Electric Field

1 Overview

The electric field vector, $\vec{\mathbf{E}}$, is a quantity assigned to a point in space. Given this quantity, we can compute the force on a charge Q will experience if it is placed at that point using the equation $\vec{\mathbf{F}} = Q\vec{\mathbf{E}}$. The direction of $\vec{\mathbf{E}}$ is also the direction a charge will begin to move if released from rest.

To find $\vec{\mathbf{E}}$ at any point in space, compute the force $\vec{\mathbf{F}}$ due to all other charges on a hypothetical (or "test") charge q_o at a point where you want to know $\vec{\mathbf{E}}$. To find $\vec{\mathbf{E}}$ at that point, divide $\vec{\mathbf{F}}$ by q_o.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_o}$$

2 Example I

Charge q_1 is at $(x, y) = (-a, -a)$. Find the electric field at $(x, y) = (a, a)$ in the form $\vec{\mathbf{E}} = E_x\hat{\boldsymbol{\imath}} + E_y\hat{\boldsymbol{\jmath}}$. Also, find E. (Note that E and $|\vec{\mathbf{E}}|$ are used interchangebly.)

Solution

To find the electric field at a point in space, we put a hypothetical "test" charge q_o at that point, compute the force on it due to all other charges, and then use

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_o}$$

The force a positive charge q_1 at $(x, y) = (-a, -a)$ exerts on a positive charge q_2 at $(x, y) = (a, a)$ was computed in a previous activity.

$$\vec{\mathbf{F}}_{q_1 \text{ on } q_2} = k\frac{|q_1 q_2|}{8a^2}(\cos 45^\circ\,\hat{\boldsymbol{\imath}} + \sin 45^\circ\,\hat{\boldsymbol{\jmath}})$$

We also found that this equation applies when q_1 and q_2 are both positive or both are negative. If q_1 was positive and q_2 was negative, or vice-versa, we found the sign changed:

$$\vec{\mathbf{F}}_{q_1 \text{ on } q_2} = -k\frac{|q_1 q_2|}{8a^2}(\cos 45^\circ\,\hat{\boldsymbol{\imath}} + \sin 45^\circ\,\hat{\boldsymbol{\jmath}})$$

Based on this, we can write a single equation for all possibilities:

$$\vec{\mathbf{F}}_{q_1 \text{ on } q_2} = k\frac{q_1 q_2}{8a^2}(\cos 45^\circ\,\hat{\boldsymbol{\imath}} + \sin 45^\circ\,\hat{\boldsymbol{\jmath}})$$

If we replace q_2 with q_o, this is

$$\vec{\mathbf{F}}_{q_1 \text{ on } q_o} = k\frac{q_1 q_o}{8a^2}(\cos 45^\circ\,\hat{\boldsymbol{\imath}} + \sin 45^\circ\,\hat{\boldsymbol{\jmath}})$$

The electric field at the location of q_o is then

$$\vec{\mathbf{E}}_{\text{at }(a,a)\text{ due to }q_1} = \frac{\vec{\mathbf{F}}}{q_o} = \frac{kq_1}{8a^2}(\cos 45^\circ\,\hat{\boldsymbol{\imath}} + \sin 45^\circ\,\hat{\boldsymbol{\jmath}}) = \frac{kq_1}{8a^2}\left[\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}\right],$$

where the fact that $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$ was used. The magnitude is $E = \sqrt{E_x^2 + E_y^2} = k|q_1|/8a^2$.

Sign check: When computing electric fields and forces, it is easy to make a sign error. The electric field vector points in the direction a positive charge will move if released there from rest. Suppose q_1 is positive. Our equation predicts that a positive charge released from rest at (a, a) will move up and to the right. Suppose q_1 is negative. Our equation predicts that a positive charge will move down and to the left. This is consistent with the fact that like charges repel and unlike charges attract.

3 Problem I

Charge q_1 is at $(x, y) = (-a, a)$. At $(x, y) = (a, 0)$, find $\vec{\mathbf{E}}$ in the form $\vec{\mathbf{E}} = E_x\hat{\boldsymbol{\imath}} + E_y\hat{\boldsymbol{\jmath}}$. Check signs of the components of $\vec{\mathbf{E}}$ using the technique used in Example I. Also, find E.

Answer:

From Problem II on the Electric Force Activity, if q_2 is at $(x, y) = (a, 0)$ (and using the arguments in the previous example to drop the absolute value sign),

$$\vec{\mathbf{F}}_{q_1\text{ on }q_2} = k\frac{q_1 q_2}{5a^2}\left(\frac{2}{\sqrt{5}}\hat{\boldsymbol{\imath}} - \frac{2}{\sqrt{5}}\hat{\boldsymbol{\jmath}}\right)$$

Replacing q_2 with a test charge q_o,

$$\vec{\mathbf{F}}_{q_1\text{ on }q_o} = k\frac{q_1 q_o}{5a^2}\left(\frac{2}{\sqrt{5}}\hat{\boldsymbol{\imath}} - \frac{2}{\sqrt{5}}\hat{\boldsymbol{\jmath}}\right)$$

$$\vec{\mathbf{E}}_{\text{at }(a,0)\text{ due to }q_1} = \frac{\vec{\mathbf{F}}}{q_o} = \frac{kq_1}{5a^2}\left(\frac{2}{\sqrt{5}}\hat{\boldsymbol{\imath}} - \frac{2}{\sqrt{5}}\hat{\boldsymbol{\jmath}}\right)$$

4 Problem II

Charge q_1 is at $(x, y) = (-a, a)$. Find the electric field at $(x, y) = (a, 0)$, find $\vec{\mathbf{E}}$ in the form $\vec{\mathbf{E}} = E_x\hat{\boldsymbol{\imath}} + E_y\hat{\boldsymbol{\jmath}}$. Check signs of the components of $\vec{\mathbf{E}}$ using the technique used in Example I. Also, find E.

Answer:

$\vec{\mathbf{E}}_{\text{at }(a,0)\text{ due to }q_1} = \dfrac{kq_1}{5a^2}\left(\dfrac{2}{\sqrt{5}}\hat{\boldsymbol{\imath}} - \dfrac{2}{\sqrt{5}}\hat{\boldsymbol{\jmath}}\right)$, which matches the solution to Problem I, as expected.