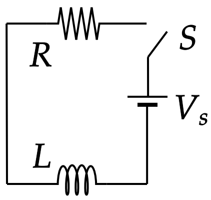
LR Circuits

1 Introduction



The above figure shows a LR series circuit consisting of an inductor of inductance L connected in series with a resistor of resistance R. The switch, S, is closed at a time $t = 0$ and then remains closed.

Using Kirchhoff's voltage law around the loop, we have

$$V_s - I(t) \cdot R - L\frac{dI(t)}{dt} = 0$$

The term $L\dfrac{dI}{dt}$ is called the induced emf.

The above differential equation can be solved for $I(t)$, the current at any time given the initial value of I, which is zero. The result is

$$I(t) = \frac{V_s}{R}\left(1 - e^{-t/(L/R)}\right)$$

After a long time, the current approaches a constant value of $I = V_s/R$ because the exponential term approaches zero and so there is effectively no time dependence in $I(t)$. How quickly the exponential term approaches zero depends on a quantity called the LR time constant defined by

$$\tau = L/R$$

which has units of seconds when L is in Henrys and R is in Ohms. With this, we can write

$$I(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right)$$

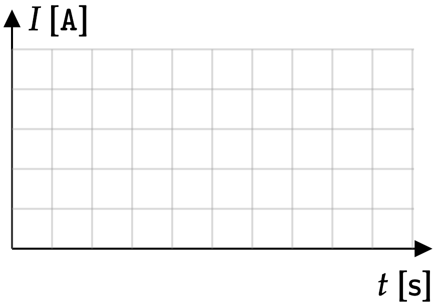
In this equation, at $t = 0$, the current is

$$I(0) = (V_s/R)\left(1 - e^{-0/\tau}\right) = (V_s/R)\,(1 - 1) = 0$$

As a result, we state that initially the inductor behaves like an open circuit because current does not flow though it.

For large t/τ, the exponential term becomes much smaller than one and so the current becomes a constant value of V_s/R. If we replace the inductor with a wire, this is the same current that we would find. As a result, we state that after a long time, the inductor behaves like a resistanceless wire.

2 Problem I



In this problem, you will consider the equation

$$I(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right)$$

that was described in the introduction.

1. If $V_s/R = 10$ A, plot dots for the values of I at $t = 0, 2, 4, 6, 12$ s with $L/R = \tau = 2$ s
2. Based on the equation, at $t = 0$ does an inductor behave like an open circuit or a wire?
3. Based on the equation, at $t \gg \tau$ does an inductor behave like an open circuit or a wire?
4. If L doubles but R remains constant

   a. does the time constant τ increase, decrease, or remain the same?;

   b. how will the points that you drew for part 1. change? (Will they move up, down, or remain the same?)

   c. Does your answer to b. make sense physically? That is, an inductor tends to impede changes in current and so is your answer to b. consistent with this?

5. The voltage across the inductor is LdI/dt. Compute dI/dt and sketch its curve on the graph above. Is this equation consistent with the statement that for large t/τ, the voltage across the inductor is zero?

3 Problem II

An inductor with an inductance of 40 mH and a resistor with a resistance of 2 Ω are connected together to form a LR series circuit. If they are connected to a 20 V DC voltage source,

1. What will be the final steady state value of the current (the current after a very long time)?

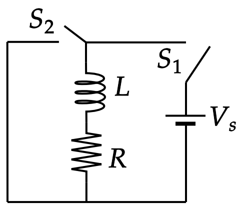
2. What is the time constant of the RL series circuit?

3. How long does it takes for the current to reach 63% of its maximum value?

4. What will be the value of the induced emf after 10 ms?

5. What will be the value of the circuit current one time constant (that is, at $t = \tau$) after the switch is closed?

4 Problem III



In the circuit above, an inductor with $L = 10$ mH and a resistor with $R = 1\ \Omega$ is connected as shown. The battery has an emf of 10 V. At $t = 0$, the switch S_1 is closed.

1. What is the current through the resistor at $t = 0$?
2. What is the current through the inductor at $t = 0$?
3. After a long time, what will the current be through the resistor and inductor?
4. After a long time, switch S_1 is opened and S_2 is closed simultaneously. Write Kirchhoff's voltage law around the new closed loop.
5. Show that the equation $I(t) = (10\ \mathrm{A})e^{-t/\tau}$ satisfies the equation in your answer to the previous question.
6. Plot $I(t)$ from $t = 0$ to $t = 0.01$ s. Assume that the switch S_1 was opened and switch S_2 was closed at $t = 0.005$ s.