1 Introduction

1.1 Definitions

The electric current in a wire is defined as

$$I = \frac{dQ}{dt}$$

where dQ is the total amount of charge that passes through a cross–section of the wire in a differential amount of time, dt.

If q is the charge (in Coulombs) of each flowing charge, n is their number per volume, v_d their average speed along the wire (called the "drift velocity"), then

$$I = n|q|v_d A$$

where A is the cross–sectional area of the wire.

We also define the average current density, $J = n|q|v_d$, which is the current per cross–sectional area:

$$J = \frac{I}{A}$$

1.2 Ohm's Law

If an electric field exists in a wire (by, for example, connecting its ends to a battery), the charges will accelerate until they collide with another particle and decelerate (collisions resist the flow). The net result will be a flow of charges with a drift velocity. Experimentally, it has been shown that in many materials, the ratio of the electric field to current density is

$$\rho = \frac{E}{J}$$

where the value of the constant ρ, called resistivity, depends on the material.

Ohm's law is

$$I = V/R$$

which means a voltage V applied to a wire will result in a current I, and this current depends on R.

For a wire of length L with a constant cross–sectional area A and made of a material with resistivity ρ,

$$R = \frac{\rho L}{A}$$

2 Problem I – Definitions and Ohm's Law

A 9–volt battery is connected to a wire of length 10 meters with a circular cross–section and radius of 0.01 meters. The wire has a resistivity of $10^{-8}\,\Omega \cdot \mathrm{m}$. The density of charge carriers is $10^{28}/\mathrm{m}^3$. Assume Ohm's law applies.

1. What is the resistance (with units) of the wire?

2. What is the current (with units) in the wire?

3. How much charge (with units) flows past a cross–section of the wire per second?

4. What is the current density (with units) that flows through the wire?

5. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6 \cdot 10^{-19}$ C.)

6. Based on the description of how charged particles flow in a wire, explain why the resistance of a cylindrical wire is proportional to its length and inversely proportional to the square of its radius.

3 Problem II – Definitions and Ohm's Law

A battery is connected to a wire of length 20 meters with a circular cross–section and a radius of 0.01 meters. The wire has a resistivity of $10^{-7}\,\Omega\cdot\mathrm{m}$. The density of charge carriers is $10^{27}/\mathrm{m}^3$. The current in the wire was measured and found to be 1 Ampere. Assume Ohm's law applies.

1. What is the resistance (with units) of the wire?

2. How much charge (with units) flows past a cross–section of the wire per second?

3. What is the current density (with units) that flows through the wire?

4. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6\cdot 10^{-19}$ C.)

4 Problem III – Current Through a Cylindrical Shell

If a cylindrical wire with an inner radius a and outer radius b carries a current I, what is J?

5 Problem IV – $I = n|q|v_d A$ derivation

Derive the relationship $I = n|q|v_d A$. Provide a diagram.