PSC7475: Introduction to Causality

Lecture 1

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Slides Updated: 2025-01-22

What is causal effect?

Factual vs. **Counterfactual**

- Does the minimum wage increase the unemployment rate?
  - Unemployment rate went up after the minimum wage increased
  - Would it have gone up if the minimum wage increase not occurred?
- Does having girls affect a judge's rulings in court?
  - A judge with a daughter gave a pro-choice ruling.
  - Would they have done that if had a son instead?
- **Fundamental problem of causal inference:**
  - Can never observe counterfactuals, must be inferred.

Criminal record experiment

- Does having a criminal record affect job prospects?
- Experimental setting:
  - Randomly assign 4 hired "confederates" (2 White, 2 Black) to apply to different jobs in Milwaukee.
  - Men were matched on physical appearance, self-presentation, age, etc.
  - Confederates would alternate indicating they had a criminal record.
- Outcome of interest: receiving a callback from a potential employer.

A tale of two applications

	Criminal Record	Callback?
Applicant 1		
Applicant 2		

- Did the first applicant not callback the applicant **because** they had a criminal record?

Notation and Jargon

- **Unit** (indexed by i): job application for employer
- **Treatment variable** T_i: criminal record or not
- **Treatment group** (treated units): applications with criminal record
- **Control group** (untreated units): applications without criminal record
- **Outcome variable** Y_i: callback

	T_i (ex-felon)	Y_i (callback)
Ex-felon applicant	1	0
Non-ex-felon applicant	0	1

Causal effects and counterfactuals

- What does "T_i causes Y_i" mean? $\rightsquigarrow$ **counterfactuals**, "what if"
- Would an employer treat criminal & noncriminal applicants differently?
- Two **potential outcomes**:
  - $Y_i(1)$: would applicant i get a callback if applied as an ex-felon?
  - $Y_i(0)$: would applicant i get a callback if applied not as an ex-felon?
- **Causal effect**: $Y_i(1)$ - $Y_i(0)$
  - $Y_i(1) - Y_i(0) = 0 \rightsquigarrow$ criminal record has no impact on callback
  - $Y_i(1) - Y_i(0) = -1 \rightsquigarrow$ criminal record prevents callback
  - $Y_i(1) - Y_i(0) = +1 \rightsquigarrow$ criminal record leads to callback

Potential Outcomes

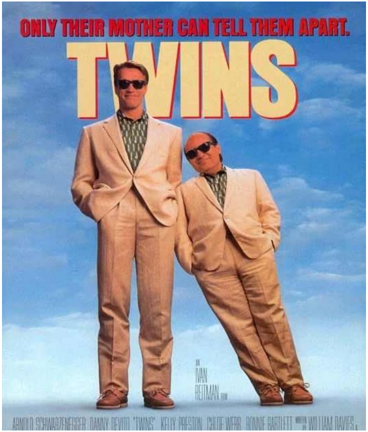
	T_i (ex-felon)	Y_i (callback)	$Y_i(1)$	$Y_i(0)$
Ex-felon applicant	1	0	0	???
Non-ex-felon applicant	0	1	???	1

- **Fundamental problem of causal inference**:
  - We only observe one of the two potential outcomes.
  - Observe $Y_i = Y_i(1)$ if $T_i = 1$ or $Y_i = Y_i(0)$ if $T_i = 0$
- To infer causal effect, we need to infer the missing counterfactuals!

How can we figure out counterfactuals?

- Find a similar unit! $\rightsquigarrow$ **matching**
  - Mill's method of difference
- Did applicant fail to get a job offer because of his criminal record?
  - $\rightsquigarrow$ find a non-ex-felon who is just like ex-felon applicant.
- NJ increased the minimum wage. Causal effect on unemployment?
  - $\rightsquigarrow$ find a state similar to NJ that didn't increase minimum wage.

Imperfect matches



- The problem: imperfect matches!
- Say we match i (treated) and j (control)
- **Selection Bias**: $Y_i(1) \neq Y_j(1)$
- Those who take treatment may be different that those who take control.
- How can we correct for that?

Break time

- Space here for a break in the action

Changing minds on gay marriage

- Question: can we effectively persuade people to change their minds?
- Hugely important question for political campaigns, companies, etc.
- Psychological studies show it isn't easy.
- **Contact Hypothesis**: outgroup hostility diminished when people from different groups interact with one another.
- Today we'll explore this question the context of support for gay marriage and contact with a member of the LGBT community.
  - $Y_i =$ support for gay marriage (1) or not (0)
  - $T_i =$ contact with member of the LGBT community (1) or not (0)

Causal effects and counterfactuals

- What does "T_i causes Y_i" mean? $\rightsquigarrow$ **counterfactuals**, "what if"
- Would citizen i have supported gay marriage if they had contact with a member of the LGBT community?
- Two **potential outcomes**:
  - $Y_i(1)$: would i have supported gay marriage if they **had** contact with a member of the LGBT community?
  - $Y_i(0)$: would i have supported gay marriage if they **didn't have** contact with a member of the LGBT community?
- **Causal effect** for citizen i: $Y_i(1) - Y_i(0)$
- **Fundamental problem of causal inference**: only one of the two potential outcomes is observable.

Sigma notation

- We will often refer to the **sample size** (number of units) as n.
- We often have n measurements of some variable: $(Y_1, Y_2, ..., Y_n)$
- We often want sums: how many in our sample support gay marriage?

$$Y_1 + Y_2 + ... + Y_n$$

* Notation is a bit clunky, so we often use the **Sigma notation**:

$$\sum_{i=1}^{n} Y_i = Y_1 + Y_2 + ... + Y_n$$

* $\sum_{i=1}^{n}$ means sum each value from Y_1 to Y_n

Averages

- The **sample average** or **sample mean** is simply the sum of all values divided by the number of values.
- Sigma notation allows us to write this in a compact way:

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

* Suppose we surveyed 6 people and 3 supported gay marriage:

$$\bar{Y} = \frac{1}{6}(1 + 1 + 1 + 0 + 0 + 0) = 0.5$$

Quantity of interest

- We want to estimate the average causal effects over all units:

$$\text{Sample Average Treatment Effect (SATE)} = \frac{1}{n}\sum_{i=1}^{n}(Y_i(1) - Y_i(0))$$

* Why can't we just calculate this quantity directly? * What we can estimate instead:

$$\text{Difference in means} = \bar{Y}_{treated} - \bar{Y}_{control}$$

- $\bar{Y}_{treated}$: observed average outcome for treated group - $\bar{Y}_{control}$: observed average outcome for control group * When will the difference-in-means be a good estimate of the SATE?

Randomized control trials (RCTs)

- **Randomize control trial**: each unit's treatment assignment is determined by chance
  - e.g., flip a coin; draw read and blue chips from a hat; etc.
- Randomization ensures **balance** between treatment and control group.
  - Treatment and control group are identical **on average**
  - Similar on both observable and unobservable characteristics.
- Control group $\approx$ what would have happened to treatment group if they had taken control
  - $\bar{Y}_{control} \approx \frac{1}{n} \sum_{i=1}^{n} Y_i(0)$
  - $\bar{Y}_{treated} - \bar{Y}_{control} \approx \text{SATE}$

Some potential problems with RCTs

- **Placebo effects**:
  - Respondents will be affected by any intervention, even if they shouldn't have any effect
- **Hawthorne effects**:
  - Respondents act differently just knowing that they are under study.

Balance checking

- Can we determine if randomization "worked"?
- If it did, we shouldn't see large differences between treatment and control group on **pretreatment variable**
  - Pretreatment variable are those that are unaffected by treatment
- We can check in the actual data for some pretreatment variable X
  - $\bar{X}_{treated}$: average value of variable for treated group
  - $\bar{X}_{control}$: average value of variable for control group
  - Under randomization, $\bar{X}_{treated} - \bar{X}_{control} \approx 0$

Multiple treatments

- Instead of 1 treatment, we might have multiple **treatment arms**:
  - Control condition
  - Treatment A
  - Treatment B
  - Treatment C, etc.
- In this case, we will look at multiple comparisons:
  - $\bar{Y}_{treated,A} - \bar{Y}_{control}$
  - $\bar{Y}_{treated,B} - \bar{Y}_{control}$
  - $\bar{Y}_{treated,A} - \bar{Y}_{treated,B}$
- If treatment arms are randomly assigned, these differences will be good estimators for each causal contrast.