Magnetic Flux

1 Introduction

The definition of magnetic flux is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\mathbf{A}$$

When the magnitude and direction of $\vec{\mathbf{B}}$ are the same at all points on the surface, the integral simplifies to

$$\Phi_B = \vec{\mathbf{B}} \cdot \mathbf{A}$$

or, equivalently,

$$\Phi_B = BA \cos \phi$$

where ϕ is the angle between the $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$ vectors.

Note that previously, electric flux was covered. The same techniques that apply to computing electric flux apply to magnetic flux. See the Electric Flux activity for additional discussion.

2 Computing Φ_B

In the following figure, a square loop of area A that can be rotated about the z axis is shown. Assume that the normal direction of the loop is as shown in the diagram. A uniform external magnetic field points in the $\hat{\boldsymbol{\jmath}}$ direction.
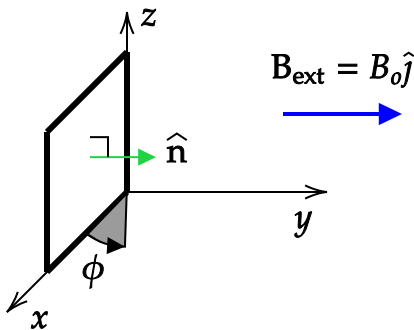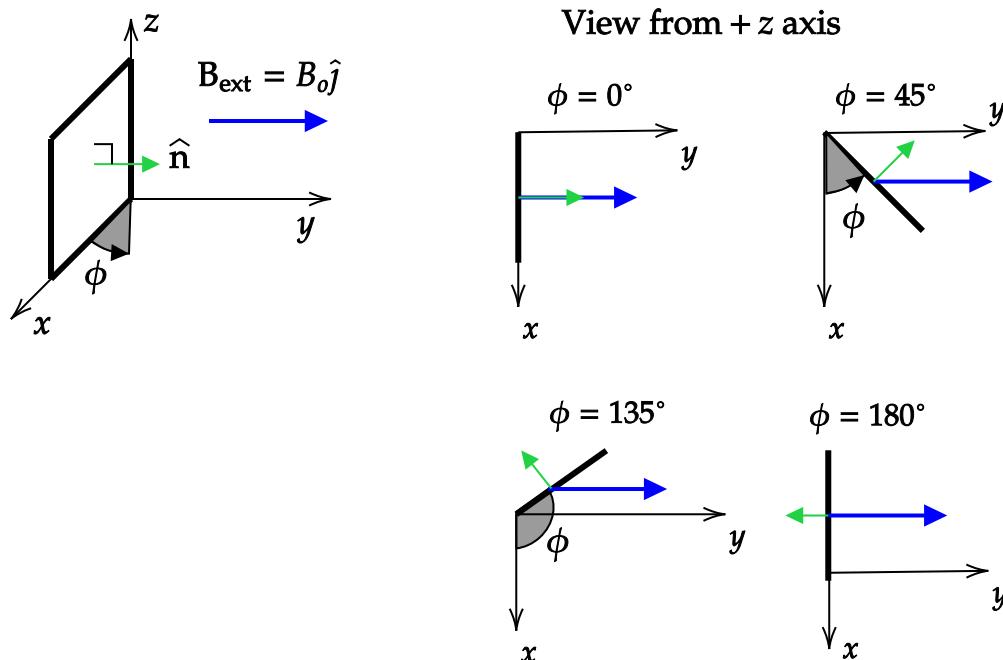

1. At what angles in the range of $\phi = [0, 360°]$ is the magnetic flux zero?

   Answer: The magnetic flux will be zero when the normal vector is perpendicular to $\vec{\mathbf{B}}_{\text{ext}}$. This corresponds to $\phi = 90°$ and $\phi = 270°$

2. Draw the loop, $\hat{\mathbf{n}}$, and $\vec{\mathbf{B}}_{\text{ext}}$ as they would appear when viewed from above (on the positive z–axis) when $\phi = 0°$, $\phi = 45°$, $\phi = 135°$, and $\phi = 180°$.

   Answer:

$B_{ext} = B_o \hat{\jmath}$

$\hat{n}$

z

y

x

ϕ

$\phi = 0°$

y

x

$\phi = 45°$

y

x

ϕ

$\phi = 135°$

y

x

ϕ

$\phi = 180°$

y

x

3. When $\phi = 45°$, is the magnetic flux positive or negative? What is its value in terms of a fraction of $B_o A$?

Answer: From the above diagram, the angle between $\hat{n}$ and $\vec{B}_{ext}$ is ϕ, so the flux will be positive. Its value will be $B_o A \cos 45° = B_o A / \sqrt{2}$.

4. When $\phi = 135°$, is the magnetic flux positive or negative? What is its value in terms of a fraction of $B_o A$?

Answer: From the above diagram, the angle between $\hat{n}$ and $\vec{B}_{ext}$ is ϕ, so the flux will be negative. Its value will be $B_o A \cos 135° = -B_o A \cos 45° = -B_o A / \sqrt{2}$.

5. Sketch a plot of the magnetic flux, Φ_B, as a function of ϕ.

From the diagram above, the angle of rotation of the plane is the same as the angle between the $\hat{n}$ and $\vec{B}_{ext}$, so the general formula is $\Phi = B_o A \cos \phi$, which is sketched below along with dots that correspond to answers to the above problems.

Φ_B

$B_o A$

$-B_o A$

$90°$ $180°$ $270°$ $360°$ ϕ
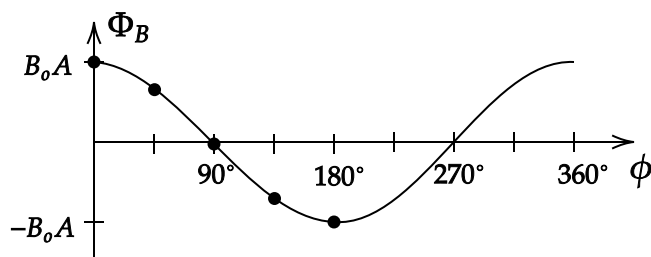
3 Computing $d\Phi_B / dt$

The time rate of change of magnetic flux through a closed loop is a quantity that will be used when Faraday's law is covered.

Suppose the loop in the previous problem rotates at a constant rate such that $\phi = \omega t$, where ω is a constant.
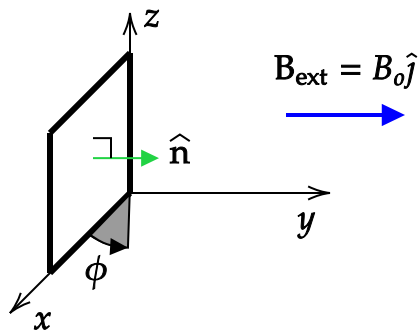
1. At what angles in the range $\phi = [0°, 360°]$ is $d\Phi_B/dt = 0$?

   The answer to part 5. of the previous problem is $\Phi_B = B_o A \cos\phi$, which is sketched below. The slope of Φ_B is zero at $\phi = 0, 180°$, and $360°$.
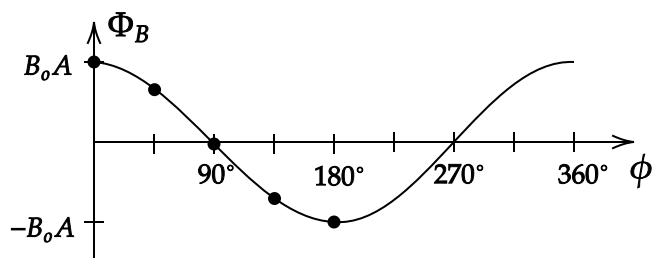
2. For what angle range is $d\Phi_B/dt$ increasing?

   From the plot of Φ_B, the slope is positive for $180° < \phi < 360°$

3. For what angle range is $d\Phi_B/dt$ decreasing?

   From the plot of Φ_B, the slope is negative for $0° < \phi < 180°$

4. What is the formula for $d\Phi_B(t)/dt$ in terms of B_o, A, t, and ω?

   $\Phi_B = B_o A \cos\phi$, so $d\Phi_B/dt = -B_o A \omega \sin\omega t$