1 Introduction

1.1 Definitions

Current is defined as the rate of change of charge, Q, per time. If you were able to count the number of charges passing a cross–section of a wire over 1 s, the current would be Q/t. If the amount of time was small, say Δt, and your labeled your count as ΔQ, then $I = \Delta Q/\Delta t$. In the limit that Δt approaches zero, we have the definition

$$I = \frac{dQ}{dt}$$

If n is the number of charges with charge q per volume and v_d is the average velocity of the charges (called the "drift velocity"), then

$$I = nqv_d A$$

where A is the cross–sectional area of the wire.

We also define a quantity called the current density, J, which is the current per unit cross–sectional area, A:

$$J = \frac{I}{A}$$

1.2 Ohm's Law

If an electric field exists in a wire (by, for example, connected its ends to a battery), the charges will accelerate until they collide with another particle and decelerate (resist the flow). The net result will be a flow of charges with a drift velocity. Experimentally, it has been shown that in many materials, the ratio of the electric field to current density is

$$\rho = \frac{E}{J}$$

where the value of the constant ρ depends on the material.

Ohm's law is

$$I = V/R$$

which means a voltage V applied to a wire will result in a current I, and this current depends on R.

The resistance is larger the longer wire and smaller for a larger cross–section of wire. The result is

$$R = \frac{\rho L}{A}$$

2 Problem I – Definitions and Ohm’s Law

A 9 Volt battery is connected to a wire of length 10 meters with circular cross section of radius of 0.01 meters. The wire has a resistivity of $10^{-8}\,\Omega\cdot\mathrm{m}$. The density of charge carriers is $10^{28}/\mathrm{m}^3$. Assume Ohm’s law applies.

1. What is the resistance (with units) of the wire?

2. How much charge (with units) flows past a cross section of the wire per second?

3. What is the current (with units) in the wire?

4. What is the current density (with units) that flows through the wire?

5. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6\cdot 10^{-19}$ C.)

6. Based on the description of how charged particles flow in a wire, explain why the resitance of a cylindrical wire is proportional to its length and inversely proportinal to the square of its radius.

3 Problem II – Definitions and Ohm's Law

A battery is connected to a wire of length 20 meters with circular cross section and a radius of 0.01 meters. The wire has a resistivity of $10^{-7}\,\Omega \cdot \mathrm{m}$. The density of charge carriers is $10^{27}/\mathrm{m}^3$. The current in the wire was measured and found to be 1 Ampere. Assume Ohm's law applies.

1. What is the resistance (with units) of the wire?

2. How much charge (with units) flows past a cross section of the wire per second?

3. What is the current (with units) in the wire?

4. What is the current density (with units) that flows through the wire?

5. What is the drift velocity of electrons in the wire? (The charge on an electron is $-1.6 \cdot 10^{-19}$ C.)

4 Problem III – $I = nqv_dA$ derivation

Derive the relationship $I = nqv_dA$. Provide a diagram.