

Magnetic Flux

1 Introduction

The definition of magnetic flux is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\mathbf{A}$$

When the magnitude and direction of $\vec{\mathbf{B}}$ are the same at all points on the surface, the integral simplifies to

$$\Phi_B = \vec{\mathbf{B}} \cdot \mathbf{A}$$

or, equivalently,

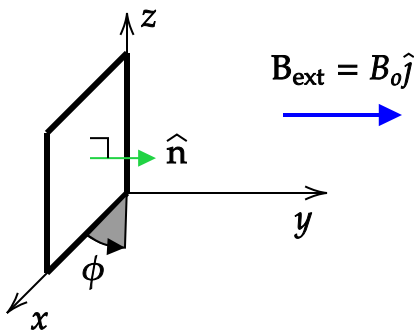
$$\Phi_B = BA \cos \phi$$

where ϕ is the angle between the $\vec{\mathbf{B}}$ and \mathbf{A} vectors.

Note that previously, electric flux was covered. The same techniques that apply to computing electric flux apply to magnetic flux. See the Electric Flux activity for additional discussion.

2 Computing Φ_B

In the following figure, a square loop of area A that can be rotated about the z axis is shown. Assume that the normal direction of the loop is as shown in the diagram. A uniform external magnetic field points in the $\hat{\mathbf{j}}$ direction.

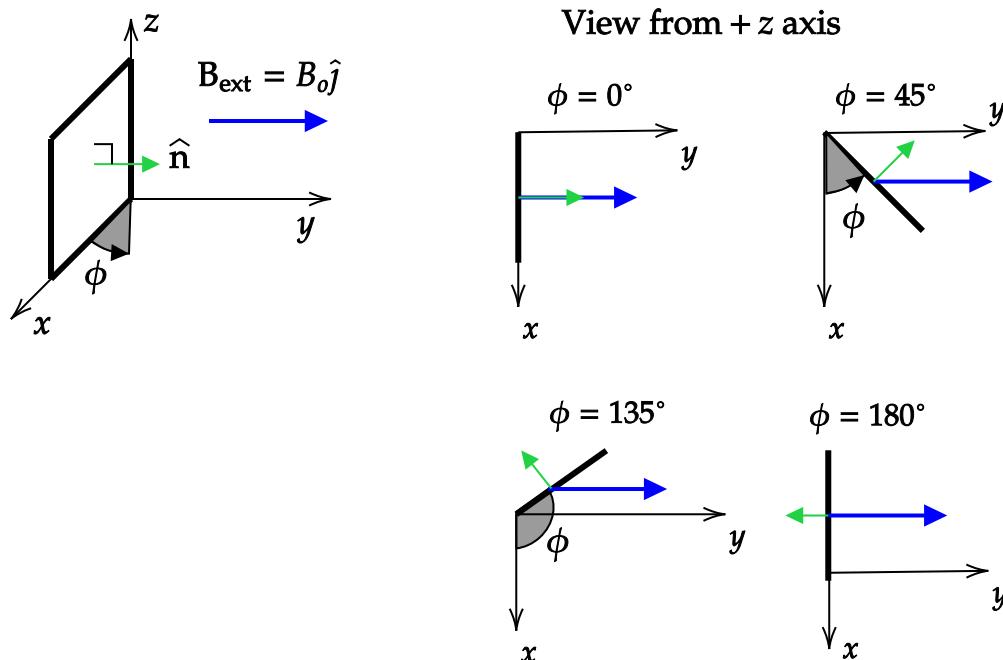


1. At what angles in the range of $\phi = [0, 360^\circ]$ is the magnetic flux zero?

Answer: The magnetic flux will be zero when the normal vector is perpendicular to $\vec{\mathbf{B}}_{\text{ext}}$. This corresponds to $\phi = 90^\circ$ and $\phi = 270^\circ$.

2. Draw the loop, $\hat{\mathbf{n}}$, and $\vec{\mathbf{B}}_{\text{ext}}$ as they would appear when viewed from large z when $\phi = 0^\circ$, $\phi = 45^\circ$, $\phi = 135^\circ$, and $\phi = 180^\circ$.

Answer:



3. When $\phi = 45^\circ$, is the magnetic flux positive or negative? What is its value in terms of a fraction of $B_o A$?

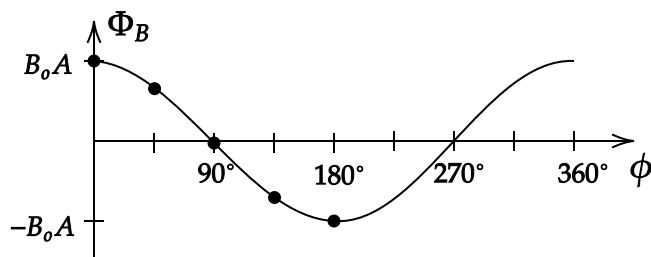
Answer: From the above diagram, the angle between \hat{n} and \vec{B}_{ext} is ϕ , so the flux will be positive. Its value will be $B_o A \cos 45^\circ = B_o A / \sqrt{2}$.

4. When $\phi = 135^\circ$, is the magnetic flux positive or negative? What is its value in terms of a fraction of $B_o A$?

Answer: From the above diagram, the angle between \hat{n} and \vec{B}_{ext} is ϕ , so the flux will be negative. Its value will be $B_o A \cos 135^\circ = -B_o A \cos 45^\circ = -B_o A / \sqrt{2}$.

5. Sketch a plot of the magnetic flux, Φ_B , as a function of ϕ .

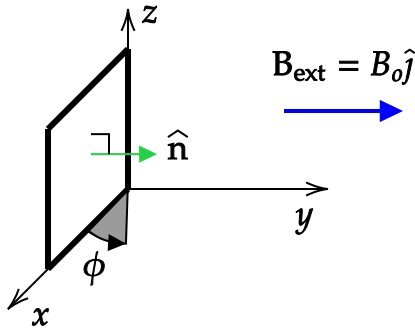
From the diagram above, the angle of rotation of the plane is the same as the angle between the \hat{n} and \vec{B}_{ext} , so the general formula is $\Phi = B_o A \cos \phi$, which is sketched below along with dots that correspond to answers to the above problems.



3 Computing $d\Phi_B/dt$

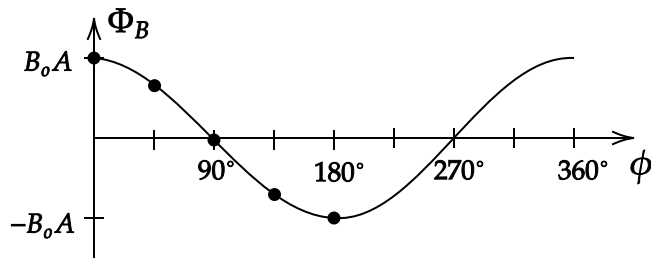
The time rate of change of magnetic flux through a closed loop is a quantity that will be used when Faraday's law is covered.

Suppose the loop in the previous problem rotates at a constant rate.



1. At what angles in the range $\phi = [0^\circ, 360^\circ]$ is $d\Phi_B/dt = 0$?

The answer to part 5. of the previous problem is $\Phi_B = B_o A \cos \phi$, which is sketched below. The slope of Φ_B is zero at $\phi = 0, 180^\circ$, and 360° .



2. For what angle range is $d\Phi_B/dt$ increasing?

From the plot of Φ_B , the slope is positive for $180^\circ < \phi < 360^\circ$

3. For what angle range is $d\Phi_B/dt$ decreasing?

From the plot of Φ_B , the slope is negative for $0^\circ < \phi < 180^\circ$

4. What is the formula for $d\Phi_B/dt$?

$$\Phi = B_o A \cos \phi, \text{ so } d\Phi_B/dt = -B_o A \sin \phi$$