

Capacitance

1 Overview

1.1 Technique

A technique for computing capacitance when Gauss's law can be used to compute the electric field is:

1. place a positive charge Q on one conductor and $-Q$ on the other conductor;
2. use Gauss's law to compute the electric field between the conductors;
3. use $V(b) - V(a) = - \int_a^b \mathbf{E} \cdot d\mathbf{l}$ to find the potential difference between the conductors. Point a is any point on the negatively charged conductor, and point b is any point on the positively charged conductor; and
4. Compute C using $C = Q/\Delta V = Q/(V(b) - V(a))$.

1.2 Review of Related Topics

Conductors: The electric field at any location in space is the superposition (vector sum) of electric fields due to every charge in the universe. For locations inside a conductor, the electric field will be zero. The reason is that charges on or in a conductor are free to move and position themselves, effectively instantaneously, on the surface in a way that makes the electric field inside the conductor zero.

1. If a conductor is isolated (meaning other charges are far enough away that we can ignore their electric fields), the electric field inside it is the superposition of the electric fields due to each charge on the conductor.
2. If a conductor is not isolated, the electric field inside it is the superposition of the electric fields due to each charge on the conductor and all other charges in the universe.

Electric Potential Energy and Electric Potential: The general formula for work is $W_{a \rightarrow b} = \int_a^b \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}$. If $\vec{\mathbf{F}}$ is a conservative force, such as the force on a charge due to an electric field, we define potential energy U according to

$$\Delta U = U(b) - U(a) \equiv -W_{a \rightarrow b}$$

The force on a charge q_o in an electric field $\vec{\mathbf{E}}$ is $\vec{\mathbf{F}} = q_o \vec{\mathbf{E}}$ and so we can write

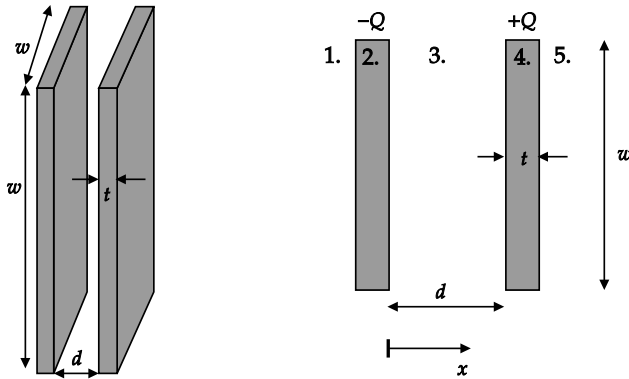
$$U(b) - U(a) = - \int_a^b q_o \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}. \text{ Dividing both sides by } q_o \text{ gives } \frac{U(b)}{q_o} - \frac{U(a)}{q_o} = - \int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

Defining electric potential as $V \equiv U/q_o$ gives $V(b) - V(a) = - \int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$. In summary, if you calculate the difference in potential between a and b (in volts), you can also determine, for a charge q_o moved from a to b ,

1. how much work (in Joules) the electric field did on the charge: $-q_o(V(b) - V(a))$, and
2. the change in electric potential energy (in Joules): $+q_o(V(b) - V(a))$.

2 Parallel Plates

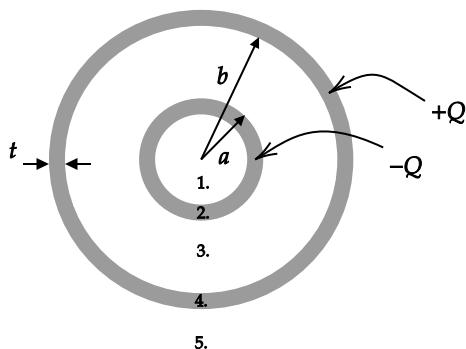
An equal and opposite amount of charge is placed on two conducting and parallel plates, as shown on the left in the following figure. On the right, a side view of the plates is shown. The area of the plates, $A = w^2$, is much larger than shown, so the width, w , is much larger than the separation distance, d . Assume Q is positive.



1. How will the charges be distributed on the plates? That is, how much charge is on each of the four large faces with area A ? Assume the charges are uniformly distributed on these four faces and that no charge appears on the thin edge faces with thickness t .
2. What is the electric field in each of the five regions? (Hint: The magnitude of the electric field due to charges uniformly distributed on a large plane is $|\sigma|/2\epsilon_0$. The electric field in each region will be the sum of the electric field due to charges on each plate. Your answer should be such that the electric field inside the conducting plates is zero!)
3. What is the electric potential difference, $V(d) - V(0)$, between the left and right plate? (Make sure the sign of your result matches your expectation based on the techniques covered in the last activity.)
4. Use your answer to 3. to find the capacitance in terms of ϵ_0 , A , and d .
5. (Review question) How much work will the electric field do on a charge q that is moved from the left plate to the right plate? What will be the change in electric potential energy?

3 Spherical

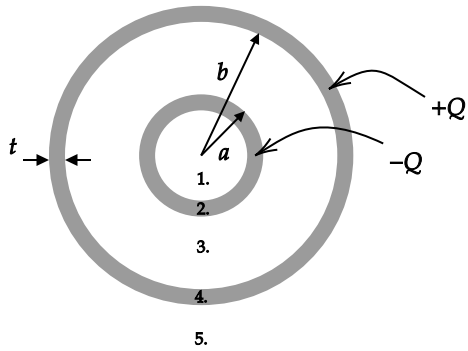
Charge is placed on two spherical conducting shells, the cross-section of which is shown. Both shells have a thickness of t . The inner shell has an outer radius of a and a net charge of $-Q$. The outer shell has an inner radius of b and a net charge of $+Q$. Assume that Q is positive.



It can be shown (see challenge questions below) that the surface at $r = a$ will have $-Q$ uniformly distributed on it, and the surface at $r = b$ will have Q uniformly distributed on it. Given this, we can conclude that the inner surface of the inner conductor and the outer surface of the outer conductor will not have any charge (why?).

1. What is the electric field in each of the five labeled regions? Region 1. is the empty volume inside of the inner conductor, region 2. is the inner conductor, region 3. is the empty volume between the conductors, region 4. is the outer conductor, and region 5. is the region outside of the outer conductor. (Hint: Use Gauss's law several times; when not zero, the electric field should be proportional to $1/r^2$.)
2. What is the potential difference, $V(b) - V(a)$?
3. Find the capacitance in terms of ϵ_o , a , and b .
4. (Review question) How much work would the electric field do on a charge q that is moved from $r = a$ to $r = b$? What will be the change in electric potential energy?

Optional challenge questions



Draw the cross-section of Gaussian surfaces you use to answer any of the following questions on the diagram above.

Use Gauss's law and the fact that the electric field inside a conductor must be zero to show

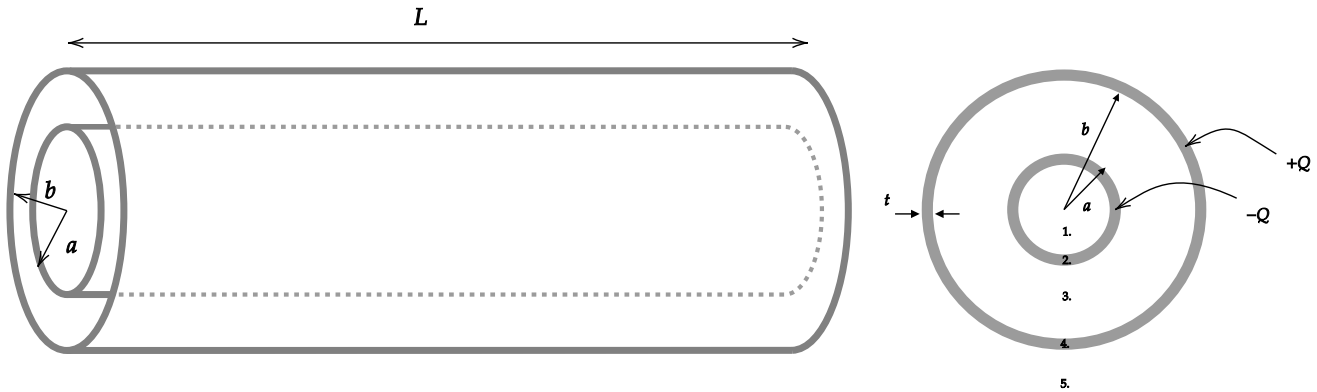
5. there can be no charge on the inner surface of the inner conductor,

6. the charge on the inner surface of the outer conductor is $+Q$, and

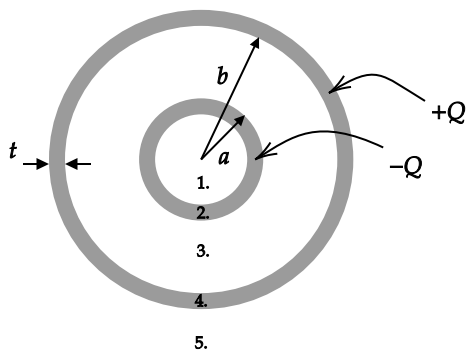
7. there is no charge on the outer surface of the outer conductor.

4 Cylindrical

Charge is placed on two long cylindrical conducting shells, the cross-section of which is shown. Both shells have a thickness of t . The inner shell has an outer radius of a and a net charge of $-Q$. The outer shell has an inner radius of b and a net charge of $+Q$. Assume that Q is positive and the cylinders have length L .



1. What is the net charge and surface charge density on each surface? Assume the charges are uniformly distributed on long surfaces and that no charge appears on end surfaces. That is, find the charge density on
 - the inner surface of the inner cylinder,
 - the outer surface of the inner cylinder,
 - the inner surface of the outer cylinder, and
 - the outer surface of the outer cylinder.



The cross-section figure from the previous page is repeated above.

2. Given your answer to part 1. of this problem, what is the electric field in each of the five regions?
(Hint: Use Gauss's law)

3. What is the electric potential difference, $V(b) - V(a)$, between the outer and inner cylinder? (Make sure the sign of your result matches your expectation based on the techniques covered in the last activity.)

4. Find the capacitance in terms of ϵ_o , L , a , and b .