Sound Intensity Level

1 Introduction

The intensity of a sound wave, I, is the average power carried across a unit area. I has units of Power/Area, which has SI units of $\mathrm{W/m^2}$. The intensity of various sources of sound is shown in the third column of the following table from Young and Freedman, 14th Edition.

TABLE 16.2 Sound Intensity Levels from Various Sources (Representative Values)

Source or Description of Sound	Sound Intensity Level, β (dB)	Intensity, I ($\mathrm{W/m^2}$)
Military jet aircraft 30 m away	140	10^{2}
Threshold of pain	120	1
Riveter	95	3.2×10^{-3}
Elevated train	90	10^{-3}
Busy street traffic	70	10^{-5}
Ordinary conversation	65	3.2×10^{-6}
Quiet automobile	50	10^{-7}
Quiet radio in home	40	10^{-8}
Average whisper	20	10^{-10}
Rustle of leaves	10	10^{-11}
Threshold of hearing at 1000 Hz	0	10^{-12}

Because of the sound intensity values span a very large range, from 0.0000000000001 to 100, we define an alternative measure of intensity called the sound intensity level, β. The equation that relates sound intensity level I, with sound intensity, β, is

$$\beta = (10\ \mathrm{dB}) \log_{10}\left(\frac{I}{I_o}\right)$$

where dB stands for “decibels”.

One advantage to using this formula is that its values span a much smaller range, from 0–140 as shown in the second column of the table above. Another advantage is that the reference value of zero corresponds to something easily interpreted: 0 dB means something that is barely audible.

2 Review of Logarithm Identities

The motivation for the base 10 logarithm is that it reduces numbers raised by a power of 10 to the power the number was raised to. So 10^2 becomes 2, 10^3 becomes 3, etc. In mathematical notation

$\log_{10}(10^x) = x$; for example $\log_{10}(10^{-5}) = -5$ and $\log_{10}(10^7) = 7$

To take the base 10 logarithm of a number that is not exactly a power of 10, use a calculator. However, you should recognize the fact that, for example, $\log_{10}(500)$ should be between $\log_{10}(10^2) = 2$ and $\log_{10}(10^3) = 3$.

Several identities follow as a result:

1. If you raise a base 10 logged number by 10, you get back the number that was logged.
   $10^{\log_{10}(x)} = x$; for example $10^{\log_{10}(7)} = 7$
2. The sum of two logged numbers is the log of the product of the numbers:

$\log_{10} y + \log_{10} x = \log_{10}(yx)$

3. The difference between two logged number is the log of the ratio of the numbers:

   $\log_{10} y - \log_{10} x = \log_{10}(y/x)$

Problems

1. What is $\log_{10}(0.000000001)$?
2. What is $\log_{10}(10,000)$?
3. What is $\log_{10}(10,000) - \log_{10}(0.000000001)$?
4. What is $\log_{10}(10,000) + \log_{10}(0.000000001)$?
5. If $x = 4\log(y)$, solve for y in terms of x.

3 Sound Intensity, I

Suppose a bird sitting on the top of a lamp post emits sound with a power of 1 W.

1. What is I at a point 5 m away?

   Answer:

   $$I = \frac{1\text{ W}}{4\pi(5\text{ m})^2} \simeq \frac{1}{314}\frac{\text{W}}{\text{m}^2}$$

2. What is I at a point 50 m away?

   Answer: If the distance increases by a factor of 10, we expect I to descrease a factor of 100 (because I is inversely proportional to area and area depends on the square of the distance). So

   $$I = \frac{1}{3140}\frac{\text{W}}{\text{m}^2}$$

A siren is emitting sound at a constant intensity level

3. If you move three times farther away from the siren, by what ratio does the sound intensity change?

   Answer:

   $I_1 = P_1/A_1$, $I_2 = P_2/A_2$. The power is constant, so $P_1 = P_2$ and we can then write

   $$\frac{I_2}{I_1} = \frac{A_1}{A_1} = \frac{4\pi d_1^2}{4\pi d_2^2} = \frac{d_1^2}{d_2^2}$$

   If I_1 is your initial position at d_1, then $d_2 = 3d_1$ and

   $$\frac{I_2}{I_1} = \frac{d_1^2}{d_2^2} = \frac{d_1^2}{(3d_1)^2} = \frac{1}{9}$$

   So decreases by a factor of 9.

4. If instead you moved four times closer to the siren, by what ratio does the sound intensity change?

   Answer: Increases by a factor of 16.

$$\frac{I_2}{I_1} = \frac{d_1^2}{d_2^2} = \frac{d_1^2}{(d_1/4)^2} = 16$$

4 Sound Intensity Level, β

1. Logarithmic scales are often used in science an engineering. Give at least one other example besides sound intensity that is a quantity that is based on a logarithmic scale.

   Answer: pH scale for acidity, Richter earthquake magnitude scale.

2. If you increase the sound intensity of a speaker on a TV from I_1 to I_2 and $I_2/I_1 = 10$, what is $\beta_2 - \beta_1$?

   In the table, each factor of 10 increase in I corresponds to an change in β by $+10$. So $\beta_2 - \beta_1 = 10$ dB. We can also use

   $$\beta_1 = (10\text{ dB})\log_{10}\left(\frac{I_1}{I_o}\right)$$

   $$\beta_2 = (10\text{ dB})\log_{10}\left(\frac{I_2}{I_o}\right)$$

   $$\beta_2 - \beta_1 = (10\text{ dB})\left[\log_{10}\left(\frac{I_2}{I_o}\right) - \log_{10}\left(\frac{I_1}{I_o}\right)\right]$$

   Using $\log_{10} y - \log_{10} x = \log_{10}(y/x)$ gives

   $$\beta_2 - \beta_1 = (10\text{ dB})\left[\log_{10}\left(\frac{I_2}{I_1}\right)\right]$$

   So if $I_2/I_1 = 10$, we have

   $$\beta_2 - \beta_1 = (10\text{ dB})\left[\log_{10}(10)\right] = 10\text{ dB}$$

3. If you increase the sound intensity level of a speaker on a TV from β_1 to β_2 and $\beta_2 - \beta_1 = 20$, what is I_2/I_1?

   Answer:

   $$20\text{ dB} = (10\text{ dB})\left[\log_{10}\left(\frac{I_2}{I_1}\right)\right]$$

   Dividing both sides by 10 dB gives

   $$2 = \log_{10}\left(\frac{I_2}{I_1}\right)$$

   Raising both sides by 10 and using the identity $10^{\log_{10} x} = x$,

   $$10^2 = \frac{I_2}{I_1}$$

   This could also have been determined using the table. The ratio of I for a whisper to I for the threshold of hearing is 100. The difference in β is 20 dB.

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4. A city council adopted a law to reduce the maximum allowed sound intensity level of leaf blowers from 95 dB to 70 dB. With the new law, what is the ratio of the new maximum allowed intensity to the previously allowed intensity?

$$\beta_2 - \beta_1 = (10\text{ dB})\left[\log_{10}\left(\frac{I_2}{I_1}\right)\right]$$

A siren is emitting sound at a constant intensity level

5. If you move three times closer to the siren, by what ratio does the sound intensity level change?
6. If instead you moved four times closer to the siren, by what ratio does the sound level intensity change?

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7. You are trying to hear a juicy conversation, but from your distance of 15.0 m it sounds like only an average whisper of 20.0 dB. How close should you move to the chatterboxes for the sound level to be 60.0 dB? Show your work.

   $$\frac{I_2}{I_1} = \frac{d_1^2}{d_2^2} = \frac{d_1^2}{(d_1/4)^2} = 16$$

   $$\beta_2 - \beta_1 = (10\text{ dB})\left[\log_{10}\left(\frac{I_2}{I_o}\right) - \log_{10}\left(\frac{I_1}{I_o}\right)\right]$$

8. Solve for I in the equation $\beta = (10\text{ dB})\log_{10}\left(\frac{I}{I_o}\right)$

   Answer:

   Divide both sides by 10 dB

   $$\beta/(10\text{ dB}) = \log_{10}\left(\frac{I}{I_o}\right)$$

   Raising both sides to the power of 10,

   $$10^{\beta/(10\text{ dB})} = 10^{\log_{10}\left(\frac{I}{I_o}\right)}$$

   Using the identity $10^{\log_{10} x} = x$,

   $$10^{\beta/(10\text{ dB})} = \frac{I}{I_o}$$

   Solving for I gives

   $$I = I_o 10^{\beta/(10\text{ dB})}$$