

Logarithms

1 Introduction – Base 10 Logarithms

The motivation for the base 10 logarithm is that it reduces numbers raised by a power of 10 to the power the number was raised to. So 10^2 becomes 2, 10^3 becomes 3, etc. The base 10 logarithm is sometimes called the “common logarithm”.

In mathematical notation,

$$\log_{10}(10^x) = x$$

For example,

$$\log_{10}(10^{-5}) = -5 \text{ and } \log_{10}(10^7) = 7$$

(To take the base 10 logarithm of a number that is not exactly a power of 10, use a calculator.)

Several identities follow as a result:

1. If you raise a base 10 logged number by 10, you get back the number that was logged.

$$10^{\log_{10}(x)} = x$$

For example,

$$10^{\log_{10}(7)} = 7 \text{ and } 10^{\log_{10}(8.8)} = 8.8$$

2. The sum of two logged numbers is the log of the product of the numbers:

$$\log_{10} y + \log_{10} x = \log_{10}(yx);$$

For example,

$$\log_{10} 10 + \log_{10} 100 = \log_{10} 10 \cdot 100 = \log_{10} 10^3 = 3$$

3. The difference between two logged number is the log of the ratio of the numbers:

$$\log_{10} y - \log_{10} x = \log_{10}(y/x)$$

For example,

$$\log_{10} 10 - \log_{10} 100 = \log_{10}(10/100) = \log_{10} 10^{-1} = -1$$

1.1 Problems

1. What is $\log_{10}(0.000000001)$?
2. What is $\log_{10}(10,000)$?
3. What is $\log_{10}(10,000) + \log_{10}(0.000000001)$?

4. What is $\log_{10}(10,000) - \log_{10}(0.000000001)$?

5. If $x = x_o \log_{10}(y/y_o)$, solve for y in terms of x .

2 Introduction – Base e Logarithm

The base 10 logarithm reduces numbers raised by a power of 10 to the power the number was raised to.

The base e logarithm reduces numbers raised by a power of e to the power the number was raised to. It is represented by $\log_e x$, or more commonly, $\ln(x)$.

“ln” represents the “natural logarithm”. The term “natural” is used because the exponential e appears in many natural problems, for example, some populations grow in proportion to $e^{t/\tau}$, where the constant τ is a growth rate.

In mathematical notation,

$\ln(e^x) = x$; for example $\ln(e^{-5}) = -5$ and $\ln(e^7) = 7$

Several identities follow as a result:

1. If you raise a base e logged number by e , you get back the number that was logged.

$$e^{\ln(x)} = x$$

2. The sum of two logged numbers is the log of the product of the numbers:

$$\ln(y) + \ln(x) = \ln(yx);$$

3. The difference between two logged number is the log of the ratio of the numbers:

$$\ln(y) - \ln(x) = \ln(y/x)$$

2.1 Problems

1. What is $\ln(e^3)$?

2. What is $\ln(1/e^3)$?

3. What is $\ln(e^{-4}) + \ln(e^3)$?

4. What is $\ln(e^3) - \ln(e^{-2})$?

5. If $x = x_o \ln(y/y_o)$, solve for y in terms of x .