Electric Potential

1 Electric Potential Energy Differences, ΔU

Mathematically, the work done by a force $\vec{\mathbf{F}}$ in moving an object from position a to position b is

$$W_{a\to b} = \int_a^b \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}$$

Another way of writing $\vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}$ is $|F|dl\cos\phi$, where ϕ is the angle between $\vec{\mathbf{F}}$ and $d\vec{\mathbf{l}}$. There are three cases that you will encounter when evaluating this integral:

1. If a force is always perpendicular to the direction of movement, the work due to that force is zero. For example, a block sliding horizontally has a gravitational force exerted on it, but the gravitational force is downward, and so is perpendicular to the direction of motion. Thus, gravity does no work.
2. When the force on an object does not change when it is moved a distance L from a to b and the direction of force is always in the same direction as the direction of movement, then

   $$W_{a\to b} = \int_a^b \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}} = (\pm)|\vec{\mathbf{F}}|L$$

   where L is positive; the $+$ sign is used for a force that is in the direction of movement, and the $-$ sign is used for a force that is in the opposite direction of movement. For example, if you lift a mass m upwards by a distance L, the force you exert is in the same direction of movement, so you do a work of mgL on the mass. The gravitational force on the mass is in the opposite direction of movement, so the work done by the gravitational force is $-mgL$. If, instead, you lower the mass, your force is upwards, and the direction of motion is downwards, so the work you do is now $-mgL$, and the work done by the gravitational force is $+mgL$.
3. When the direction of force relative to the direction of movement changes (so the dot product changes) and/or the magnitude of force changes. This is covered on page 755 of the textbook.

If $\vec{\mathbf{F}}$ is a special kind of force, called a *conservative* force, we do not need to perform integration to every time that we want to compute the work. For each conservative force, there is an equation for U (called potential energy, or PE) such that one needs to only know U at b and a. In this case,

$$W_{a\to b}^{\text{cons}} \equiv -\Delta U = -(U_b - U_a)$$

where the symbol $\equiv$ is used to indicate a definition and the superscript cons indicates that the equation applies only to a conservative force.

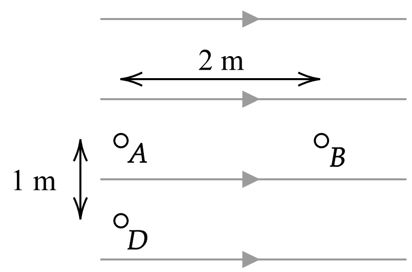
In mechanics, you encountered conservative forces of

1. A force that is constant in magnitude and direction (e.g., the force on a small mass near Earth's surface)
2. A force that varies according to $\hat{\mathbf{r}}/r^2$ (e.g., the gravitational force between two objects separated by a large distance)

In E&M, we encounter these same two types of conservative forces.

1.1 Problem – Uniform Field

The following diagram shows a region of space where the electric field is constant and has a value of $3\ \mathrm{N/C}$ and points to the right. Field lines are shown as lines with arrows.



1. A charge of $+3$ C is placed at point A. What happens to that charge when it is released from rest?

2. A charge of $+3$ C is moved from A to B. (a) How much work was done by the electric field? (b) By how much has the potential energy of the charge changed?

3. A charge of -3 C is placed at point A. What happens to that charge when it is released from rest?

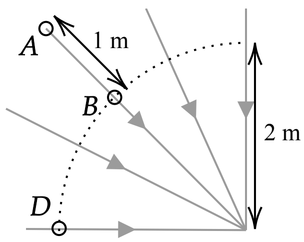
4. A charge of -3 C is moved from A to B. (a) How much work was done by the electric field? (b) By how much has the potential energy of the charge changed?

5. A charge of -3 C is moved straight downward from A to D. (a) How much work was done by the electric field? (b) By how much has the potential energy of the charge changed?

6. If a charge of -3 C is moved from A to D on a path that is not a straight line, will your answers to the previous problem change? If no, explain why. If yes, provide new answers.

1.2 Problem – Radial Field

In the previous problem, a charge was moved in a region of space where the electric field was constant and so the calculation of work did not require integration. In this problem, the electric field is not constant and so integration is required. The integration that must be performed to compute work in this case is given by Equation 23.8 in the textbook.



There is a charge of -6 C at the origin. Some electric field lines for this charge are shown. To simplify the calculations, use $k = 9 \cdot 10^9\ \mathrm{N \cdot m^2/C^2}$.

1. A charge of $+3$ C is moved from A to B. (a) How much work was done by the electric field? (b) By how much has the potential energy of the moved charge changed?

2. A charge of -3 C is moved from A to B. (a) How much work was done by the electric field? (b) By how much has the potential energy of the moved charge changed?

3. A charge of -3 C is moved from B to D along the dotted curve. (a) How much work was done by the electric field? (b) By how much has the potential energy of the moved charge changed?

4. A charge of -3 C is moved from from D to B but along a path that deviates from the dotted curve. (a) How much work was done by the electric field? (b) By how much has the potential energy of the moved charge changed?

2 Electric potential difference, ΔV

In the previous section, we considered moving an arbitrary amount of charge (either positive or negative) from point a to point b and computed its change in potential energy ΔU.

An electric potential difference ΔV is defined to be the change in electric potential energy of a test charge, q_o when it is moved from point a to point b divided by q_o.

As a result, the only difference between the ΔU calculations performed previously and ΔV calculations is that we first compute ΔU for a $+1$ C charge. To get ΔV, we simply divide by ΔU by $+1$ C.

The definition of electric potential is similar to the definition of the electric field in that they both involve consideration of a test charge. That is, the electric field is the force on a test charge divided by the magnitude of the test charge:

$$\vec{\mathbf{E}} = \vec{\mathbf{F}}/q_o$$

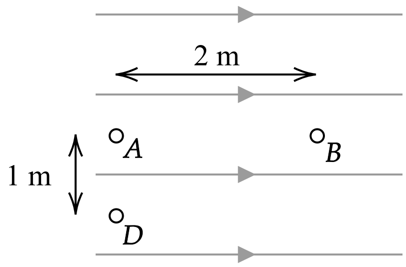
A change in electric potential is the change in electric potential energy of a positive test charge divided by the magnitude of the test charge's charge:

$$\Delta V = \Delta U/q_o$$

The advantage of using changes in electric potential (ΔV) as opposed to changes in electric potential energy (ΔU) of a specific amount of charge is that once the electric potential difference ΔV between two points is known for a test charge, the change in potential energy for an arbitrary amount of charge Q can be computed by simply multiplying ΔV by Q. This is similar to the advantage of the electric field. If we know the electric field at a given point, we can find the force on an arbitrary charge Q at that point by multiplying $\vec{\mathbf{E}}$ by Q.

2.1 Problem

The following diagram shows a region of space where the electric field is constant and has a value of 3~N/C.



1. What is the difference in electric potential $\Delta V = V_B - V_A$.

2. A charge of $+3$ C is moved from A to B. By how much has the electric potential energy of the moved charge changed?

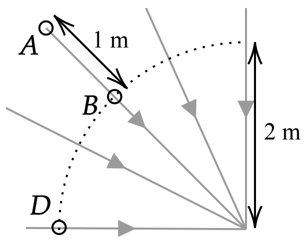
3. A charge of -3 C is moved from A to B. By how much has the electric potential energy of the moved charge changed?

4. A charge of -3 C is moved from B to D. By how much has the electric potential energy of the moved charge changed?

5. What is the difference in electric potential $\Delta V = V_B - V_D$.

2.2 Problem

There is a charge of -6 C at the origin. Some electric field lines for this charge are shown. To simplify the math, use $k = 9 \cdot 10^9\ \mathrm{N \cdot m^2/C^2}$.



1. What is the difference in potential $\Delta V = V_B - V_A$?

2. A charge of -3 C is moved from A to B. By how much has the electric potential energy of the moved charge changed?

3. Aa charge of -3 C is moved from B to D. By how much has the electric potential energy of the moved charge changed?

4. What is difference in electric potential $\Delta V = V_D - V_A$.

3 U and V and Superposition

The electric potential energy of a charge q_0 that is a distance of r_1 from a charge q_1 is defined to be

$$U = k\frac{q_0 q_1}{r_1}$$

This corresponds to the work required to move q_0 from infinity to r_1. In this formula, if the charges have opposite signs then U is negative; if they have the same sign then U is positive. Note that there is a sign associated with the potential energy, but the direction of the vector that connects the charges does not matter; the equation for U only involves the values of the charges and the magnitude of the separation distance between them. As a result, we can also state that the formula above corresponds to the work required to move q_1 from infinity to a distance r_1 from q_0.

Consider next the potential energy of charge q_0 when it is a distance r_1 from charge q_1 and a distance r_2 from charge q_2. The potential energy of q_0 is the sum

$$U = k\frac{q_0 q_1}{r_1} + k\frac{q_0 q_2}{r_2}$$

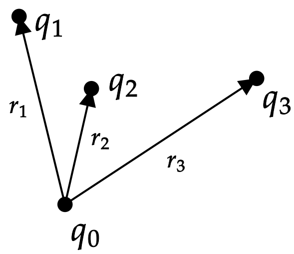
More generally, if q_o is among a group of N other charges, the potential energy of q_0 is

$$U = kq_0 \sum_{i=1}^{N} \frac{q_i}{r_i}$$

Dividing by q_o gives the electric potential at a point in space due to a group of N charges:

$$V = k \sum_{i=1}^{N} \frac{q_i}{r_i}$$

3.1 Problem



1. What is the electric potential energy of the charge q_0 in the diagram shown?

2. What is the electric potential at the position of q_0 if q_0 was not there?

3. Can you find the potential energy at the position of q_0 if that charge was not there? Why or why not?

4. Explain the difference between potential and potential energy.

3.2 Problem

Given a point charge q_1 at the origin:

1. Write the general equation for the electric potential at a distance r from q_1

2. Find the electric potential, V_1, at $(x, y) = (-d, 0)$ due to q_1.

3. If a charge q_2 is placed at $(x, y) = (d, 0)$, find the electric potential, V, at $(x, y) = (-d, 0)$ (hint – it is the sum of the electric potentials at due to q_1 and q_2).

4. How much work is required to place charge q_3 at $(x, y) = (-d, 0)$?

5. What is the potential energy, U, of q_3 when it is at $(x, y) = (-d, 0)$?

4 Energy to Assemble a Collection of Charges

In the previous problem you computed the work required to move q_3 to $(x, y) = (-d, 0)$ after q_2 was in place. The total work required to assemble the system of three charges is larger than this work because it also took work to move q_2 into place. Given a point charge q_1 at origin, as in the previous question,

1. how much work is required to move q_2 to $(x, y) = (d, 0)$?;

2. how much work is required to move q_3 to $(x, y) = (-d, 0)$ if only q_1 is present?;

3. how much work is required to move q_3 to $(x, y) = (-d, 0)$ if only q_2 is present?

4. The total work required to assemble the system of three charges is the sum of the work from parts 1.-3.. Write the equation for this sum in terms of the given variables. (This sum is known as the total potential energy of the system of charges – see equation 23.11 of the textbook.)