

Electric Force

This activity covers topics in [Section 21.3 of Young and Freedman 2015, 14th Edition](#). If you need to review vectors, see [sections 1.6-1.8 of Young and Freedman 2015, 14th Edition](#) and [Vectors at Khan Academy](#).

1 Coulomb's Law

Magnitude

$$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = k \frac{|q_1 q_2|}{r^2}$$

where r is the distance between q_1 and q_2 . To simplify notation, we are using k in place of $1/4\pi\epsilon_0$. Note that by definition, the magnitude of a vector is positive, which is the reason for the use of the absolute value.

Direction: Along line that connects q_1 and q_2 . Direction depends on signs of q_1 and q_2 . (Likes repel, opposites attract.).

2 Example

Charge q_1 is at $(x, y) = (-a, -a)$ and charge q_2 is at (a, a) . Both charges have a charge of q .

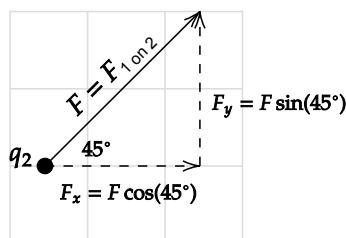
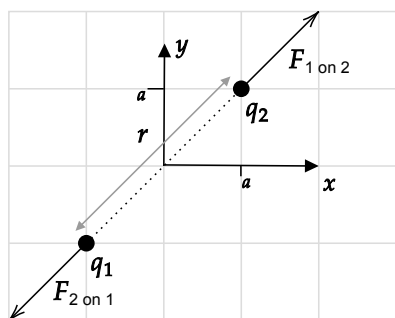
1. Find the magnitude and direction of the force of q_1 on q_2 .
2. Write the force of q_1 on q_2 in the form $\vec{F} = F_x \hat{i} + F_y \hat{j}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

Solution

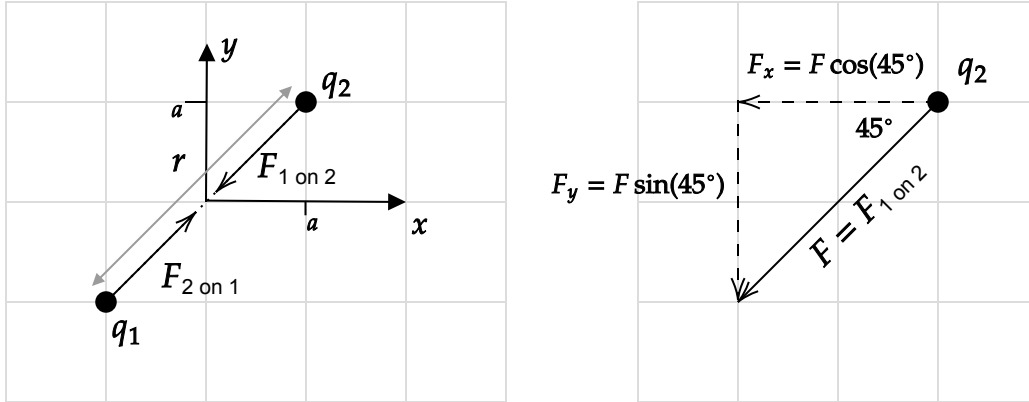
1. The distance between the charges is $r = \sqrt{(2a)^2 + (2a)^2} = \sqrt{8a^2}$, so

$$F_{1 \text{ on } 2} = k \frac{|q_1 q_2|}{r^2} = \frac{k|qq|}{(\sqrt{8a^2})^2} = \frac{kq^2}{8a^2}$$

The charges will repel each other, so the direction of forces of one on the other will be as shown in the left part of the following diagram.



- Let $F = F_{1 \text{ on } 2}$ from part 1. to simplify notation. The right part of the above diagram shows the calculation of the components F_x and F_y , from which it follows that $\vec{\mathbf{F}} = F \cos 45^\circ \hat{\mathbf{i}} + F \sin 45^\circ \hat{\mathbf{j}}$.
- The magnitude will not change (it is by definition a positive number). The force vectors will reverse direction as shown on the left in the following diagram. The diagram on the right shows the calculation of $\vec{\mathbf{F}}_{1 \text{ on } 2}$, from which it follows that $\vec{\mathbf{F}}_{1 \text{ on } 2} = -F \cos 45^\circ \hat{\mathbf{i}} - F \sin 45^\circ \hat{\mathbf{j}}$. Note that reversing the direction of a vector is the same as multiplying each of its components by -1 .



3 Problem I

Charge q_1 is at $(x, y) = (-a, a)$ and charge q_2 is at $(a, -a)$. Both charges have a charge of q . Draw this charge configuration and then using the steps in the previous example,

- Find the magnitude and direction of the force of q_1 on q_2 .
- Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x \hat{\mathbf{i}} + F_y \hat{\mathbf{j}}$.
- If the charges have opposite signs, how will your answers to 1. and 2. change?



Solution

- $F_{1 \text{ on } 2} = k \frac{|q_1 q_2|}{r^2} = \frac{k|qq|}{(\sqrt{8a^2})^2} = \frac{kq^2}{8a^2}$
- $\vec{\mathbf{F}}_{1 \text{ on } 2} = F_{1 \text{ on } 2} (\cos 45^\circ \hat{\mathbf{i}} - \sin 45^\circ \hat{\mathbf{j}})$

3. 1.: No change; 2. $\vec{F}_{1 \text{ on } 2} = F_{1 \text{ on } 2}(-\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$

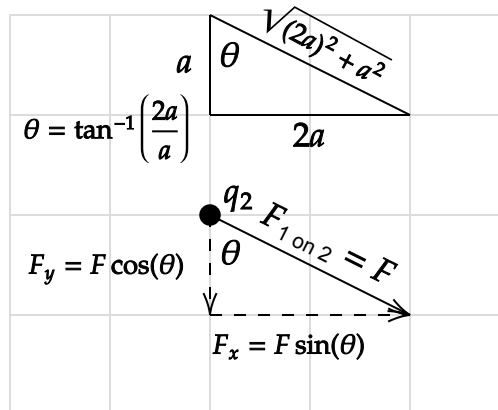
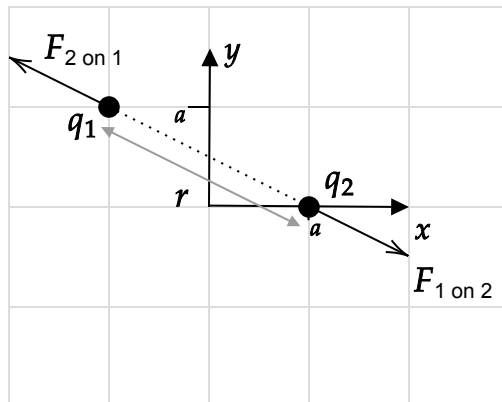
4 Problem II

Charge q_1 is at $(x, y) = (-a, a)$ and charge q_2 is at $(a, 0)$. Charge q_1 has a charge of $+q$. Charge q_2 has a charge of $+q$, where q is a positive number. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of q_1 on q_2 .
2. Write the force of q_1 on q_2 in the form $\vec{F} = F_x \hat{i} + F_y \hat{j}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?



Solution



$$1. F_{1 \text{ on } 2} = k \frac{|q_1 q_2|}{r^2} = \frac{k|qq|}{(\sqrt{(2a)^2 + a^2})^2} = \frac{kq^2}{5a^2}$$

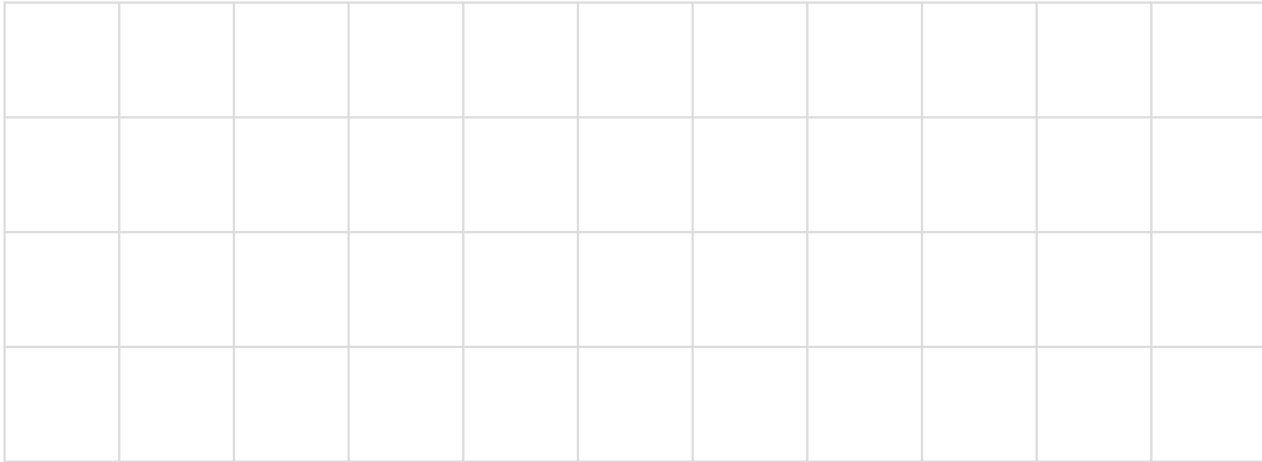
$$2. \vec{F}_{1 \text{ on } 2} = F_{1 \text{ on } 2}(\sin \theta^\circ \hat{i} - \cos \theta^\circ \hat{j}), \text{ where } \theta = \tan^{-1}(2) = 63.4^\circ.$$

$$3. 1.: \text{No change; } 2. \vec{F}_{1 \text{ on } 2} = F_{1 \text{ on } 2}(-\sin \theta^\circ \hat{i} + \cos \theta^\circ \hat{j}); \theta = 63.4^\circ$$

5 Problem III

Charge q_1 is at $(x, y) = (-a, 0)$ and charge q_2 is at $(0, 3a)$. Charge q_1 has a charge of $-q$. Charge q_2 has a charge of $+q$, where q is a positive number. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of q_1 on q_2 .
2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x \hat{\mathbf{i}} + F_y \hat{\mathbf{j}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?



Solution

1. $r = \sqrt{a^2 + (3a)^2}$, $F = k|q(-q)|/r^2 = kq^2/10a^2$
2. $\vec{\mathbf{F}} = -F \sin \theta \hat{\mathbf{i}} - F \cos \theta \hat{\mathbf{j}}$, where $\theta = \tan^{-1}(1/3) = 18.4^\circ$
3. 1.: No change; 2.: $\vec{\mathbf{F}} = +F \sin \theta \hat{\mathbf{i}} + F \cos \theta \hat{\mathbf{j}}$