Loops, Prediction, and Regression

PSC7475: Week 5

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Villanova University

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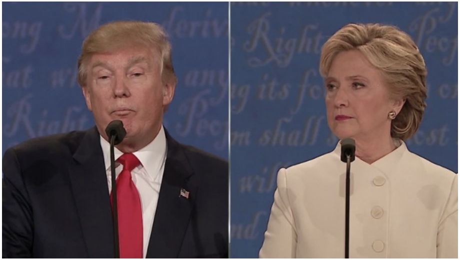
2016 US Presidential Election



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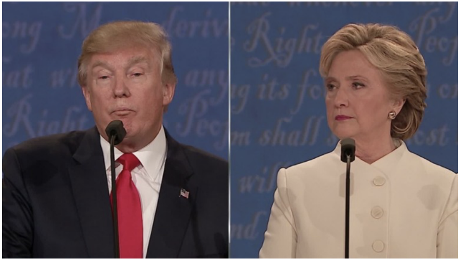

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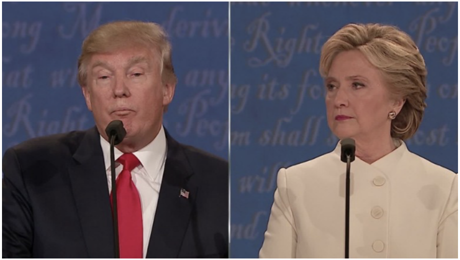

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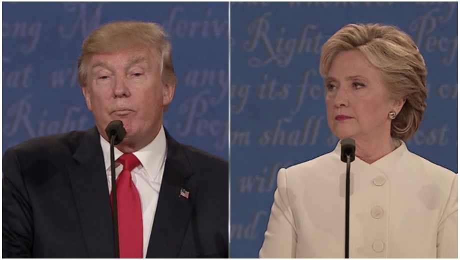

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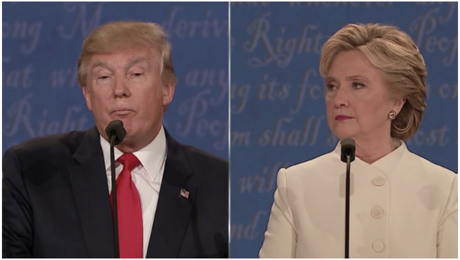

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- Election determined by 77,744 votes (margins in WI, MI, PA)
  - 0.056% of the electorate (~136million)

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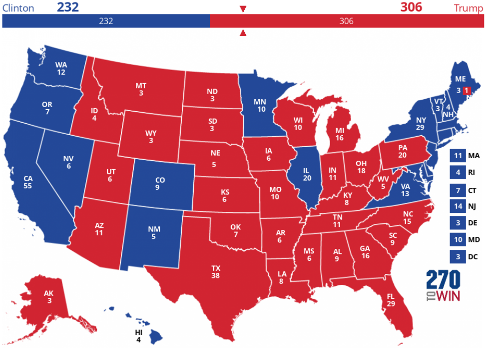

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- Sounds like a lot of subsets, ugh...

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  - Easy in R: `values * 2`
  - Pretend you didn't know this approach

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values <- c(2,4,6)
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```

```
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```

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  - `expression`: a set of expressions that will be repeatedly evaluated.
  - `{ }`: curly braces to define beginning and end of the loop.
- Indentation is important for readability of the code.

Loop example:

```
values <- c(2,4,6)
n <- length(values)
results <- rep(NA, times = n)

## begin loop
for (i in 1:n) {
  results[i] <- values[i] * 2
  print(str_c(values[i], " times 2 is equal to ", results[i]))
}
```

```
## [1] "2 times 2 is equal to 4"
## [1] "4 times 2 is equal to 8"
## [1] "6 times 2 is equal to 12"
```

2016 polling prediction

- Election data: `pres.csv`

Name	Description
`state_abb`	abbreviated name of state
`clinton`	Clinton's vote share (percentage)
`trump`	Trump's vote share (percentage)

- Polling data `polls16.csv`

Name	Description
`state`	abbreviated name of state in which polls was conducted
`middate`	middate of the period when polls was conducted
`daysleft`	number of days between middate and election day
`pollster`	name of organization conducting poll
`clinton`	predicted support for Clinton (percentage)
`trump`	predicted support for Trump (percentage)

Some preprocessing

```
## download; don't forget to setwd()
pres16 <- read_csv("../data/pres2016.csv")
polls16 <- read_csv("../data/polls2016.csv")

## calculate Trump's margin of victory
polls16 <- polls16 %>%
  mutate(margin = Trump - Clinton)
pres16 <- pres16 %>%
  mutate(margin = Trump - Clinton)
```

What does the data look like?

```r
head(polls16)
```

```
## # A tibble: 6 x 8
##      id state Clinton Trump days_to_election electoral_votes
##   <dbl> <chr>   <dbl> <dbl>             <dbl>           <dbl>
## 1 26255 TX         38    41                24              38
## 2 26253 WI         48    44                23              10
## 3 26252 VA         54    41                23              13
## 4 26251 NV         47    40                19               6
## 5 26250 TX         46    48                23              38
## 6 26249 NH         50    43                23               4
## # i 2 more variables: population <chr>, margin <dbl>
```

Poll prediction for each state

```r
## place holder
poll.pred <- rep(NA, 51)

## get list of unique state names to iterate over
state_names <- unique(polls16$state)

## add labels to place holder
names(poll.pred) <- state_names
```

Poll prediction for each state

```r
for (i in seq_along(state_names)) {
```

Poll prediction for each state

```
for (i in seq_along(state_names)) {
  ## subset the ith state
  state.data <- polls16 %>%
    filter(state == state_names[i])
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for (i in seq_along(state_names)) {
  ## subset the ith state
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  ## pull out the closest date (minimum days to election)
  min_days <- min(state.data$days_to_election)
```

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for (i in seq_along(state_names)) {
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  min_days <- min(state.data$days_to_election)

  ## subset only the latest polls within the state
  state.data <- state.data %>%
    filter(days_to_election == min_days)
```

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  ## subset only the latest polls within the state
  state.data <- state.data %>%
    filter(days_to_election == min_days)

  ## compute the mean of the latest polls and store it
  poll.pred[i] <- mean(state.data$margin)
}
head(poll.pred)
```

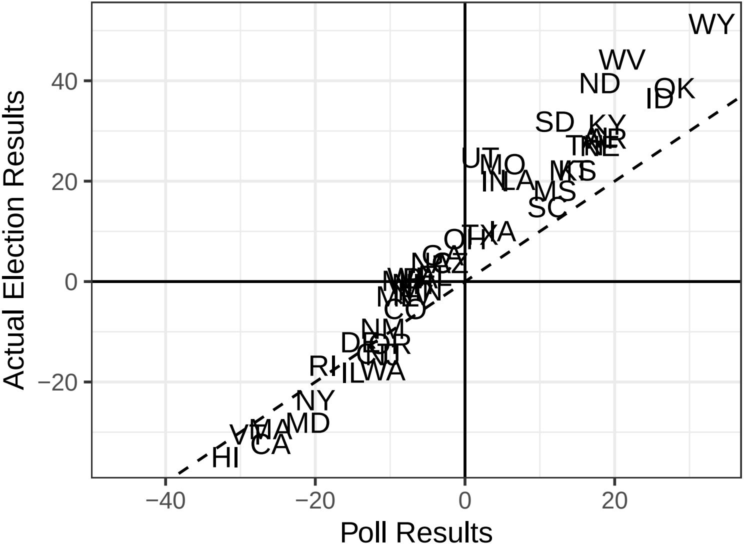
```
##  TX  WI  VA  NV  NH  PA
##   2  -8 -15  -7  -7  -6
```

Poll prediction for each state (my way)

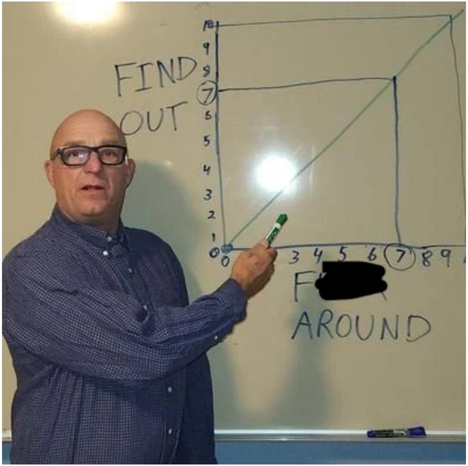
```
poll.list <- list()
state_names <- unique(polls16$state)

for (i in seq_along(state_names)) {
  state.data <- polls16 %>%
    filter(state == state_names[i]) %>%
    filter(days_to_election == min(days_to_election)) %>%
    mutate(margin_poll = mean(margin)) %>%
    select(state, margin_poll)
  poll.list[[i]] <- state.data
  print(i)
}
PollPred <- do.call(rbind,poll.list)
head(PollPred)
```

Comparing polls to outcomes



Let’s talk about regression



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- Market prices of each contract fluctuate based on its sales
- Why might we expect betting markets like Intrade to accurately predict outcomes of elections?

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- Terminology:
  - **Dependent/outcome variable**: what we want to predict (election margin).
  - **Independent/explanatory variable**: what we're using to predict (market margin).

We'll use two datasets: `intrade08.csv` & `pres08.csv`

Name	Description
`day`	Date of the session
`statename`	Full name of each state (including DC in 2008)
`state`	Abbreviation of each state (including DC in 2008)
`PriceD`	Closing price (predicted vote share) of Democratic Nominee's market
`PriceR`	Closing price (predicted vote share) of Republican Nominee's market
`VolumeD`	Total session trades of Democratic Party Nominee's market
`VolumeR`	Total session trades of Republican Party Nominee's market

- `intrade08.csv`: Each row represents daily trading information about the contracts for either the Democratic or Republican Party nominee's victory in a particular state.

Presidential voting data from 2008

Name	Description
`state.name`	Full name of state (only in `pres2008`)
`state`	Two letter state abbreviation
`Obama`	Vote percentage for Obama
`McCain`	Vote percentage for McCain
`EV`	Number of electoral college votes for this state

Predicting Elections Using Betting Markets and Linear Models

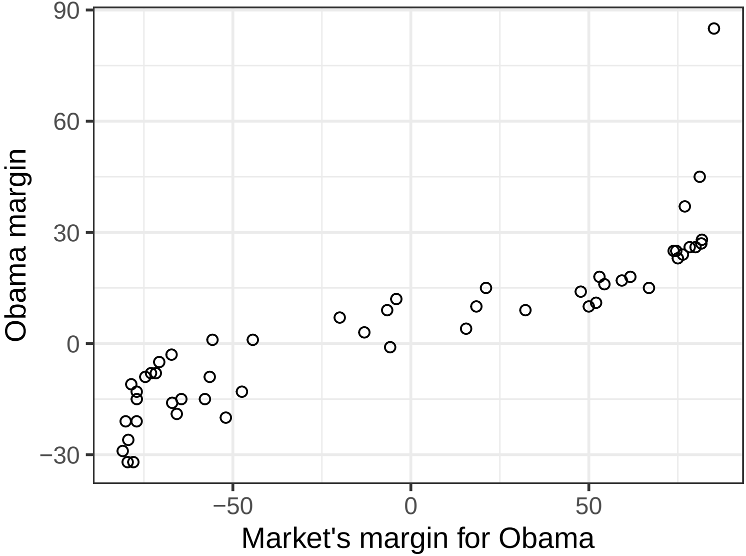
Predicting Elections Using Betting Markets and Linear Models

- Load the data

```
library(tidyverse)
intrade08 <- read.csv("../data/intrade08.csv")
pres08 <- read.csv("../data/pres08.csv")

## merge datasets and calculate margins for DV and IV
intresults08 <- inner_join(intrade08,pres08) %>%
  mutate(obama.intmarg = PriceD - PriceR,
         obama.actmarg = Obama - McCain)
```

Plot bivariate relationship



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  - Some points will be above the line, some below.
  - Need a way to account for **chance variation** away from the line.

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- Model for the line of best fit

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- Useful fiction: this model represents the **data generating process**
  - George Box: "all models are wrong, some are useful"

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- **Slope** β: average change in Y when X increases by one unit
    - Average increase in Obama margin for each additional margin increase by the market.
- But we don't know α or β. How can we estimate them? Next time...
    - Or now if we still have time!

Linear Regression Model (skip if same day)

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Estimate coefficients

Estimate coefficients

- Parameters: α, β
  - Unknown features of the data-generating process.
  - Chance error makes these impossible to observe directly.

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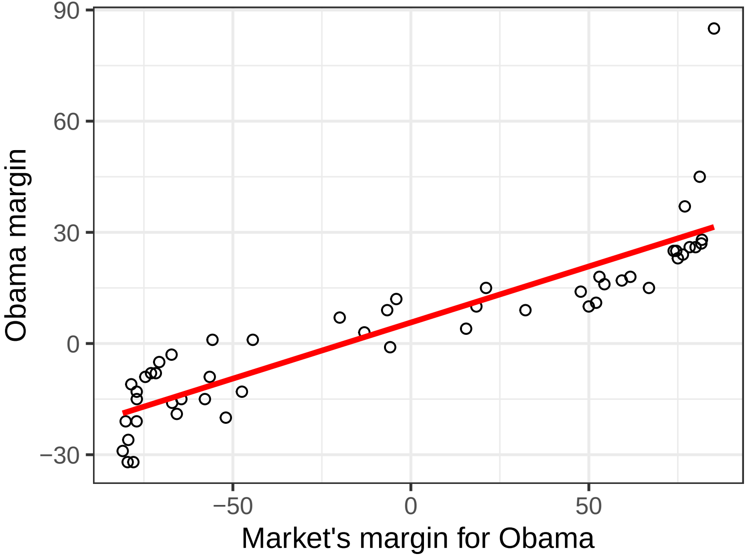
  - An estimate is our best guess about some parameter.
- Regression line:

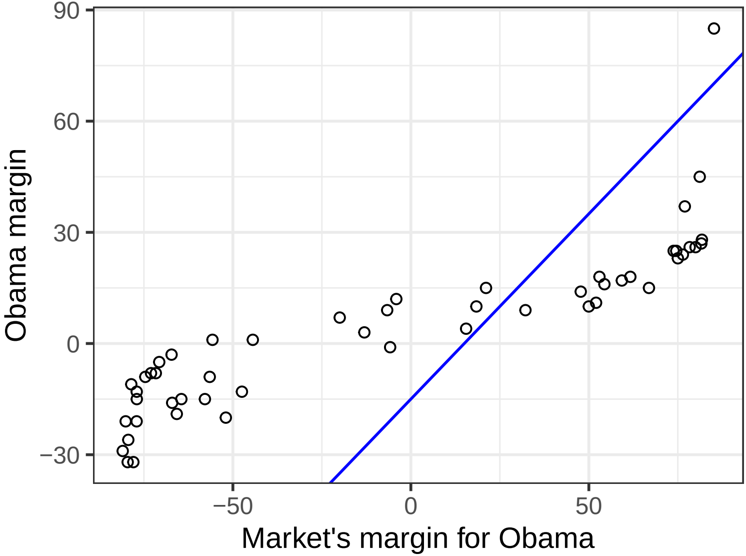
$$\hat{Y} = \hat{\alpha} + \hat{\beta} * x$$

- Average value of Y when X is x
- Represents the best guess or **predicted value** of the outcome at x.

Line of best fit



Why not this line?



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- Finds the line that minimizes the magnitude of the prediction errors!

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  - `y` is the name of the dependent variable
  - `x` is the name of the independent variable
  - `mydata` is the data.frame where they live

Linear Regression in R

```
fit <- lm(obama.actmarg ~ obama.intmarg, data = intresults08)
fit
```

```
##
## Call:
## lm(formula = obama.actmarg ~ obama.intmarg, data = intresults08)
##
## Coefficients:
##   (Intercept)  obama.intmarg
##        5.5681         0.2799
```

Coefficients and fitted values

- Use coef() to extract estimated coefficients:

```
coef(fit)
```

```
##   (Intercept) obama.intmarg
##     5.5681423     0.2799326
```

- R can show you each of the fitted values as well:

```
head(fitted(fit))
```

```
##        1        2        3        4        5        6
## 5.568142 5.568142 5.568142 5.568142 5.568142 5.568142
```

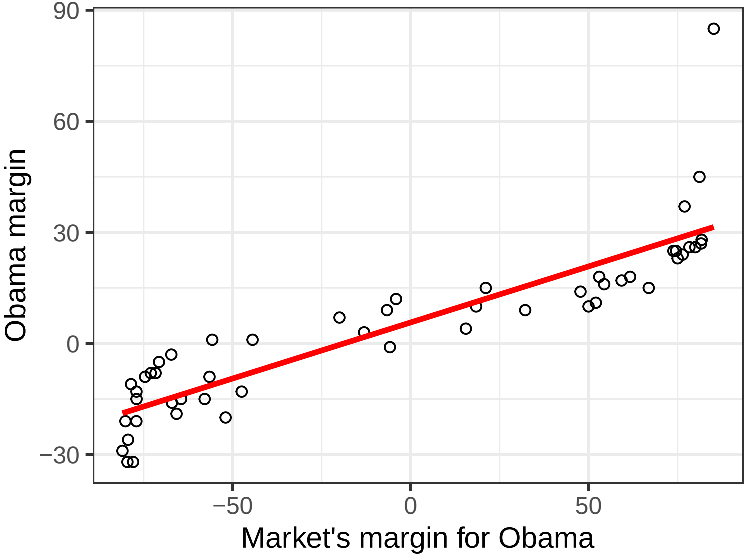
Properties of least squares

- Least squares line always goes through $(\bar{X}, \bar{Y})$
- Estimated slope is related to correlation:

$$\hat{\beta} = (\text{correlation of } X \text{and } Y) \times \frac{\text{SD of } Y}{\text{SD of } X}$$

- Mean of residuals is always 0

Visual components of least squares



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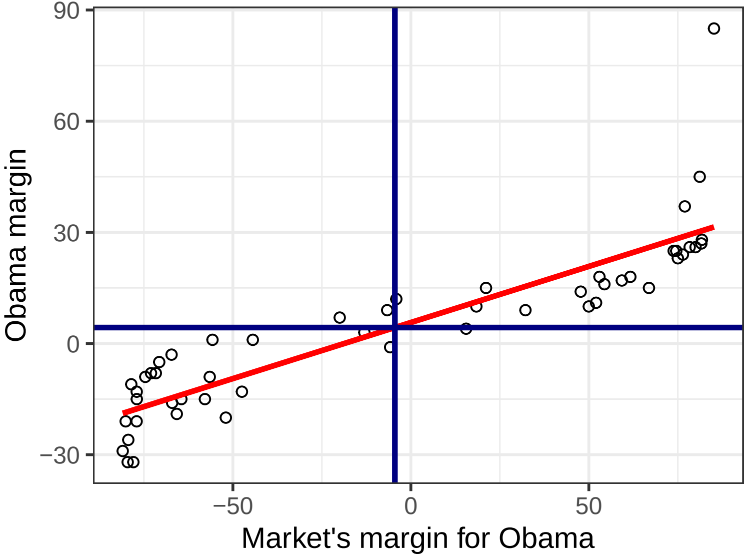

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