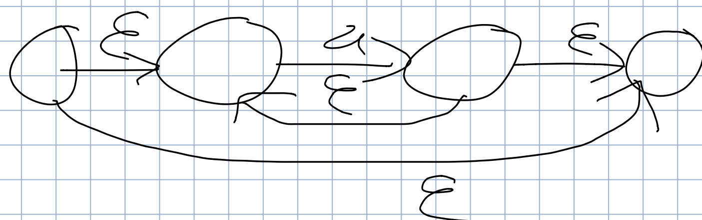


e). $R_1 \circ R_2$ R_1, R_2 symbols $\{a, b\}$

$a \circ b \Rightarrow$



f). R^*

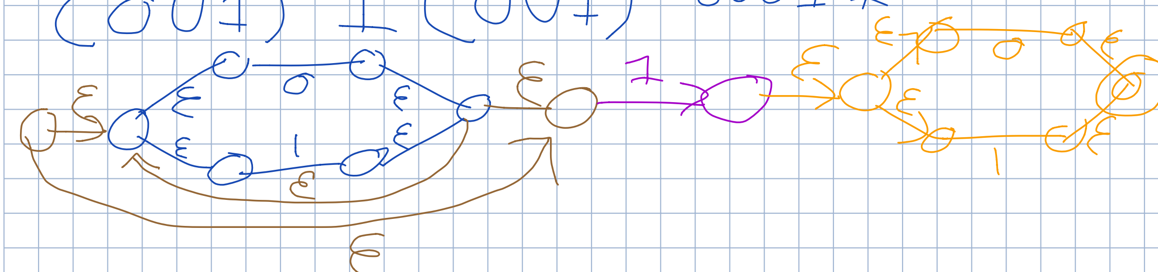


Hopcroft Ex 3.8

$(0 \cup 1)^* 1 (0 \cup 1)$

00010

00011



Pumping Lemma

If A is a reg lang, there is a Number p where is s is a STRING IN A of AT LEAST length p, then s may be divided INTO 3 pieces $s = xyz$, satisfying the following conditions

1) For all $i \geq \emptyset$, $xy^iz \in A$
2) $y \neq \epsilon$
3) $|xy| \leq p$

Example:

$L = \{0^n 1^n \mid n \geq \emptyset\}$

1). $w = 0^n 1^n = xyz$

$|xy| \leq n$ so $x = \emptyset, y = \emptyset$

$y \neq \varepsilon$ $|y| > 0$

2). $x = 0^a$ $a + b \leq n$ r3

$y = 0^b$ $b \geq 1$ r2

$z = 0^c 1^n$ $a + b + c = n$

3). but if $i = \emptyset$

$0^a 0^b 0^c 1^n$

$0^a 0^c 1^n = 0^{a+c} 1^n$

but $a + c \neq n$

because $b \geq 1$