

Magnetic Flux

1 Introduction

The definition of magnetic flux is

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

When the magnitude and direction of $\vec{\mathbf{B}}$ is the same at all points on the surface, the integral simplifies to

$$\Phi_B = \mathbf{B} \cdot \mathbf{A}$$

or, equivalently,

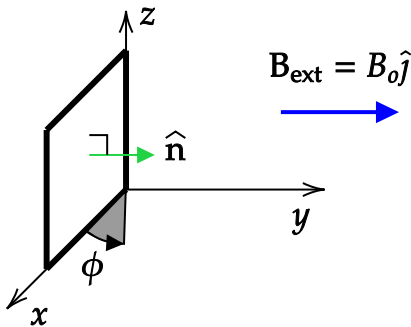
$$\Phi_B = BA \cos \phi$$

where θ is the angle between the $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$ vectors.

Note that previously electric flux was covered. The same techniques that apply to computing electric flux apply to magnetic flux. See the Electric Flux activity for additional discussion.

2 Computing Φ_B

In the following figure, a square loop of area A that can be rotated about the z axis is shown. Assume that the normal direction of the loop is as shown in the diagram. A uniform external magnetic field points in the $\hat{\mathbf{j}}$ direction.

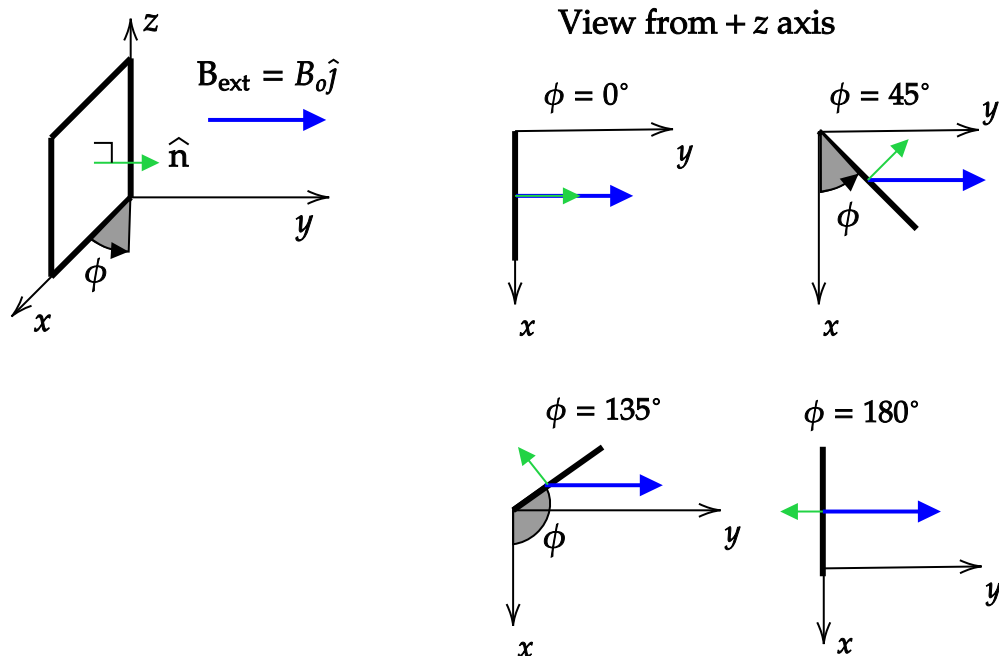


1. At what angles in the range of $\phi = [0, 360^\circ]$ is the magnetic flux zero?

Answer: The magnetic flux will be zero when the normal vector is perpendicular to $\vec{\mathbf{B}}_{\text{ext}}$. This corresponds to $\phi = 90^\circ$ and $\phi = 270^\circ$.

2. Draw the loop, $\hat{\mathbf{n}}$, and $\vec{\mathbf{B}}_{\text{ext}}$ as they would appear when viewed from large z when $\phi = 0^\circ$, $\phi = 45^\circ$, $\phi = 135^\circ$, and $\phi = 180^\circ$.

Answer:



3. When $\phi = 45^\circ$, is the magnetic flux positive or negative? What is its value in terms of $B_o A$?

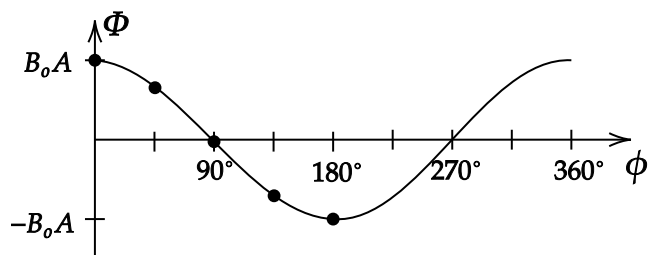
Answer: As shown in the diagram in the solution to 2., the dot product of the normal vector and \mathbf{B}_{ext} will be positive when $\phi = 45^\circ$, so the flux will be positive. Its value will be $B_o A \cos 45^\circ = B_o A / \sqrt{2}$.

4. When $\phi = 135^\circ$, is the magnetic flux positive or negative? What is its value in terms of $B_o A$?

Answer: As shown in the diagram above, when $\phi = 135^\circ$ the dot product of the normal vector and \mathbf{B}_{ext} will be negative, so the flux will be negative. Its value will be $B_o A \cos 135^\circ = -B_o A \cos 45^\circ = -B_o A / \sqrt{2}$.

5. Sketch a plot of the magnetic flux, Φ , as a function of ϕ .

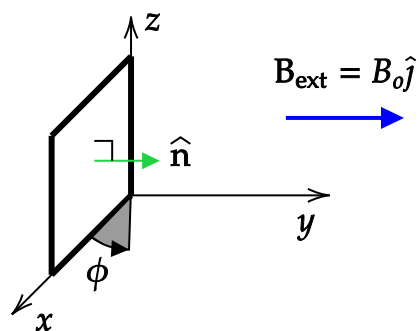
The curve is $\Phi = B_o A \cos \phi$ is sketched below along with dots that were used to help determine the shape of the curve.



3 Computing $d\Phi_B/dt$

The time rate of change of magnetic flux through a closed loop is a quantity that will be used when Faraday's law is covered.

Suppose the loop in the previous problem rotates at a constant rate.



1. At what angles is $d\Phi_B/dt = 0$?
2. For what angle range is $d\Phi_B/dt$ increasing?
3. For what angle range is $d\Phi_B/dt$ decreasing?
4. What is the formula for $d\Phi_B/dt$?