PHYS 305 Make-up Final Exam | December 15th, 2021

- Write your name on all pages.
- Solve all five problems.
- Each problem is equally weighted.

Equations

$\mathbf{P} = \epsilon_o \chi_e \mathbf{E}$

$\mathbf{M} = \dfrac{\chi_m}{\mu_o(1+\chi_m)}\mathbf{B}$

$\nabla f = \frac{\partial f}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial f}{\partial s}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$

$\boldsymbol{\nabla} \cdot \mathbf{U} = \frac{1}{s}\frac{\partial (sU_s)}{\partial s} + \frac{1}{s}\frac{\partial U_\phi}{\partial \phi} + \frac{\partial U_z}{\partial z}$

$\boldsymbol{\nabla} \times \mathbf{U} = \left(\frac{1}{s}\frac{\partial U_z}{\partial \phi} - \frac{\partial U_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial U_s}{\partial z} - \frac{\partial U_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left(\frac{\partial (sU_\phi)}{\partial s} - \frac{\partial U_s}{\partial \phi}\right)\hat{\mathbf{z}}$

$\boldsymbol{\nabla} f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$

$\boldsymbol{\nabla} \cdot \mathbf{U} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 U_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(U_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial U_\phi}{\partial \phi}$

$\boldsymbol{\nabla} \times \mathbf{U} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}\left(U_\phi \sin\theta\right) - \frac{\partial U_\theta}{\partial \phi}\right)\hat{\mathbf{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial U_r}{\partial \phi} - \frac{\partial}{\partial r}\left(rU_\phi\right)\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rU_\theta\right) - \frac{\partial U_r}{\partial \theta}\right)\hat{\boldsymbol{\phi}}$

1 Ampere's Law

In cylindrical coordinates with cylindrical unit vectors, the magnetic field can be written in the form

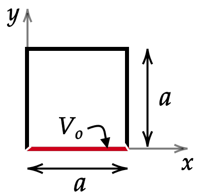
$\mathbf{B}(s,\phi,z) = B_s(s,\phi,z)\hat{\mathbf{s}} + B_\phi(s,\phi,z)\hat{\boldsymbol{\phi}} + B_z(s,\phi,z)\hat{\mathbf{z}}$

To find B_ϕ due to current flowing along an infinitely long wire that runs along the z-axis, Ampere's law can be used.

1. Explain why B_s and B_z are zero.
2. When using Ampere's law to find B_ϕ, a justification must be made for why B_ϕ is independent of ϕ.
   1. Provide this justification.
   2. Explain where this justification is used.

2 Boundary Value Problem

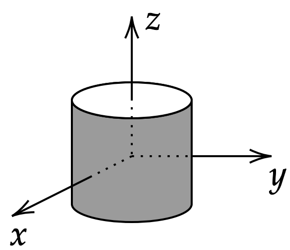
The cross–section of a long conducting duct is shown in the following figure. Three of the sides are held at $V = 0$ and the bottom side is held at $V = V_o$.



Find $V(x, y)$.

3 Charge on Cylinder

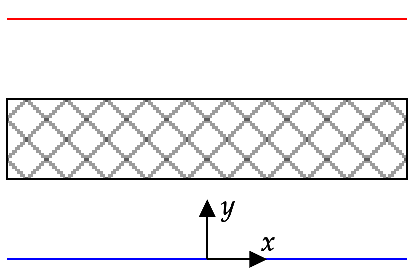
Charge is uniformly distributed on the curved surface of a cylinder of length h and radius R. The cylinder is centered on the origin, aligned with the z–axis, and has a charge density of σ_o.



Find an equation for $\mathbf{E}$ on the z–axis in terms of a single integral with an integrand that depends only on dz', z', z, and R. You do not need to evaluate the integral.

4 Magnetizable Object

A slab of magnetizable material is placed between two infinite current–carrying sheets shown in the following figure. The cross section below is in the $z = 0$ plane.



The slab is between $y = t$ and $y = 2t$ and is infinite in extent in the x and z directions. The bottom sheet is in the $y = 0$ plane and carries a surface current with density of $K_o\hat{\mathbf{x}}$. The top sheet is in the $y = 3t$ plane and carries a surface current with density of $-K_o\hat{\mathbf{x}}$.

You may assume without proof that $\mathbf{J}_b = 0$.

1. Find and plot $B(y)$ if the slab has a magnetic susceptibility of $\chi_m = 0$.
2. Find and plot $B(y)$ if the slab has $\chi_m = 0.5$.

5 Current–Carrying Slab

The slab shown on the left in the following figure carries a current density of $J_o\hat{\mathbf{x}}$. A cross section of the slab is shown on the right. Assume $w \gg t$ so that the slab can be treated as infinite in the x and y directions.

1. What is the direction of $\mathbf{B}$ outside of the slab ($z > t/2$ and $z < -t/2$)?
2. What is $\mathbf{B}$ in the x–y plane?
3. Find $\mathbf{B}(z)$