Electric Force

This activity covers topics in Section 21.3 of Young and Freedman 2015, 14th Edition. If you need to review vectors, see sections 1.6-1.8 of Young and Freedman 2015, 14th Edition and Vectors at Khan Academy.

1 Coulomb's Law

Magnitude

$$F_{1\text{ on }2} = F_{2\text{ on }1} = k\frac{|q_1 q_2|}{r^2}$$

where r is the distance between q_1 and q_2. To simplify notation, we are using k in place of $1/4\pi\epsilon_o$. Note that by definition, the magnitude of a vector is positive, which is the reason for the use of the absolute value.

Direction: Along line that connects q_1 and q_2. Direction depends on signs of q_1 and q_2. (Likes repel, opposites attract.).

2 Example

Charge q_1 is at $(x, y) = (-a, -a)$ and charge q_2 is at (a, a). Both charges have a charge of q.

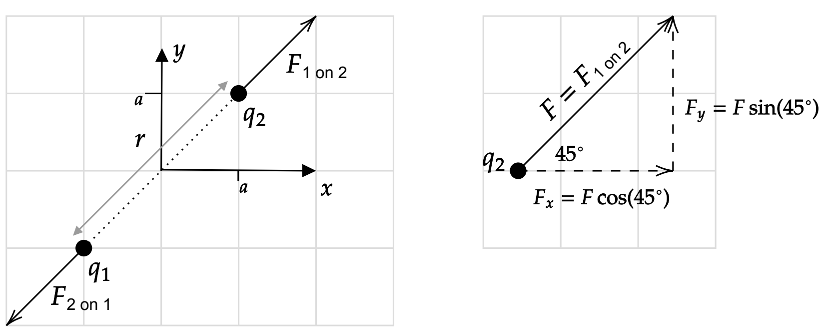
1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

Solution

1. The distance between the charges is $r = \sqrt{(2a)^2 + (2a)^2} = \sqrt{8a^2}$, so

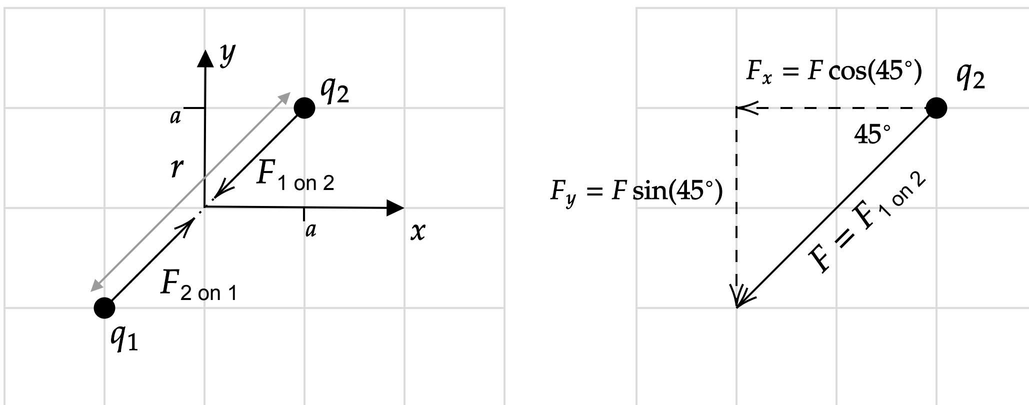
   $$F_{1\text{ on }2} = k\frac{|q_1 q_2|}{r^2} = \frac{k|qq|}{(\sqrt{8a^2})^2} = \frac{kq^2}{8a^2}$$

   The charges will repel each other, so the direction of forces of one on the other will be as shown in the left part of the following diagram.



2. Let $F = F_{1\text{ on }2}$ from part 1. to simplify notation. The right part of the above diagram shows the calculation of the components F_x and F_y, from which it follows that $\vec{\mathbf{F}} = F\cos 45°\,\hat{\boldsymbol{\imath}} + F\sin 45°\,\hat{\boldsymbol{\jmath}}$.

3. The magnitude will not change (it is by definition a positive number). The force vectors will reverse direction as shown on the left in the following diagram. The diagram on the right shows the calculation of $\vec{\mathbf{F}}_{1\text{ on }2}$, from which it follows that $\vec{\mathbf{F}}_{1\text{ on }2} = -F\cos 45°\,\hat{\boldsymbol{\imath}} - F\sin 45°\,\hat{\boldsymbol{\jmath}}$. Note that reversing the direction of a vector is the same as multiplying each of its components by -1.



3 Problem I

Charge q_1 is at $(x, y) = (-a, a)$ and charge q_2 is at $(a, -a)$. Both charges have a charge of q. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

4 Problem II

Charge q_1 is at $(x, y) = (-a, a)$ and charge q_2 is at $(a, 0)$. Charge q_1 has a charge of $+q$. Charge q_2 has a charge of $+q$, where q is a positive number. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x \hat{\boldsymbol{\imath}} + F_y \hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

5 Problem III

Charge q_1 is at $(x, y) = (-a, 0)$ and charge q_2 is at $(0, 3a)$. Charge q_1 has a charge of $-q$. Charge q_2 has a charge of $+q$, where q is a positive number. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x \hat{\boldsymbol{\imath}} + F_y \hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?