RC Circuits

1 Notation and Equations

- We use the the letter "C" as a label for a capacitor and the units of charge. When used as a label, C is usually written in italics. Thus

  "$Q_C = 10$ C" means "the capacitor labeled C has a charge of 10 Coulombs."
- Lower case letters are used for electrical quantities that vary in time. In circuits constant current, charge, and voltages, we used I, Q, and V. In the circuits considered in this activity, the currents and voltages vary in time, so we use $i(t)$, $q(t)$, and $v(t)$.
- In this activity, we use the the relationship $i(t) = dq(t)/dt$ between the current $i(t)$ in the wires connected to a capacitor and the charge $q(t)$ on the capacitor.

2 Discharging Capacitor

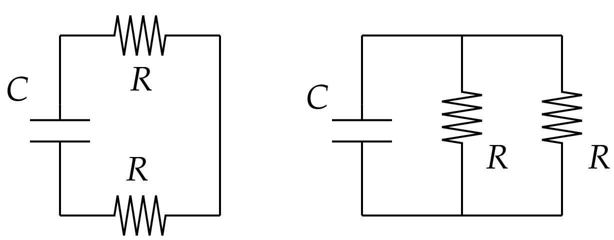
If one capacitor with capacitance C is in a circuit with other resistors (and no emfs), the charge on the capacitor changes with time according to

$q(t) = Q_o e^{-t/\tau}$,

where $\tau \equiv RC$, Q_o is the charge on the capacitor at $t = 0$, and R is the equivalent resistance. The quantity τ is called the "RC time constant." The equation for $q(t)$ is derived by solving the differential equation that follows from using Kirchhoff's voltage law.

2.1 Problem

Find τ in seconds for the following two circuits. Use $R = 10$ kΩ and $C = 1$ μF. Note that $1\ \Omega \cdot \text{F} = 1$ second.

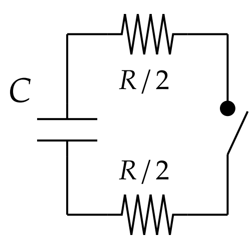


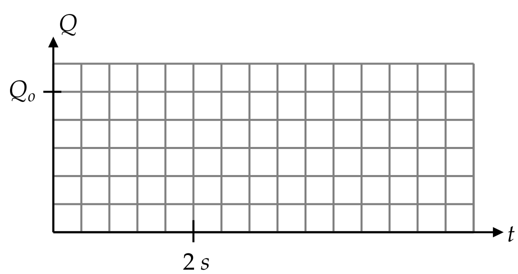
2.2 Problem

For the following circuit, find the charge q on the capacitor at $t = 2$ s if the switch is closed at $t = 0$ and the capacitor has an initial charge of Q_o for the following three cases.

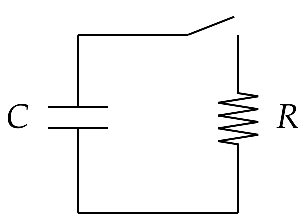
1. $RC = 1$ s $\quad q(2\text{ s}) = \boxed{}$
2. $RC = 2$ s $\quad q(2\text{ s}) = \boxed{}$
3. $RC = 4$ s $\quad q(2\text{ s}) = \boxed{}$

4. Next, sketch a plot of $q(t)$ from $t = 0$ to $t = 6$ s for each of these three cases. Used a solid, dashed, and dotted line for case 1., 2., and 3., respectively. (Hint: For all curves $q(0) = Q_o$. Use the $q(2)$ values from parts 1.–3. to help sketch the curve.)





2.3 Problem



In the circuit above, the values of R and C are such that $RC = 1$ s.

1. If the capacitor has a charge of 10 nC and the switch is closed, how long will it take for the charge on the capacitor to fall to half of this value?
2. If the capacitor instead had a charge of 20 nC and the switch is closed, how long will it take for the charge on the capacitor to fall to half of this value?

3 Charging a Capacitor

If a single capacitor is in series with other resistors and a DC voltage source, the charge of the capacitor varies according to

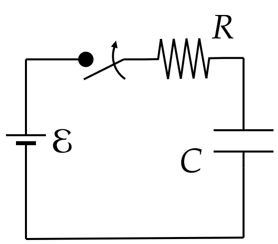
$$q(t) = Q_f(1 - e^{-t/\tau})$$

where $\tau \equiv RC$, R is the equivalent resistance, and Q_f is the final charge on the capacitor, that is, the charge on the capacitor as $t \to \infty$ (it is technically more accurate to say $t \gg \tau$ instead of $t \to \infty$; why?).

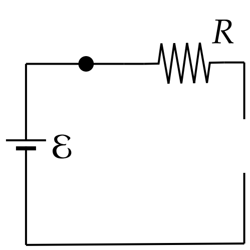
To find Q_f, replace the capacitor with an open circuit and use Kirchhoff's voltage law to find the voltage across the capacitor, V_f, and then use the relationship $Q_f = CV_f$.

3.1 Example

Find $q(t)$ for the following circuit, assuming the switch is closed at $t = 0$ and the capacitor is initially uncharged.



Answer: When the switch is closed, charge builds up on the capacitor. This build–up continues until the charge on the capacitor is such that no current flows in the circuit. If no current flows through the capacitor, the circuit is equivalent to one in which the capacitor is replaced with an open circuit, as shown below.



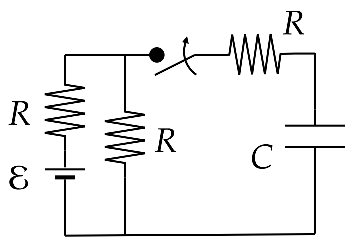
Kirchhoff's voltage Law gives $\mathcal{E} - i(t)R - v_C(t) = 0$, where $v_C(t)$ is the voltage across the capacitor. For large t, $i(t) = 0$, so $v_C(t) = \mathcal{E}$. This voltage is for large t, so we relabel it as V_f. Using $Q_f = CV_f$ gives $Q_f = C\mathcal{E}$, so

$$q(t) = C\mathcal{E}(1 - e^{-t/RC})$$

Note that the general formula $q(t) = Q_f(1 - e^{-t/\tau})$ that applies for all t is derived by solving the differential equation $\mathcal{E} - i(t)R - v_C(t) = 0$ using $i(t) = dq(t)/dt$ and $v_C(t) = q(t)/C$.

3.2 Problem

For a circuit with any number of capacitors, DC voltage sources, and resistors, finding the equation for $q(t)$ for each capacitor requires solving a system of differential equations, which is not covered in this course. However, you can find the currents and charges on the capacitor after a long time by replacing all capacitors with open circuits and then using KVL.

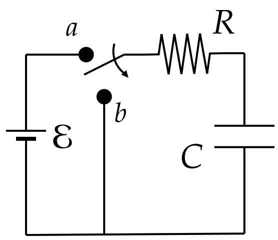


(In the above circuit, the equation used in the previous problem does not apply and so you are only asked for the state of the circuit after a long time.)

1. Find the charge on the capacitor a long time after the switch is closed.
2. Find the current in the left, middle, and right resistors a long time after the switch is closed.

4 Charge/Discharge Problem

4.1 Part I



The switch in the above circuit has been in position a for a long time. At $t = 0$, the switch is moved instantaneously to position b. The values of the circuit elements are $\mathcal{E} = 12$ V, $C = 10$ mF, and $R = 20\ \Omega$.

Let $t = 0^-$ correspond to the time just before the switch is moved from a to b. Let $t = 0^+$ correspond to the time just after the switch is moved from a to b.

1. What is the current through the resistor at $t = 0^-$?

2. What is the charge on the capacitor at $t = 0^-$?

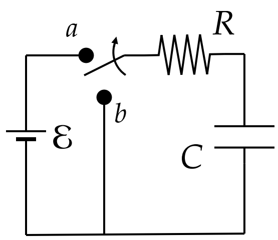
3. What is the charge on the capacitor at $t = 0^+$?

4. What is the voltage across the capacitor at $t = 0^+$?

5. What is the current through the resistor at $t = 0^+$?

6. What is the charge on the capacitor at $t = 200$ ms?

4.2 Part II



The switch in the above circuit has been in position b for a long time. At $t = 0$, the switch is moved instantaneously to position a. The values of the circuit elements are $\mathcal{E} = 12$ V, $C = 10$ mF, and $R = 20\ \Omega$.

Let $t = 0^-$ correspond to the time just before the switch is moved from b to a. Let $t = 0^+$ correspond to the time just after the switch is moved from b to a.

1. What is the current through the resistor at $t = 0^-$?

2. What is the voltage across the capacitor at $t = 0^-$?

3. What is the charge on the capacitor at (a) $t = 0^-$ and (b) $t = 0^+$?

4. What is the voltage across the capacitor at $t = 0^+$?

5. What is the current through the resistor at $t = 0^+$?

6. What is the charge on the capacitor at time $t = 200$ ms?