

Example: sinusoid at frequency $F_c = f_c F_s$ Hz.

$$x[n] = \cos(2\pi f_c n), \quad n = 0, \dots, N-1$$

$$\begin{aligned} X_{DTFT}(f) &= \sum_{n=0}^{N-1} \cos(2\pi f_c n) e^{-j2\pi f n} = \frac{1}{2} \sum_{n=0}^{N-1} (e^{j2\pi f_c n} + e^{-j2\pi f_c n}) e^{-j2\pi f n} \\ &= \frac{1}{2} g_N(f - f_c) + \frac{1}{2} g_N(f + f_c) \end{aligned}$$

$$g_N(\nu) = \sum_{n=0}^{N-1} e^{-j2\pi \nu n}$$

Use geometric series formula $\sum_{n=0}^M a^n = (1 - a^{M+1})/(1 - a)$ to obtain

$$g_N(\nu) = N e^{-j\pi \nu (N-1)} \underbrace{\frac{\sin(\pi \nu N)}{N \sin(\pi \nu)}}_{\text{Dirichlet kernel}}$$