1 Overview

We generally describe vector equations using two representations:

1. Simple – An equation for the magnitude is given and words are used to describe the direction.
2. Compact – A single equation is given for the vector and additional equations are given for parts of the equation.

For example, in Section 21.3 of the textbook, the equation for Coulomb's Law was given in the simple form. In Section 21.4, the equation for the electric field due to a point charge was given in the compact form.

In PHYS 260, you will need to be able to solve problems similar to the examples given here quickly. If you found the problems in this activity to be difficult, review Sections 1.7-1.8 in the textbook and see the Khan Acadamy's comprehensive introduction to vectors.

2 Coulomb's Law in Simple Form

Magnitude

$$F_{1\text{ on }2} = k\frac{|q_1 q_2|}{r^2}$$

where r is the distance between q_1 and q_2. To simplify notation, we are using k in place of $1/4\pi\epsilon_o$.

Direction

Along line that connects q_1 and q_2. Direction depends on signs of q_1 and q_2. (Likes repel, opposites attract.).

2.1 Example

Charge q_1 is at $(x, y) = (-a, -a)$ and charge q_2 is at (a, a). Both charges are positive and have a charge of q.

1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\mathbf{F} = F_x\hat{\imath} + F_y\hat{\jmath}$.

Solution

1. From the drawing, the distance between the charges is $r = 2\sqrt{2}a$, so

$$F_{1\text{ on }2} = \frac{kq^2}{8a^2}$$

   The charges will repel each other, so the direction of forces of one on the other will be as shown in the diagram. The direction of the force vector is $45°$ upwards from horizontal.

2. Let $F = F_{1\text{ on }2}$ from part 1. to simplify notation. Then

$\mathbf{F} = F\cos 45° \hat{\imath} + F\sin 45° \hat{\jmath}$. Given that $\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$, we can also write

$$\mathbf{F} = F\left[\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right]$$

2.2 Problem

Charge q_1 is at $(x, y) = (a, a)$ and charge q_2 is at $(-a, -a)$. Both charges are positive and have a charge of q.

1. Find the magnitude and direction of the force of q_1 on q_2.
2. Write the force of q_1 on q_2 in the form $\mathbf{F} = F_x\hat{\imath} + F_y\hat{\jmath}$.

3 Coulomb's Law in Compact Form

$$\mathbf{F}_{1\text{ on }2} = kq_1q_2\frac{\hat{\mathbf{r}}}{r^2}$$

$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ is the vector from the position of q_1 to the position of q_2, $\mathbf{r}_1$ is a vector from the origin to the location of q_1, and $\mathbf{r}_2$ is a vector from the origin to the location of q_2.

$r = |\mathbf{r}| = |\mathbf{r}_2 - \mathbf{r}_1|$ is the distance from q_1 to q_2.

$\hat{\mathbf{r}} = \dfrac{\mathbf{r}}{r}$ is the unit vector pointing from the position of q_1 to the position of q_2.

Although this form looks more complex, it requires basic steps.

3.1 Example

Charge q_1 is at $(x, y) = (-a, -a)$ and charge q_2 is at (a, a). Both charges are positive and have a charge of q.

Using the steps in the previous example,

1. Write the force of q_1 on q_2 in the form $\mathbf{F} = F_x\hat{\imath} + F_y\hat{\jmath}$.
2. Find the magnitude and direction of the force of q_1 on q_2.

Solution

1. The vector from the origin to the location of q_1 is $\mathbf{r}_1 = -a\hat{\imath} - a\hat{\jmath}$

   The vector from the origin to the location of q_2 is $\mathbf{r}_2 = a\hat{\imath} + a\hat{\jmath}$

   The distance vector is $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = 2a\hat{\imath} + 2a\hat{\jmath}$

   The length of the distance vector is

   $$r = |\mathbf{r}_2 - \mathbf{r}_1| = \sqrt{(2a)^2 + (2a)^2} = 2\sqrt{2}a$$

The unit vector pointing from the position of q_1 to the position of q_2 is

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$$

Using the above results in

$$\mathbf{F}_{1 \text{ on } 2} = kq_1q_2\frac{\hat{\mathbf{r}}}{r^2}$$

gives

$$\mathbf{F}_{1 \text{ on } 2} = kq^2\frac{\left[\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right]}{(2\sqrt{2}a)^2} \quad \text{or} \quad \mathbf{F}_{1 \text{ on } 2} = \frac{kq^2}{8a^2}\left[\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right]$$

2. Let $\mathbf{F} = \mathbf{F}_{1 \text{ on } 2}$ from part 1. to simplify notation. Then

$$F = |\mathbf{F}| = \frac{kq^2}{8a^2}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{kq^2}{8a^2}$$

The angle is

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{\frac{F}{\sqrt{2}}}{\frac{F}{\sqrt{2}}}\right) = 45°$$

3.2 Problem

Charge q_1 is at $(x, y) = (a, a)$ and charge q_2 is at $(-a, -a)$. Both charges are positive and have a charge of q.

Using the steps in the previous example,

1. Write the force of q_1 on q_2 in the form $\mathbf{F} = F_x\hat{\imath} + F_y\hat{\jmath}$.
2. Find the magnitude and direction of the force of q_1 on q_2.