1 Introduction

1.1 Definitions

The electric current in a wire is defined as

$$I = \frac{dQ}{dt}$$

where dQ is the total amount of charge that passes through a cross–section of the wire in a differential amount of time, dt.

If q is the charge (in Coulombs) of each flowing charge, n is their number per volume (“number density”), v_d their average speed along the wire (called the “drift velocity”), then

$I = n|q|v_d A$, where A is the cross–sectional area of the wire.

The average current density is defined as

$J = \frac{I}{A}$, which can be written as $J = n|q|v_d$.

1.2 Ohm’s Law

If an electric field exists in a wire (by, for example, connecting its ends to a battery), the charges will accelerate until they collide with another particle and decelerate (collisions resist the flow). The net result will be current – a flow of charges with an average drift velocity. Experimentally, it has been shown that in many materials, the electric field is proportional to the current density:

$$E = \rho J$$

where the value of the proportionality constant ρ, called resistivity, depends on the material. This is one version of Ohm’s law.

For a wire of length L and constant cross–sectional area A, Ohm’s law can also be written as $V = (\rho L/A)I$. If we define resistance as $R = \rho L/A$, then we have another relationship that is also referred to as Ohm’s law:

$$V = IR.$$

In this form, the interpretation is that the voltage across a wire is proportional to the current in the wire, with the proportionality constant of R.

2 Problem I – Definitions and Ohm's Law

A 9–volt power source is connected to a wire of length 10 meters with a circular cross–section and radius of 0.01 meters. The wire has a resistivity of $10^{-8}\ \Omega \cdot \mathrm{m}$. The number density of charge carriers is $10^{28}/\mathrm{m}^3$. Assume Ohm's law applies and the charges that flow are electrons.

1. What is the resistance (with units) of the wire?

   Answer: $R = \dfrac{\rho L}{A} = \dfrac{(10^{-8}\ \Omega \cdot \mathrm{m})(10\ \mathrm{m})}{\pi (0.01\ \mathrm{m})^2} = \dfrac{10^{-3}}{\pi}\ \Omega$. When solving circuit problems, the resistors involved typically have a much larger than this, which is why we neglect the resistance of the wires.

2. What is the current (with units) in the wire?

   Answer: $I = \dfrac{9\ \mathrm{v}}{\frac{10^{-3}}{\pi}\ \Omega} = 9,000\pi\ \mathrm{A}$. This is a huge current. If you look at the back of an electronic device, you will see a rating on the order of $1\ \mathrm{A}$. Household circuit breakers are set to break at approximately $15\ \mathrm{A}$. Most power sources cannot supply current at this rate; even if the power source could supply this current, the amount of heat created would lead to a fire or melting of the wire.

3. How much charge (in Coulombs) flows past a cross–section of the wire per second?

   Answer: $9,000\pi\ \mathrm{C}$

4. How many electrons flow past a cross–section of the wire per second?

5. What is the current density (with units)?

   Answer: $J = \dfrac{I}{A} = 9 \cdot 10^7 \dfrac{\mathrm{A}}{\mathrm{m}^2}$

6. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6 \cdot 10^{-19}$ C.)

   Answer: $v_d = J/n|q| \simeq 6\ \mathrm{cm/s}$

7. Based on the description of how charged particles flow in a wire, explain why the resistance of a cylindrical wire is proportional to its length and inversely proportional to the square of its radius.

   Answer: See the description in your textbook.

3 Problem II – Definitions and Ohm's Law

A power source is connected to a wire of length 20 meters with a circular cross–section and a radius of 0.01 meters. The wire has a resistivity of $10^{-7}\ \Omega \cdot \mathrm{m}$. The number density of charge carriers is $10^{27}/\mathrm{m}^3$. The current in the wire was measured and found to be $1\ \mathrm{A}$. Assume Ohm's law applies and the charges that flow are electrons.

1. What is the resistance (with units) of the wire?

   Answer: $(.02/\pi)\ \Omega$

2. How much charge (with units) flows past a cross–section of the wire per second?

   Answer: $1\ \mathrm{C}$

3. What is the current density (with units)?

   Answer: $J = \dfrac{10^4}{\pi} \dfrac{\mathrm{A}}{\mathrm{m}^2}$

4. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6 \cdot 10^{-19}$ C.)

   Answer: $v_d = J/n|q| = 0.02 \text{ mm/s}$

4 Problem III – Current Through a Cylindrical Shell

If a cylindrical wire with an inner radius a and outer radius b carries a current I, what is J?

Answer: $J = \dfrac{I}{\pi(b^2 - a^2)}$

5 Problem IV – $I = n|q|v_d A$ derivation

Derive the relationship $I = n|q|v_d A$. Provide a diagram.

Answer: See textbook.