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THE ORIGIN OF THE SOLAR FLARE WAITING-TIME DISTRIBUTION

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ABSTRACT

It was recently pointed out that the distribution of times between solar flares (the flare waiting-time distribution) follows a power law for long waiting times. Based on 25 years of soft X-ray flares observed by *Geostationary Operational Environmental Satellite* instruments, it is shown that (1) the waiting-time distribution of flares is consistent with a time-dependent Poisson process and (2) the fraction of time the Sun spends with different flaring rates approximately follows an exponential distribution. The second result is a new phenomenological law for flares. It is shown analytically how the observed power-law behavior of the waiting times originates in the exponential distribution of flaring rates. These results are argued to be consistent with a nonstationary avalanche model for flares.

Subject headings: MHD — Sun: activity — Sun: corona — Sun: flares — X-rays

1. INTRODUCTION

The distribution of times between flares ("waiting times") gives information about whether flares occur as independent events and provides a test for models for flare statistics. For example, the avalanche model for flares (Lu & Hamilton 1991; Lu et al. 1993) is a model designed to reproduce the observed power-law distributions of flare energy and duration. Flares are described as redistribution events in a cellular automaton (CA) that is driven to a self-organized critical state. Because the system is driven at a constant (mean) rate and flares occur as independent events, the model makes the specific prediction that the flare waiting-time distribution (WTD) is a simple exponential, consistent with a Poisson process.

Observational determinations of the flare WTD have given varying results. Determinations based on hard X-ray observations have focused on the distribution of short waiting times (seconds–hours). Biesecker (1994) found the WTD for hard X-ray bursts observed by the Burst and Transient Source Experiment on the *Compton Gamma Ray Observatory* to be consistent with a time-dependent Poisson process, i.e., one in which the mean flaring rate is time-varying. This result is consistent with a nonstationary avalanche model for flares (an avalanche model driven with a nonconstant rate). However, Wheatland, Sturrock, & McTiernan (1998) found an overabundance of short waiting times (by comparison with a time-dependent Poisson process) in hard X-ray bursts observed by the *International Cometary Explorer* spacecraft. (For another determination of the WTD based on hard X-ray, see Pearce, Rowe, & Yeung 1993 and Crosby 1996.)

Recently, the distribution of times between soft X-ray flares observed by the *Geostationary Operational Environmental Satellite* (*GOES*) sensors between 1976 and 1996 was examined by Boffeta et al. (1999). The advantage of the *GOES* data is that it provides a long sequence of data with few gaps, and so the flare WTD can be examined for long waiting times. Boffeta et al. found that the distribution follows a power law for waiting times greater than a few hours. They argued that this result is inconsistent with the avalanche model and that the appearance of a power law suggests a turbulence model for the origin of flares.

In this Letter the *GOES* data is reexamined. It is shown that the observed, power-law–like WTD is consistent with a piecewise-constant Poisson process and hence with the nonstationary avalanche model. Furthermore, it is shown that the time distribution of rates of the *GOES* flares averaged over several solar cycles is approximately exponential. This is a new phenomenological law for flaring. Finally, it is shown analytically how a piecewise-constant Poisson process with an exponential distribution of rates has a WTD that is power-law–distributed for long waiting times, consistent with the observations.

2. DATA ANALYSIS

The data examined here are the catalog of flares observed during 1975–1999 by the 1–8 Å *GOES* sensors (see Garcia 1994 for details of the *GOES* instrument). The chosen period of time covers three solar cycles (21, 22, and 23). Because the soft X-ray background rises with the solar cycle, flares are undercounted near solar maximum, by comparison with solar minimum. Hence, only those flares with a peak flux greater than a threshold value (10^{-6} W m^{-2}, corresponding to a *GOES* C1.0 class flare) are included in the study. This leaves a total of 32,563 flares.

Figure 1 shows the WTD for the *GOES* events included in this study (*histogram*), constructed from differences between start times of flares. The figure agrees well with Figure 1 in Boffeta et al. (1999) and in particular shows the same power-law–like behavior for waiting times greater than a few hours. The index of the power law is about -2.16 ± 0.05 (for waiting times >10 hr), which may be compared with Boffeta et al.'s estimate of -2.4 ± 0.1. The difference in power-law indices is due to the restriction to flares greater than class C1; Boffeta et al. included all flares in their determination of the WTD. Error bars are plotted on the histogram in Figure 1, corresponding to the square root of the number of waiting times in each bin. The meaning of the solid and dashed curves in the figure is explained below.

To compare the observed occurrence of flares with that expected from a time-dependent Poisson process, it is necessary to determine the mean rate of flaring as a function of time. For this step, a Bayesian procedure devised by Scargle (1998) was used. (The same procedure was used in Wheatland et al. 1998.) The method takes a sequence of times of events and determines a decomposition into intervals of time when the observed event occurrence is consistent with a (constant rate) Poisson process. These intervals are characterized by a duration t_i and a rate λ_i and are referred to as "Bayesian blocks." The procedure has

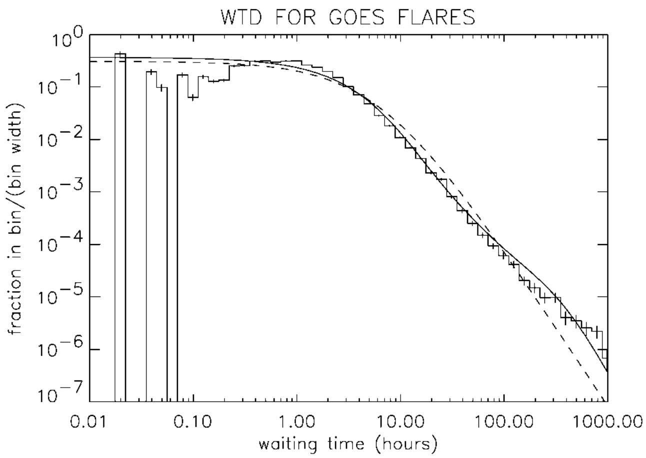

FIG. 1.—WTD for the *GOES* flares (*histogram*). The solid curve shows the result for a time-dependent Poisson process with a distribution of rates estimated from the data. The dashed curve is a theoretical distribution corresponding to independent events with an exponential distribution of rates.

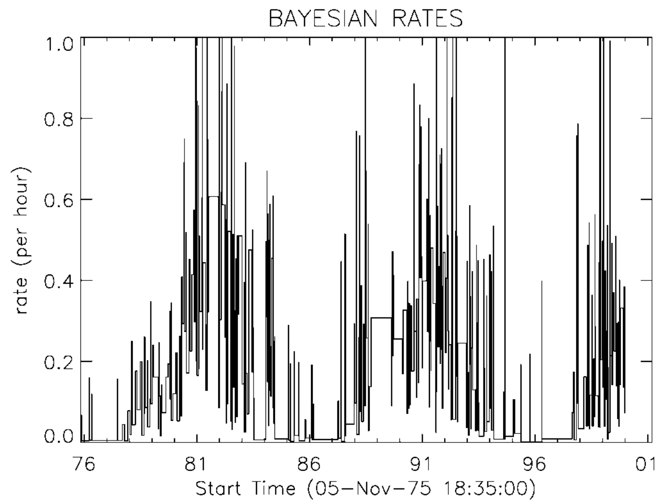

FIG. 2.—Bayesian blocks decomposition of the rate of occurrence of the *GOES* flares.

only one free parameter, a "prior odds ratio," which disfavors further segmentation of intervals when the single-rate and dual-rate Poisson model are almost equally likely (see Scargle 1998 for further details). However, very similar results are achieved for different choices of this ratio. In the following analysis, the value of prior odds = 2 is used.

Figure 2 shows the results of the application of the Bayesian procedure to the *GOES* data. The method has decomposed the 25 years of flaring into 390 Bayesian blocks. The rate of flaring is observed to vary with the solar cycle, as expected, and also exhibits short timescale variations. Relatively long intervals with a constant rate are also observed.

A Poisson process with a constant rate λ has a WTD given by $P(\Delta t) = \lambda \exp(-\lambda\Delta t)$, where Δt describes a waiting time. The WTD for a piecewise-constant Poisson process with rates λ_i and intervals t_i may be approximated by

$$P(\Delta t) \approx \sum_i \varphi_i \lambda_i \exp(-\lambda_i \Delta t), \quad (1)$$

where

$$\varphi_i = \frac{\lambda_i t_i}{\sum_j \lambda_j t_j} \quad (2)$$

is the fraction of events associated with a given rate λ_i.

The rates and intervals shown in Figure 2 were used to construct a model WTD from equation (1). The result is plotted in Figure 1 as the solid curve. It is clear that there is good qualitative agreement between the observed and model WTDs. In particular, the model distribution reproduces power-law–like behavior for long waiting times and is relatively constant for short waiting times. There is some discrepancy between the curves, e.g., there are too few observed short waiting times and too many observed waiting times near the rollover in the distribution. However, it is likely that there are errors in the observational determination of the WTD. For example, short waiting times are likely to be missed as a result of the overlap of flares close in time. Also, the Bayesian method for determining rates from the data may produce some erroneous rates and intervals, and equation (1) and decomposition into a piecewise-constant Poisson process involve approximations that are difficult to precisely quantify. The good qualitative agreement of the model and observed distributions is taken as strong evidence that the *GOES* flares occur as a time-varying Poisson process.

3. THE ORIGIN OF THE POWER-LAW BEHAVIOR

The Bayesian procedure decomposed the *GOES* time series into a large number of Bayesian blocks. For a piecewise-constant Poisson process involving a large number of rates, the summation in equation (1) may be replaced by an integral:

$$P(\Delta t) = \frac{1}{\lambda_0}\int_0^\infty f(\lambda)\lambda^2 e^{-\lambda\Delta t}\, d\lambda, \quad (3)$$

where $f(\lambda)d\lambda$ is the fraction of time that the flaring rate is in the range $(\lambda, \lambda + d\lambda)$ and

$$\lambda_0 = \int_0^\infty \lambda f(\lambda) d\lambda \quad (4)$$

is the mean rate of flaring.

Equation (3) for the WTD of a piecewise-constant Poisson process depends on only the time distribution of the rates of flaring, $f(\lambda)$. Figure 3 shows this distribution (*histogram*), constructed from the rates and intervals shown in Figure 2. This figure reveals the remarkable fact that the rate of flaring—effectively averaged over several solar cycles—follows an exponential distribution, a result that does not appear to have been noted in the literature before. The observed distribution may be approximated by

$$f(\lambda) = \lambda_0^{-1} \exp(-\lambda/\lambda_0), \quad (5)$$

where $\lambda_0 \approx 0.15\ \mathrm{hr}^{-1}$ is obtained from the total number of flares divided by the total observing time. Equation (5) is shown by the straight line in Figure 3. The observed distribution does not agree exactly with the exponential form. The cumulative probability distribution corresponding to Figure 3 (this distribution is preferable to the differential distribution because it

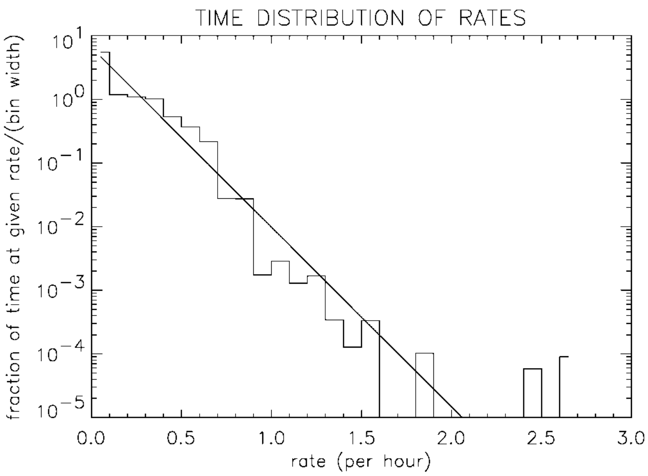

FIG. 3.—Distribution of flaring rates, based on the Bayesian rate estimates of Fig. 2.

involves no binning) was compared with the model distribution corresponding to equation (5) using the Kolmogorov-Smirnov test (Babu & Feigelson 1996). This test excludes the possibility that the two distributions are the same at a high level of significance. However, the observationally inferred distribution of rates is somewhat uncertain, and the exponential model clearly provides a good first approximation to the observed distribution.

Substituting equation (5) into equation (3), the integral may be evaluated to give

$$P(\Delta t) = \frac{2\lambda_0}{(1 + \lambda_0 \Delta t)^3}. \qquad (6)$$

Equation (6) is plotted in Figure 1 as the dashed curve. For short waiting times ($\Delta t \ll \lambda_0^{-1}$), equation (6) approaches the value $P(\Delta t) = 2\lambda_0$. For long waiting times ($\Delta t \gg \lambda_0^{-1}$), the distribution has the power-law form $P(\Delta t) \sim 2\lambda_0^{-2}(\Delta t)^{-3}$. Hence, equation (6) accounts for the qualitative behavior of the WTD: in particular, the power-law behavior for large waiting times, the location of the rollover to power-law form ($\Delta t \approx \lambda_0^{-1}$), and the approximate index of the power-law tail. The behavior of the observed WTD is seen to originate from a time-dependent Poisson process with an approximately exponential distribution of rates.

4. DISCUSSION

In this Letter the WTD for 25 years of *GOES* soft X-ray flares (of greater than C1.0 class) has been investigated. The observed WTD is found to be qualitatively consistent with a piecewise-constant Poisson process, with a time history of rates determined from the data using a Bayesian procedure. This result indicates that the *GOES* flares are independent, random events. There does not appear to be good evidence in the *GOES* events for flare sympathy or for long-term correlations in the times of flare occurrence.

The *GOES* WTD displays a power-law tail for long waiting times, as pointed out by Boffeta et al. (1999) and confirmed here. In this Letter the power-law behavior is demonstrated to originate from two basic assumptions that are well supported by the data: (1) that flare process is Poisson and (2) that the distribution of flaring rates follows an approximate exponential. Subject to only these assumptions, the theoretical WTD is equation (6), which reproduces the qualitative features of the observed WTD, including the power-law tail. There is some discrepancy between equation (5) and the observationally determined WTD. For example, the observational determination of the power-law index of the tail of the distribution is around -2.2 (Boffeta et al. 1999 found -2.4 ± 0.1), whereas equation (6) predicts an index of -3. This difference is most likely due to the departure of the observed distribution of flaring rates from a simple exponential form, particularly for low flaring rates (which influence the behavior of the WTD for long waiting times).

This Letter presents the new result that the probability of flare occurrence per unit time, when averaged over the solar cycle, follows an approximate exponential distribution (see Fig. 3). This is a new phenomenological law for flaring that must be explained by any theory for the origin of flare energy. The rate of flare occurrence reflects the total rate of energy release in flaring, which must match the rate of energy supply to the system. Hence, it follows that the rate at which energy is supplied to the corona for flaring also follows an exponential distribution. It is clear from Figure 2 that this new law does not hold instantaneously; e.g., at times of maxima of the cycle there are few low flaring rates. For certain periods of time during each solar cycle the flaring rate is approximately constant. From these points it also follows that the observed flare WTD is time-dependent and may have a different form depending on the interval of observation. If the WTD is constructed for a short period of observation, during which time the rate of flaring is approximately constant, then the distribution will resemble an exponential. The power-law tail of the WTD appears in the *GOES* data taken over several solar cycles, during which time there is wide variation in the flaring rate. For shorter periods of observation the power-law form might not appear, depending on whether there is sufficient variation in the flaring rate. The time dependence and cycle dependence of the rate and WTDs will be investigated in more detail in future work.

In this Letter, waiting times between flares from all active regions present on the Sun have been considered, so that the Sun is treated as a single flaring system. Boffeta et al. (1999) also considered flares in individual active regions, as identified (in the *GOES* catalog) from Hα events. The distribution for waiting times in individual active regions was found to be similar to that from all active regions. In future work the WTD in individual active regions will be considered in more detail.

The results presented in this Letter are consistent with the avalanche model for flares. Although avalanche cellular automatons produce an exponential WTD when driven with a constant rate, if the rate of driving is varied so that the distribution of rates is exponential, then the resulting model (referred to here as a nonstationary avalanche model) should reproduce the qualitative features of the observed WTD. There is no need to consider models that produce a power-law WTD through long-term correlations between events (e.g., models of MHD turbulence; cf. Boffeta et al. 1999), because the WTD is seen to be a simple consequence of the statistics of independent flare events together with an exponential distribution of flaring rates.

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