1 Introduction

1.1 Definitions

Current is defined as the rate of change of charge, Q, per time. If you were able to count the number of charges passing a cross–section of a wire over 1 s, the current would be Q/t. If the amount of time was small, say Δt, and your labeled your count as ΔQ, then $I = \Delta Q/\Delta t$. In the limit that Δt approaches zero, we have the definition

$$I = \frac{dQ}{dt}$$

If n is the number of charges with charge q per volume and v_d is the average velocity of the charges (called the “drift velocity”), then

$$I = nqv_d A$$

where A is the cross–sectional area of the wire.

We also define a quantity called the current density, J, which is the current per unit cross–sectional area, A:

$$J = \frac{I}{A}$$

1.2 Ohm’s Law

If an electric field exists in a wire (by, for example, connected its ends to a battery), the charges will accelerate until they collide with another particle and decelerate (resist the flow). The net result will be a flow of charges with a drift velocity. Experimentally, it has been shown that in many materials, the ratio of the electric field to current density is

$$\rho = \frac{E}{J}$$

where the value of the constant ρ depends on the material.

Ohm’s law is

$$I = V/R$$

which means a voltage V applied to a wire will result in a current I, and this current depends on R.

The resistance is larger the longer wire and smaller for a larger cross–section of wire. The result is

$$R = \frac{\rho L}{A}$$

2 Problem I – Definitions and Ohm's Law

A 9 Volt battery is connected to a wire of length 10 meters with circular cross section of radius of 0.01 meters. The wire has a resistivity of $10^{-8}\,\Omega \cdot \mathrm{m}$. The density of charge carriers is $10^{28}/\mathrm{m}^3$. Assume Ohm's law applies.

1. What is the resistance (with units) of the wire?

   Answer:

2. How much charge (with units) flows past a cross section of the wire per second?

   Answer:

3. What is the current (with units) in the wire?

   Answer:

4. What is the current density (with units) that flows through the wire?

   Answer:

5. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6 \cdot 10^{-19}$ C.)

   Answer:

6. Based on the description of how charged particles flow in a wire, explain why the resitance of a cylindrical wire is proportional to its length and inversely proportinal to the square of its radius.

   Answer:

3 Problem II – Definitions and Ohm's Law

A battery is connected to a wire of length 20 meters with circular cross section and a radius of 0.01 meters. The wire has a resistivity of $10^{-7}\,\Omega \cdot \mathrm{m}$. The density of charge carriers is $10^{27}/\mathrm{m}^3$. The current in the wire was measured and found to be 1 Ampere. Assume Ohm's law applies.

1. What is the resistance (with units) of the wire?

   Answer:

2. How much charge (with units) flows past a cross section of the wire per second?

   Answer:

3. What is the current (with units) in the wire?

   Answer:

4. What is the current density (with units) that flows through the wire?

   Answer:

5. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6\cdot 10^{-19}$ C.)

   Answer:

4 Problem III – $I = nqv_d A$ derivation

Derive the relationship $I = nqv_d A$. Provide a diagram.

Answer: See textbook.