

Electric Flux

1 Overview

Several calculations must be performed to use Gauss's law to find the electric field for a system of charges (if possible). First, one must find the electric flux through a closed surface. Second, one must find the amount of charge inside of a closed surface. In this activity, you will compute the electric flux through both open and closed surfaces.

Electric flux, Φ_E , is the integral of $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$ over a surface:

$$\Phi_E = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

When the magnitude and direction of $\vec{\mathbf{E}}$ is the same at all points on the surface, the integral simplifies to

$$\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}A$$

where $\hat{\mathbf{n}}$ is a unit vector that is perpendicular to the surface with area A . Electric flux is a scalar quantity because it results from the dot product of two vectors (similar to work, which is the dot product of a force vector and displacement vector).

The textbook also discusses several other forms for Φ_E . The equation $\Phi_E = \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}A$ can also be written as

$$\Phi_E = E_{\perp}A$$

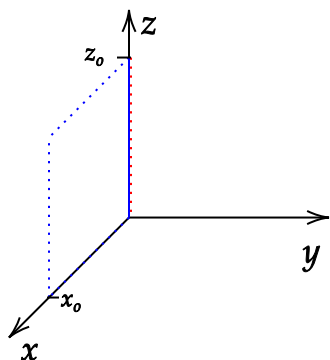
where E_{\perp} is the component of $\vec{\mathbf{E}}$ that is perpendicular to A . If the perpendicular component of $\vec{\mathbf{E}}$ is in the same direction as the normal direction for $\vec{\mathbf{A}}$, the flux is positive. If the perpendicular component of $\vec{\mathbf{E}}$ is in the opposite direction as the normal direction for $\vec{\mathbf{A}}$, the flux is negative. A final form is

$$\Phi_E = EA \cos \theta$$

where θ is the angle between the $\vec{\mathbf{E}}$ and $\vec{\mathbf{A}}$ vectors. Any of these equations can be used, and students are encouraged to use the one they are more comfortable with and ideally understand the relationship between the different equations. In general, when the calculation for θ is simple based on a diagram, $EA \cos \theta$ is easiest to use. When $\vec{\mathbf{E}}$ and/or $\hat{\mathbf{n}}$ as three components, $\vec{\mathbf{E}} \cdot \hat{\mathbf{n}}A$ is usually easier to use.

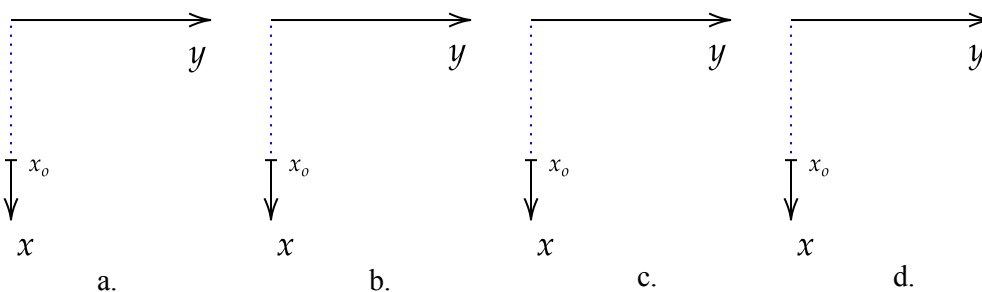
2 Φ_E Through Open Surface

2.1 Problem I



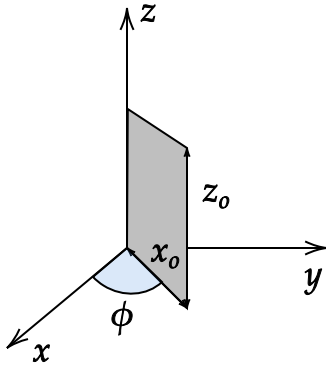
1. a. Draw an area vector $\vec{A} = \hat{n}A$ on the figure above.
 b. Is there only one normal direction to this area? Discuss your reasoning with your group.

2. For each of the following \vec{E} vectors a.-d., draw \vec{E} and \hat{n} on the following diagram, which shows the area in the previous figure when viewed from a point on the positive z -axis that is far from the origin. Then find θ and compute Φ_E .

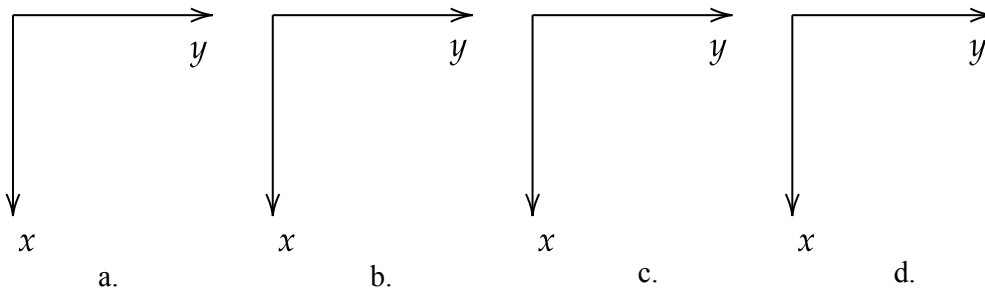


- a. $\vec{E} = E_o \hat{i}$ $\theta =$ $\Phi_E =$
- b. $\vec{E} = E_o \hat{j}$ $\theta =$ $\Phi_E =$
- c. $\vec{E} = E_o \hat{k}$ $\theta =$ $\Phi_E =$
- d. $\vec{E} = \frac{E_o}{\sqrt{2}} \hat{i} + \frac{E_o}{\sqrt{2}} \hat{j}$ $\theta =$ $\Phi_E =$

2.2 Problem II



If the area from the previous problem is rotated by $\phi = 45^\circ$ around the z -axis, draw the area as it would look from a point on the positive z -axis that is far from the origin (that is, draw the projection onto the x - y plane). Then draw $\hat{\mathbf{n}}$ and $\vec{\mathbf{E}}$ for each of the electric fields a.–d.



- a. $\vec{\mathbf{E}} = E_o \hat{\mathbf{i}}$ $\theta =$ $\Phi_E =$
- b. $\vec{\mathbf{E}} = E_o \hat{\mathbf{j}}$ $\theta =$ $\Phi_E =$
- c. $\vec{\mathbf{E}} = E_o \hat{\mathbf{k}}$ $\theta =$ $\Phi_E =$
- d. $\vec{\mathbf{E}} = \frac{E_o}{\sqrt{2}} \hat{\mathbf{i}} + \frac{E_o}{\sqrt{2}} \hat{\mathbf{j}}$ $\theta =$ $\Phi_E =$

3 Φ_E Through Closed Surface

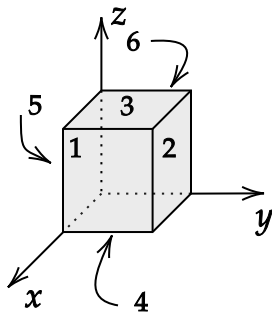
In the previous problem, you computed the flux through an open surface. You should have noted that one can associate two area vectors to an open surface – imagine your hand being an open surface. You can put the a pencil (1) on the top of your hand with the tip pointing up or (2) in your palm with the tip pointing down. The pencil represents the vector and the tip indicates the direction.

Gauss's law, which involves electric flux, always involves a closed surface (if you put water inside a closed surface, it would not leak out). For Gauss's law, there is a convention for which area vector to choose – it is the one that points outwards from the volume that the surface encloses.

In the following example, the electric flux is computed through a closed surface (a cube) by finding the flux through each of the faces of the cube. The total electric flux is the sum of the fluxes through each cube.

With Coulomb's law, we are given the location and values of charges and we compute the electric field anywhere in space. With Gauss's law, we can do the reverse – given an electric field on the surface of a small volume of space, we can compute the charge in the volume. (If the volume is large, we can only compute the amount of charge enclosed in the volume; however, if the closed surface volume approaches zero, we can compute the amount of charge at a point in space.)

3.1 Example



Find the flux through the six labeled faces of the cube with side length a when the electric field is everywhere in the $+z$ direction with magnitude E_o .

Answer This example is similar to [Example 22.2a in the textbook](#). The electric field is parallel to surfaces 1, 2, 5, and 6. For each of these surfaces, $\theta = 90^\circ$, and $\cos(90^\circ) = 0$. Or, thinking in terms of the analogy of the electric field representing lines of flow, the flux is zero through these faces.

$$\Phi_E^1 = \Phi_E^2 = \Phi_E^5 = \Phi_E^6 = 0$$

By convention, the normal direction for surface 3 is outwards from the volume, which is in the $+z$ -direction. The electric field is in the same direction, so $\theta = 0$ and

$$\Phi_E^3 = E_o A \cos 0^\circ = E_o a^2$$

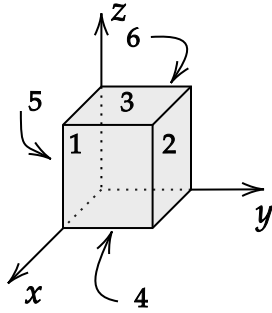
The normal direction for the bottom surface is downwards, which is in the opposite direction as the electric field, so $\theta = 180^\circ$ and

$$\Phi_E^4 = E_o A \cos 180^\circ = -E_o a^2$$

The total flux through the cube, $\Phi_E^1 + \dots + \Phi_E^6$, is zero. Thinking again in terms of the electric field representing flow lines, every electric field line that enters the cube exits, so the flow in equals the flow out. (Perhaps confusingly, flow out of a volume corresponds to a positive flux. The reason for this convention for flux is that from Gauss's law, a net positive flow out of a closed surface corresponds to a net positive charge inside the surface.)

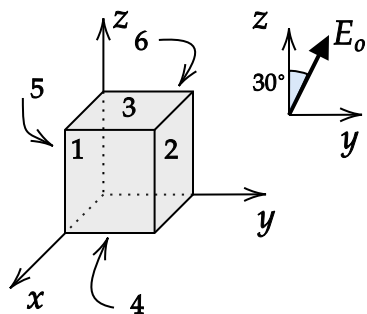
How much charge is inside the cube? The net flux through the cube's surface is zero, so it follows from Gauss's law that the total charge enclosed is zero.

3.2 Problem



Find the flux through the six labeled faces of the cube with side length a when the electric field is everywhere in the $+y$ direction.

3.3 Problem



Find the flux through the six labeled faces of the cube with side length a when the electric field is as shown in the diagram. Provide diagrams to justify your equations.