

Kirchhoff's Circuit Laws

1 Introduction

To find the current through each resistor in a circuit with only resistors and batteries, Kirchhoff's Current Law and Kirchhoff's Voltage Law can be used.

1. Kirchhoff's Current Law (KCL): The sum of all currents entering and exiting a junction is zero. Or, equivalently, the current flowing into a junction is equal to the current flowing out of a junction.
2. Kirchhoff's Voltage Law (KVL): The sum of all voltage changes around a closed loop is zero.

General procedure

To find the values of currents in a circuit,

1. assume directions of current,
2. write equations for KCL for junctions,
3. write equations for KVL for loops, and
4. solve for currents. If you get a negative value for a current, the actual direction of current flow is opposite to what you assumed in 1.

To apply KVL, draw a closed loop and choose a direction to step around the loop.

1. If stepping around the loop requires a step across a battery with emf \mathcal{E} and the $-$ side is encountered first, the voltage change is $+\mathcal{E}$. If the $+$ side is encountered first, the voltage change is $-\mathcal{E}$. *The direction of current through the battery does not matter.*
2. If stepping around the loop requires a step across a resistor R and the direction of current I through the resistor is the same as the step direction, the voltage change is $-IR$. If the current is opposite the step direction, the voltage change is $+IR$.

A common error is to assume every loop equation requires an emf.

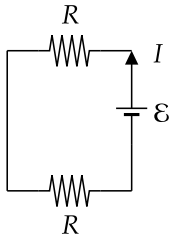
Note on Equivalent Resistances

You may have already had experience solving for currents in circuits using equivalent resistances. In this activity, do not use equivalent resistances unless asked. The motivation is that you will better understand their interpretation and derivation after solving for currents without using equivalent resistances.

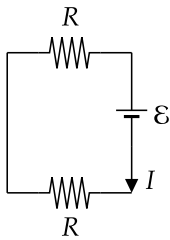
2 Single Loop

Only KVL is needed to find the current in a single-loop circuit.

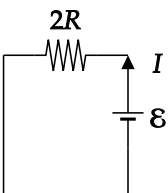
1. The direction of the unknown current I in the following circuit is assumed to be counterclockwise. Write the equation for KVL by stepping around the loop counterclockwise; then solve for I in terms of \mathcal{E} and R .



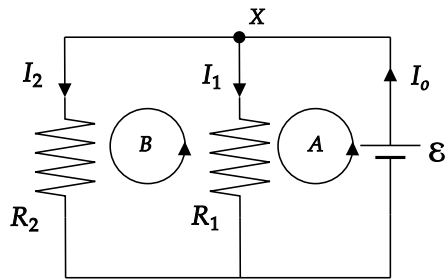
2. The direction of current I in the following circuit was assumed to be clockwise. Write the equation for KVL stepping around the loop counterclockwise; then solve for I in terms of \mathcal{E} and R .



3. Which value for I found above is correct?
4. In part 1., a counterclockwise current was assumed, and you stepped in the counterclockwise direction. If you stepped clockwise, would your answer for I be different?
5. For the circuit in part 1., if you replace the bottom resistor with a wire and replace the top resistor with a resistor with resistance $2R$, the following circuit results. What is the current I for this circuit?



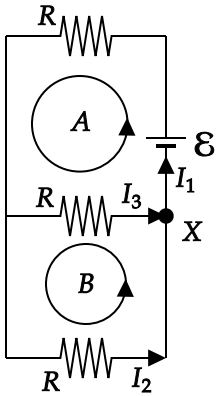
3 Multiple Loops I



1. Use KCL at junction X and KVL for loops A and B to show that $I_o = \mathcal{E} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$.
2. If $R_2 > R_1$, which resistor will have more current?
3. If $R_1 \rightarrow 0$, what happens to I_o , I_1 , and I_2 ?

4 Multiple Loops II

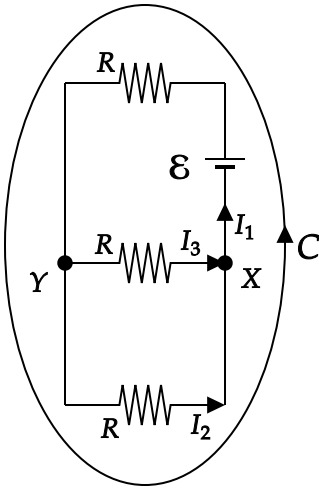
Assume the direction of currents I_1 , I_2 , and I_3 in the following circuit are as shown.



1. Write the equation for KVL for loop A .
2. Write the equation for KVL for loop B .
3. Write the equation for KCL for junction X .
4. Use the three equations found above to solve for the three unknowns, I_1 , I_2 , and I_3 in terms of \mathcal{E} and R .
5. Check your answers by plugging your values for I_1 , I_2 , and I_3 into the equations that you wrote for parts 1.-3.

5 Multiple Loops II

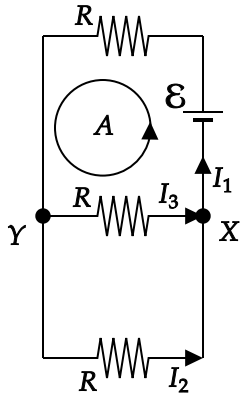
In the circuit for the previous problem, there are three possible loops. The third loop is loop C , which is indicated below.



1. Write the equation for KVL for loop C .
2. Use the equation for KVL for loop B and the KCL equation for junction X along with the KVL equation for loop C to find I_1 , I_2 , and I_3 in terms of \mathcal{E} and R . (You should get the same answers as the previous problem.)

6 Redundant Equations

When solving circuit problems with multiple loops, you will generally find that you can use KVL and KCL to write more equations than there are unknown currents. If you attempt to solve a problem with N unknowns by writing N equations based on KVL and KCL but cannot solve for the unknowns, the reason is that two or more of the equations that you wrote were not independent. To demonstrate this, for the following circuit,



1. write the KCL equation for junction X ,
2. write the KCL equation for junction Y ,
3. write the KVL equation for loop A , and
4. Is it possible to use the above three equations to solve for I_1 , I_2 , and I_3 in terms of \mathcal{E} and R ? If yes, do it. If no, explain why.