Electric Field

1 Overview

The electric field vector, $\vec{\mathbf{E}}$, is a quantity assigned to a point in space. Given this quantity, we can compute the force a charge Q will experience if it is placed at that point using the equation $\vec{\mathbf{F}} = Q\vec{\mathbf{E}}$. The direction of $\vec{\mathbf{E}}$ is also the direction a positive charge will move if released from rest.

To find $\vec{\mathbf{E}}$ at any point in space, compute the force $\vec{\mathbf{F}}$ due to all other charges on a hypothetical (or "test") charge q_o that has an infinitesimal charge and size at a point where you want to know $\vec{\mathbf{E}}$. To find $\vec{\mathbf{E}}$ at that point, divide $\vec{\mathbf{F}}$ by q_o.

$$\vec{\mathbf{E}}_{\text{due to } q} = \frac{\vec{\mathbf{F}}_{q \text{ on } q_o}}{q_o}$$

2 Example I

A positive charge q_1 is at $(x, y) = (-a, -a)$. Find the electric field at $(x, y) = (a, a)$ in the form $\vec{\mathbf{E}} = E_x\hat{\boldsymbol{\imath}} + E_y\hat{\boldsymbol{\jmath}}$. Also, find E. (Note that E and $|\vec{\mathbf{E}}|$ are used interchangeably.)

Solution

To find the electric field at a point in space, we put a hypothetical and positive "test" charge q_o at that point, compute the force on it due to all other charges, and then use

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_o}$$

The force a positive charge q_1 at $(x, y) = (-a, -a)$ exerts on a positive charge q_2 at $(x, y) = (a, a)$ was computed in a previous activity. The result was

$$\vec{\mathbf{F}}_{q_1 \text{ on } q_2} = k\frac{|q_1 q_2|}{8a^2}(\cos 45^\circ\,\hat{\boldsymbol{\imath}} + \sin 45^\circ\,\hat{\boldsymbol{\jmath}})$$

Because both charges are positive, we can drop the absolute value in the above equation, giving

$$\vec{\mathbf{F}}_{q_1 \text{ on } q_2} = k\frac{q_1 q_2}{8a^2}(\cos 45^\circ\,\hat{\boldsymbol{\imath}} + \sin 45^\circ\,\hat{\boldsymbol{\jmath}})$$

If we replace q_2 with q_o, this is

$$\vec{\mathbf{F}}_{q_1 \text{ on } q_o} = k\frac{q_1 q_o}{8a^2}(\cos 45^\circ\,\hat{\boldsymbol{\imath}} + \sin 45^\circ\,\hat{\boldsymbol{\jmath}})$$

The electric field at the location of q_o is then

$$\vec{\mathbf{E}}_{\text{at } (a,a) \text{ due to } q_1} = \frac{\vec{\mathbf{F}}}{q_o} = \frac{kq_1}{8a^2}(\cos 45^\circ\,\hat{\boldsymbol{\imath}} + \sin 45^\circ\,\hat{\boldsymbol{\jmath}}) = \frac{kq_1}{8a^2}\left[\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}\right],$$

where the fact that $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$ was used. The magnitude is $E = k|q_1|/8a^2$.

Sign check: When computing electric fields and forces, it is easy to make a sign error. The electric field vector points in the direction a positive charge will move if released there from rest. q_1 was given to be positive. Our equation predicts that a positive charge released from rest at (a, a) will move up and to the right. Suppose q_1 is negative. Our equation predicts that a positive charge will move down and to the left. This is consistent with the fact that like charges repel and unlike charges attract. (Note that we only derived the equation for $\vec{\mathbf{E}}$ when q_1 was positive, but it turns out that the equation we arrive at is correct when q_1 is negative.)

3 Problem I

A positive charge q_1 is at $(x, y) = (-a, a)$. At $(x, y) = (a, 0)$, find $\vec{\mathbf{E}}$ in the form $\vec{\mathbf{E}} = E_x\hat{\boldsymbol{\imath}} + E_y\hat{\boldsymbol{\jmath}}$. Check signs of the components of $\vec{\mathbf{E}}$ using the technique used in Example I. Also, find E.

Answer:

From Problem II on the Electric Force Activity, if q_2 is at $(x, y) = (a, 0)$ (and using the arguments in the previous example to drop the absolute value sign),

$$\vec{\mathbf{F}}_{q_1 \text{ on } q_2} = k\frac{q_1 q_2}{5a^2}\left(\frac{2}{\sqrt{5}}\hat{\boldsymbol{\imath}} - \frac{1}{\sqrt{5}}\hat{\boldsymbol{\jmath}}\right)$$

Replacing q_2 with a test charge q_o,

$$\vec{\mathbf{F}}_{q_1 \text{ on } q_o} = k\frac{q_1 q_o}{5a^2}\left(\frac{2}{\sqrt{5}}\hat{\boldsymbol{\imath}} - \frac{1}{\sqrt{5}}\hat{\boldsymbol{\jmath}}\right)$$

$$\vec{\mathbf{E}}_{\text{at } (a,0) \text{ due to } q_1} = \frac{\vec{\mathbf{F}}}{q_o} = \frac{kq_1}{5a^2}\left(\frac{2}{\sqrt{5}}\hat{\boldsymbol{\imath}} - \frac{1}{\sqrt{5}}\hat{\boldsymbol{\jmath}}\right)$$

Check: Based on a diagram, we expect a positive charge at $$(x,y)=(a,0) *tohaveaforceonitthatistotherightanddown, whichisconsistentwiththesignsofthecomponentsof* \bfvec{E}$.

$E = k|q_1|/5a^2$

4 Problem II

To find the electric field at a point in space in Example I and Problem I, we first computed the force on a charge at that point and then divided the result by the charge.

An alternative approach is to start with the equation for the magnitude of the elctric field due to a charge q at a point that is a distance r from the charge:

$E_{\text{due to }q} = k|q|/r^2$

Charge q_1 is at $(x, y) = (a, a)$. Find the magnitude of the electric field, E, at $(x, y) = (0, 0)$ using the above formula and then find $\vec{\mathbf{E}}$ in the form $\vec{\mathbf{E}} = E_x\hat{\boldsymbol{\imath}} + E_y\hat{\boldsymbol{\jmath}}$.

Answer:

$E = k|q_1|/2a^2$

$\vec{\mathbf{E}}_{\text{at (0,0) due to }q_1} = -\dfrac{kq_1}{2a^2}\left(\dfrac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} + \dfrac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}\right)$

Check: Based on a diagram, we expect a positive charge at the origin to have a force on it that is to the left and down, which is consistent with the signs of the components of $\vec{\mathbf{E}}$.