Lorentz Force on Wires

1 Forces on Straight Wires

A wire carrying a current contains many moving charges. If a current–carrying wire is in a region of space with a magnetic field, the forces on the moving charges in the wire results in force on the wire.

Each charge q moving with a velocity $\vec{v}$ in a segment of wire will have a force on it of

$$\vec{F} = q\vec{v} \times \vec{B}$$

This equation can be transformed into an equation involving the current I and length L of the wire segment using $q\vec{v} = \vec{I}L$:

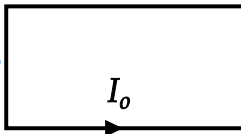$$\vec{F} = L\vec{I} \times \vec{B}$$

or, equivalently, $q\vec{v} = I\vec{L}$.

$$\vec{F} = I\vec{L} \times \vec{B}$$

where length vector $\vec{L}$ is defined to be a vector in the direction of $\vec{I}$ with a magnitude of the length of the wire segment.

These equations apply to (1) straight wires with (2) the same $\vec{B}$ on all segments of wire.

1.1 Example

$$\mathbf{F} = I_o(-6L_o\hat{y}) \times (B_o\hat{z})$$
$$= -6L_o I_o B_o \hat{x}$$

In the figure above, a magnetic field of magnitude B_o is pointing out of the page and the rectangle has a height of $6L_o$ and a width of $12L_o$.

Determine the magnitude and direction of the force on each segment of the wire and the net force on the loop.

Answer: In this problem, a coordinate system was not given. We are free to use any coordinate system directions provided that it is a right–handed coordinate system. (In a right–handed coordinate system, $\hat{\imath} \times \hat{\jmath} = \hat{k}$. If we choose the $+x$ direction to be to the right and $+y$ up, then the $+z$ direction must be out of the page in order for $\hat{\imath} \times \hat{\jmath} = \hat{k}$. A common error in a problem such as this is to choose the $+z$ direction into the page. In this case, you answers will be be off by a factor of -1.)

- Left wire: The length of the left wire is $6L_0$ and the current is flowing downward (in the $-\hat{\jmath}$ direction), so $\vec{L} = -6L_o\hat{\jmath}$.

$$\vec{F} = I\vec{L} \times \vec{B} = I_o(-6L_o\hat{\jmath}) \times B_o\hat{k} = -6I_oL_oB_o\hat{\jmath} \times B_o\hat{k} = -6I_oL_oB_o\hat{\imath}$$

- Right wire: The length is $6L_0$ and the current is flowing upward (in the $+\hat{\jmath}$ direction), so $\vec{L} = +6L_o\hat{\jmath}$.
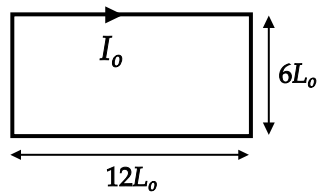
$$\vec{F} = I_o(+6L_o\hat{\jmath}) \times (B_o\hat{k}) = +6I_oL_oB_o\hat{\imath}$$

- Bottom wire: $\vec{F} = I_o(12L_o\hat{\imath}) \times (B_o\hat{k}) = -12I_oL_oB_o\hat{\jmath}$.

- Top wire: $\vec{F} = I_o(-12L_o\hat{\imath}) \times (B_o\hat{k}) = +12I_oL_oB_o\hat{\jmath}$.
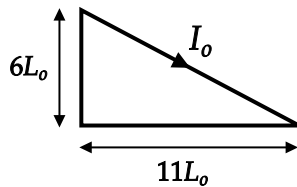
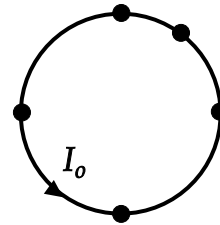Note that the sum of the forces is zero.

1.2 Problem

a.

b.

c.

1. For the current loops in a. and b. in the figure above, determine the magnitude and direction of the force on each segment of the wire and the net force on the loop given that a magnetic field of magnitude B_o points out of the page. Draw the vector on the line segments and show your calculation on the figure or in the space below.

2. For the current loop in c., determine **only** the direction of the force at each of the five points shown as solid dots. Draw a vector on the diagram to indicate the direction of the force at that point. What is

the net force on this loop?

2 Torques on Current Loops

In the previous section, you considered forces on the current loops when $\vec{\mathbf{B}} = B_o\hat{\mathbf{k}}$. The forces on all of the current loops that you drew in Problem 1.1 should have been such that the loop would not tend to translate (the net force should have been zero). The general result is that a closed loop in a *constant* magnetic field will not have a net force.

However, a closed loop in a constant magnetic field can have a net torque (it will tend to rotate).

The general equation for computing the torque on a current loop that lies in a plane is:

$$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$$

where $\vec{\mu}$ is the magnetic moment defined as $\vec{\mu} = I\vec{\mathbf{A}}$ and $\vec{\mathbf{A}}$ is the area vector for a loop. The magnitude of the area vector is simply the area of the loop. The direction of the area vector is determined by a right–hand rule: wrap your fingers along the direction of current and your thumb points in the direction of the area vector.

The equation $\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$ predicts that when $\vec{\mathbf{A}}$ is perpendicular to $\vec{\mathbf{B}}$, the torque magnitude is the largest. To see this, note that $\vec{\mu} \times \vec{\mathbf{B}}$ can be equivalently written as $\mu A \sin\phi$, where ϕ is the angle between $\vec{\mu}$ and $\vec{\mathbf{B}}$. When $\phi = \pm 90°$, the cross product will have the largest magnitude.
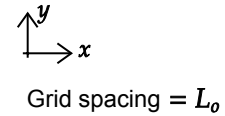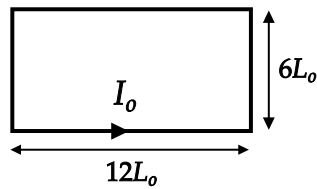
Interpreting the torque unit vector

- A torque in the $\hat{\mathbf{k}}$ direction corresponds to rotation in the same sense as rotation of the $+x$-axis towards the $+y$ axis;

- a torque in the $\hat{\mathbf{\imath}}$ direction corresponds to rotation in the same sense as rotation of the $+y$-axis towards the $+z$ axis;

- a torque in the $\hat{\mathbf{\jmath}}$ direction corresponds to rotation in the same sense as rotation of the $+z$-axis towards the $+x$ axis.

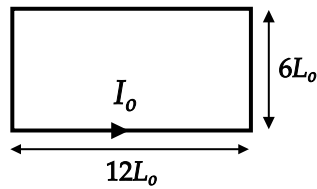You do not need to memorize these – they follow from table of cross products discussed in the last activity.

2.1 Problem – $\vec{\mathbf{B}} \perp$ to Loop

Although the forces on the current loops shown in Problem 1.1 were such that the loop would not have a net force, a current loop in a constant magnetic field can experience a net torque. To see this, consider loops a. and c. in the following figure (loop b. is a copy of loop a. to allow you to draw force vectors for two different magnetic field directions). The magnetic field is pointing to the right so that $\vec{\mathbf{B}} = B_o\hat{\mathbf{\imath}}$.

a.

$6L_o$

I_o

$12L_o$

$\uparrow y$
$\llcorner\!\!\rightarrow x$

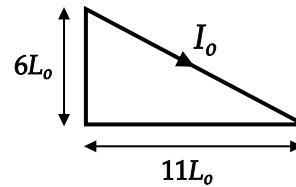Grid spacing $= L_o$

c.

b.

$6L_o$

I_o

$12L_o$

$6L_o$

I_o

$11L_o$

1. Will the loop a. tend to rotate? If so, what axis will it tend to rotate about? Determine this by drawing the direction of the force (determined using the right–hand rule) on each segment of the wire.

2. If the current direction is reversed, will loop a. tend to rotate? If so, what axis will it tend to rotate about?

3. Will loop c. tend to rotate? If so, what axis will it tend to rotate about?

4. If the current direction is reversed, will loop c. tend to rotate? If so, what axis will it tend to rotate about?

5. What is the magnetic moment, μ, of loops a. and c.?

6. What is the torque, τ, on loops a. and c.? Verify that this torque vector corresponds to an axis of

rotation that matches your previous answers.

7. For loop b., compute τ when $\vec{B} = (B_o/\sqrt{2})\hat{\imath} + (B_o/\sqrt{2})\hat{\jmath}$ and verify that this torque vector matches what you would expect from using the right-hand rule to determine the direction of the force on each segment of the loop.

8. For loop c., compute τ when $\vec{B} = (B_o/\sqrt{2})\hat{\imath} + (B_o/\sqrt{2})\hat{\jmath}$ and verify that this torque vector matches what you would expect from using the right-hand rule to determine the direction of the force on each segment of the loop.

2.2 Problem – $\vec{B}$ not $\perp$ to Loop
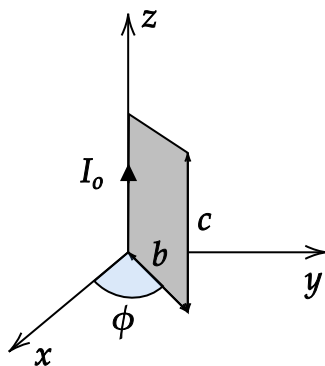
If $\vec{B} = B_o\hat{\imath}$,

1. Find the force on each segment of the loop.

2. Find the net force on the loop.

3. Find the net torque on the loop.