

# RC Circuits

## 1 Notation and Equations

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- We use the letter “C” as a label for a capacitor and the units of charge. When used as a label,  $C$  is usually written in italics. Thus  
“ $Q_C = 10\text{ C}$ ” means “the capacitor labeled  $C$  has a charge of 10 Coulombs.”
- Lower case letters are used for electrical quantities that vary in time. In circuits with only emfs and resistors, currents and voltages are constant, and we used  $I$  and  $V$ . In the circuits considered in this activity, the currents and voltages vary in time, so we use  $i(t)$  and  $v(t)$ .
- In this activity, we use the relationship  $i(t) = dq(t)/dt$  between the current  $i(t)$  in the wires connected to a capacitor and the charge  $q(t)$  on the capacitor.

## 2 Discharging Capacitor

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If one capacitor with capacitance  $C$  is in a circuit with other resistors (and no emfs), the charge on the capacitor changes with time according to

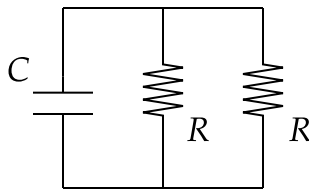
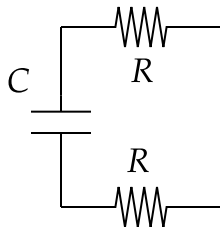
$$q(t) = Q_o e^{-t/\tau},$$

where  $\tau \equiv RC$ ,  $Q_o$  is the charge on the capacitor at  $t = 0$ , and  $R$  is the equivalent resistance. The quantity  $\tau$  is called “ $RC$  time constant.” The equation for  $q(t)$  follows from solving the differential equation that follows from using Kirchhoff’s voltage law.

### 2.1 Problem

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Find  $\tau$  in seconds for the following two circuits. Use  $R = 10\text{ k}\Omega$  and  $C = 1\text{ }\mu\text{F}$ . Note that  $1\text{ }\Omega \cdot \text{F} = 1\text{ second}$ .



## 2.2 Problem

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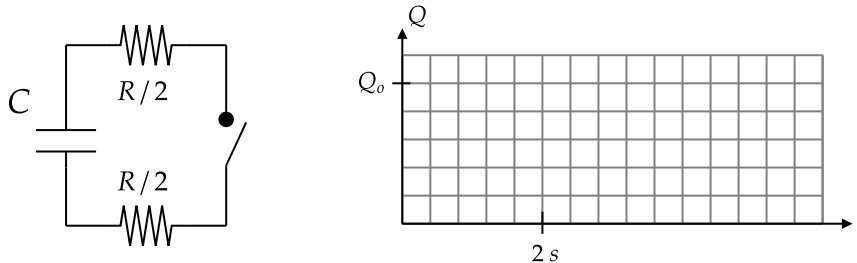
For the following circuit, find the charge  $q$  on the capacitor at  $t = 2$  s if the switch is closed at  $t = 0$  and the capacitor has an initial charge of  $Q_o$  for the following three cases.

1.  $RC = 1$  s       $q(2 \text{ s}) =$

2.  $RC = 2$  s       $q(2 \text{ s}) =$

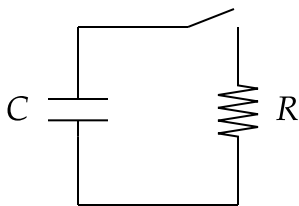
3.  $RC = 4$  s       $q(2 \text{ s}) =$

4. Next, sketch a plot of  $q(t)$  from  $t = 0$  to  $t = 6$  s for each of these three cases. Used a solid, dashed, and dotted line for case 1., 2., and 3., respectively. (Hint: For all curves  $q(0) = Q_o$ . Use the  $q(2)$  values from parts 1.–3. to help sketch the curve.)



## 2.3 Problem

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In the circuit above, the values of  $R$  and  $C$  are such that  $RC = 1$  s.

1. If the capacitor has a charge of 10 nC and the switch is closed, how long will it take for the charge on the capacitor to fall to half of this value?
2. If the capacitor instead had a charge of 20 nC and the switch is closed, how long will it take for the charge on the capacitor to fall to half of this value?

### 3 Charging a Capacitor

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If a single capacitor is in series with other resistors and a DC voltage source, the charge of the capacitor varies according to

$$q(t) = Q_f(1 - e^{-t/\tau})$$

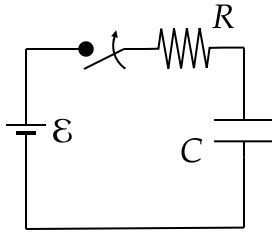
where  $\tau \equiv RC$ ,  $R$  is the equivalent resistance, and  $Q_f$  is the final charge on the capacitor, that is, the charge on the capacitor as  $t \rightarrow \infty$  (it is technically more accurate to say  $t \gg \tau$  instead of  $t \rightarrow \infty$ ; why?).

To find  $Q_f$ , replace the capacitor with an open circuit and use Kirchhoff's voltage law to find the voltage across the capacitor,  $V_f$ . Then use  $Q_f = CV_f$ .

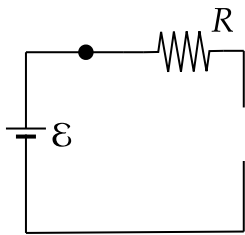
#### 3.1 Example

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Find  $q(t)$  for the following circuit, assuming the switch is closed at  $t = 0$  and the capacitor is initially uncharged.



**Answer:** When the switch is closed, charge builds up on the capacitor. This build-up continues until the charge on the capacitor is such that no current flows in the circuit. If no current flows through the capacitor, the circuit is equivalent to one in which the capacitor is replaced with an open circuit, as shown below.



Kirchhoff's voltage Law gives  $\mathcal{E} - i(t)R - v_C(t) = 0$ , where  $v_C(t)$  is the voltage across the capacitor. For large  $t$ ,  $i(t) = 0$ , leaving  $v_C(t) = \mathcal{E}$ . This voltage is for large  $t$ , so we relabel it as  $V_f$ . Using  $Q_f = CV_f$  gives  $Q_f = C\mathcal{E}$ , so

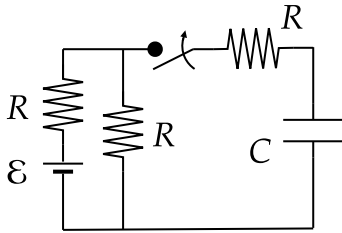
$$q(t) = C\mathcal{E}(1 - e^{-t/RC})$$

Note that the general formula  $q(t) = Q_f(1 - e^{-t/\tau})$  that applies for all  $t$  is derived by solving the differential equation  $\mathcal{E} - i(t)R - v_C(t) = 0$  using  $i(t) = dq(t)/dt$  and  $v_C(t) = q(t)/C$ .

### 3.2 Problem

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For a circuit with any number of capacitors, DC voltage sources, and resistors, finding the equation for  $q(t)$  for each capacitor requires solving a system of differential equations, which is not covered in this course. However, you can find the currents and charges on the capacitor after a long time by replacing all capacitors with open circuits and then using KVL.



(In the above circuit, the equation used in the previous problem does not apply and so you are only asked for the state of the circuit after a long time.)

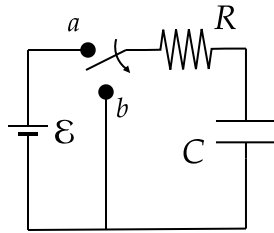
1. Find the charge on the capacitor a long time after the switch is closed.
2. Find the current in the left, middle, and right resistors after long time after the switch is closed.

## 4 Charge/Discharge Problem

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### 4.1 Part I

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The switch in the above circuit has been in position  $a$  for a long time. At  $t = 0$ , the switch is moved instantaneously to position  $b$ . The values of the circuit elements are  $\mathcal{E} = 12 \text{ V}$ ,  $C = 10 \text{ mF}$ , and  $R = 20 \Omega$ .

Let  $t = 0^-$  correspond to the time just before the switch is moved from  $a$  to  $b$ . Let  $t = 0^+$  correspond to the time just after the switch is moved from  $a$  to  $b$ .

1. What is the current through the resistor at  $t = 0^-$ ?

2. What is the charge on the capacitor at  $t = 0^-$ ?

3. What is the charge on the capacitor at  $t = 0^+$ ?

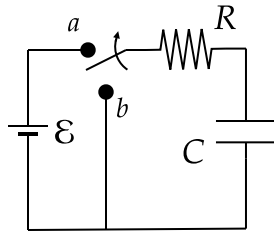
4. What is the voltage across the capacitor at  $t = 0^+$ ?

5. What is the current through the resistor at  $t = 0^+$ ?

6. What is the charge on the capacitor at  $t = 200 \text{ ms}$ ?

## 4.2 Part II

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The switch in the above circuit has been in position  $b$  for a long time. At  $t = 0$ , the switch is moved instantaneously to position  $a$ . The values of the circuit elements are  $\mathcal{E} = 12 \text{ V}$ ,  $C = 10 \text{ mF}$ , and  $R = 20 \Omega$ .

Let  $t = 0^-$  correspond to the time just before the switch is moved from  $b$  to  $a$ . Let  $t = 0^+$  correspond to the time just after the switch is moved from  $b$  to  $a$ .

1. What is the current through the resistor at  $t = 0^-$ ?
2. What is the voltage across the capacitor at  $t = 0^-$ ?
3. What is the charge on the capacitor at (a)  $t = 0^-$  and (b)  $t = 0^+$ ?
4. What is the voltage across the capacitor at  $t = 0^+$ ?
5. What is the current through the resistor at  $t = 0^+$ ?
6. What is the charge on the capacitor at time  $t = 200 \text{ ms}$ ?