Electric Force

This activity covers topics in Section 21.3 of Young and Freedman 2015, 14th Edition. If you need to review vectors, see sections 1.6-1.8 of Young and Freedman 2015, 14th Edition and Vectors at Khan Academy.

1 Coulomb's Law

Magnitude

$$F_{1\text{ on }2} = F_{2\text{ on }1} = k\frac{|q_1 q_2|}{r^2}$$

where r is the distance between q_1 and q_2. To simplify notation, we are using k in place of $1/4\pi\epsilon_o$. Note that by definition, the magnitude of a vector is positive, which is the reason for the use of the absolute value.

Direction: Along line that connects q_1 and q_2. Direction depends on signs of q_1 and q_2. (Likes repel, opposites attract.).

2 Example

Charge q_1 is at $(x, y) = (-a, -a)$ and charge q_2 is at (a, a). Both charges have a charge of q.

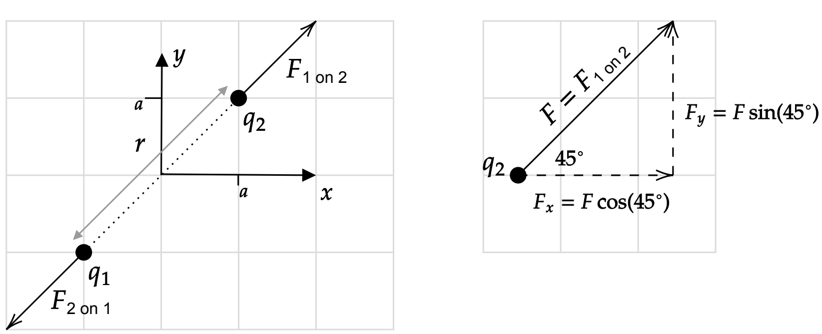
1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

Solution

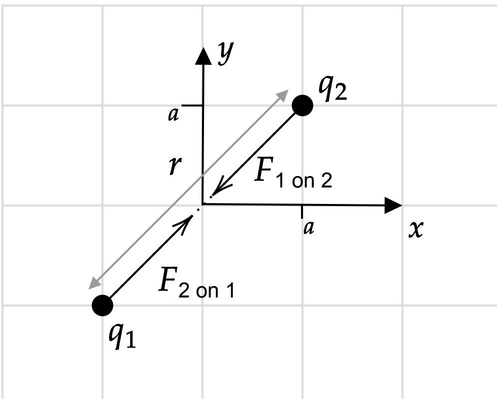
1. The distance between the charges is $r = \sqrt{(2a)^2 + (2a)^2} = \sqrt{8a^2}$, so

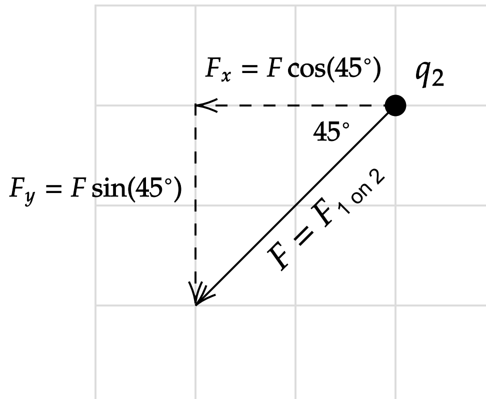
   $$F_{1\text{ on }2} = k\frac{|q_1 q_2|}{r^2} = \frac{k|qq|}{(\sqrt{8a^2})^2} = \frac{kq^2}{8a^2}$$

   The charges will repel each other, so the direction of forces of one on the other will be as shown in the left part of the following diagram.



2. Let $F = F_{1\text{ on }2}$ from part 1. to simplify notation. The right part of the above diagram shows the calculation of the components F_x and F_y, from which it follows that $\vec{\mathbf{F}} = F\cos 45°\,\hat{\boldsymbol{\imath}} + F\sin 45°\,\hat{\boldsymbol{\jmath}}$.
3. The magnitude will not change (it is by definition a positive number). Assume "Opposite signs" means that one is positive and one is negative and still $|q_1| = |q_2| = q$. The force vectors will reverse direction as shown on the left in the following diagram. The diagram on the right shows the calculation of $\vec{\mathbf{F}}_{1\text{ on }2}$, from which it follows that $\vec{\mathbf{F}}_{1\text{ on }2} = -F\cos 45°\,\hat{\boldsymbol{\imath}} - F\sin 45°\,\hat{\boldsymbol{\jmath}}$. Note that reversing the direction of a vector is the same as multiplying each of its components by -1.





3 Problem I

Charge q_1 is at $(x, y) = (-a, a)$ and charge q_2 is at $(a, -a)$. Both charges have a charge of q. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

Solution

1. $F_{1\text{ on }2} = k\dfrac{|q_1 q_2|}{r^2} = \dfrac{k|qq|}{(\sqrt{8a^2})^2} = \dfrac{kq^2}{8a^2}$
2. $\vec{\mathbf{F}}_{1\text{ on }2} = F_{1\text{ on }2}(\cos 45°\,\hat{\boldsymbol{\imath}} - \sin 45°\,\hat{\boldsymbol{\jmath}})$
3. 1.: No change; 2. Assuming "Opposite signs" means that one is positive and one is negative and still $|q_1| = |q_2| = q$, $\vec{\mathbf{F}}_{1\text{ on }2} = F_{1\text{ on }2}(-\cos 45°\,\hat{\boldsymbol{\imath}} + \sin 45°\,\hat{\boldsymbol{\jmath}})$

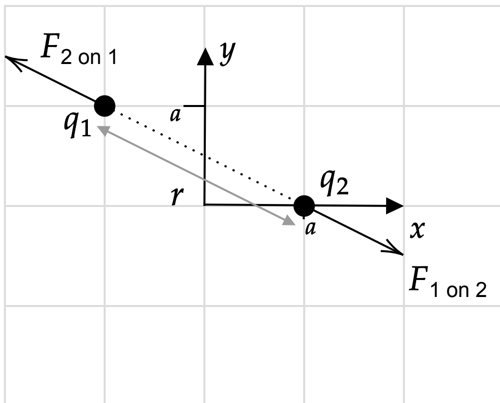
4 Problem II

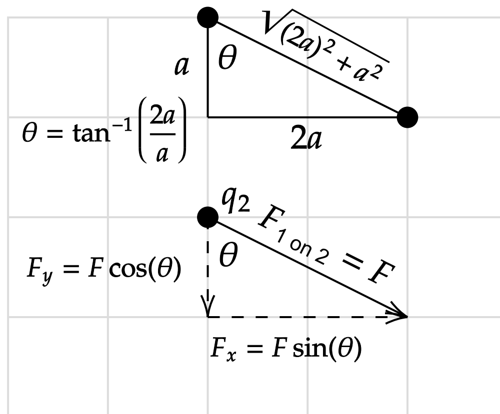
Charge q_1 is at $(x, y) = (-a, a)$ and charge q_2 is at $(a, 0)$. Charge q_1 has a charge of $+q$. Charge q_2 has a charge of $+q$, where q is a positive number. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of q_1 on q_2.

2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

Solution





1. $F_{1\text{ on }2} = k\dfrac{|q_1q_2|}{r^2} = \dfrac{k|qq|}{(\sqrt{(2a)^2+a^2})^2} = \dfrac{kq^2}{5a^2}$
2. $\vec{\mathbf{F}}_{1\text{ on }2} = F_{1\text{ on }2}(\sin\theta\hat{\boldsymbol{\imath}} - \cos\theta\hat{\boldsymbol{\jmath}})$, where $\theta = \tan^{-1}(2) = 63.4°$.

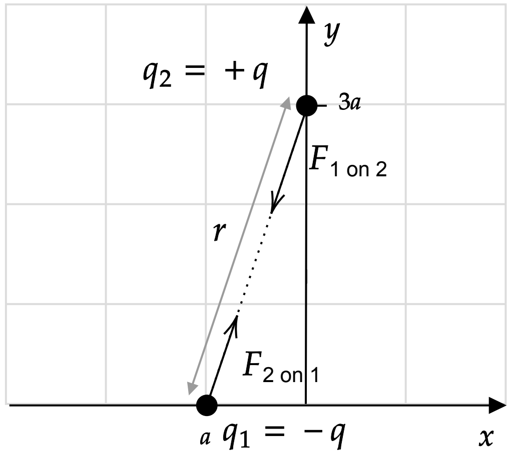
   Alternatively, from the diagram on the right, $\sin\theta = 2a/\sqrt{5}a$ and $\cos\theta = 1a/\sqrt{5}a$, so $\vec{\mathbf{F}}_{1\text{ on }2} = F_{1\text{ on }2}\left(\dfrac{2}{\sqrt{5}}\hat{\boldsymbol{\imath}} - \dfrac{1}{\sqrt{5}}\hat{\boldsymbol{\jmath}}\right)$.
3. 1.: No change; 2. Assuming "Opposite signs" means that one is positive and one is negative and still $|q_1| = |q_2| = q$, $\vec{\mathbf{F}}_{1\text{ on }2} = F_{1\text{ on }2}(-\sin\theta\hat{\boldsymbol{\imath}} + \cos\theta\hat{\boldsymbol{\jmath}})$; $\theta = 63.4°$.

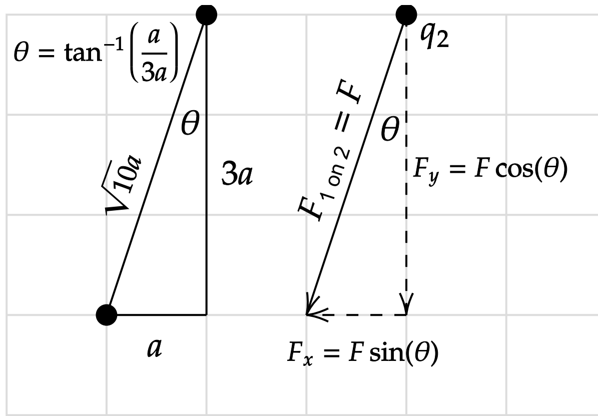
5 Problem III

Charge q_1 is at $(x, y) = (-a, 0)$ and charge q_2 is at $(0, 3a)$. Charge q_1 has a charge of $-q$. Charge q_2 has a charge of $+q$, where q is a positive number. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

Solution





1. $r = \sqrt{a^2 + (3a)^2}$, $F = k|q(-q)|/r^2 = kq^2/10a^2$
2. $\vec{\mathbf{F}} = -F\sin\theta\hat{\imath} - F\cos\theta\hat{\jmath}$, where $\theta = \tan^{-1}(1/3) = 18.4°$.

   Alternatively, from the diagram, $\sin\theta = a/\sqrt{10}a$ and $\cos\theta = 3a/\sqrt{10}a$, so $\vec{\mathbf{F}}_{1\text{ on }2} = F\left(-\frac{1}{\sqrt{10}}\hat{\imath} - \frac{3}{\sqrt{10}}\hat{\jmath}\right)$.
3. 1.: No change; 2.: Note: the problem statement should have been "if both charges have the *same* sign" (the charges were given to have opposite signs). In this case: $\vec{\mathbf{F}}_{1\text{ on }2} = +F\sin\theta\hat{\imath} + F\cos\theta\hat{\jmath}$.

6 Problem IV

Charge q_1 is at $(x, y) = (x_1, y_1)$ and charge q_2 is at (x_2, y_2). Find the magnitude of the force of q_1 on q_2.

Solution: $F = kq^2/\left((x_2 - x_1)^2 + (y_2 - y_1)^2\right)$. Make sure that you can justify this with a diagram.