Magnetic Flux

1 Introduction

The definition of magnetic flux is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\mathbf{A}$$

When the magnitude and direction of $\vec{\mathbf{B}}$ are the same at all points on the surface, the integral simplifies to

$$\Phi_B = \vec{\mathbf{B}} \cdot \mathbf{A}$$

or, equivalently,

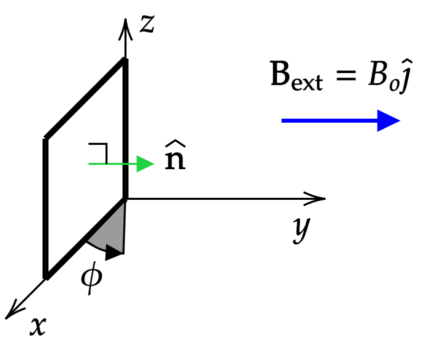
$$\Phi_B = BA\cos\phi$$

where ϕ is the angle between the $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$ vectors.

Note that previously, electric flux was covered. The same techniques that apply to computing electric flux apply to magnetic flux. See the Electric Flux activity for additional discussion.

2 Computing Φ_B

In the following figure, a square loop of area A that can be rotated about the z axis is shown. Assume that the normal direction of the loop is as shown in the diagram. A uniform external magnetic field points in the $\hat{\jmath}$ direction.

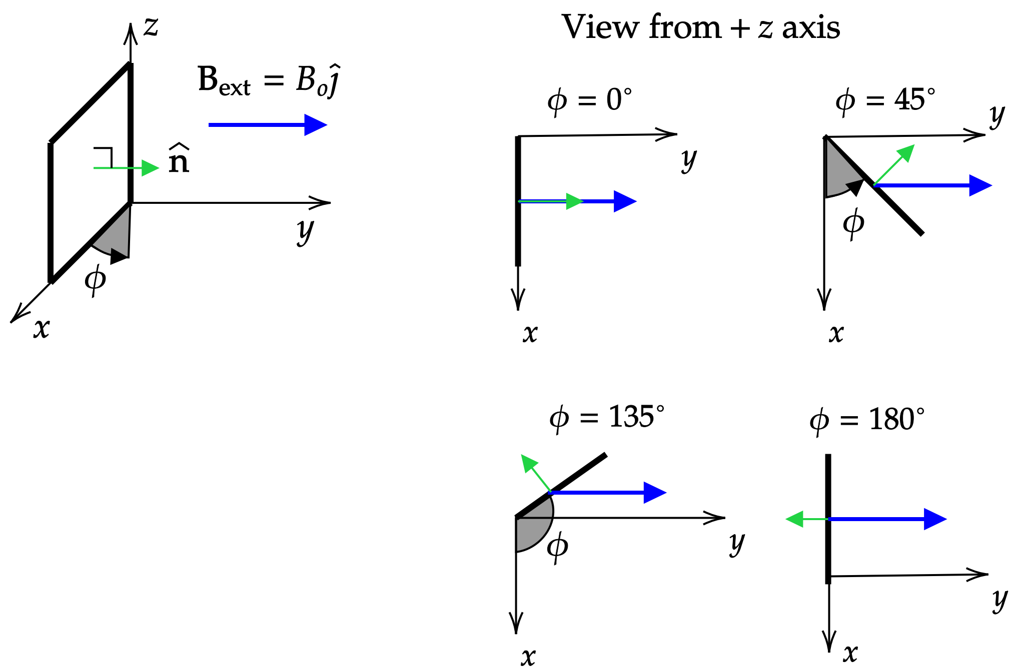


1. At what angles in the range of $\phi = [0, 360°]$ is the magnetic flux zero?

   Answer: The magnetic flux will be zero when the normal vector is perpendicular to $\vec{\mathbf{B}}_{\text{ext}}$. This corresponds to $\phi = 90°$ and $\phi = 270°$

2. Draw the loop, $\hat{\mathbf{n}}$, and $\vec{\mathbf{B}}_{\text{ext}}$ as they would appear when viewed from above (on the positive z–axis) when $\phi = 0°$, $\phi = 45°$, $\phi = 135°$, and $\phi = 180°$.

   Answer:



3. When $\phi = 45^\circ$, is the magnetic flux positive or negative? What is its value in terms of a fraction of $B_o A$?

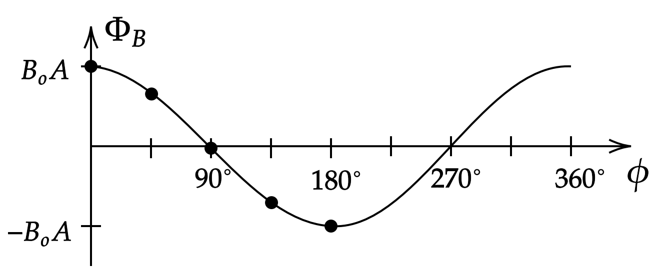
   Answer: From the above diagram, the angle between $\hat{\mathbf{n}}$ and $\vec{\mathbf{B}}_{\text{ext}}$ is ϕ, so the flux will be positive. Its value will be $B_o A \cos 45^\circ = B_o A/\sqrt{2}$.

4. When $\phi = 135^\circ$, is the magnetic flux positive or negative? What is its value in terms of a fraction of $B_o A$?

   Answer: From the above diagram, the angle between $\hat{\mathbf{n}}$ and $\vec{\mathbf{B}}_{\text{ext}}$ is ϕ, so the flux will be negative. Its value will be $B_o A \cos 135^\circ = -B_o A \cos 45^\circ = -B_o A/\sqrt{2}$.

5. Sketch a plot of the magnetic flux, Φ_B, as a function of ϕ.

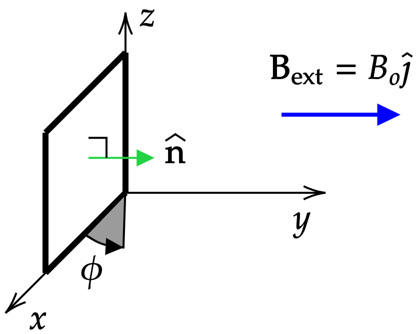
   From the diagram above, the angle of rotation of the plane is the same as the angle between the $\hat{\mathbf{n}}$ and $\vec{\mathbf{B}}_{\text{ext}}$, so the general formula is $\Phi = B_o A \cos\phi$, which is sketched below along with dots that correspond to answers to the above problems.



3 Computing $d\Phi_B/dt$

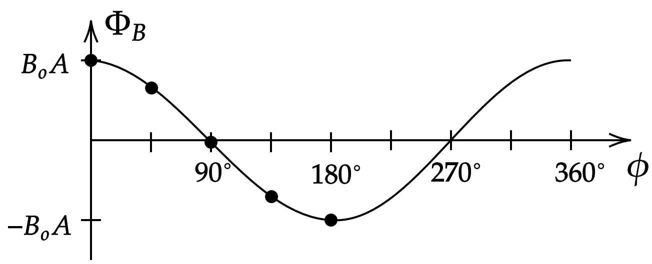
The time rate of change of magnetic flux through a closed loop is a quantity that will be used when Faraday's law is covered.

Suppose the loop in the previous problem rotates at a constant rate such that $\phi = \omega t$, where ω is a contant.



1. At what angles in the range $\phi = [0^\circ, 360^\circ]$ is $d\Phi_B/dt = 0$?

   The answer to part 5. of the previous problem is $\Phi_B = B_o A \cos\phi$, which is sketched below. The slope of Φ_B is zero at $\phi = 0, 180^\circ$, and 360°.

2. For what angle range is $d\Phi_B/dt$ increasing?

   From the plot of Φ_B, the slope is positive for $180^\circ < \phi < 360^\circ$

3. For what angle range is $d\Phi_B/dt$ decreasing?

   From the plot of Φ_B, the slope is negative for $0^\circ < \phi < 180^\circ$

4. What is the formula for $d\Phi_B(t)/dt$ in terms of B_o, A, t, and ω?

   $\Phi_B = B_o A \cos\phi$, so $d\Phi_B/dt = -B_o A\omega \sin\omega t$