Superposition and Symmetry

1 Overview

In previous activities, only one charge was responsible for creating the electric field. When there are more charges, superposition can be used to find the total electric field by summing $\vec{\mathbf{E}}$ due to each charge. Superposition can also be used to find the total electric force on a charge due to two or more charges.

Superposition problems are often simplified by recognizing a symmetry. For example, if we want to know the electric field at the origin due to charges $+q$ at $(x, y) = (\pm a, 0)$, we can state the answer is zero without computing the fields due to each charge – we know they will be equal and opposite.

2 Problem

Charge $q_1 = +q$ is at $(x, y) = (a, 0)$, charge $q_2 = +q$ is at $(x, y) = (-a, 0)$, and charge $q_3 = -q$ is at $(x, y) = (0, a)$. Assume that q is a positive number.

1. Draw this charge configuration below.

2. Why does it not make sense to ask what the electric *force* is at the origin?

In the following,

3. Find the electric field at the origin due to q_1. Write your answer in the form $\vec{\mathbf{E}}_1 = E_{x1}\hat{\boldsymbol{\imath}} + E_{y1}\hat{\boldsymbol{\jmath}}$.

4. Find the electric field at the origin due to q_2. Write your answer in the form $\vec{\mathbf{E}}_2 = E_{x2}\hat{\boldsymbol{\imath}} + E_{y2}\hat{\boldsymbol{\jmath}}$.

5. Find the electric field at the origin due to q_3. Write your answer in the form $\vec{\mathbf{E}}_3 = E_{x3}\hat{\boldsymbol{\imath}} + E_{y3}\hat{\boldsymbol{\jmath}}$.

6. Find the total electric field at the origin by adding $\vec{\mathbf{E}}_1$, $\vec{\mathbf{E}}_2$, and $\vec{\mathbf{E}}_3$. Write your answer in the form $\vec{\mathbf{E}} = E_x\hat{\boldsymbol{\imath}} + E_y\hat{\boldsymbol{\jmath}}$.

7. Will your answers to 3.–6. change if the problem had asked for the electric field at a different position? If so, which answers?

8. Find the electric field at the origin if charge $q_1 = 2q$ (instead of q).

9. Find the electric field at the origin if charge $q_1 = -2q$ (instead of q).