PSC4375: Interactions and Nonlinearities

Week 8 & 9

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Heterogeneous treatment effects

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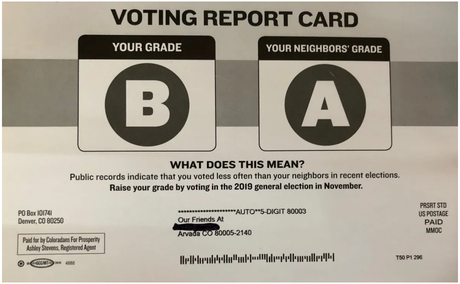
Social pressure experiment



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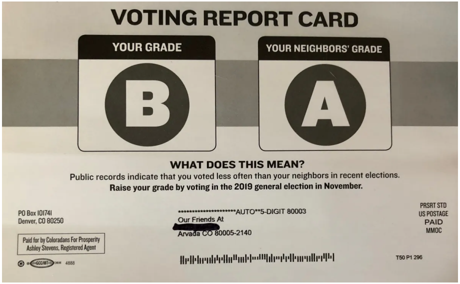

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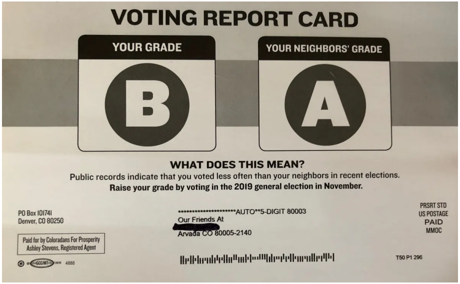

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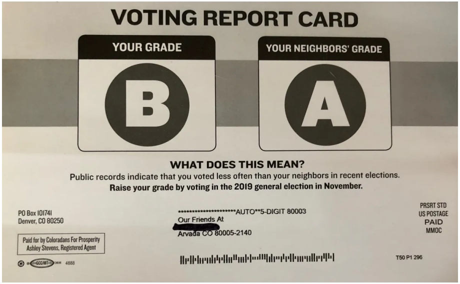

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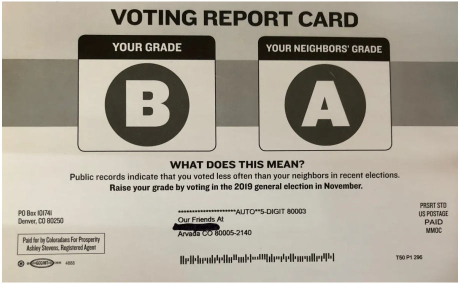

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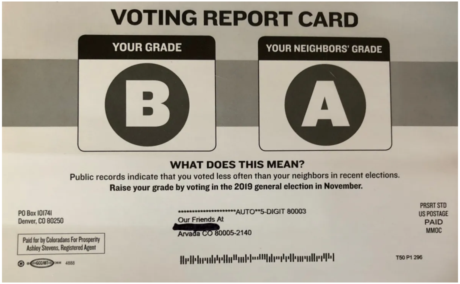

- `primary2004` whether the person voted in 2004, before the experiment.
- Do 2004 voters react differently to social pressure mailer than nonvoters?
- Two approaches:
  - Subsets, subsets, subsets.
  - Interaction terms in regression.

Subset approach

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Subset approach

- Easy way to estimate heterogeneous effects: our old friend, `filter()`, `group_by()`, and `summarize()`. Woo!
  - First, get the data

```
data(social, package="qss")
```

Subset approach

- Now, estimate the ATE for the **voters**:

Subset approach

- Now, estimate the ATE for the **voters**:

```
VotersATE <- social %>%
  filter(primary2004 == 1,
         messages %in% c("Control","Neighbors")) %>%
  group_by(messages) %>%
  summarize(primary2006_mean = mean(primary2006)) %>%
  pivot_wider(names_from = "messages",
              values_from = "primary2006_mean") %>%
  mutate(ate_v = Neighbors - Control) %>%
  select(ate_v)
VotersATE
```

```
## # A tibble: 1 x 1
##    ate_v
##    <dbl>
## 1 0.0965
```

Filter approach

- Now, estimate the ATE for the **nonvoters**:

Filter approach

- Now, estimate the ATE for the **nonvoters**:

```
NonvotersATE <- social %>%
  filter(primary2004 == 0,
         messages %in% c("Control","Neighbors")) %>%
  group_by(messages) %>%
  summarize(primary2006_mean = mean(primary2006)) %>%
  pivot_wider(names_from = "messages",
              values_from = "primary2006_mean") %>%
  mutate(ate_nv = Neighbors - Control) %>%
  select(ate_nv)
NonvotersATE
```

```
## # A tibble: 1 x 1
##   ate_nv
##    <dbl>
## 1 0.0693
```

Difference in effects

- How much does the estimated treatment effect differ between groups?

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```
VotersATE$ate_v - NonvotersATE$ate_nv
```

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## [1] 0.02722908
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- How much does the estimated treatment effect differ between groups?

```
VotersATE$ate_v - NonvotersATE$ate_nv
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```
## [1] 0.02722908
```

- Any easier way to allow for different effects of treatment by groups?

Interaction terms

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$$
\begin{aligned}
\text{turnout}_i = \alpha + \beta_1 \text{primary2004}_i + \beta_2 \text{neighbors}_i + \\
\beta_3(\text{primary2004}_i \times \text{neighbors}_i) + \varepsilon_i
\end{aligned}
$$

Interaction terms

- Can allow for different effects of a variable with an interaction term:

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- Primary 2004 variable multiplied by the neighbors variable.
  - Equal to 1 if voted in 2004 (primary2004 $==$ 1) and received neighbors mailer (neighbors $==$ 1)

Interaction terms

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- Primary 2004 variable multiplied by the neighbors variable.
  - Equal to 1 if voted in 2004 (primary2004 $==1$) and received neighbors mailer (neighbors $==1$)
- Easiest to understand by investigating predicted values.

Predicted values from non-interacted model

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	Control ($Z_i = 0$)	Neighbors ($Z_i = 1$)
non-voter ($X_i = 0$)		
voter ($X_i = 1$)		

-
-

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non-voter ($X_i = 0$)	$\hat{\alpha}$	
voter ($X_i = 1$)		

Predicted values from non-interacted model

- Let X_i = primary2004$_i$ and Z_i = neighbors$_i$:

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	Control ($Z_i = 0$)	Neighbors ($Z_i = 1$)
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- Effect of Neighbors for voters: $(\hat{\alpha} + \hat{\beta}_1 + \hat{\beta}_2) - (\hat{\alpha} + \hat{\beta}_1) = \hat{\beta}_2$

Predicted from interacted model

- Now for the interacted model:

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta}_1 X_i + \hat{\beta}_2 Z_i + \hat{\beta}_3 X_i Z_i$$

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Interpreting coefficients

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta}_1 \text{primary2004}_i + \hat{\beta}_2 \text{neighbors}_i + \hat{\beta}_3 (\text{primary2004}_i \times \text{neighbors}_i)$$

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	Control Group	Neighbors Group
2004 primary non-voter	$\hat{\alpha}$	$\hat{\alpha} + \hat{\beta}_2$
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- $\hat{\alpha}$: turnout rate for 2004 nonvoters in control group.

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- $\hat{\alpha}$: turnout rate for 2004 nonvoters in control group.
- $\hat{\beta}_1$: avg difference in turnout between 2004 voters and nonvoters.
- $\hat{\beta}_2$: effect of neighbors for 2004 nonvoters.
- $\hat{\beta}_3$: difference in the effect of neighbors mailer between 2004 voters and nonvoters.

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social.neighbor <- social %>%
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```

```
fit <- lm(primary2006 ~ primary2004 + neighbors +
            primary2004:neighbors, data = social.neighbor)
coef(fit)
```

```
##           (Intercept)           primary2004
##            0.23710990            0.14869507
##             neighbors primary2004:neighbors
##            0.06929617            0.02722908
```

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- Compare coefficients to earlier approach:

```r
NonvotersATE$ate_nv
```

```
## [1] 0.06929617
```

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- Compare coefficients to earlier approach:

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NonvotersATE$ate_nv
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## [1] 0.06929617
```

```r
VotersATE$ate_v - NonvotersATE$ate_nv
```

```
## [1] 0.02722908
```

Interactions with Continuous Variables

Interactions with Continuous Variables

- Create an age variable for the Michigan **social pressure get-out-the-vote experiment**:

```
social.neighbor <- social.neighbor %>%
  mutate(age = 2006 - yearofbirth)
summary(social.neighbor$age)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   20.00   41.00   50.00   49.82   59.00  106.00
```

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- Remarkable, the same **interaction term** will work here too!

$$Y_i = \alpha + \beta_1 \text{age}_i + \beta_2 \text{neighbors}_i + \beta_3 (\text{age}_i \times \text{neighbors}_i) + \varepsilon_i$$

Predicted values from non-interacted model

- Let $X_i = \text{age}_i$ and $Z_i = \text{neighbors}_i$

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	Control ($Z_i = 0$)	Neighbors ($Z_i = 1$)
25 year-old ($X_i = 25$)		
26 year-old ($X_i = 26$)		

-
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	Control ($Z_i = 0$)	Neighbors ($Z_i = 1$)
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	Control ($Z_i = 0$)	Neighbors ($Z_i = 1$)
25 year-old ($X_i = 25$)	$\hat{\alpha} + \hat{\beta}_1 25$	$\hat{\alpha} + \hat{\beta}_1 25 + \hat{\beta}_2$
26 year-old ($X_i = 26$)	$\hat{\alpha} + \hat{\beta}_1 26$	$\hat{\alpha} + \hat{\beta}_1 26 + \hat{\beta}_2$

-
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Predicted values from non-interacted model

- Let $X_i = \text{age}_i$ and $Z_i = \text{neighbors}_i$

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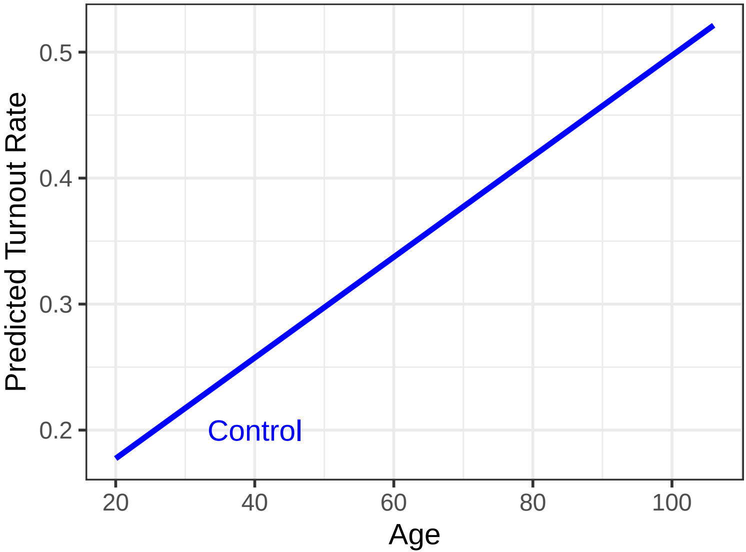
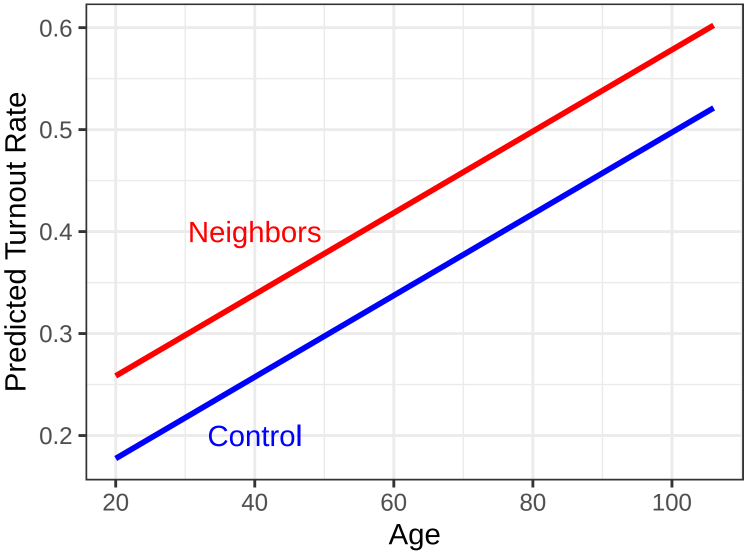
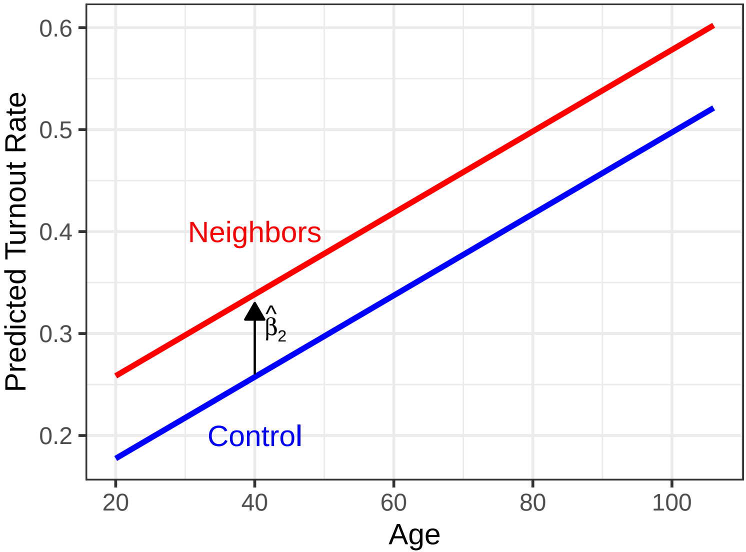
Visualizing the regression



Visualizing the regression



Visualizing the regression



Predicted values from interacted model

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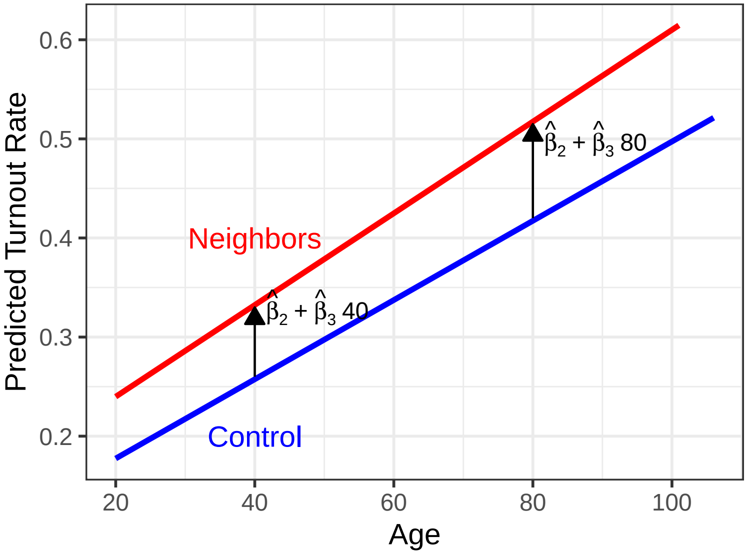
- Effect of Neighbors for a x year-old: $\hat{\beta}_2 + \hat{\beta}_3 x$

Visualizing the interaction



Interpretting coefficients

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$$Y_i = \alpha + \beta_1 \text{age}_i + \beta_2 \text{neighbors}_i + \beta_3 (\text{age}_i \times \text{neighbors}_i)$$

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  - Change in effect: $\hat{\beta}_3$

Interactions in R

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- You can use the `:` way to create interaction terms like last time:

```
int.fit <- lm(primary2006 ~ age + neighbors + age:neighbors, d
coef(int.fit)
```

```
##   (Intercept)           age     neighbors age:neighbors
##  0.0974732574  0.0039982107  0.0498294321  0.0006283079
```

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```
##   (Intercept)            age      neighbors age:neighbors
##  0.0974732574  0.0039982107  0.0498294321  0.0006283079
```

- Or you can use the `var1 * var2` shortcut, which will add both variable and their interaction:

```
int.fit2 <- lm(primary2006 ~ age*neighbors, data = social.neig
coef(int.fit2)
```

```
##   (Intercept)            age      neighbors age:neighbors
##  0.0974732574  0.0039982107  0.0498294321  0.0006283079
```

General interpretation of interactions

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Nonlinear relationships

Nonlinear relationships

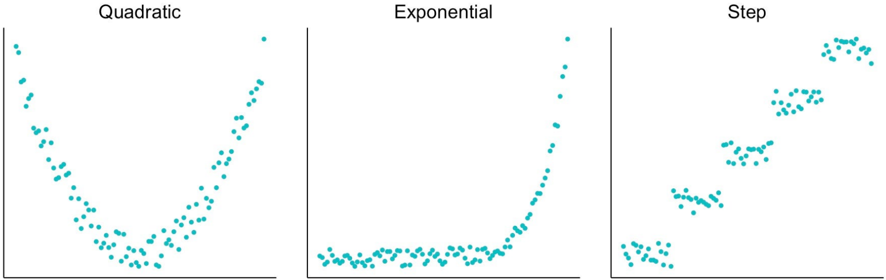

Figure 1: Types of Non-linear Relationships

Linear regression are linear

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta}_1 X_i$$

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$$\hat{Y}_i = \hat{\alpha} + \hat{\beta}_1 X_i$$

- Standard linear regression can only pick up **linear** relationships.
- What if the relationship between X_i and Y_i is nonlinear?

Adding a squared term

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- To allow for nonlinearity in age, add a squared term to the model

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$$\hat{Y}_i = \hat{\alpha} + \hat{\beta}_1 \text{age}_i + \hat{\beta}_2 \text{age}_i^2$$

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- To allow for nonlinearity in age, add a squared term to the model

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- In R, we need to wrap the squared term in `I()`:

```
fit.sq <- lm(primary2006 ~ age + I(age^2), data = social.neigh
coef(fit.sq)
```

```
##  (Intercept)          age     I(age^2)
## -0.080067046  0.012154358 -0.000079999
```

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```
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## -0.080067046  0.012154358 -0.000079999
```

- $\hat{\beta}_2$: how the effect of age increases as age increases

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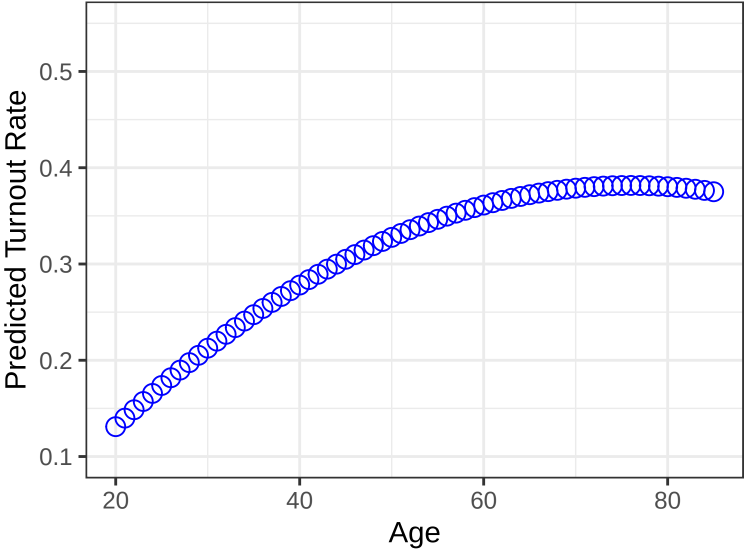
- Plot the predictions:

```
ggplot(age.plot,aes(x = age.vals, y = age.preds)) +
  geom_point(color = "blue", size = 3, shape = 1) + ylim(0.1, 0.55)
  labs(x = "Age", y = "Predicted Turnout Rate") + theme_bw()
```

Plotting predicted values

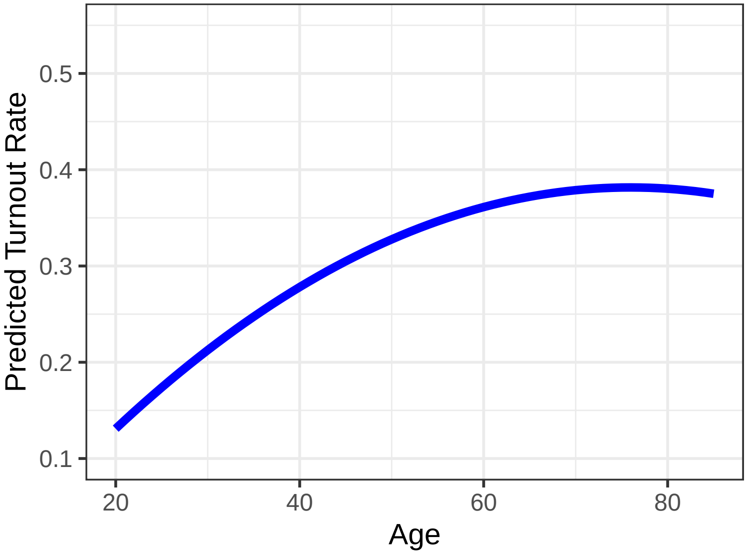


Plotting lines instead of points:

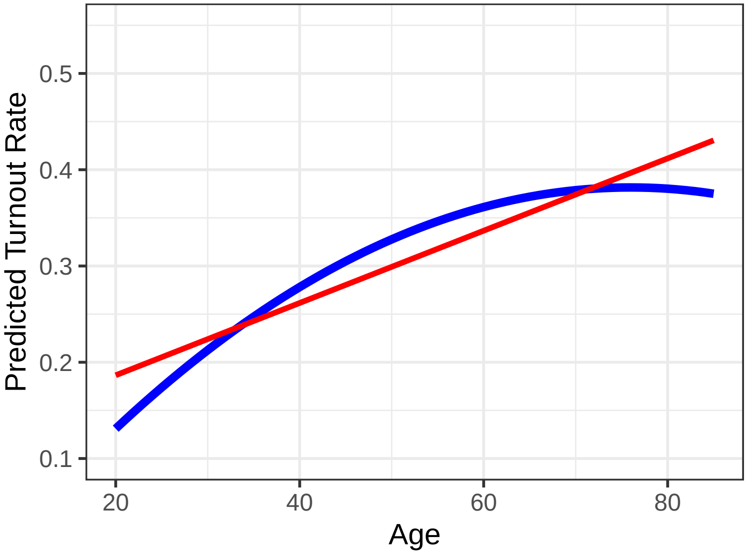
- If you want to connect the dots in your scatterplot, you can use `geom_line()`:

```
ggplot(age.plot, aes(x = age.vals, y = age.preds)) +
  geom_line(color = "blue", size = 1.5) +
  ylim(0.1, 0.55) +
  labs(x = "Age", y = "Predicted Turnout Rate") +
  theme_bw()
```

Plotting predicted values



Comparing to linear fit



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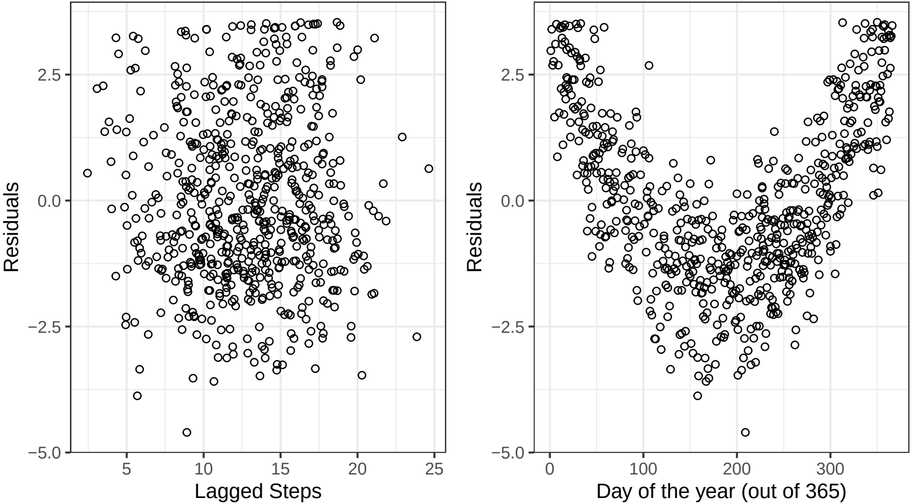
- With multiple independent variables, harder to diagnose.
- One useful tool: scatterplot of residuals versus independent variables.
- Example: let's talk about walking and health

```
health <- read.csv("../data/health.csv")

w.fit <- lm(weight ~  steps.lag + dayofyear, data = health)
```

Residual plot

Residual plot



Add a squared term for a better fit

```r
w.fit.sq <- lm(weight ~ steps.lag + dayofyear +
                  I(dayofyear^2), data = health)
coef(w.fit.sq)
```

```
##    (Intercept)      steps.lag      dayofyear I(dayofyear^2)
##   1.749194e+02  -2.509427e-03  -5.288116e-02   1.439635e-04
```

Residual plot, redux