1 Introduction

1.1 Definitions

The electric current in a wire is defined as

$$I = \frac{dQ}{dt}$$

where dQ is the total amount of charge that passes through a cross–section of the wire in a differential amount of time, dt.

If q is the charge (in Coulombs) of each flowing charge, n is their number per volume ("number density"), v_d their average speed along the wire (called the "drift velocity"), then

$$I = n|q|v_d A$$

where A is the cross–sectional area of the wire.

We also define the average current density, $J = n|q|v_d$, which is the current per cross–sectional area:

$$J = \frac{I}{A}$$

1.2 Ohm's Law

If an electric field exists in a wire (by, for example, connecting its ends to a battery), the charges will accelerate until they collide with another particle and decelerate (collisions resist the flow). The net result will be current – a flow of charges with an average drift velocity. Experimentally, it has been shown that in many materials, the electric field is proportional to the current density:

$$E = \rho J$$

where the value of the proportionality constant ρ, called resistivity, depends on the material. This is one version of Ohm's law.

For a wire of length L and constant cross–sectional area A, Ohm's law can also be written as

$$V = \frac{\rho L}{A} I$$

If we define

$$R = \frac{\rho L}{A}$$

then we have another relationship that is also referred to as Ohm's law: $V = IR$.

In this form, the interpretation is that the voltage across a wire is proportional to the current in the wire, with the proportionality constant of R.

2 Problem I – Definitions and Ohm's Law

A 9–volt power source is connected to a wire of length 10 meters with a circular cross–section and radius of 0.01 meters. The wire has a resistivity of $10^{-8}\ \Omega\cdot\mathrm{m}$. The number density of charge carriers is $10^{28}/\mathrm{m}^3$. Assume Ohm's law applies.

1. What is the resistance (with units) of the wire?

   Answer: $R = \dfrac{\rho L}{A} = \dfrac{(10^{-8}\ \Omega\cdot\mathrm{m})(10\ \mathrm{m})}{\pi(0.01\ \mathrm{m})^2} = \dfrac{10^{-3}}{\pi}\ \Omega$. When solving circuit problems, the resistors involved are typically much larger than this, so we neglect the resistance of the wires.

2. What is the current (with units) in the wire?

   Answer: $I = \dfrac{9\ \mathrm{v}}{\frac{10^{-3}}{\pi}\ \Omega} = 9,000\pi\ \mathrm{A}$. This is a huge current. If you look at the back of an electronic device, you will see a rating on the order of 1 A. Household circuit breakers are set to break at approximately 15 A. Most power sources cannot supply current at this rate; even if the power source could supply this current, the amount of heat created would lead to a file.

3. How much charge (in Coulombs) flows past a cross–section of the wire per second?

   Answer: $9,000\pi\ \mathrm{C}$

4. What is the current density (with units) that flows through the wire?

   Answer: $J = \dfrac{I}{A} = 9\cdot 10^7 \dfrac{\mathrm{A}}{\mathrm{m}^2}$

5. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6\cdot 10^{-19}$ C.)

   Answer: $v_d = J/n|q| = 6\ \mathrm{cm/s}$

6. Based on the description of how charged particles flow in a wire, explain why the resistance of a cylindrical wire is proportional to its length and inversely proportional to the square of its radius.

   Answer: See the description in your textbook.

3 Problem II – Definitions and Ohm's Law

A power source is connected to a wire of length 20 meters with a circular cross–section and a radius of 0.01 meters. The wire has a resistivity of $10^{-7}\ \Omega\cdot\mathrm{m}$. The number density of charge carriers is $10^{27}/\mathrm{m}^3$. The current in the wire was measured and found to be 1 A. Assume Ohm's law applies.

1. What is the resistance (with units) of the wire?

   Answer: $(.02/\pi)\ \Omega$

2. How much charge (with units) flows past a cross–section of the wire per second?

   Answer: 1 C

3. What is the current density (with units) that flows through the wire?

   Answer: $J = \dfrac{10^4}{\pi}\dfrac{\mathrm{A}}{\mathrm{m}^2}$

4. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6\cdot 10^{-19}$ C.)

Answer: $v_d = J/n|q| = 0.02\ \mathrm{mm/s}$

4 Problem III – Current Through a Cylindrical Shell

If a cylindrical wire with an inner radius a and outer radius b carries a current I, what is J?

Answer: $J = \dfrac{I}{\pi(b^2 - a^2)}$

5 Problem IV – $I = n|q|v_d A$ derivation

Derive the relationship $I = n|q|v_d A$. Provide a diagram.

Answer: See textbook.