Electric Field Activity

1 Overview

This activity covers topics in Section 21.3-4 of Young and Freedman 2015, 14th Edition.

Electric Force

Coulomb's Law in compact form is

$$\vec{\mathbf{F}}_{1\text{ on }2} = kq_1q_2\frac{\hat{\mathbf{r}}}{r^2}$$

where $\hat{\mathbf{r}}$ is the unit vector that points from the position of q_1 to q_2 and r is the distance between q_1 and q_2.

Electric Field

The electric field vector, $\vec{\mathbf{E}}$, is a quantity that we assign to a point in space. Given this quantity, we can compute the force on a charge Q will experience if it is placed at that point in space using the equation $\vec{\mathbf{F}} = Q\vec{\mathbf{E}}$.

To find $\vec{\mathbf{E}}$ at any point in space, compute the force $\vec{\mathbf{F}}$ due to all other charges on a hypothetical (or "test") charge q_o at a point where you want to know $\vec{\mathbf{E}}$. To find $\vec{\mathbf{E}}$ at that point, divide $\vec{\mathbf{F}}$ by q_o.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_o}$$

2 Example

Charge q_1 is at $(x, y) = (-a, -a)$.

Find the electric field at $(x, y) = (a, a)$ in the form $\vec{\mathbf{E}} = E_x\hat{\imath} + E_y\hat{\jmath}$.

Solution

To find the electric field at a point in space, we put a hypothetical "test" charge q_o at that point, compute the force on it due to all other charges, and then use

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_o}$$

The force a charge q_1 at $(x, y) = (-a, -a)$ exerts on a charge q_2 at $(x, y) = (a, a)$ was computed in a previous activity. We can use the answer after the replacement of q_2 with q_o. The result is

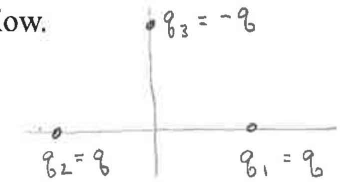
$\vec{\mathbf{F}} = k\frac{q_1q_o}{8a^2}\left[\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right]$. The electric field is then $\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_o} = k\frac{q_1}{8a^2}\left[\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right]$

3 Problem

In the previous example, there was only one charge responsible for creating the electric field $\vec{\mathbf{E}}$. To find the electric field when there are more charges, superposition can be used.

Charge $q_1 = +q$ is at $(x, y) = (a, 0)$, charge $q_2 = +q$ is at $(x, y) = (-a, 0)$, and charge $q_3 = -q$ is at $(x, y) = (0, a)$. Assume that the quantity associated with q is positive.

1. Draw this charge configuration below.



2. Why does it not make sense to ask what the electric force is at the origin?

Forces act on objects. Nothing at origin.

In the following,

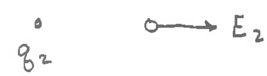
3. Find the electric field at the origin due to q_1. Write your answer in the form $\vec{\mathbf{E}}_1 = E_{x1}\hat{\boldsymbol{\imath}} + E_{y1}\hat{\boldsymbol{\jmath}}$.

$E_{x1} = -\frac{kq}{a^2}$ $\quad E_{y1} = 0$

$\vec{E}_1 = -\frac{kq}{a^2}\hat{\imath}$

4. Find the electric field at the origin due to q_2. Write your answer in the form $\vec{\mathbf{E}}_2 = E_{x2}\hat{\boldsymbol{\imath}} + E_{y2}\hat{\boldsymbol{\jmath}}$.

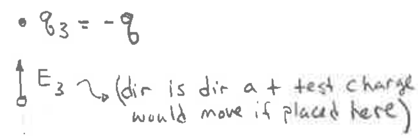
$E_{x2} = \frac{kq}{a^2}$ $\quad E_{y2} = 0$



$\vec{E}_2 = +\frac{kq}{a^2}\hat{\imath}$

5. Find the electric field at the origin due to q_3. Write your answer in the form $\vec{\mathbf{E}}_3 = E_{x3}\hat{\boldsymbol{\imath}} + E_{y3}\hat{\boldsymbol{\jmath}}$.

$E_{x3} = 0$ $\quad E_{y3} = +\frac{kq}{a^2}$



$\vec{E}_3 = +\frac{kq}{a^2}\hat{\jmath}$

6. Find the electric field at the origin. Write your answer in the form $\vec{\mathbf{E}} = E_x\hat{\boldsymbol{\imath}} + E_y\hat{\boldsymbol{\jmath}}$.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = -\frac{kq}{a^2}\hat{\imath} + \frac{kq}{a^2}\hat{\imath} + \frac{kq}{a^2}\hat{\jmath} = \frac{kq}{a^2}\hat{\jmath}$$

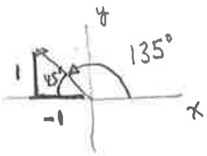
7. Will your answers to 3.-6. change if the problem had asked for the electric field at a different position? If so, which answers?

All change b/c distance from charge changes.

8. Find the electric field at the origin if charge $q_1 = 2q$ (instead of q).

$$E_{1x} = -\frac{2kq}{a^2}$$

E_1 q_1



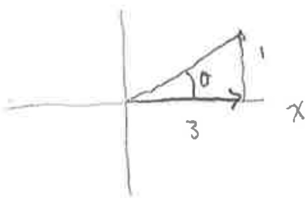
$$\vec{E} = -\frac{2kq}{a^2}\hat{i} + \frac{kq}{a^2}\hat{i} + \frac{kq}{a^2}\hat{j} = -\frac{kq}{a^2}\hat{i} + \frac{kq}{a^2}\hat{j}$$

$$|\vec{E}| = \frac{kq}{a^2}\sqrt{2}$$

9. Find the electric field at the origin if charge $q_1 = -2q$ (instead of q).

$$\vec{E} = \frac{3kq}{a^2}\hat{i} + \frac{kq}{a^2}\hat{j}$$

E_1 q_1



$$|\vec{E}| = \frac{kq}{a^2}\sqrt{10}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right) = 18.4°$$