# Electric Force

This activity covers topics in Section 21.3 of Young and Freedman 2015, 14th Edition. If you need to review vectors, see sections 1.6-1.8 of Young and Freedman 2015, 14th Edition and Vectors at Khan Academy.

## 1 Coulomb's Law

*Magnitude*

$$F_{1\text{ on }2} = F_{2\text{ on }1} = k\frac{|q_1 q_2|}{r^2}$$

where $r$ is the distance between $q_1$ and $q_2$. To simplify notation, we are using $k$ in place of $1/4\pi\epsilon_o$. Note that by definition, the magnitude of a vector is positive, which is the reason for the use of the absolute value.

*Direction*: Along line that connects $q_1$ and $q_2$. Direction depends on signs of $q_1$ and $q_2$. (Likes repel, opposites attract.).

## 2 Example

Charge $q_1$ is at $(x, y) = (-a, -a)$ and charge $q_2$ is at $(a, a)$. Both charges have a charge of $q$.

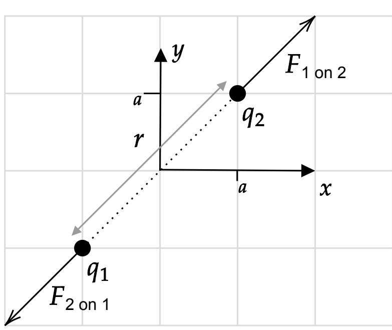
1. Find the magnitude and direction of the force of $q_1$ on $q_2$.
2. Write the force of $q_1$ on $q_2$ in the form $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

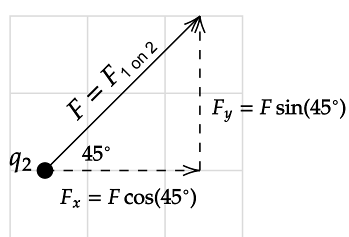
**Solution**

1. The distance between the charges is $r = \sqrt{(2a)^2 + (2a)^2} = \sqrt{8a^2}$, so

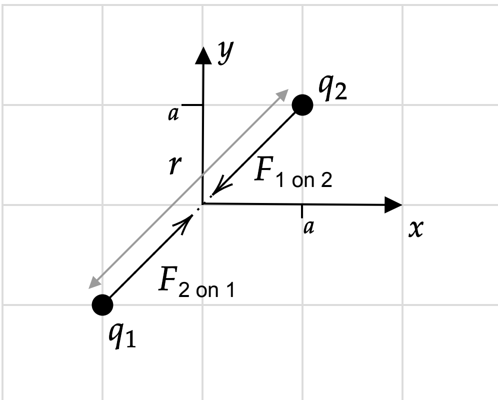
   $$F_{1\text{ on }2} = k\frac{|q_1 q_2|}{r^2} = \frac{k|qq|}{(\sqrt{8a^2})^2} = \frac{kq^2}{8a^2}$$

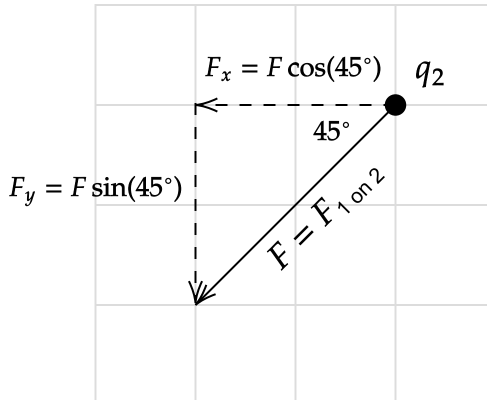
   The charges will repel each other, so the direction of forces of one on the other will be as shown in the left part of the following diagram.



2. Let $F = F_{1\text{ on }2}$ from part 1. to simplify notation. The right part of the above diagram shows the calculation of the components $F_x$ and $F_y$, from which it follows that $\vec{\mathbf{F}} = F\cos 45^\circ\,\hat{\boldsymbol{\imath}} + F\sin 45^\circ\,\hat{\boldsymbol{\jmath}}$.
3. The magnitude will not change (it is by definition a positive number). The force vectors will reverse direction as shown on the left in the following diagram. The diagram on the right shows the calculation of $\vec{\mathbf{F}}_{1\text{ on }2}$, from which it follows that $\vec{\mathbf{F}}_{1\text{ on }2} = -F\cos 45^\circ\,\hat{\boldsymbol{\imath}} - F\sin 45^\circ\,\hat{\boldsymbol{\jmath}}$. Note that reversing the direction of a vector is the same as multiplying each of its components by $-1$.




## 3 Problem I

Charge $q_1$ is at $(x, y) = (-a, a)$ and charge $q_2$ is at $(a, -a)$. Both charges have a charge of $q$. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of $q_1$ on $q_2$.
2. Write the force of $q_1$ on $q_2$ in the form $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

**Solution**

1. $F_{1\text{ on }2} = k\dfrac{|q_1q_2|}{r^2} = \dfrac{k|qq|}{(\sqrt{8a^2})^2} = \dfrac{kq^2}{8a^2}$
2. $\vec{\mathbf{F}}_{1\text{ on }2} = F_{1\text{ on }2}(\cos 45^\circ\,\hat{\boldsymbol{\imath}} - \sin 45^\circ\,\hat{\boldsymbol{\jmath}})$
3. 1.: No change; 2. $\vec{\mathbf{F}}_{1\text{ on }2} = F_{1\text{ on }2}(-\cos 45^\circ\,\hat{\boldsymbol{\imath}} + \sin 45^\circ\,\hat{\boldsymbol{\jmath}})$

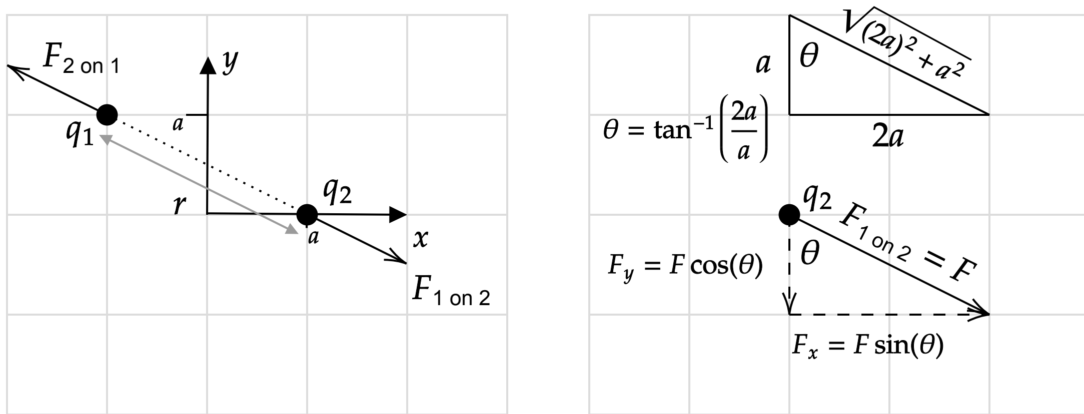
## 4 Problem II

Charge $q_1$ is at $(x, y) = (-a, a)$ and charge $q_2$ is at $(a, 0)$. Charge $q_1$ has a charge of $+q$. Charge $q_2$ has a charge of $+q$, where $q$ is a positive number. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of $q_1$ on $q_2$.
2. Write the force of $q_1$ on $q_2$ in the form $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$.

3. If the charges have opposite signs, how will your answers to 1. and 2. change?

**Solution**




1. $F_{1 \text{ on } 2} = k\dfrac{|q_1 q_2|}{r^2} = \dfrac{k|qq|}{(\sqrt{(2a)^2 + a^2})^2} = \dfrac{kq^2}{5a^2}$
2. $\vec{\mathbf{F}}_{1 \text{ on } 2} = F_{1 \text{ on } 2}(\sin\theta\hat{\boldsymbol{\imath}} - \cos\theta\hat{\boldsymbol{\jmath}})$, where $\theta = \tan^{-1}(2) = 63.4°$.

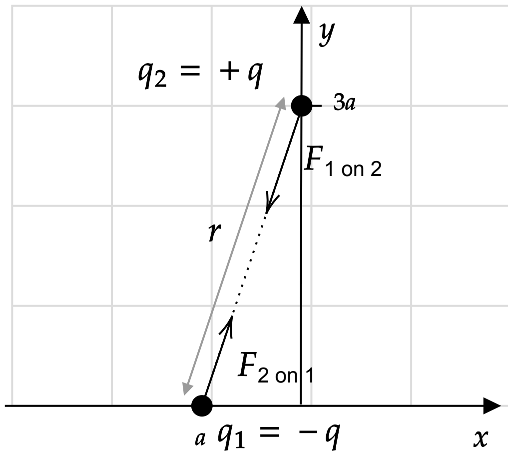
   Alternatively, from the diagram on the right, $\sin\theta = 2a/\sqrt{5}a$ and $\cos\theta = 1a/\sqrt{5}a$, so
   $\vec{\mathbf{F}}_{1 \text{ on } 2} = F_{1 \text{ on } 2}\left(\dfrac{2}{\sqrt{5}}\hat{\boldsymbol{\imath}} - \dfrac{1}{\sqrt{5}}\hat{\boldsymbol{\jmath}}\right)$
3. 1.: No change; 2. $\vec{\mathbf{F}}_{1 \text{ on } 2} = F_{1 \text{ on } 2}(-\sin\theta\hat{\boldsymbol{\imath}} + \cos\theta\hat{\boldsymbol{\jmath}})$; $\theta = 63.4°$

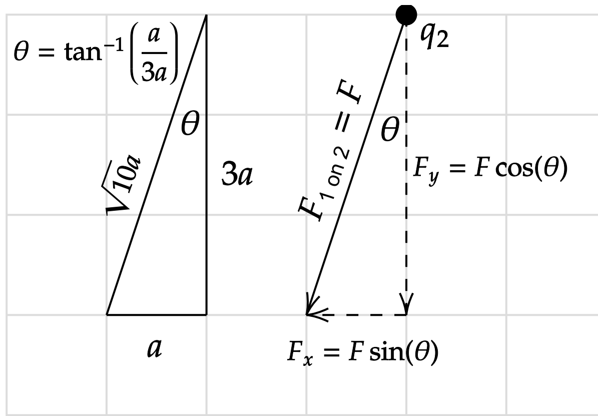
## 5 Problem III

Charge $q_1$ is at $(x, y) = (-a, 0)$ and charge $q_2$ is at $(0, 3a)$. Charge $q_1$ has a charge of $-q$. Charge $q_2$ has a charge of $+q$, where $q$ is a positive number. Draw this charge configuration and then using the steps in the previous example,

1. Find the magnitude and direction of the force of $q_1$ on $q_2$.
2. Write the force of $q_1$ on $q_2$ in the form $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$.
3. If the charges have opposite signs, how will your answers to 1. and 2. change?

**Solution**






1. $r = \sqrt{a^2 + (3a)^2}$, $F = k|q(-q)|/r^2 = kq^2/10a^2$
2. $\vec{\mathbf{F}} = -F\sin\theta\hat{\imath} - F\cos\theta\hat{\jmath}$, where $\theta = \tan^{-1}(1/3) = 18.4°$.

   Alternatively, from the diagram, $\sin\theta = a/\sqrt{10}a$ and $\cos\theta = 3a/\sqrt{10}a$, so $\vec{\mathbf{F}}_{1\text{ on }2} = F\left(-\frac{1}{\sqrt{10}}\hat{\imath} - \frac{3}{\sqrt{10}}\hat{\jmath}\right)$.
3. 1.: No change; 2.: Note: the problem statement should have been "if both charges have the *same* sign" (the charges were given to have opposite signs). In this case: $\vec{\mathbf{F}}_{1\text{ on }2} = +F\sin\theta\hat{\imath} + F\cos\theta\hat{\jmath}$.

## 6 Problem IV

Charge $q_1$ is at $(x, y) = (x_1, y_1)$ and charge $q_2$ is at $(x_2, y_2)$. Find the magnitude of the force of $q_1$ on $q_2$.

**Solution**: $F = kq^2/\left((x_2 - x_1)^2 + (y_2 - y_1)^2\right)$. Make sure that you can justify this with a diagram.