

# Electric Field

## 1 Overview

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The electric field vector,  $\vec{E}$ , is a quantity assigned to a point in space. Given this quantity, we can compute the force on a charge  $Q$  will experience if it is placed at that point using the equation  $\vec{F} = Q\vec{E}$ . The direction of  $\vec{E}$  is also the direction a charge will begin to move if released from rest.

To find  $\vec{E}$  at any point in space, compute the force  $\vec{F}$  due to all other charges on a hypothetical (or “test”) charge  $q_o$  that has an infinitesimal charge and size at a point where you want to know  $\vec{E}$ . To find  $\vec{E}$  at that point, divide  $\vec{F}$  by  $q_o$ .

$$\vec{E}_{\text{due to } q} = \frac{\vec{F}_{q \text{ on } q_o}}{q_o}$$

## 2 Example I

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A positive charge  $q_1$  is at  $(x, y) = (-a, -a)$ . Find the electric field at  $(x, y) = (a, a)$  in the form  $\vec{E} = E_x\hat{i} + E_y\hat{j}$ . Also, find  $E$ . (Note that  $E$  and  $|\vec{E}|$  are used interchangeably.)

### Solution

To find the electric field at a point in space, we put a hypothetical and positive “test” charge  $q_o$  at that point, compute the force on it due to all other charges, and then use

$$\vec{E} = \frac{\vec{F}}{q_o}$$

The force a positive charge  $q_1$  at  $(x, y) = (-a, -a)$  exerts on a positive charge  $q_2$  at  $(x, y) = (a, a)$  was computed in a previous activity. The result was

$$\vec{F}_{q_1 \text{ on } q_2} = k \frac{|q_1 q_2|}{8a^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$$

Because both charges are positive, we can drop the absolute value in the above equation, giving

$$\vec{F}_{q_1 \text{ on } q_2} = k \frac{q_1 q_2}{8a^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$$

If we replace  $q_2$  with  $q_o$ , this is

$$\vec{F}_{q_1 \text{ on } q_o} = k \frac{q_1 q_o}{8a^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$$

The electric field at the location of  $q_o$  is then

$$\vec{E}_{\text{at } (a,a) \text{ due to } q_1} = \frac{\vec{F}}{q_o} = \frac{kq_1}{8a^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}) = \frac{kq_1}{8a^2} \left[ \frac{1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{2}} \hat{j} \right],$$



## 4 Problem II

To find the electric field at a point in space in Example I and Problem I, we first computed the force on a charge at that point and then divided the result by the charge.

An alternative approach is to start with the equation for the magnitude of the electric field due to a charge  $q$  at a point that is a distance  $r$  from the charge:

$$E_{\text{due to } q} = k|q|/r^2$$

Charge  $q_1$  is at  $(x, y) = (a, a)$ . Find the magnitude of the electric field,  $E$ , at  $(x, y) = (0, 0)$  using the above formula and then find  $\vec{\mathbf{E}}$  in the form  $\vec{\mathbf{E}} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}}$ .