

# Logarithms

## 1 Introduction – Base 10 Logarithms

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The motivation for the base 10 logarithm is that it reduces numbers raised by a power of 10 to the power the number was raised to. So  $10^2$  becomes 2,  $10^3$  becomes 3, etc. The base 10 logarithm is sometimes called the “common logarithm”.

In mathematical notation,

$$\log_{10}(10^x) = x$$

For example,

$$\log_{10}(10^{-5}) = -5 \text{ and } \log_{10}(10^7) = 7$$

(To take the base 10 logarithm of a number that is not exactly a power of 10, use a calculator.)

Several identities follow as a result:

1. If you raise a base 10 logged number by 10, you get back the number that was logged.

$$10^{\log_{10}(x)} = x$$

For example,

$$10^{\log_{10}(7)} = 7 \text{ and } 10^{\log_{10}(8.8)} = 8.8$$

2. The sum of two logged numbers is the log of the product of the numbers:

$$\log_{10} y + \log_{10} x = \log_{10}(yx);$$

For example,

$$\log_{10} 10 + \log_{10} 100 = \log_{10} 10 \cdot 100 = \log_{10} 10^3 = 3$$

3. The difference between two logged number is the log of the ratio of the numbers:

$$\log_{10} y - \log_{10} x = \log_{10}(y/x)$$

For example,

$$\log_{10} 10 - \log_{10} 100 = \log_{10}(10/100) = \log_{10} 10^{-1} = -1$$

### 1.1 Problems

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1. What is  $\log_{10}(0.000000001)$ ?

**Answer:**

2. What is  $\log_{10}(10,000)$ ?

**Answer:**

3. What is  $\log_{10}(10,000) + \log_{10}(0.000000001)$ ?

**Answer:**

4. What is  $\log_{10}(10,000) - \log_{10}(0.000000001)$ ?

**Answer:**

5. If  $x = x_o \log_{10}(y/y_o)$ , solve for  $y$  in terms of  $x$ .

**Answer:**

## 2 Introduction – Base $e$ Logarithm

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The base 10 logarithm reduces numbers raised by a power of 10 to the power the number was raised to.

The base  $e$  logarithm reduces numbers raised by a power of  $e$  to the power the number was raised to. It is represented by  $\log_e x$ , or more commonly,  $\ln(x)$ .

“ $\ln$ ” represents the “natural logarithm”. The term “natural” is used because the exponential  $e$  appears in many natural problems, for example, some populations grow in proportion to  $e^{t/\tau}$ , where the constant  $\tau$  is a growth rate.

In mathematical notation,

$\ln(e^x) = x$ ; for example  $\ln(e^{-5}) = -5$  and  $\ln(e^7) = 7$

Several identities follow as a result:

1. If you raise a base  $e$  logged number by  $e$ , you get back the number that was logged.

$$e^{\ln(x)} = x$$

2. The sum of two logged numbers is the log of the product of the numbers:

$$\ln(y) + \ln(x) = \ln(yx);$$

3. The difference between two logged number is the log of the ratio of the numbers:

$$\ln(y) - \ln(x) = \ln(y/x)$$

### 2.1 Problems

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1. What is  $\ln(e^3)$ ?

**Answer:**

2. What is  $\ln(1/e^3)$ ?

**Answer:**

3. What is  $\ln(e^{-4}) + \ln(e^3)$ ?

**Answer:**

4. What is  $\ln(e^3) - \ln(e^{-2})$ ?

**Answer:**

5. If  $x = x_o \ln(y/y_o)$ , solve for  $y$  in terms of  $x$ .

**Answer:**

