

# Continuous Charge Distributions

## 1 Overview

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Previously, you found the electric field at a location in space due to one or more point charges by finding the electric field due to each charge and then vectorially summing the fields to get the total field (this is “using superposition”).

This process requires a significant amount of calculation if there are many charges. To reduce the number of calculations when charges are closely spaced, we sometimes assume they are continuously distributed; in this case, to compute the electric field, we need to evaluate an integral rather than a sum with many terms.

Section 21.5 of the textbook gives three examples for charges that are continuously distributed:

1. charges uniformly distributed along a straight line,
2. charges uniformly distributed on a circle, and
3. charges uniformly distributed on a disk.

If you read the textbook examples and the lecture notes, you should be able to identify the following steps (not necessarily in this order).

1. Identify answer features
2. Find  $d\vec{E}$  (or  $dE$  and its direction) for a  $dQ$  on the charged object
3. Find  $dQ$  in terms of coordinates (e.g.,  $dx$ ,  $dy$ ,  $r$ ,  $d\theta$ , etc.)
4. Simplify  $d\vec{E}$  (if possible) using symmetry arguments
5. Integrate  $d\vec{E}$
6. Check answer features

In this activity, you will explicitly address all of these steps for charges that are uniformly and continuously distributed along a straight line.

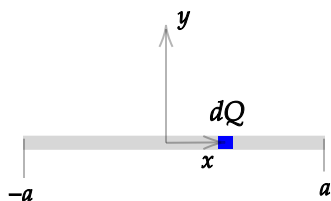
## 2 Finite Line of Charge

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The following diagram shows a differential charge  $dQ$  at a location on the  $x$ -axis. Recall that the general equation for the electric field due to a point charge (with  $q$  replaced with  $dQ$  and  $\vec{E}$  replaced with  $d\vec{E}$ ) is

$$d\vec{E} = k \frac{dQ}{r^2} \hat{r}, \text{ which has magnitude } dE = k \frac{dQ}{r^2}$$

(In a previous activity, you found the components of  $\vec{E}$  using two methods that are used in the textbook. You may use either method for this problem.)



1. Draw the expected directions of  $dE_x$  and  $dE_y$  at a location on the  $+y$ -axis on the diagram given the location of  $dQ$  shown on the figure (assume  $dQ$  is positive).
2. In the diagram, the differential charge is at a positive  $x$ . At the same location on the  $+y$ -axis, draw the expected direction of  $dE_x$  and  $dE_y$  on the diagram with dotted lines if  $dQ$  is at  $-x$ .
3. Draw the expected direction of  $dE_x$  and  $dE_y$  on the diagram at a location on the  $-y$ -axis for  $dQ$  at the position in the figure.
4. Find equations for the electric field components  $dE_x$  and  $dE_y$  at any location on the  $y$ -axis in terms of  $dQ$ ,  $x$ ,  $y$ , and  $k$ .
5. Do your equations for  $dE_x$  and  $dE_y$  give the directions predicted by your answers to 1.–3.? Try plugging in values of  $y = \pm a$  and  $x = \pm a$ .
6. If the differential charge  $dQ$  is at  $x = 0$ , do you expect  $dE_x$  or  $dE_y$  to be zero? Is your answer consistent with the result of plugging in  $x = 0$  into your equations for  $dE_x$  and  $dE_y$ ?

If the charge per unit length on the line is  $\lambda$ , then we can write  $dQ = \lambda dx$ . To find  $E_x$  and  $E_y$ , integrate  $dE_x$  and  $dE_y$  from  $x = -a$  to  $a$ .

7. Write down the integrals that must be evaluated to find  $E_x$  and  $E_y$ . Do you expect either of the integrals to be zero?

8. From an integral table (or using trig substitutions), we know  $\int \frac{du}{(u^2 + a^2)^{3/2}} = \frac{u}{a^2 \sqrt{u^2 + a^2}} + C$ .  
Use this to find  $E_y$ .

9. If  $y$  is negative (for example  $y = -a$ ), is  $E_y$  positive or negative? Is this consistent with your answer to 3.?

10. The total charge on the line is  $Q = 2a\lambda$  (length  $\times$  charge/length). Suppose  $y \gg a$  so that you can replace  $a^2 + y^2$  with  $y^2$ . Is your equation for  $E_y$  consistent with the electric field for a point charge  $Q = 2a\lambda$  at the origin?

### 3 Long Line of Charge

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Using the result of the previous problem, we can find  $E_y$  when  $a \gg y$ , which corresponds to a long line of charge. The result is

$$E_y = \frac{2\lambda k}{y}$$

This equation can be written more generally as

$$E = \frac{2\lambda k}{r}$$

where  $r$  is the perpendicular distance from the line and the direction of  $E$  is perpendicular the line with a direction that depends on the sign of the charge density  $\lambda$ .

A long line of charge lies along the line  $y = b$ .

1. Find the electric field magnitude and direction at the origin.

2. Find the electric field magnitude and direction at  $(x, y) = (b, 0)$ .