1 Introduction

1.1 Definitions

The electric current in a wire is defined as

$$I = \frac{dQ}{dt}$$

where dQ is the total amount of charge that passes through a cross–section of the wire in a differential amount of time, dt.

If q is the charge (in Coulombs) of each flowing charge, n is their number per volume, v_d their average speed along the wire (called the "drift velocity"), then

$$I = n|q|v_d A$$

where A is the cross–sectional area of the wire.

We also define the current density, $J = n|q|v_d$, which is the current per cross–sectional area:

$$J = \frac{I}{A}$$

1.2 Ohm's Law

If an electric field exists in a wire (by, for example, connecting its ends to a battery), the charges will accelerate until they collide with another particle and decelerate (collisions resist the flow). The net result will be a flow of charges with a drift velocity. Experimentally, it has been shown that in many materials, the ratio of the electric field to current density is

$$\rho = \frac{E}{J}$$

where the value of the constant ρ depends on the material.

Ohm's law is

$$I = V/R$$

which means a voltage V applied to a wire will result in a current I, and this current depends on R.

For a wire of length L with a constant cross–sectional area A, resistance is proportional to L and inversely proportional A:

$$R = \frac{\rho L}{A}$$

2 Problem I – Definitions and Ohm's Law

A 9–volt battery is connected to a wire of length 10 meters with a circular cross–section and radius of 0.01 meters. The wire has a resistivity of $10^{-8}\Omega \cdot \mathrm{m}$. The density of charge carriers is $10^{28}/\mathrm{m}^3$. Assume Ohm's law applies.

1. What is the resistance (with units) of the wire?

   Answer:

2. What is the current (with units) in the wire?

   Answer:

3. How much charge (with units) flows past a cross–section of the wire per second?

   Answer:

4. What is the current density (with units) that flows through the wire?

   Answer:

5. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6 \cdot 10^{-19}$ C.)

   Answer:

6. Based on the description of how charged particles flow in a wire, explain why the resistance of a cylindrical wire is proportional to its length and inversely proportional to the square of its radius.

   Answer:

3 Problem II – Definitions and Ohm's Law

A battery is connected to a wire of length 20 meters with a circular cross–section and a radius of 0.01 meters. The wire has a resistivity of $10^{-7}\Omega \cdot \mathrm{m}$. The density of charge carriers is $10^{27}/\mathrm{m}^3$. The current in the wire was measured and found to be 1 Ampere. Assume Ohm's law applies.

1. What is the resistance (with units) of the wire?

   Answer:

2. How much charge (with units) flows past a cross–section of the wire per second?

   Answer:

3. What is the current density (with units) that flows through the wire?

   Answer:

4. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6 \cdot 10^{-19}$ C.)

   Answer:

4 Problem III – Current Through a Cylindrical Shell

If a cylindrical wire with an inner radius a and outer radius b carries a current I, what is J?

Answer:
$J = \dfrac{I}{\pi(b^2 - a^2)}$

5 Problem IV – $I = nqv_dA$ derivation

Derive the relationship $I = n|q|v_dA$. Provide a diagram.

Answer: See textbook.