Capacitance

1 Introduction

Note that we use the the letter “C” as a label for a capacitor and in the units of capacitance. When used as a label, C is usually written in itallics. Thus

“$C = 10$ C” means “the capacitor labeled C has a capacitance of 10 Coulombs.

2 Discharging Capacitor

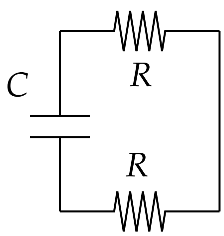
If a single charged capacitor with capacitance C is in a circuit with other resistors (and no batteries), the charge on the capacitor changes with time according to

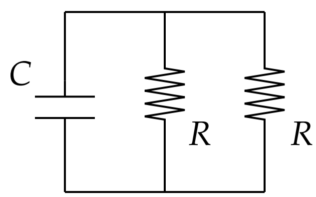
$$q(t) = Q_o e^{-t/\tau},$$

where $\tau \equiv RC$, Q_o is the charge on the capacitor at $t = 0$, and R is the equivalent resistance. The quantity τ is often referred to as the “RC time constant”.

2.1 Problem

Find τ for the following two circuits. Use $R = 10$ kΩ and $C = 1$ μF.



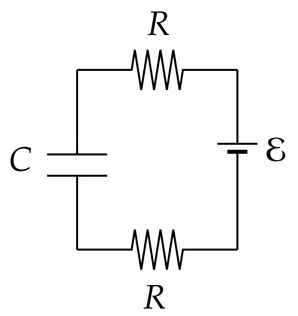


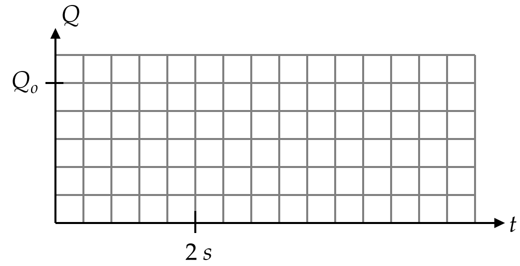
2.2 Problem

For the following circuit, find the charge Q on the capacitor at $t = 2$ s if the switch is closed at $t = 0$ and the capacitor has an inital charge of Q_o for the following three cases.

1. $RC = 1$ s $\quad q(t = 2\text{ s}) = \boxed{\quad\quad}$
2. $RC = 2$ s $\quad q(t = 2\text{ s}) = \boxed{\quad\quad}$
3. $RC = 4$ s $\quad q(t = 2\text{ s}) = \boxed{\quad\quad}$

Next sketch a plot of $q(t)$ from $t = 0$ to $t = 6$ s for each of these three cases. Used a solid, dashed, and dotted line for case 1., 2., and 3., respectively.





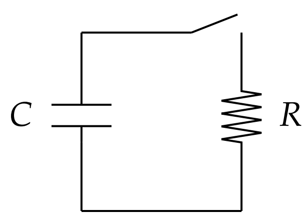
Answer:

$q(2) = Q_o e^{-2/RC}$, assuming RC has units of seconds. So

1. $q(2) = Q_o e^{-2/1} = Q_o/e^2 \simeq 0.14Q_o$
2. $q(2) = Q_o e^{-2/2} = Q_o/e \simeq 0.37Q_o$
3. $q(2) = Q_o e^{-2/4} = Q_o/\sqrt{e} \simeq 0.61Q_o$

All curves decay exponentially, start at Q_o, and pass through one of the values above $t = 2$.

2.3 Problem



In the circuit above, the values of R and C are such that $RC = \tau = 1$ s.

1. If the capacitor has a charge of 10 nC and the switch is closed, how long will it take for the charge on the capacitor to fall to half of its value?
2. If the capacitor instead had a charge of 20 nC and $RC = 1$ s and the switch is closed, how long will it take for the charge on the capacitor to fall to half of its value?

Answer:

$q(t) = Q_o e^{-t/\tau}$ and we want t when $q(t) = Q_o/2$, so need to solve $Q_o/2 = Q_o e^{-t/\tau}$, or $1/2 = e^{-t/\tau}$. Taking the natural log of boths sides, and using $\ln(1/x) = -\ln x$, gives

$$-\ln(2) = \ln e^{-t/\tau} = -\frac{t}{\tau}$$

so $t = \tau \ln(2) \simeq 0.70\tau$.

3 Charging a Capacitor

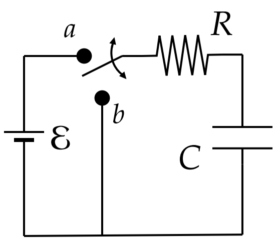
If a single capacitor is in a circuit with other resistors and a DC voltage source, the charge of the capacitor varies according to

$$Q(t) = Q_o(1 - e^{-t/\tau})$$

where $\tau \equiv RC$, Q_o is the charge on the capacitor at $t = 0$, and R is the equivalent resistance.

4 Charge/Discharge Problem

4.1 Part I



The switch in the above circuit has been in position a for a long time. At time $t = 0$ the switch is moved to position b. The values of circuit elements are: $\mathcal{E} = 12$ V, $C = 10$ mF, and $R = 20\ \Omega$.

Let $t = 0^-$ correspond to the time just before the switch is moved from a to b. Let $t = 0^+$ correspond to the time just after the switch is moved from a to b.

1. What is the current through the resistor at $t = 0^-$?

   Answer: Just before the switch is thrown, the current is zero. By Kirchhoff's loop rule, the voltage across the capacitor, V_C, must be equal to the voltage of the power source, so $V_C = \mathcal{E} = 12$ V.

2. What is the current through the resistor at $t = 0^+$?

   Answer: Immediately after the switch is moved to b, the capacitor starts to discharge though the resistor. By Kirchhoff's loop rule, the voltage across the resistor equals the voltage across the capacitor, V_C. The current through the resistor is $I = V_C/R = 12\text{ V}/20\ \Omega = 0.6$ A.

3. What is the charge on the capacitor at $t = 0^-$?

   Answer: Just before the switch is thrown the charge on the capacitor is:
   $Q = CV_C = (10\text{ mF})(12\text{ V}) = 120$ mC.

4. What is the charge on the capacitor at $t = 0^+$?

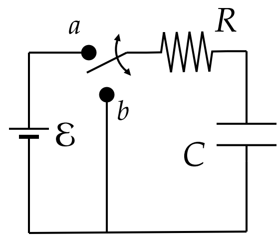
   Answer: 120 mC

5. What is the charge on the capacitor at $t = 200$ ms?

   Answer: The time constant for the RC circuit is $RC = (10\text{ mF})(20\ \Omega) = 200$ ms. At $t = 200$ ms after the switch was moved, the capacitor is in the process of discharging. The charge on the capacitor varies according to $q(t) = Q_o e^{-t/RC}$, so

   $q(200\text{ ms}) = Q_0 e^{-200\text{ ms}/RC} = 120\text{ mC} \cdot e^{-200\text{ ms}/200\text{ ms}} \simeq 120\text{ mC} \cdot 0.37 \simeq 44$ mC.

4.2 Part II



The switch in the above circuit has been in position b for a long time. At time $t = 0$ the switch is moved to position a. The values of circuit elements are: $\mathcal{E} = 12$ V, $C = 10$ mF, and $R = 20\ \Omega$.

1. What is the current through the resistor just **before** the switch is thrown?

   Answer: Since the switch is on position b for a long time, the charge across the capacitor is 0 (fully discharged). There is no current in the circuit, that is, the current through the resistor is 0.

2. What is the current through the resistor just **after** the switch is thrown? When

   Answer: Just after the switch is thrown to position a, the voltage across the capacitor is 0, and the voltage across the resistor is V_b. From Ohm's law, the current through the resistor is $I = V_b/R = 12V/20\Omega = 0.6A$.

3. What is the charge on the capacitor just **before** the switch is thrown?

   Answer: Before the switch is thrown, the charge on the capacitor is 0.

4. What is the charge on the capacitor just **after** the switch is thrown?

   Answer: The charge on the capacitor just after the switch is thrown remains zero.

5. What is the charge on the capacitor at time t = 0.3 msec after the switch is thrown?

   Answer: After the switch is thrown, the capacitor is in a charging process. The time constant is RC=200 msec. At time=0.3, the charge accumulated on the capacitor is:

   $Q = CV_b[1 - e^{t/RC}] = 10mF \times 12V[1 - e^{-0.3/200}] = 0.18mC$