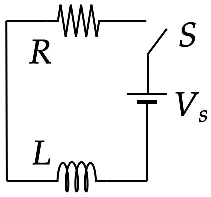
RL Circuits

1 Introduction



The above figure shows a RL series circuit consisting of an inductor of inductance L connected in series with a resistor of resistance R. The switch, S, is closed at a time $t = 0$ and remains closed.

Using Kirchhoff's voltage law around the loop, we have

$$V_s - I(t) \cdot R - L\frac{dI(t)}{dt} = 0$$

This differential equation can be solved for $I(t)$, the current at any time. If the current at $t = 0$ is zero, the solution is

$$I(t) = \frac{V_s}{R}\left(1 - e^{-t/(L/R)}\right)$$

After a long time* (specifically, $t \gg \tau$), the current approaches a constant value of $I = V_s/R$ because the exponential term approaches zero. How quickly the exponential term approaches zero depends on a quantity called the RL time constant defined by

$$\tau \equiv L/R$$

which has units of seconds when L is in Henrys (H) and R is in Ohms (Ω). Using τ, we have

$$I(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right)$$

When $t/\tau \simeq 0$, $e^{-t/\tau} \simeq e^0 = 1$, so $I(t) \simeq (V_s/R)(1 - 1) = 0$. As a result, we state that initially the inductor behaves like an open circuit because the current through it is nearly zero.

When $t/\tau \gg 1$, the exponential term $e^{-t/\tau}$ becomes much smaller than one, so $I(t) \simeq (V_s/R)(1 - 0) = V_s/R$. If we replace the inductor with a wire, this is the same current that we would find. As a result, we state that after a long time, an inductor behaves like a resistanceless wire.

* Informally, we often use phrases such as "after a long time". This phrase is ambiguous because a reference length of time is not given. To be specific, one should instead state $t \gg t_{\text{ref}}$, where t_{ref} is a reference length of time.

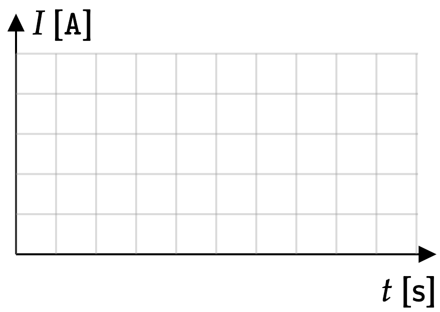
2 Problem I

In this problem, you will consider the circuit and equation

$$I(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right)$$

that was described in the introduction.

1. If $V_s = 10$ V and $R = 1\ \Omega$, plot dots for the values of I at $t = 0, 2, 4, 6, 12$ s using $L/R = \tau = 2$ s



2. Based on the equation, at $t = 0$ does the inductor behave like an open circuit or a resistanceless wire?

3. Based on the equation, at $t \gg \tau$ does an inductor behave like an open circuit or a resistanceless wire?

4. If L doubles but R remains constant

   a. does the time constant τ increase, decrease, or remain the same?;

   b. how will the position of the points that you drew for part 1. change? (Will they move up, down, or remain the same?)

   c. Does your answer to b. make sense physically? That is, an inductor tends to impede changes in current and so is your answer to b. consistent with this?

5. The voltage across the inductor is LdI/dt. Compute dI/dt and sketch its curve on the graph above. Is this equation consistent with the statement that for large t/τ, the voltage across the inductor is zero?

3 Problem II

If the circuit in the introduction has $L = 40$ mH, $R = 2\ \Omega$, and $V_s = 20$ V,

1. What is the current after a very long time after the switch is closed?

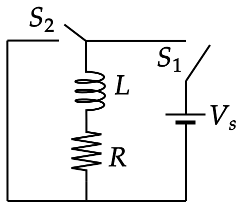
2. What is the time constant of this RL series circuit?

3. How long does it take for the current to reach 63% of its maximum value?

4. What is the voltage across the inductor at $t = 10$ ms?

5. What is the current at $t = \tau$ after the switch is closed?

4 Problem III



In the circuit above, an inductor with $L = 10$ mH and a resistor with $R = 1\ \Omega$ is connected as shown. The battery has an emf of 10 V. At $t = 0$, the switch S_1 is closed.

1. What is the current through the resistor at $t = 0$?

2. What is the current through the inductor at $t = 0$?

3. After a long time, what is the current through the resistor and inductor?

4. S_1 is opened and S_2 is closed simultaneously at $t = t_o$. Write Kirchhoff's voltage law around the new closed loop.

5. Show that the equation $I(t) = I_o e^{-(t-t_o)/\tau}$ satisfies the equation in your answer to the previous question.

6. If switch S_1 was opened and switch S_2 was closed at $t_o = 5$ ms, plot $I(t)$ from $t = 0$ to $t = 10$ ms.