Magnetic Field of a Moving Charge

1 Introduction

In previous activities, you computed the force on moving charges in a region of space where there is a magnetic field. No mention was made of how the magnetic field was created.

In this activity, you compute the magnetic field created by moving charges.

The magnetic field due to a point charge q moving with velocity $\vec{\mathbf{v}}$ (when $|\vec{\mathbf{v}}|$ is small compared to the speed of light) is

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

where $\hat{\mathbf{r}}$ is the unit vector that points from the position of q to the point in space where we want to know $\vec{\mathbf{B}}$, and r is the distance between q and that point.

To find $\hat{\mathbf{r}}$ (see also the $\hat{\mathbf{r}}$ Unit Vector activity),

1. draw a vector, $\vec{\mathbf{r}}$, from q to the point in space where you want to know $\vec{\mathbf{B}}$;
2. Write $\vec{\mathbf{r}}$ in the form $\vec{\mathbf{r}} = r_x\hat{\boldsymbol{\imath}} + r_y\hat{\boldsymbol{\jmath}}$; then
3. $\hat{\mathbf{r}} = \vec{\mathbf{r}}/r$, where $r = \sqrt{r_x^2 + r_y^2}$.

In this activity, the examples and solutions are given using the above approach for computing $\vec{\mathbf{B}}$. An alternative is to use the fact that $\vec{\mathbf{v}} \times \hat{\mathbf{r}} = |\vec{\mathbf{v}}| \sin\phi = v\sin\phi$, where ϕ is the angle between $\vec{\mathbf{v}}$ and $\hat{\mathbf{r}}$ and $0 \le \phi \le 180^\circ$. With this, the magnitude of the magnetic field is

$$B = \frac{\mu_o}{4\pi}\frac{|q|v\sin\phi}{r^2}$$

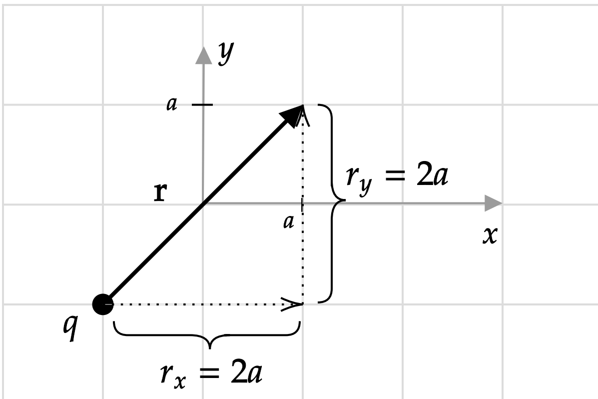
and the right–hand rule can be used to determine the direction of $\vec{\mathbf{B}}$. See the Cross Products activity for a discussion of when and how to compute the cross-product using this method.

2 Example

If q is at $(x,y)=(-a,-a)$ and has a velocity of $\vec{\mathbf{v}}=v_o\hat{\boldsymbol{\imath}}$, find $\vec{\mathbf{B}}$ at $(x,y)=(a,a)$.

Solution

To find $\hat{\mathbf{r}}$, we draw a vector from q to the point where we want to compute $\vec{\mathbf{B}}$.



Based on the diagram, $\vec{\mathbf{r}}=2a\hat{\boldsymbol{\imath}}+2a\hat{\boldsymbol{\jmath}}$ and $r=\sqrt{(2a)^2+(2a)^2}=2\sqrt{2}a$, so

$$\hat{\mathbf{r}}=\frac{\vec{\mathbf{r}}}{r}=\left[\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}}+\frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}\right]$$

The cross-product is

$$\vec{\mathbf{v}}\times\hat{\mathbf{r}}=v_o\hat{\boldsymbol{\imath}}\times\left[\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}}+\frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}\right]=\frac{v_o}{\sqrt{2}}(\hat{\boldsymbol{\imath}}\times\hat{\boldsymbol{\jmath}})=\frac{v_o}{\sqrt{2}}\hat{\boldsymbol{k}}$$

Substitution into

$$\vec{\mathbf{B}}=\frac{\mu_o}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2}$$

gives

$$\vec{\mathbf{B}}(a,a)=\frac{\mu_o}{4\pi}\frac{q\frac{v_o}{\sqrt{2}}\hat{\boldsymbol{k}}}{(2\sqrt{2}a)^2}=\frac{\mu_o}{4\pi}\frac{qv_o}{(8\sqrt{2})a^2}\hat{\boldsymbol{k}}$$

Check: Use the right–hand rule for cross products on $\vec{\mathbf{v}}\times\hat{\mathbf{r}}$ to verify that the result is out of the page. (Why do we know that the $\hat{\boldsymbol{k}}$ direction is out of the page?)

3 Problem I

If q is at $(x, y) = (a, a)$ and has a velocity of $\vec{\mathbf{v}} = v_o \hat{\boldsymbol{\imath}}$, find $\vec{\mathbf{B}}$ at $(x, y) = (-a, -a)$.

4 Problem II

If q is at $(x, y) = (a, 0)$ and has a velocity of $\vec{\mathbf{v}} = v_o \hat{\boldsymbol{\jmath}}$, find $\vec{\mathbf{B}}$ vector at $(x, y) = (a, a)$.

5 Problem III

If q is at $(x, y) = (a, 2a)$ and has a velocity of $\vec{\mathbf{v}} = v_o\hat{\boldsymbol{\jmath}}$, find $\vec{\mathbf{B}}$ at $(x, y) = (-a, -a)$.

6 Problem IV

If q is at $(x, y) = (x_o, y_o)$ and has a velocity of $\vec{\mathbf{v}} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}}$,

$$\vec{\mathbf{B}}(x, y) = \frac{\mu_o}{4\pi}\frac{1}{r^3}\left[v_x(y - y_o) - v_y(x - x_o)\right]\hat{\boldsymbol{k}}$$

where

$$r = \sqrt{(x - x_o)^2 + (y - y_o)^2}$$

1. Explain why $\vec{\mathbf{B}}$ only has a $\hat{\boldsymbol{k}}$ component.
2. Use this formula to find $\vec{\mathbf{B}}$ for the example problem in section 2.
3. Derive this formula.