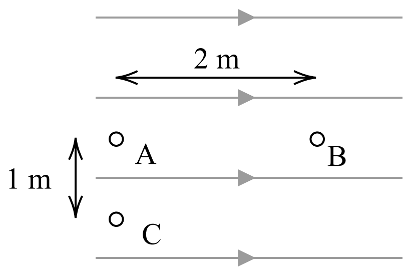
1 Electric Potential Energy Differences, ΔU

1.1 Problem – Uniform Field

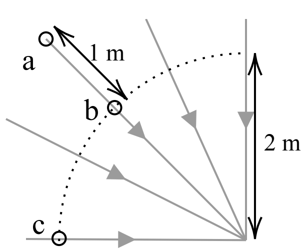
The following diagram shows a region of space where the electric field is constant and has a value of $3\ \mathrm{N/C}$.



1. You place a charge of $+3$ C at point A. What happens to that charge when it is released?
2. You move charge of $+3$ C from a to b. (a) How much work did you do? (b) How much work was done by the electric field? (c) By how much has the potential energy of the charge changed?
3. You place a charge of -3 C at point a. What happens to that charge when it is released?
4. A charge of -3 C is moved from a to b. How much work did you do? (b) How much work was done by the electric field? (c) By how much has the potential energy of the charge changed?
5. You move a charge of $+3$ C straight downward from a to c. (a) How much work did you do? (b) How much work was done by the electric field? (c) By how much has the potential energy of the charge changed?
6. If you move a charge of -3 C from a to c but deviate from a straight line, will your answers to the previous problem change? If no, explain why. If yes, provide new answers.

1.2 Problem – Radial Field

In the previous problem, a charge was moved in a region of space where the electric field was constant and so the calculation of work did not require integration. In this problem, the electric field is not constant and so integration is required. The type of integration that must be performed to compute work in this case is given by Equation 23.8 in the textbook.



There is a charge of -6 C at the origin. Some electric field lines for this charge are shown. To simplify the math, use $k = 9 \cdot 10^9\ \mathrm{N \cdot m^2/C^2}$.

1. You move a charge of $+3$ C from a to b. (a) How much work did you do? (b) How much work was

done by the electric field? (c) By how much has the potential energy of the moved charge changed?

This is the answer for (b). Note that the work is positive as expected – the force on the charge is in the same direction as its movement. The work that you do is equal and opposite to the work that the field does. The change in potential is equal and opposite to the work done by the field. In summary, (a) $-27 \cdot 10^9$ J, (b) $+27 \cdot 10^9$ J, (c) $-27 \cdot 10^9$ J. (Use the techniques given in the previous problem to make sure that the signs of these answers are correct.) Note that the answer of $27 \cdot 10^9$ J is unphysically large; it is the amount of energy that you would need to lift $27 \cdot 10^9$ kg (about 5 million elephants) by 1 m.

2. You move a charge of -3 C from a to b. (a) How much work did you do? (b) How much work was done by the electric field? (c) By how much has the potential energy of the moved charge changed?
3. You move a charge of -3 C from b to c along the dotted line. (a) How much work did you do? (b) How much work was done by the electric field? (c) By how much has the potential energy of the moved charge changed?
4. You move a charge of -3 C from c to b but deviate from the dotted line. (a) How much work did you do? (b) How much work was done by the electric field? (c) By how much has the potential energy of the moved charge changed?

2 Electric potential difference, ΔV

In the previous section, we considered moving an arbitrary amount of charge (either positive or negative) from point a to point b and computed its change in potential energy ΔU.

An electric potential difference ΔV is defined to be the change in electric potential energy of a (positive by convention) test charge when it is moved from point a to point b divided by the charge on the test charge.

As a result, the only difference between the ΔU calculations performed previously and ΔV calculations is that we first compute ΔU for a $+1$ C charge. To get ΔV, we simply divide by ΔU by $+1$ C.

The definition of electric potential is similar to the definition of electric field in that they both involve consideration of a test charge. That is, the electric field is the force on a test charge divided by the magnitude of the test charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q_o}$$

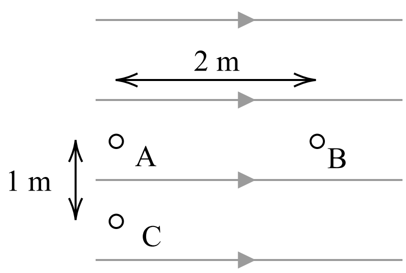
A change in potential is the change in potential of a test charge divided by the magnitude of the test charge's charge:

$$\Delta V = \frac{\Delta U}{q_o}$$

The advantage of using changes in electric potential (ΔV) as opposed to changes in electric potential energy (ΔU) of a specific amount of charge is that once the electric potential difference ΔV is known for a test charge, the change in potential energy for an arbitrary amount of charge Q can be computed by simply multiplying ΔV and Q. This is similar to the advantage of the electric field. If we know the electric field at a given point, we can find the force on an arbitrary charge Q by multiplying $\mathbf{E}$ and Q.

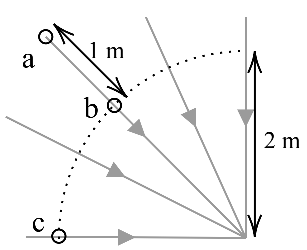
2.1 Problem

The following diagram shows a region of space where the electric field is constant and has a value of 3 N/C.



1. What is difference in potential $\Delta V = V_B - V_A$.
2. What is the difference in potential $\Delta V = V_B - V_C$.
3. You move a charge of $+3$ C from a to b. By how much has the electric potential energy of the moved charge changed?
4. You move a charge of -3 C from a to b. By how much has the electric potential energy of the moved charge changed?
5. You move a charge of -3 C from b to c. By how much has the electric potential energy of the moved charge changed?

2.2 Problem



There is a charge of -6 C at the origin. Some electric field lines for this charge are shown. To simplify the math, use $k = 9 \cdot 10^9\ \mathrm{N \cdot m^2/C^2}$.

1. What is difference in potential $\Delta V = V_b - V_a$?
2. What is difference in potential $\Delta V = V_c - V_a$.
3. You move a charge of -3 C from a to b. By how much has the potential energy of the moved charge changed?
4. You move a charge of -3 C from b to c. By how much has the potential energy of the moved charge changed?

3 ΔU and ΔV and Superposition

The electric potential energy of a charge q_0 that is a distance of r_1 from a charge q_1 is defined to be

$$U = k\frac{q_0 q_1}{r_1}$$

In this formula, if the charges have opposite signs then U is negative; if they have the same sign then U

is positive. Note that there is a sign associated with the potential energy but the direction of the vector that connects the charges does not matter; the equation for U only involves the values of the charges and the magnitude of the separation distance between them.

Consider next the potential energy of charge q_0 when it is a distance r_1 from charge q_1 and a distance r_2 from charge q_2. Because potential energy is a scalar and not a vector, the potential energy of q_0 is the **algebraic** sum, rather than the vector sum, of the potential energies due to q_1 and q_2

$$U = k\frac{q_0 q_1}{r_1} + k\frac{q_0 q_2}{r_2}$$

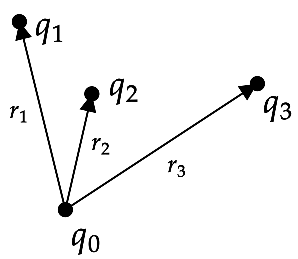
More generally, if there is a group of N charges, the potential energy of charge q_0 is

$$U = kq_0 \sum_{i=1}^{N} \frac{q_i}{r_i}$$

Similarly, the electric potential at a point in space due to a group of N charges is the **algebraic** sum of the potentials due to each of the charges at that point in space

$$V = k \sum_{i=1}^{N} \frac{q_i}{r_i}$$

3.1 Problem



1. What is the potential energy of the charge q_0 in the diagram shown?
2. What is the potential at the position of q_0 if that charge were to be removed (i.e., the potential due to charges q_1, q_2, and q_3)?
3. Can you find the potential energy at the position of q_0 if that charge were to be removed? Why or why not?
4. Explain the difference between potential and potential energy.

3.2 Problem

Given a point charge q_1 at the origin:

1. Write the general equation for the potential of a charge q.
2. Find the electric potential, V, at $(x, y) = (d, 0)$.
3. If a charge q_2 is placed at $(x, y) = (d, 0)$, find the electric potential, V, at $(x, y) = (-d, 0)$ (hint - it is the sum of the potentials due to q_1 and q_2).
4. How much work is required to place q_3 at $(x, y) = (-d, 0)$? Note that the work required is also

referred to as the potential energy, U, of q_3.

In summary, to find the work required to put a charge Q at point P (or, equivalently, the electric potential energy U of a single charge Q when it is at point P), find the potential V at point P due to all of the other charges and then $U = QV$.

4 Work Required to Assemble a System of Charges

In the previous problem you computed the work required to move q_3 to $(x, y) = (-d, 0)$ after q_2 was in place. The total work required to assemble the system of three charges is larger than this work because it also took work to move q_2 into place. Given a point charge q_1 at origin, as in the previous question:

a. how much work is required to move q_2 to (x,y) = (d,0)?;

b. how much work is required to move q_3 to $(x, y) = (-d, 0)$ if only q_1 is present?;

c. how much work is required to move q_3 to $(x, y) = (-d, 0)$ if only q_2 is present?

d. The total work required to assemble the system of three charges is the sum of the work from parts a., b., and c. Write the equation for this sum. This sum is known as the total potential energy of the system of charges.