

# Magnetic Flux

## 1 Introduction

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The definition of magnetic flux is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\mathbf{A}$$

When the magnitude and direction of  $\vec{\mathbf{B}}$  are the same at all points on the surface, the integral simplifies to

$$\Phi_B = \vec{\mathbf{B}} \cdot \mathbf{A}$$

or, equivalently,

$$\Phi_B = BA \cos \phi$$

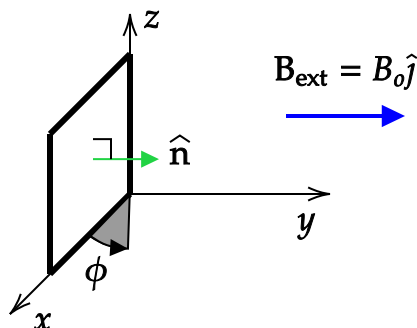
where  $\phi$  is the angle between the  $\vec{\mathbf{B}}$  and  $\vec{\mathbf{A}}$  vectors.

Note that previously, electric flux was covered. The same techniques that apply to computing electric flux apply to magnetic flux. See the Electric Flux activity for additional discussion.

## 2 Computing $\Phi_B$

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In the following figure, a square loop of area  $A$  that can be rotated about the  $z$  axis is shown. Assume that the normal direction of the loop is as shown in the diagram. A uniform external magnetic field points in the  $\hat{j}$  direction.



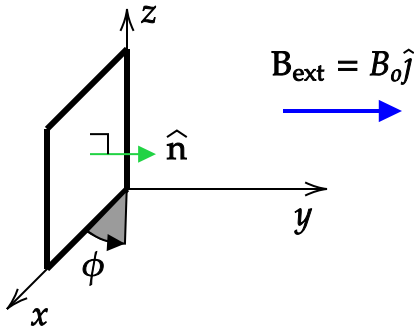
1. At what angles in the range of  $\phi = [0, 360^\circ]$  is the magnetic flux zero?
2. Draw the loop,  $\hat{n}$ , and  $\vec{B}_{\text{ext}}$  as they would appear when viewed from above (on the positive  $z$ -axis) when  $\phi = 0^\circ$ ,  $\phi = 45^\circ$ ,  $\phi = 135^\circ$ , and  $\phi = 180^\circ$ .
3. When  $\phi = 45^\circ$ , is the magnetic flux positive or negative? What is its value in terms of a fraction of  $B_0 A$ ?
4. When  $\phi = 135^\circ$ , is the magnetic flux positive or negative? What is its value in terms of a fraction of  $B_0 A$ ?
5. Sketch a plot of the magnetic flux,  $\Phi_B$ , as a function of  $\phi$ .

### 3 Computing $d\Phi_B/dt$

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The time rate of change of magnetic flux through a closed loop is a quantity that will be used when Faraday's law is covered.

Suppose the loop in the previous problem rotates at a constant rate such that  $\phi = \omega t$ , where  $\omega$  is a constant.



1. At what angles in the range  $\phi = [0^\circ, 360^\circ]$  is  $d\Phi_B/dt = 0$ ?
2. For what angle range is  $d\Phi_B/dt$  increasing?
3. For what angle range is  $d\Phi_B/dt$  decreasing?
4. What is the formula for  $d\Phi_B(t)/dt$  in terms of  $B_o$ ,  $A$ ,  $t$ , and  $\omega$ ?