

Electric Field

1 Overview

The electric field vector, \vec{E} , is a quantity assigned to a point in space. Given this quantity, we can compute the force a charge Q will experience if it is placed at that point using the equation $\vec{F} = Q\vec{E}$. The direction of \vec{E} is also the direction a positive charge will begin to move if released from rest.

To find \vec{E} at any point in space, compute the force \vec{F} due to all other charges on a hypothetical (or “test”) charge q_o that has an infinitesimal charge and size at a point where you want to know \vec{E} . To find \vec{E} at that point, divide \vec{F} by q_o .

$$\vec{E}_{\text{due to } q} = \frac{\vec{F}_{q \text{ on } q_o}}{q_o}$$

2 Example I

A positive charge q_1 is at $(x, y) = (-a, -a)$. Find the electric field at $(x, y) = (a, a)$ in the form $\vec{E} = E_x\hat{i} + E_y\hat{j}$. Also, find E . (Note that E and $|\vec{E}|$ are used interchangeably.)

Solution

To find the electric field at a point in space, we put a hypothetical and positive “test” charge q_o at that point, compute the force on it due to all other charges, and then use

$$\vec{E} = \frac{\vec{F}}{q_o}$$

The force a positive charge q_1 at $(x, y) = (-a, -a)$ exerts on a positive charge q_2 at $(x, y) = (a, a)$ was computed in a previous activity. The result was

$$\vec{F}_{q_1 \text{ on } q_2} = k \frac{|q_1 q_2|}{8a^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$$

Because both charges are positive, we can drop the absolute value in the above equation, giving

$$\vec{F}_{q_1 \text{ on } q_2} = k \frac{q_1 q_2}{8a^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$$

If we replace q_2 with q_o , this is

$$\vec{F}_{q_1 \text{ on } q_o} = k \frac{q_1 q_o}{8a^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$$

The electric field at the location of q_o is then

$$\vec{E}_{\text{at } (a,a) \text{ due to } q_1} = \frac{\vec{F}}{q_o} = \frac{kq_1}{8a^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}) = \frac{kq_1}{8a^2} \left[\frac{1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{2}} \hat{j} \right],$$

4 Problem II

To find the electric field at a point in space in Example I and Problem I, we first computed the force on a charge at that point and then divided the result by the charge.

An alternative approach is to start with the equation for the magnitude of the electric field due to a charge q at a point that is a distance r from the charge:

$$E_{\text{due to } q} = k|q|/r^2$$

Charge q_1 is at $(x, y) = (a, a)$. Find the magnitude of the electric field, E , at $(x, y) = (0, 0)$ using the above formula and then find $\vec{\mathbf{E}}$ in the form $\vec{\mathbf{E}} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}}$.