PSC4375: Linear Regression

Week 5: Lecture 10 (& 11)

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- People trade contracts such as “Obama to win the electoral votes in Florida’ ’
- Market prices of each contract fluctuate based on its sales
- Why might we expect betting markets like Intrade to accurately predict outcomes of elections?

Linear Regression: Prediction using bivariate relationships

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  - **Dependent/outcome variable**: what we want to predict (election margin).
  - **Independent/explanatory variable**: what we're using to predict (market margin).

We'll use two datasets: `intrade08.csv` & `pres08.csv`

Name	Description
`day`	Date of the session
`statename`	Full name of each state (including District of Columbia in 2008)
`state`	Abbreviation of each state (including District of Columbia in 2008)
`PriceD`	Closing price (predicted vote share) of Democratic Nominee's market
`PriceR`	Closing price (predicted vote share) of Republican Nominee's market
`VolumeD`	Total session trades of Democratic Party Nominee's market
`VolumeR`	Total session trades of Republican Party Nominee's market

- `intrade08.csv`: Each row represents daily trading information about the contracts for either the Democratic or Republican Party nominee's victory in a particular state.

Presidential voting data from 2008

Name	Description
`state.name`	Full name of state (only in `pres2008`)
`state`	Two letter state abbreviation
`Obama`	Vote percentage for Obama
`McCain`	Vote percentage for McCain
`EV`	Number of electoral college votes for this state

Predicting Elections Using Betting Markets and Linear Models

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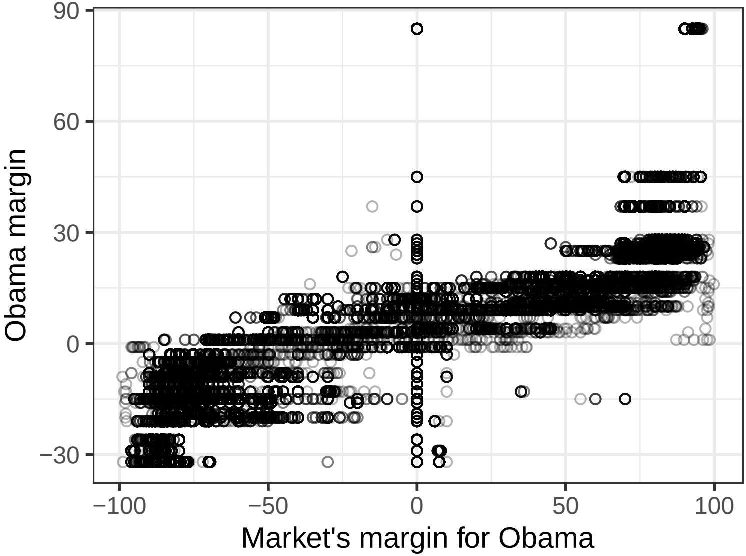
- Load the data

```
library(tidyverse)
intrade08 <- read.csv("../data/intrade08.csv")
pres08 <- read.csv("../data/pres08.csv")

## merge datasets and calculate margins for DV and IV
intresults08 <- inner_join(intrade08,pres08) %>%
  mutate(obama.intmarg = PriceD - PriceR,
         obama.actmarg = Obama - McCain)
```

Plot bivariate relationship



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  - Need a way to account for **chance variation** away from the line.

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- Model for the line of best fit

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- Useful fiction: this model represents the **data generating process*
    - George Box: "all models are wrong, some are useful"

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  - Or now if we still have time!

Linear Regression Model (skip if same day)

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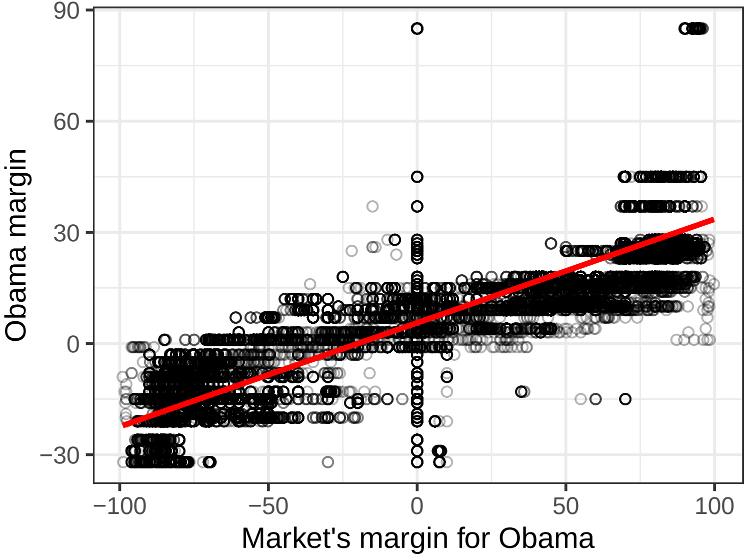
  - An estimate is our best guess about some parameter.
- Regression line:

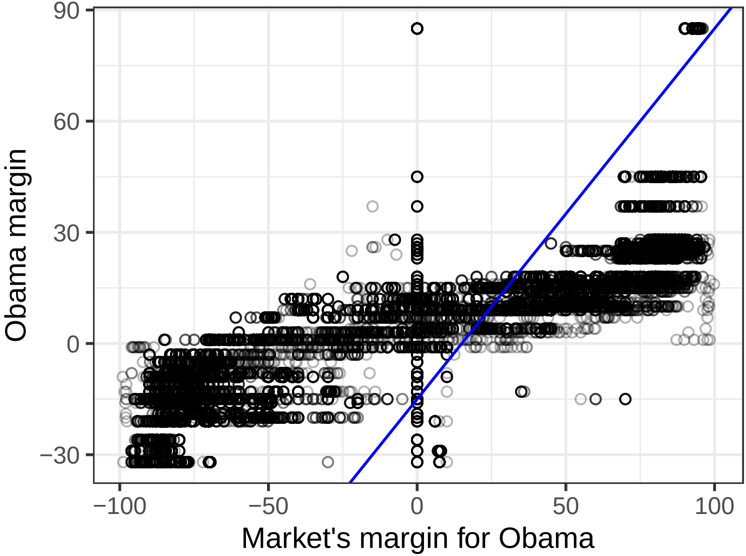
$$\hat{Y} = \hat{\alpha} + \hat{\beta} \times x$$

- Average value of Y when X is x - Represents the best guess or **predicted value** of the outcome at x.

Line of best fit



Why not this line?



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- Finds the line that minimizes the magnitude of the prediction errors!

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  - Syntax: `lm(y ~ x, data = mydata)`
  - `y` is the name of the dependent variable
  - `x` is the name of the independent variable
  - `mydata` is the data.frame where they live

Linear Regression in R

```r
fit <- lm(obama.actmarg ~ obama.intmarg, data = intresults08)
fit
```

```
##
## Call:
## lm(formula = obama.actmarg ~ obama.intmarg, data = intresul
##
## Coefficients:
##   (Intercept)  obama.intmarg
##        5.5681         0.2799
```

Coefficients and fitted values

- Use `coef()` to extrac estimated coefficients:

```r
coef(fit)
```

```
##   (Intercept) obama.intmarg
##     5.5681423     0.2799326
```

- R can show you each of the fitted values as well:

```r
head(fitted(fit))
```

```
##        1        2        3        4        5        6
## 5.568142 5.568142 5.568142 5.568142 5.568142 5.568142
```

Properties of leasat squares

- Least squares line always goes through $(\bar{X}, \bar{Y})$
- Estimated slope is related to correlation:

$$\hat{\beta} = (\text{correlation of } X \text{and } Y) \times \frac{\text{SD of } Y}{\text{SD of } X}$$

- Mean of residuals is always 0

Visual components of least squares