Magnetic Field of a Moving Charge

1 Introduction

In previous activities, you computed the force on moving charges in a region of space where there is a magnetic field. No mention was made of how the magnetic field was created.

In this activity, you compute the magnetic field created by moving charges.

The magnetic field due to a point charge q moving with velocity $\vec{\mathbf{v}}$ (when $|\vec{\mathbf{v}}|$ is small compared to the speed of light) is

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

where $\hat{\mathbf{r}}$ is the unit vector that points from the position of q to the point in space where we want to know $\vec{\mathbf{B}}$, and r is the distance between q and that point.

To find $\hat{\mathbf{r}}$ (see also the $\hat{\mathbf{r}}$ Unit Vector activity),

1. draw a vector, $\vec{\mathbf{r}}$ from q to the point in space where you want to know $\vec{\mathbf{B}}$;
2. Write $\vec{\mathbf{r}}$ in the form $\vec{\mathbf{r}} = r_x\hat{\boldsymbol{\imath}} + r_y\hat{\boldsymbol{\jmath}}$; then
3. $\hat{\mathbf{r}} = \vec{\mathbf{r}}/r$, where $r = \sqrt{r_x^2 + r_y^2}$.

In this activity, the examples and solutions are given using the above approach.

An alternative approach for computing $\vec{\mathbf{B}}$ is to use the fact that $\vec{\mathbf{v}} \times \hat{\mathbf{r}} = |\vec{\mathbf{v}}|\sin\phi = v\sin\phi$, where ϕ is the angle between $\vec{\mathbf{v}}$ and $\hat{\mathbf{r}}$ and $0 \leq \phi \leq 180°$. With this, the magnitude of the magnetic field is

$$B = \frac{\mu_o}{4\pi}\frac{|q|v\sin\phi}{r^2}$$

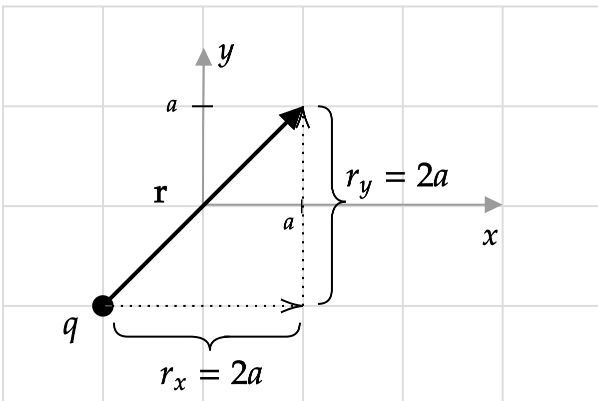
and the right–hand rule is used to determine the dirction of $\vec{\mathbf{B}}$. See the Cross Products activity for a discussion of when and how to compute the cross-product using this method.

2 Example

If q is at $(x, y) = (-a, -a)$ and has a velocity of $\vec{\mathbf{v}} = v_o\hat{\boldsymbol{\imath}}$, find the magnetic field vector at $(x, y) = (a, a)$.

Solution

To find $\hat{\mathbf{r}}$, we draw a vector from q to the point where we want to compute $\vec{\mathbf{B}}$.



Based on the diagram, $\vec{\mathbf{r}} = 2a\hat{\boldsymbol{\imath}} + 2a\hat{\boldsymbol{\jmath}}$ and $r = \sqrt{(2a)^2 + (2a)^2} = 2\sqrt{2}a$, so

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r} = \left[\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}\right]$$

The cross-product is

$$\vec{\mathbf{v}} \times \hat{\mathbf{r}} = v_o\hat{\boldsymbol{\imath}} \times \left[\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}\right] = \frac{v_o}{\sqrt{2}}(\hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\jmath}}) = \frac{v_o}{\sqrt{2}}\hat{\boldsymbol{k}}$$

Substitution into

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

gives

$$\vec{\mathbf{B}}(a, a) = \frac{\mu_o}{4\pi}\frac{q\frac{v_o}{\sqrt{2}}\hat{\boldsymbol{k}}}{(2\sqrt{2}a)^2} = \frac{\mu_o}{4\pi}\frac{qv_o}{(8\sqrt{2})a^2}\hat{\boldsymbol{k}}$$

Check: Use the right–hand rule for cross products on $\vec{\mathbf{v}} \times \hat{\mathbf{r}}$ to verify that the result is out of the page. (Why do we know that the $\hat{\boldsymbol{k}}$ direction is out of the page?)

3 Problem I

If q is at $(x, y) = (a, a)$ and has a velocity of $\vec{\mathbf{v}} = v_o\hat{\boldsymbol{\imath}}$, find the magnetic field vector at $(x, y) = (-a, -a)$.

4 Problem II

If q is at $(x, y) = (a, 0)$ and has a velocity of $\vec{\mathbf{v}} = v_o\hat{\boldsymbol{\jmath}}$, find the magnetic field vector at $(x, y) = (a, a)$.

5 Problem III

If q is at $(x, y) = (a, 2a)$ and has a velocity of $\vec{\mathbf{v}} = v_o \hat{\boldsymbol{\jmath}}$, find the magnetic field vector at $(x, y) = (-a, -a)$.

6 Problem IV

If q is at $(x, y) = (x_o, y_o)$ and has a velocity of $\vec{\mathbf{v}} = v_x \hat{\boldsymbol{\imath}} + v_y \hat{\boldsymbol{\jmath}}$, show

$$\vec{\mathbf{B}}(x, y) = \frac{\mu_o}{4\pi} \frac{1}{r^3} \left[v_x (y - y_o) - v_y (x - x_o) \right] \hat{\boldsymbol{k}}$$

where

$$r = \sqrt{(x - x_o)^2 + (y - y_o)^2}$$