Electric Force Activity

1 Overview

We generally describe vector equations in two ways:

1. Simple – An equation for the magnitude is given, and words are used to describe the direction.
2. Compact – A single equation is given for the vector, and additional equations are given for parts of the equation.

For example, in Section 21.3 of the textbook, the equation for Coulomb's Law was given in the simple form. In Section 21.4, the equation for the electric field due to a point charge was given in the compact form.

In PHYS 260, you will need to be able to solve problems similar to the examples given here quickly. If you found the problems in this activity to be difficult, review Sections 1.7-1.8 in the textbook and see Khan Academy's comprehensive introduction to vectors.

2 Coulomb's Law in Simple Form

Magnitude

$$F_{1\text{ on }2} = k\frac{|q_1 q_2|}{r^2}$$

where r is the distance between q_1 and q_2. To simplify notation, we are using k in place of $1/4\pi\epsilon_o$.

Direction

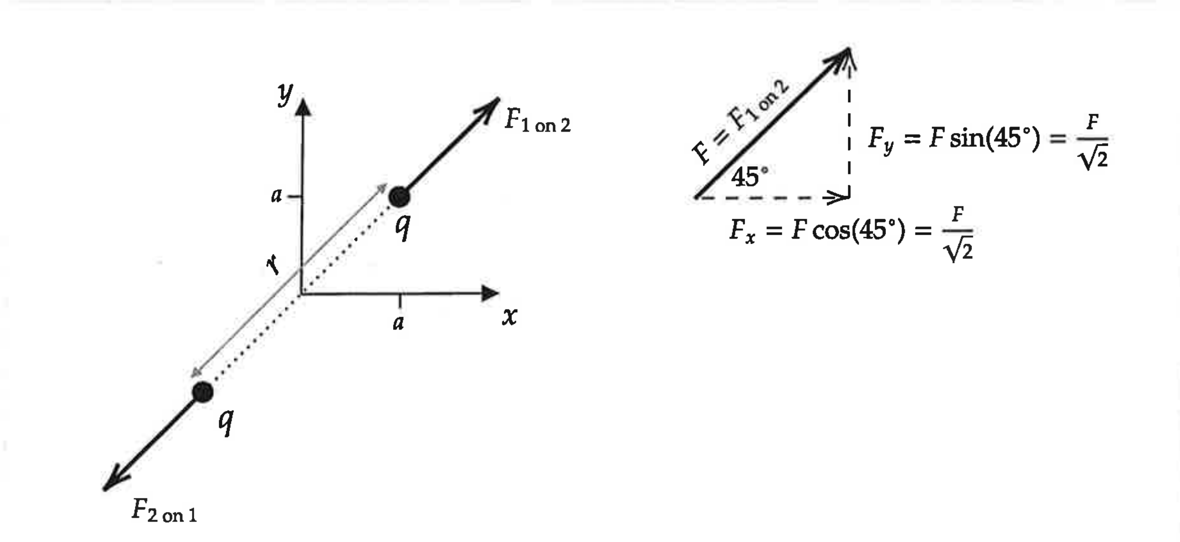
Along line that connects q_1 and q_2. Direction depends on signs of q_1 and q_2. (Likes repel, opposites attract.).

2.1 Example

Charge q_1 is at $(x, y) = (-a, -a)$ and charge q_2 is at (a, a). Both charges have a charge of q.

1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x\hat{\imath} + F_y\hat{\jmath}$.

Solution



1. The distance between the charges is $r = 2\sqrt{2}a$, so

$$F_{1 \text{ on } 2} = k\frac{|q_1 q_2|}{r^2} = \frac{k|qq|}{(2\sqrt{2}a)^2} = \frac{kq^2}{8a^2}$$

The charges will repel each other, so the direction of forces of one on the other will be as shown in the diagram. The direction of the force vector on q_2 is shown in the diagram.

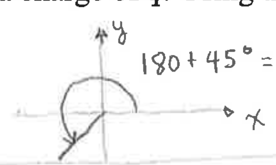
2. Let $F = F_{1 \text{ on } 2}$ from part 1. to simplify notation. Then

$\vec{\mathbf{F}} = F\cos 45°\,\hat{\imath} + F\sin 45°\,\hat{\jmath}$. Given that $\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$, we can also write

$$\vec{\mathbf{F}} = F\left[\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right]$$

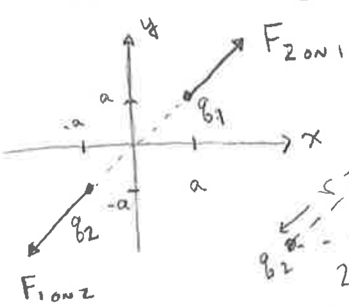
2.2 Problem

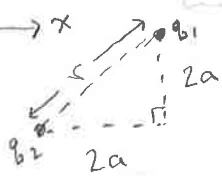
Charge q_1 is at $(x, y) = (a, a)$ and charge q_2 is at $(-a, -a)$. Both charges have a charge of q. Using the steps in the previous example,



$180 + 45° = 225°$

1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x\hat{\imath} + F_y\hat{\jmath}$.





$r^2 = (2a)^2 + (2a)^2$
$= 8a^2$

1. $\boxed{F_{1 on 2} = \frac{K|q_1 q_2|}{r^2} = \frac{k|qq|}{8a^2} = \frac{Kq^2}{8a^2} \quad \text{direction is } 225°}$

2. $\boxed{F_x = \frac{Kq^2}{8a^2}\cos 225°, \; F_y = \frac{Kq^2}{8a^2}\sin 225°}$

or

$F_x = -\frac{Kq^2}{8a^2}\cos 45°, \; F_y = -\frac{Kq^2}{8a^2}\sin 45°$

2. Let $\vec{\mathbf{F}} = \vec{\mathbf{F}}_{1\text{ on }2}$ from part 1. to simplify notation. Then

$$F = |\vec{\mathbf{F}}| = \frac{kq^2}{8a^2}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{kq^2}{8a^2}$$

The angle is $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{\frac{F}{\sqrt{2}}}{\frac{F}{\sqrt{2}}}\right) = 45°$

See the margin note on page 16 of the textbook for an issue that may arise when using this formula to compute an angle.

3.2 Problem

Charge q_1 is at $(x, y) = (a, a)$ and charge q_2 is at $(-2a, -2a)$. Both charges have a charge of q. Using the steps in the previous example,

1. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x\hat{\imath} + F_y\hat{\jmath}$.
2. Find the magnitude and direction of the force of q_1 on q_2.

$\vec{r}_1 = a\hat{i} + a\hat{j}$

$\vec{r}_2 = -2a\hat{i} - 2a\hat{j}$

$\vec{r} = \vec{r}_2 - \vec{r}_1 = -3a\hat{i} - 3a\hat{j}$

$r^2 = (3a)^2 + (3a)^2 = 18a^2$

$\hat{r} = \frac{\vec{r}}{r} = \frac{-3a\hat{i} - 3a\hat{j}}{\sqrt{18}\,a}$

$\vec{F} = kq_1q_2\frac{\hat{r}}{r^2} = kq^2\frac{(-3\cancel{a}\hat{i} - 3\cancel{a}\hat{j})/\sqrt{18}\,\cancel{a}}{18a^2}$

1. $\boxed{\vec{F} = kq^2\left(\frac{-3\hat{i} - 3\hat{j}}{18^{3/2}a^2}\right) = \frac{kq^2}{18a^2}\left(-\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right) \Rightarrow F_x = -\frac{kq^2}{18\sqrt{2}a^2},\ F_y = -\frac{kq^2}{18\sqrt{2}a^2}}$

2. $|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \boxed{\frac{kq^2}{18a^2}}$!

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}(1) = 45°$ but both F_x + F_y are negative $\Rightarrow \theta = 180° + 45° = \boxed{225°}$

3 Coulomb's Law in Compact Form

$$\vec{\mathbf{F}}_{1\text{ on }2} = kq_1q_2\frac{\hat{\mathbf{r}}}{r^2}$$

$\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$ is the vector from the position of q_1 to the position of q_2, $\vec{\mathbf{r}}_1$ is a vector from the origin to the location of q_1, and $\vec{\mathbf{r}}_2$ is a vector from the origin to the location of q_2.

$r = |\vec{\mathbf{r}}| = |\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1|$ is the distance from q_1 to q_2.

$\hat{\mathbf{r}} = \dfrac{\vec{\mathbf{r}}}{r}$ is the unit vector pointing from the position of q_1 to the position of q_2.

Although this form looks more complex, it requires basic steps to use.

3.1 Example

Charge q_1 is at $(x, y) = (-a, -a)$ and charge q_2 is at (a, a). Both charges have a charge of q.

1. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x\hat{\imath} + F_y\hat{\jmath}$.
2. Find the magnitude and direction of the force of q_1 on q_2.

Solution

1. The vector from the origin to the location of q_1 is $\vec{\mathbf{r}}_1 = -a\hat{\imath} - a\hat{\jmath}$

   The vector from the origin to the location of q_2 is $\vec{\mathbf{r}}_2 = a\hat{\imath} + a\hat{\jmath}$

   The distance vector is $\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = 2a\hat{\imath} + 2a\hat{\jmath}$

   The length of the distance vector is

   $$r = |\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1| = \sqrt{(2a)^2 + (2a)^2} = 2\sqrt{2}a$$

   The unit vector pointing from the position of q_1 to the position of q_2 is

   $$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r} = \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$$

   Inserting the above results into the equation

   $$\vec{\mathbf{F}}_{1\text{ on }2} = kq_1q_2\frac{\hat{\mathbf{r}}}{r^2} \quad \text{gives}$$

   $$\vec{\mathbf{F}}_{1\text{ on }2} = kq^2\frac{\left[\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right]}{(2\sqrt{2}a)^2} \quad \text{or} \quad \vec{\mathbf{F}}_{1\text{ on }2} = \frac{kq^2}{8a^2}\left[\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right]$$

   (Note that one can avoid the need to compute $\hat{\mathbf{r}}$ by using the equivalent formula $\vec{\mathbf{F}}_{1\text{ on }2} = kq_1q_2\vec{\mathbf{r}}/r^3$.)