Kirchoff's Circuit Laws

1 Introduction

To find the current through each resistor in a circuit with only batteries and resistors, Kirchhoff's Current Law and Kirchhoff's Voltage Rule can be used.

1. Kirchhoff's Voltage Law (KVL): The sum of all voltage changes around a closed loop must equal zero.
2. Kirchhoff's Current Law (KCL): The sum of all currents entering and exiting a junction must equal zero.

General procedure

1. Assume directions of current.
2. Write equations for KCL for nodes.
3. Write equations for KVL for loops using the assumed directions of current and the chosen loop direction.
4. Solve for currents.

Sign conventions

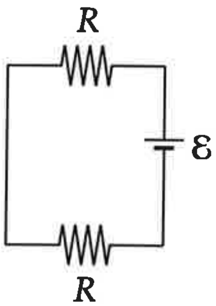
If you get a negative number for a current, your assumed direction was wrong.

When you write KVL, you must choose a direction that you move around the loop. If you "step" across a resistor R in the direction of an assumed current i, the voltage change is $-iR$. If you "step" across in the direction opposite of i, the voltage change is $+iR$.

If you step across a battery with emf $\mathcal{E}$ from the $-$ to the $+$, the voltage change is $+\mathcal{E}$. If you step across a battery with emf $\mathcal{E}$ from the $+$ to the $-$, the voltage change is $-\mathcal{E}$. *The direction of the assumed current does not matter.*

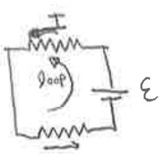
2 Single Loop

In a single loop circuit, only KVL is needed to find the current.



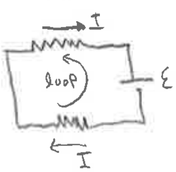
1. Assume the direction of current I in the above circuit is counterclockwise. Write the equation for KVL stepping around the loop in the counterclockwise direction; then solve for I in terms of $\mathcal{E}$ and R.
2. Assume the direction of current I in the following circuit is clockwise. Write the equation for KVL stepping around the loop in the counterclockwise direction; then solve for I in terms of $\mathcal{E}$ and R.
3. Which value for I found above is correct?
4. If you removed the bottom resistor and replaced the top resistor with a resistor with resistance $2R$, would I change?
5. If you repeated 1. but stepped in a clockwise direction, would your answer change?

1.



$+\mathcal{E} - IR - IR = 0 \Rightarrow I = \mathcal{E}/2R$

2.

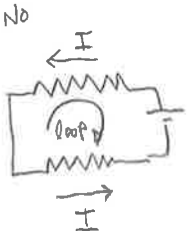


$+\mathcal{E} + IR + IR = 0 \Rightarrow I = -\mathcal{E}/2R$

3. Both correct. Negative means current flows in dir opposite to that assumed

4. NO

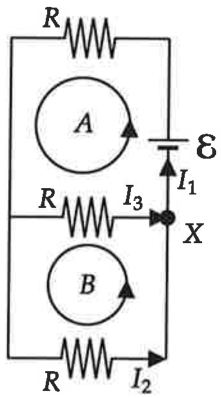
5. NO



$-\mathcal{E} + IR + IR = 0 \Rightarrow I = \mathcal{E}/2R$

3 Multiple Loops I

Assume the direction of currents I_1, I_2, and I_3 in the following circuit are as shown.



1. Write the equation for KVL for loop A. $+\mathcal{E} - I_1 R - I_3 R = 0$
2. Write the equation for KVL for loop B. $-I_2 R + I_3 R = 0$
3. Write the equation for KCL for node X. $I_3 + I_2 - I_1 = 0$
4. Use the three equations found above to solve for the three unknowns, I_1, I_2, and I_3 in terms of $\mathcal{E}$ and R.
5. Check your answers by pluggin your values for I_1, I_2, and I_3 into the equations that you wrote for 1.-3.

4. $-I_2 R + I_3 R = 0 \Rightarrow I_2 = I_3$, Plug into 3. $I_3 + I_3 - I_1 = 0 \Rightarrow I_3 = I_1/2$

Plug into 1. $+\mathcal{E} - I_1 R - \frac{I_1}{2} R = 0 \Rightarrow I_1 = \frac{2}{3}\mathcal{E}/R$

$$\boxed{I_1 = \frac{2}{3}\frac{\mathcal{E}}{R},\ I_2 = \frac{1}{3}\frac{\mathcal{E}}{R},\ I_3 = \frac{1}{3}\frac{\mathcal{E}}{R}}$$

5. 1. $\mathcal{E} - \left(\frac{2}{3}\frac{\mathcal{E}}{R}\right)R - \left(\frac{1}{3}\frac{\mathcal{E}}{R}\right)R \overset{?}{=} 0$

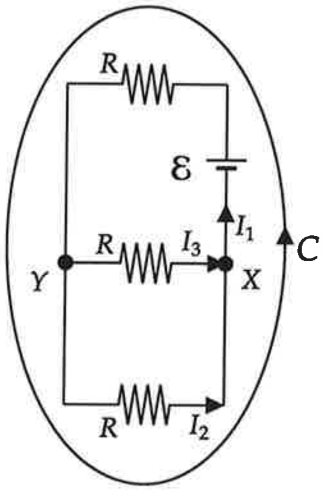
$\mathcal{E} - \frac{2}{3}\mathcal{E} - \frac{1}{3}\mathcal{E} = 0$ ✓

2. $-\left(\frac{1}{3}\frac{\mathcal{E}}{\cancel{R}}\right)\cancel{R} + \left(\frac{1}{3}\frac{\mathcal{E}}{\cancel{R}}\right)\cancel{R} = 0$ ✓

3. $\frac{1}{3}\frac{\mathcal{E}}{R} + \frac{1}{3}\frac{\mathcal{E}}{R} - \frac{2}{3}\frac{\mathcal{E}}{R} = 0$ ✓

4 Multiple Loops II

In the circuit for the previous problem, there are three possible loops. The third loop is loop C. indicated below.



1. Write the equation for KVL for loop C. $-I_2R + \mathcal{E} - I_1R = 0$
2. Use the equation for KVL for loop B. and the KCL equation for node X from the previous problem along with the KVL equation for loop C to find I_1, I_2, and I_3 in terms of $\mathcal{E}$ and R. (You should get the same answers.)

B: $-I_2R + I_3R = 0 \Rightarrow I_3 = I_2$

X: $I_3 + I_2 - I_1 = 0 = I_2 + I_2 - I_1 \Rightarrow I_2 = I_1/2$

plug these into 1.

$-\frac{I_1}{2}R + \mathcal{E} - I_1R = 0 \Rightarrow \boxed{I_1 = \frac{2}{3}\mathcal{E}/R}$

using this and $I_2 = \frac{I_1}{2} \Rightarrow I_2 = \frac{1}{3}\frac{\mathcal{E}}{R}$

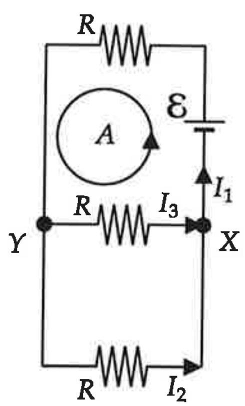
from B: $I_3 = I_2 = \frac{1}{3}\mathcal{E}/R$

Same Answers

5 Redundant Equations

When solving circuit problems with multiple loops, you will generally find that you can use KVL and KCL to write more equations than there are unknowns. If you encounter a situation where you wrote N equations based on KVL and KCL but cannot find N unknows, the reason is that two or more of the N equations that you wrote were not independent. To demonstrate this, for the circuit below,

1. Write the KCL equation for node X
2. Write the KCL equation for node Y
3. Write the KVL equation for loop A
4. Attempt to use the above three equations to solve for I_1, I_2, and I_3 in terms of $\mathcal{E}$ and R.



X: $I_3 + I_2 - I_1 = 0$

Y: $I_1 - I_2 - I_3 = 0$

multiply eqn Y by (-1)

Y: $-I_1 + I_2 + I_3 = 0$

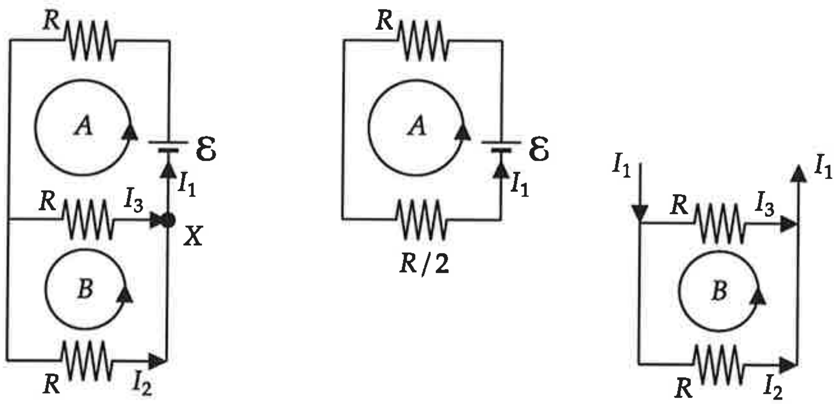
this is same as X.

3. $-I_3 R + \mathcal{E} - I_1 R = 0$

we have 2. independent equations + 3 unknowns.
can't solve.

6 Using Equivalent Resistances

The analysis of the circuit in the following figure on the left could have been simplified if we had combined the lower two resistors into an equivalent resistor. In this case, we can easily solve for I_1 using the equivalent circuit in the middle.



1. For the middle circuit, find I_1 in terms of $\mathcal{E}$ and R. $I_1 = \frac{2}{3}\,\mathcal{E}/R$
2. Use KVL for loop B and $I_1 = I_2 + I_3$ to find I_2 and I_3 in terms of $\mathcal{E}$ and R.

B. $-I_2 R + I_3 R \Rightarrow I_2 = I_3$

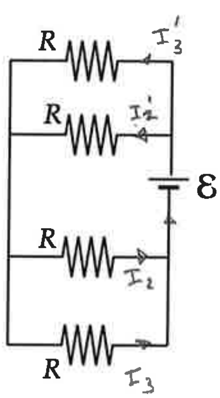
$\Rightarrow I_1 = 2I_2 \Rightarrow I_2 = I_1/2$

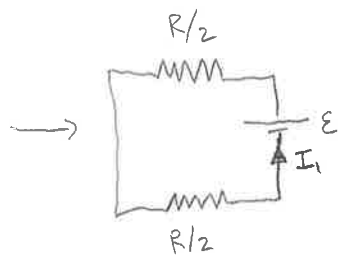
$I_3 = I_1/2$

An alternative to the last step is to recognize that the voltage drop across both of the bottom two resistors is $V = I_1 R/2$. Then the current through both resistors is $I_2 = I_3 = V/R$. Use this equation to check your answer to 2.

7 Using Equivalent Resistances

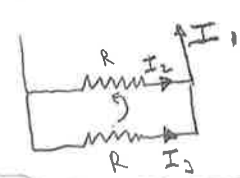
Find the current through all resistors in the following circuit in terms of $\mathcal{E}$ and R.





$+\mathcal{E} - I_1\frac{R}{2} - I_1\frac{R}{2} = 0 \Rightarrow \boxed{I_1 = \mathcal{E}/R}$

bottom



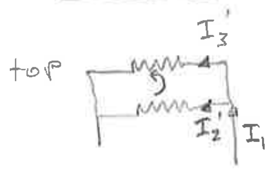
$I_1 = I_2 + I_3$

$+I_2 R - I_3 R = 0 \Rightarrow I_2 = I_3$

$\Rightarrow \boxed{I_2 = I_1/2 = \frac{1}{2}\,\mathcal{E}/R}$

$\boxed{I_3 = \frac{1}{2}\,\mathcal{E}/R}$

top



$I_1 = I_2' + I_3'$

$-I_3'R + I_2'R = 0$

$\Rightarrow I_2' = \mathcal{E}/2R$

$I_3' = \mathcal{E}/2R$