Sound Intensity Level

1 Introduction

The intensity of a sound wave, I, is the average power rate per unit area of energy from a sound wave that passes through or into a surface,

$$I = \frac{P}{A}$$

I has units of Power/Area, which has SI units of $\mathtt{Watts}/\mathrm{m}^2$. The intensity of various sources of sound is shown in the third column of the following table from Young and Freedman, 14th Edition. The second column is discussed in section 3 of this activity.

TABLE 16.2 Sound Intensity Levels from Various Sources (Representative Values)

Source or Description of Sound	Sound Intensity Level, β (dB)	Intensity, I (W/m^2)
Military jet aircraft 30 m away	140	10^2
Threshold of pain	120	1
Riveter	95	3.2×10^{-3}
Elevated train	90	10^{-3}
Busy street traffic	70	10^{-5}
Ordinary conversation	65	3.2×10^{-6}
Quiet automobile	50	10^{-7}
Quiet radio in home	40	10^{-8}
Average whisper	20	10^{-10}
Rustle of leaves	10	10^{-11}
Threshold of hearing at 1000 Hz	0	10^{-12}

If the sound wave is created by a point source with power P, is emitted uniformly in all direction (isotropically), and there are no reflections or obstructions, the intensity varies with the distance r from the point source according to

$$I = \frac{P}{4\pi r^2}$$

Because I is proportional to r^2, if the disance from the source doubles, the intensity decreases by a factor of four.

2 Sound Intensity, I

Suppose a bird sitting on the top of a lamp post emits sound with a power of 1 W; assume that the relationship $I = P/4\pi r^2$ applies.

1. What is I at a point 5 m away?

   Answer:

   $$I = \frac{1\ \mathrm{W}}{4\pi(5\ \mathrm{m})^2} \simeq \frac{1}{314}\frac{\mathrm{W}}{\mathrm{m}^2}$$

2. What is I at a point 50 m away?

   Answer: If the distance increases by a factor of 10, we expect I to descrease a factor of 100 (because I is inversely proportional to area and area depends on the square of the distance). So

   $$I = \frac{1}{3140}\frac{\mathrm{W}}{\mathrm{m}^2}$$

A siren is emitting sound at a constant intensity level; assume assume that the relationship $I = P/4\pi r^2$ applies.

3. If you move three times farther away from the siren, by what ratio does the sound intensity change?

   Answer:

   Because intensity is proportional to $1/r^2$, moving 3 time farther away will decrease the intensity by a factor of $1/9$. This can be shown in more detail by noting that $I_1 = P_1/A_1$, $I_2 = P_2/A_2$ and the power is constant, so $P_1 = P_2$. Thus,

   $$\frac{I_2}{I_1} = \frac{A_1}{A_1} = \frac{4\pi d_1^2}{4\pi d_2^2} = \frac{d_1^2}{d_2^2}$$

   If I_1 is your initial position at d_1, then $d_2 = 3d_1$ and

   $$\frac{I_2}{I_1} = \frac{d_1^2}{d_2^2} = \frac{d_1^2}{(3d_1)^2} = \frac{1}{9}$$

   So decreases by a factor of 9.

4. If instead you moved four times closer to the siren, by what ratio does the sound intensity change?

   Answer: Increases by a factor of 16.

   $$\frac{I_2}{I_1} = \frac{d_1^2}{d_2^2} = \frac{d_1^2}{(d_1/4)^2} = 16$$

3 Sound Intensity Level, β

Because of the sound intensity values of human hearing span a very large range, from 0.0000000000001 to 100 $\mathrm{Watts/m^2}$ (see the table in the introduction), we define an alternative measure of intensity called the sound intensity *level*, β. The equation that relates sound intensity level I, with sound intensity, β, is

$$\beta = (10\text{ dB})\log_{10}\left(\frac{I}{I_o}\right)$$

where dB stands for "decibels".

One advantage to using this formula is that its values span a much smaller range, from 0–140, as shown in the second column of the table in the introduction. Another advantage is that the reference value of zero corresponds to something easily interpreted: 0 dB means something that is barely audible by humans.

1. Logarithmic scales are often used in science an engineering. Give at least one other example besides sound intensity level that is a quantity that is based on a logarithmic scale.

   Answer: pH scale for acidity, Richter earthquake magnitude scale.

2. If you increase the sound intensity of a speaker on a TV from I_1 to I_2 and $I_2/I_1 = 10$, what is $\beta_2 - \beta_1$?

   In the table, each factor of 10 increase in I corresponds to an change in β by $+10$. So $\beta_2 - \beta_1 = 10$ dB. We can also use

   $$\beta_1 = (10\text{ dB})\log_{10}\left(\frac{I_1}{I_o}\right)$$

   $$\beta_2 = (10\text{ dB})\log_{10}\left(\frac{I_2}{I_o}\right)$$

   $$\beta_2 - \beta_1 = (10\text{ dB})\left[\log_{10}\left(\frac{I_2}{I_o}\right) - \log_{10}\left(\frac{I_1}{I_o}\right)\right]$$

   Using $\log_{10} y - \log_{10} x = \log_{10}(y/x)$ gives

   $$\beta_2 - \beta_1 = (10\text{ dB})\left[\log_{10}\left(\frac{I_2}{I_1}\right)\right]$$

   So if $I_2/I_1 = 10$, we have

   $$\beta_2 - \beta_1 = (10\text{ dB})\left[\log_{10}(10)\right] = 10\text{ dB}$$

3. If you increase the sound intensity level of a speaker on a TV from β_1 to β_2 and $\beta_2 - \beta_1 = 20$ dB, what is I_2/I_1?

   Answer:

   $$20\text{ dB} = (10\text{ dB})\left[\log_{10}\left(\frac{I_2}{I_1}\right)\right]$$

   Dividing both sides by 10 dB gives

   $$2 = \log_{10}\left(\frac{I_2}{I_1}\right)$$

Raising both sides by 10 and using the identity $10^{\log_{10} x} = x$ gives

$$10^2 = \frac{I_2}{I_1}$$

This could also have been determined using the table in the introduction. The difference in β for a whisper (20 dB) to β for the threshold of hearing (0 dB) is 20 dB. The ratio of I for a whisper ($10^{-10}\ \mathrm{W/m^2}$) to I for the threshold of hearing ($10^{-12}\ \mathrm{W/m^2}$) is $10^{-10}/10^{-12} = 100$.

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4. A city council adopted a law to reduce the maximum allowed sound intensity level of leaf blowers from 95 dB to 70 dB. With the new law, what is the ratio of the new maximum allowed intensity to the previously allowed intensity?

   Answer:

   From the solution to problem 3.2,

   $$\beta_2 - \beta_1 = (10\ \mathrm{dB}) \left[\log_{10}\left(\frac{I_2}{I_1}\right)\right]$$

   If β_2 corresponds to the new maximum sound intensity level and β_1 the old,

   $$(70 - 95)\ \mathrm{dB} = (10\ \mathrm{dB}) \log_{10}\left(\frac{I_2}{I_1}\right)$$

   $$-2.5 = \left(\frac{I_2}{I_1}\right)$$

   $$10^{-2.5} = \frac{I_2}{I_1}$$

   $$\frac{I_2}{I_1} = \frac{1}{10^{2.5}} \simeq \frac{1}{316}$$

A siren is emitting sound at a constant intensity level

5. If you move three times closer to the siren, what is the change in the sound intensity level?
6. If instead you moved four times closer to the siren,, what is the change in the sound intensity level ?

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7. You are trying to hear a juicy conversation, but from your distance of 15.0 m it sounds like only an average whisper of 20.0 dB. How close should you move to the chatterboxes for the sound level to be 60.0 dB? Show your work.

   From the solution to problem 3.2,

   $$\beta_2 - \beta_1 = (10\ \mathrm{dB}) \left[\log_{10}\left(\frac{I_2}{I_1}\right)\right]$$

   From the solution to problem 2.2,

   $$\frac{I_2}{I_1} = \frac{d_1^2}{d_2^2}$$

8. Solve for I in the equation $\beta = (10\ \mathrm{dB}) \log_{10}\left(\frac{I}{I_o}\right)$

Answer:

Divide both sides by 10 dB

$$\beta/(10 \text{ dB}) = \log_{10}\left(\frac{I}{I_o}\right)$$

Raising both sides to the power of 10 gives

$$10^{\beta/(10 \text{ dB})} = 10^{\log_{10}\left(\frac{I}{I_o}\right)}$$

Using the identity $10^{\log_{10} x} = x$,

$$10^{\beta/(10 \text{ dB})} = \frac{I}{I_o}$$

Solving for I gives

$$I = I_o 10^{\beta/(10 \text{ dB})}$$

9. Solve for I_2/I_1 in the equation $\beta_2 - \beta_1 = (10 \text{ dB}) \log_{10}\left(\frac{I_2}{I_1}\right)$

   Answer:

   $$\frac{I_2}{I_1} = 10^{(\beta_2 - \beta_1)/(10 \text{ dB})}$$