## **Electric Field**

### 1 Overview

The electric field vector,  $\vec{\mathbf{E}}$ , is a quantity assigned to a point in space. Given this quantity, we can compute the force on a charge Q will experience if it is placed at that point using the equation  $\vec{\mathbf{F}} = Q\vec{\mathbf{E}}$ . The direction of  $\vec{\mathbf{E}}$  is also the direction a charge will begin to move if released from rest.

To find  $\vec{\mathbf{E}}$  at any point in space, compute the force  $\vec{\mathbf{F}}$  due to all other charges on a hypothetical (or "test") charge  $q_o$  at a point where you want to know  $\vec{\mathbf{E}}$ . To find  $\vec{\mathbf{E}}$  at that point, divide  $\vec{\mathbf{F}}$  by  $q_o$ .

$$ec{\mathbf{E}} = rac{ec{\mathbf{F}}}{q_o}$$

# 2 Example I

Charge  $q_1$  is at (x, y) = (-a, -a). Find the electric field at (x, y) = (a, a) in the form  $\vec{\mathbf{E}} = E_x \hat{\imath} + E_y \hat{\jmath}$ . Also, find E. (Note that E and  $|\vec{\mathbf{E}}|$  are used interchangebly.)

#### **Solution**

To find the electric field at a point in space, we put a hypothetical "test" charge  $q_o$  at that point, compute the force on it due to all other charges, and then use

$$ec{\mathbf{E}} = rac{ec{\mathbf{F}}}{q_o}$$

The force a positive charge  $q_1$  at (x, y) = (-a, -a) exerts on a positive charge  $q_2$  at (x, y) = (a, a) was computed in a previous activity.

$$ec{\mathbf{F}}_{q_1 ext{ on } q_2} = k rac{|q_1 q_2|}{8a^2} (\cos 45^\circ \hat{m{\imath}} + \sin 45^\circ \hat{m{\jmath}})$$

We also found that this equation applies when  $q_1$  and  $q_2$  are both positive or both are negative. If  $q_1$  was positive and  $q_2$  was negative, or vice-versa, we found the sign changed:

$$ec{\mathbf{F}}_{q_1 \; ext{on} \; q_2} = -k rac{|q_1 q_2|}{8a^2} (\cos 45^{\circ} \hat{m{\imath}} + \sin 45^{\circ} \hat{m{\jmath}})$$

Based on this, we can write a single equation for all possibilities:

$$ec{\mathbf{F}}_{q_1 \,\, ext{on} \,\, q_2} = k rac{q_1 q_2}{8 a^2} (\cos 45^{\circ} \hat{m{\imath}} + \sin 45^{\circ} \hat{m{\jmath}})$$

If we replace  $q_2$  with  $q_o$ , this is

$$ec{\mathbf{F}}_{q_1 ext{ on } q_o} = k rac{q_1 q_o}{8a^2} (\cos 45^\circ \hat{m{\imath}} + \sin 45^\circ \hat{m{\jmath}})$$

The electric field at the location of  $q_o$  is then

$$ec{f E}_{{
m at}\,(a,a)\,{
m due}\,{
m to}\,q_1} = rac{ec{f F}}{q_o} = rac{kq_1}{8a^2}(\cos 45^\circ \hat{m \imath} + \sin 45^\circ \hat{m \jmath}) = rac{kq_1}{8a^2}\left[rac{1}{\sqrt{2}}\hat{m \imath} + rac{1}{\sqrt{2}}\hat{m \jmath}
ight],$$

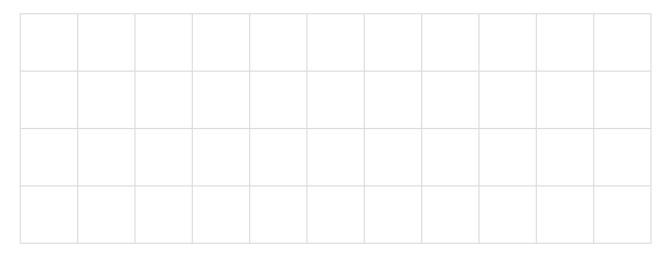
where the fact that  $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$  was used. The magnitude is  $E = \sqrt{E_x^2 + E_y^2} = k|q_1|/8a^2$ 

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Sign check: When computing electric fields and forces, it is easy to make a sign error. The electric field vector points in the direction a positive charge will move if released there from rest. Suppose  $q_1$  is positive. Our equation predicts that a positive charge released from rest at (a, a) will move up and to the right. Suppose  $q_1$  is negative. Our equation predicts that a positive charge will move down and to the left. This is consistent with the fact that like charges repel and unlike charges attract.

### 3 Problem I

Charge  $q_1$  is at (x, y) = (-a, a). At (x, y) = (a, 0), find  $\vec{\mathbf{E}}$  in the form  $\vec{\mathbf{E}} = E_x \hat{\imath} + E_y \hat{\jmath}$ . Check signs of the components of  $\vec{\mathbf{E}}$  using the technique used in Example I. Also, find E.



#### Answer:

From Problem II on the Electric Force Activity, if  $q_2$  is at (x, y) = (a, 0) (and using the arguments in the previous example to drop the absolute value sign),

$$ec{\mathbf{F}}_{q_1 ext{ on } q_2} = k rac{q_1 q_2}{5a^2} \left( rac{2}{\sqrt{5}} \hat{m{\imath}} - rac{2}{\sqrt{5}} \hat{m{\jmath}} 
ight)$$

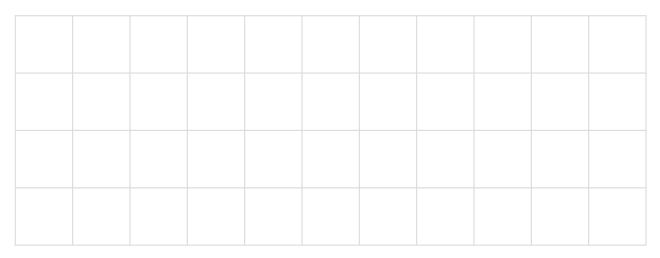
Replacing  $q_2$  with a test charge  $q_o$ ,

$$ec{\mathbf{F}}_{q_1 ext{ on } q_o} = k rac{q_1 q_o}{5a^2} \left( rac{2}{\sqrt{5}} \hat{\pmb{\imath}} - rac{2}{\sqrt{5}} \hat{\pmb{\jmath}} 
ight)$$

$$ec{\mathbf{E}}_{\mathrm{at}\,(a,0)\,\mathrm{due\,to}\,q_1} = rac{ec{\mathbf{F}}}{q_o} = rac{kq_1}{5a^2}\left(rac{2}{\sqrt{5}}\hat{m{\imath}} - rac{2}{\sqrt{5}}\hat{m{\jmath}}
ight)$$

# 4 Problem II

Charge  $q_1$  is at (x,y)=(-a,a). Find the electric field at (x,y)=(a,0), find  $\vec{\bf E}$  in the form  $\vec{\bf E}=E_x\hat{\imath}+E_y\hat{\jmath}$ . Check signs of the components of  $\vec{\bf E}$  using the technique used in Example I. Also, find E.



Answer:

 $\vec{\mathbf{E}}_{\mathrm{at}\,(a,0)\,\mathrm{due\,to}\,q_1}=rac{kq_1}{5a^2}\left(rac{2}{\sqrt{5}}\hat{\pmb{\imath}}-rac{2}{\sqrt{5}}\hat{\pmb{\jmath}}
ight)$ , which matches the solution to Problem I, as expected.