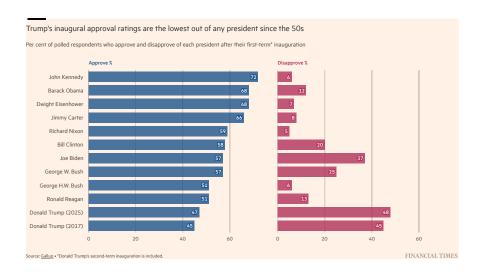
Regression Part II: Model Fit and Variable type/quantity PSC7475: Week 6

Prof. Weldzius

Villanova University

Slides Updated: 2025-02-26

Presidential Popularity and the Midterms



Presidential Popularity and the Midterms

 Does popularity of the president or recent changes in the economy better predict midterm election outcomes?

Name	Description
year	midterm election year
president	name of president
party	Democrat or Republican
approval	Gallup approval rating at midterms
rdi.change	change in real disposable income over the year before midterms
seat.changechange in the number of House seats for the president's party	

Loading the data:

```
library(tidyverse)
midterms <- read.csv("../data/midterms.csv")
head(midterms)</pre>
```

```
##
          president party approval seat.change rdi.change
    1946
               Truman
                           D
                                    33
                                                -55
                                                             NΑ
                                                            8.2
   2 1950
               Truman
                           D
                                    39
                                                -29
    1954 Eisenhower
                                    61
                                                            1.0
                           R.
                                                 -4
     1958 Eisenhower
                                    57
                                                -47
                                                            1.1
                           R.
   5 1962
              Kennedy
                           D
                                    61
                                                -4
                                                            5.0
              Johnson
                                                -47
   6 1966
                           D
                                    44
                                                            5.3
```

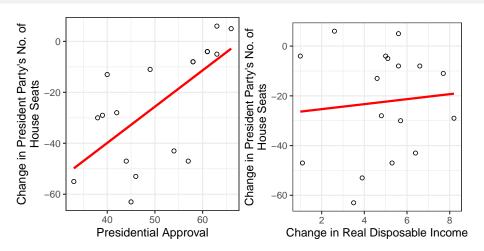
Fitting the Approval Model

```
fit.app <- lm(seat.change ~ approval, data = midterms)</pre>
fit.app
##
## Call:
## lm(formula = seat.change ~ approval, data = midterms)
##
## Coefficients:
## (Intercept) approval
## -96.845
                     1.424
```

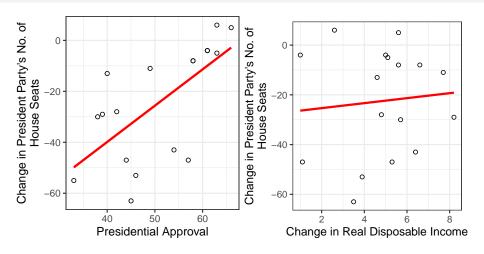
Fitting the Income Model

```
fit.rdi <- lm(seat.change ~ rdi.change, data = midterms)
fit.rdi
##
## Call:
## lm(formula = seat.change ~ rdi.change, data = midterms)
##
## Coefficients:
## (Intercept) rdi.change
## -27.354
                     1.004
```

Comparing Models

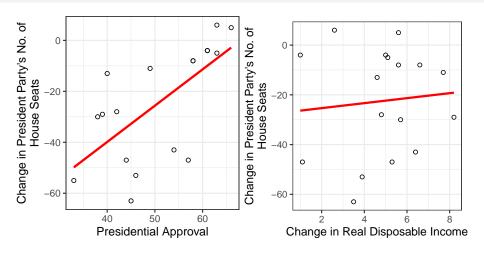


Comparing Models



• How well do the models "fit the data"?

Comparing Models



- How well do the models "fit the data"?
 - How well does the model predict the outcome variable in the data?

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 - Measure of the **proportional reduction in error** by the model.

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$$R^2 = \frac{\mathsf{TSS} - \mathsf{SSR}}{\mathsf{TSS}}$$

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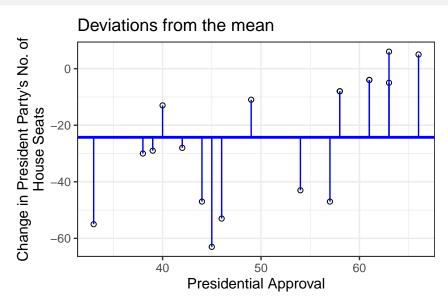
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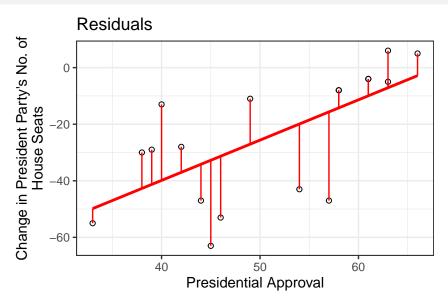
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• Roughly: proportion of the variation in Y_i "explained by" X_i

Total sum of squares vs. Sum of squared residuals



Total sum of squares vs. Sum of squared residuals



Model Fit in R

• To access R^2 from the lm() output, use the summary() function:

```
fit.app.sum <- summary(fit.app)
fit.app.sum$r.squared</pre>
```

```
## [1] 0.4307133
```

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## [1] 0.4307133
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• Compare to fit using change in income:

```
fit.rdi.sum <- summary(fit.rdi)
fit.rdi.sum$r.squared</pre>
```

```
## [1] 0.008529029
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• Which does a better job predicting midterm election outcomes?

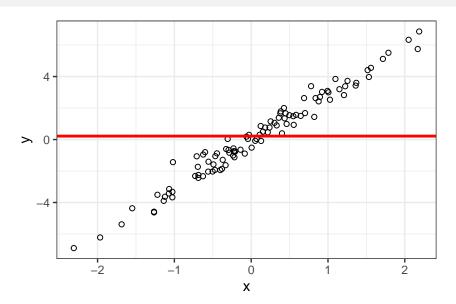
• Little hard to see what's happening in that example.

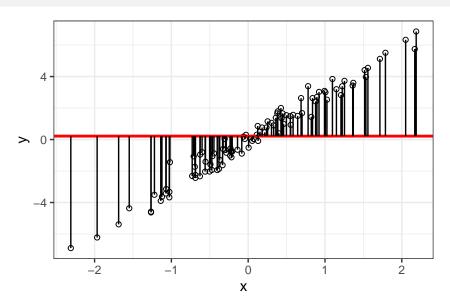
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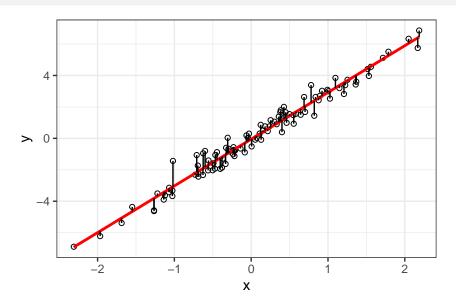
- Little hard to see what's happening in that example.
- Let's look at fake variables x and y:

$$fit.x \leftarrow lm(y \sim x)$$

• Very good model fit: $R^2 \approx 0.95$

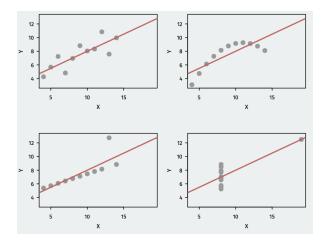






Is R-squared useful?

• Can be very misleading. Each of these samples have the same R^2 even though they are vastly different:



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 - Until 2016, gender was a perfect predictor of who wins the primary.

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 - Example: predicting winner of Democratic presidential primary with gender of the candidate.
 - Until 2016, gender was a **perfect** predictor of who wins the primary.
 - Prediction for 2016 based on this: Bernie Sanders as Dem. nominee.
 - Bad out-of-sample prediction due to overfitting!

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- Better predictions (at least in-sample).
 - Better interpretation as ceteris paribus relationships:
 - β_1 is the relationship between approval and seat.change holding rdi.change constant.

##

```
mult.fit <- lm(seat.change ~ approval + rdi.change, data = midterms)</pre>
mult.fit
##
## Call:
## lm(formula = seat.change ~ approval + rdi.change, data = midterms)
##
## Coefficients:
## (Intercept) approval rdi.change
     -120.436
                  1.572
                                  3.334
```

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• $\hat{\alpha} = -120.4$: average seat change president has 0% approval and no change in income levels.

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- $\hat{\alpha} = -120.4$: average seat change president has 0% approval and no change in income levels.
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- $\hat{\beta}_1 = 1.57$: average increase in seat change for additional percentage point of approval, **holding RDI change fixed**
- $\hat{\beta}_1 = 3.334$: average increase in seat change for each additional percentage point increase of RDI, **holding approval fixed**

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 Find the coefficients that minimizes the sum of the squared residuals:

$$SSR = \sum_{i=1}^{n} \hat{\epsilon}_{i}^{2} = (Y_{i} - \hat{\alpha} - \hat{\beta}_{1}X_{i1} - \hat{\beta}_{2}X_{i2})^{2}$$

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 - But this could be overfitting!!
- Solution: penalize regression models with more variables.
 - Occam's razor: simpler models are preferred
- Adjusted R^2 : lowers regular R^2 for each additional covariate.
 - If the added covariates don't help predict, adjusted R^2 goes down!

```
summary(fit.app)$r.squared
```

```
## [1] 0.4307133
```

summary(mult.fit)\$r.squared

```
summary(fit.app)$r.squared
## [1] 0.4307133
```

```
## [1] 0.4448387
```

```
summary(fit.app)$r.squared

## [1] 0.4307133

summary(mult.fit)$r.squared
```

```
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```

```
summary(mult.fit)$adj.r.squared
```

```
## [1] 0.3655299
```

Binary and Categorical Predictors



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- Political effects of government programs
 - Progresa: Mexican conditional cash transfer program (CCT) from ~2000
 - Welfare \$ given if kids enrolled in schools, get regular check-ups, etc.
 - Do these programs have political effects?
 - Program had support from most parties.
 - Was implemented in a nonpartisan fashion.
 - Would the incumbent presidential party be rewarded?

The Data

- Randomized roll-out of the CCT program:
 - treatment: receive CCT 21 months before 2000 election
 - control: receive CCT 6 months before 2000 election
 - Does having CCT longer mobilize voters for incumbent PRI party?

Name	Description
treatment pri2000s t2000	early Progresa (1) or late Progresa (0) PRI votes in the 2000 election as a share of adults in precinct turnout in the 2000 election as share of adults in precinct

```
cct <- read.csv("../data/progresa.csv")</pre>
```

Does CCT affect turnout?

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```
cct.turn.ate <- cct %>% group_by(treatment) %>%
  summarize(t2000_mean = mean(t2000)) %>%
  pivot_wider(names_from = treatment, values_from = t2000_mean) %>%
  mutate(turnout_ate = `1` - `0`)
cct.turn.ate$turnout_ate
```

```
## [1] 4.269676
```

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```

```
## [1] 3.622496
```

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• If there are other independent variables, this becomes the difference-in-means controlling for those covariates.

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cct.pri.ate$pri_ate
## [1] 3.622496
lm(pri2000s ~ treatment, data = cct)
##
## Call:
## lm(formula = pri2000s ~ treatment, data = cct)
##
## Coefficients:
   (Intercept) treatment
        34 489
                      3.622
```

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Unit	Party	Democrat	Republican	Independent
1	Democrat	1	0	0
2	Democrat	1	0	0
3	Independent	0	0	1
4	Republican	0	1	0
:	<u>:</u>	:	:	÷

- We often have categorical variables:
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 - Partisanship: Democrat, Republican, Independent
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:	:	:	:	÷

• Then include all but one of these binary variables:

 $turnout_i = \alpha + \beta_1 \text{Republican}_i + \beta_2 \text{Independent}_i + \epsilon_i$

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 - ullet \hat{eta}_1 : average difference in turnout between Republicans and Democrats

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- $\hat{\alpha}$: average outcome in the **omitted group/baseline** (Democrats).
- $\hat{\beta}$ coefficients: average difference between each group and the baseline.
 - \hat{eta}_1 : average difference in turnout between Republicans and Democrats
 - $\hat{\beta}_2$: average difference in turnout between Independents and Democrats