Regression Part II: Model Fit and Variable type/quantity

PSC7475: Week 6

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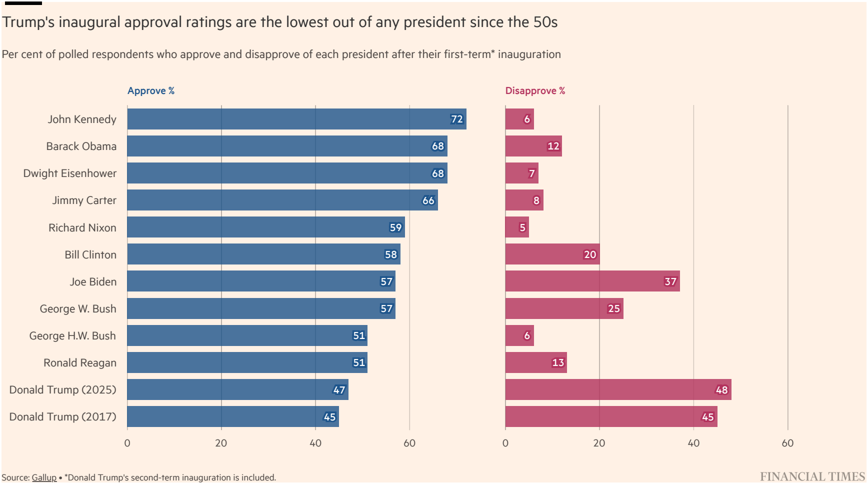
Presidential Popularity and the Midterms



Trump's inaugural approval ratings are the lowest out of any president since the 50s

Per cent of polled respondents who approve and disapprove of each president after their first-term* inauguration

Source: Gallup • *Donald Trump's second-term inauguration is included.

Presidential Popularity and the Midterms

- Does popularity of the president or recent changes in the economy better predict midterm election outcomes?

Name	Description
`year`	midterm election year
`president`	name of president
`party`	Democrat or Republican
`approval`	Gallup approval rating at midterms
`rdi.change`	change in real disposable income over the year before midterms
`seat.change`	change in the number of House seats for the president’s party

Loading the data:

```r
library(tidyverse)
midterms <- read.csv("../data/midterms.csv")
head(midterms)
```

```
##   year  president party approval seat.change rdi.change
## 1 1946     Truman     D       33         -55         NA
## 2 1950     Truman     D       39         -29        8.2
## 3 1954 Eisenhower     R       61          -4        1.0
## 4 1958 Eisenhower     R       57         -47        1.1
## 5 1962    Kennedy     D       61          -4        5.0
## 6 1966    Johnson     D       44         -47        5.3
```

Fitting the Approval Model

```r
fit.app <- lm(seat.change ~ approval, data = midterms)
fit.app
```

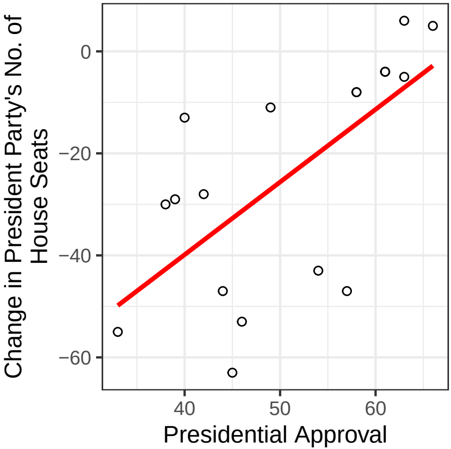
```
##
## Call:
## lm(formula = seat.change ~ approval, data = midterms)
##
## Coefficients:
## (Intercept)     approval
##     -96.845        1.424
```

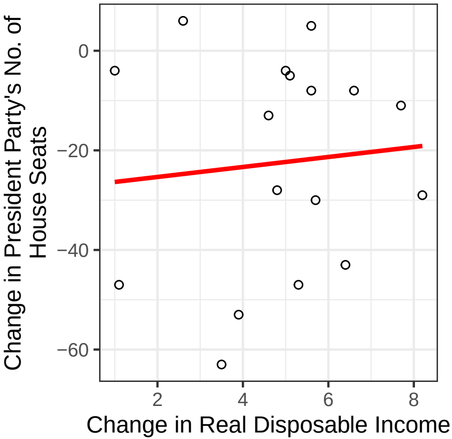
Fitting the Income Model

```r
fit.rdi <- lm(seat.change ~ rdi.change, data = midterms)
fit.rdi
```

```
##
## Call:
## lm(formula = seat.change ~ rdi.change, data = midterms)
##
## Coefficients:
## (Intercept)   rdi.change
##     -27.354        1.004
```

Comparing Models





- How well do the models “fit the data”?
    - How well does the model predict the outcome variable in the data?

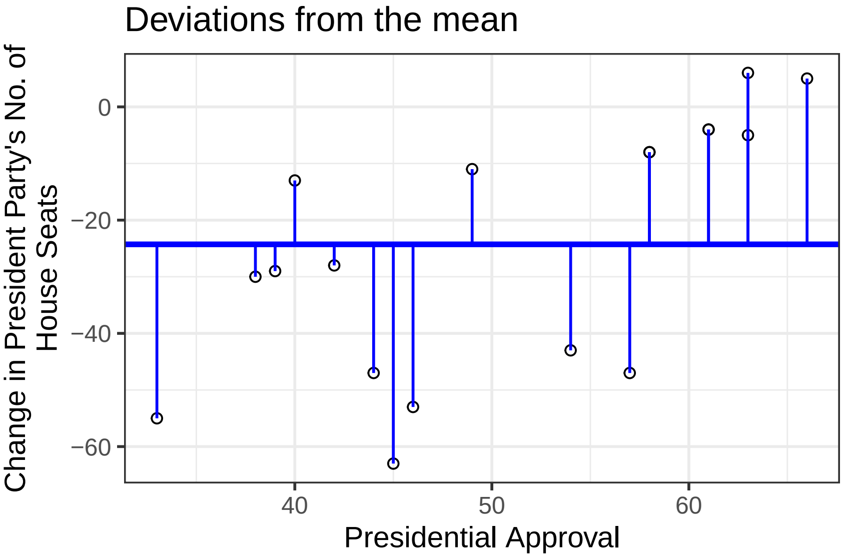
Model Fit

- One number summary of model fit: R^2 or **coefficient of determination**.
    - Measure of the **proportional reduction in error** by the model.
- Prediction error compared to what?
    - Baseline prediction error: **Total sum of squares**
        - $\text{TSS} = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$
    - Model prediction error: **Sum of squared residuals**
        - $\text{SSR} = \sum_{i=1}^{n} \epsilon_i^2$
    - TSS - SSR: reduction in prediction error by the model.
- R^2 is this reduction in error divided by the baseline error:

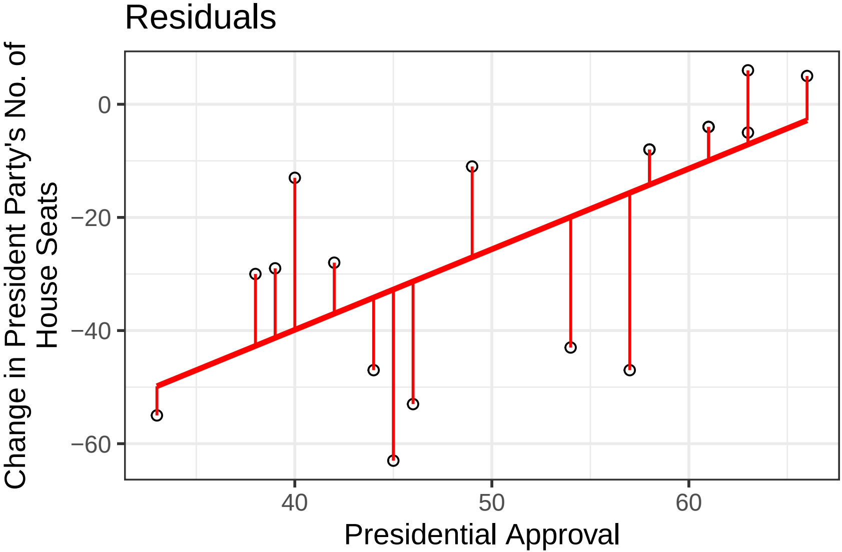
$$R^2 = \frac{\text{TSS} - \text{SSR}}{\text{TSS}}$$

- Roughly: proportion of the variation in Y_i “explained by” X_i

Total sum of squares vs. Sum of squared residuals



Total sum of squares vs. Sum of squared residuals



Model Fit in R

- To access R^2 from the `lm()` output, use the `summary()` function:

```
fit.app.sum <- summary(fit.app)
fit.app.sum$r.squared
```

```
## [1] 0.4307133
```

- Compare to fit using change in income:

```
fit.rdi.sum <- summary(fit.rdi)
fit.rdi.sum$r.squared
```

```
## [1] 0.008529029
```

- Which does a better job predicting midterm election outcomes?

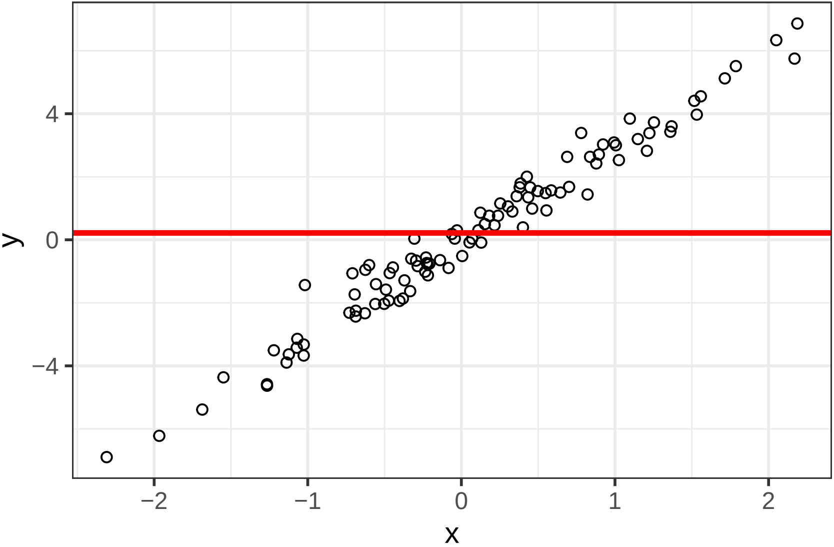
Fake data, better fit

- Little hard to see what's happening in that example.
- Let's look at fake variables x and y:

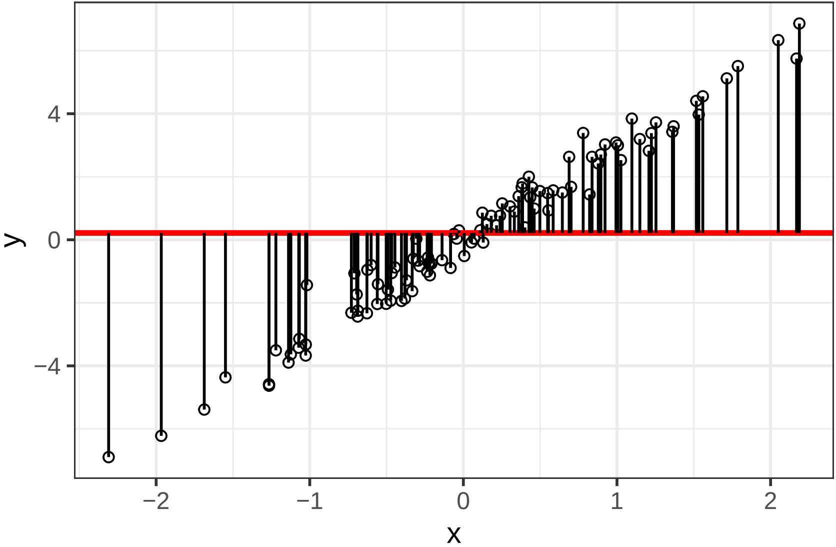
```
fit.x <- lm(y ~ x)
```

- Very good model fit: $R^2 \approx 0.95$

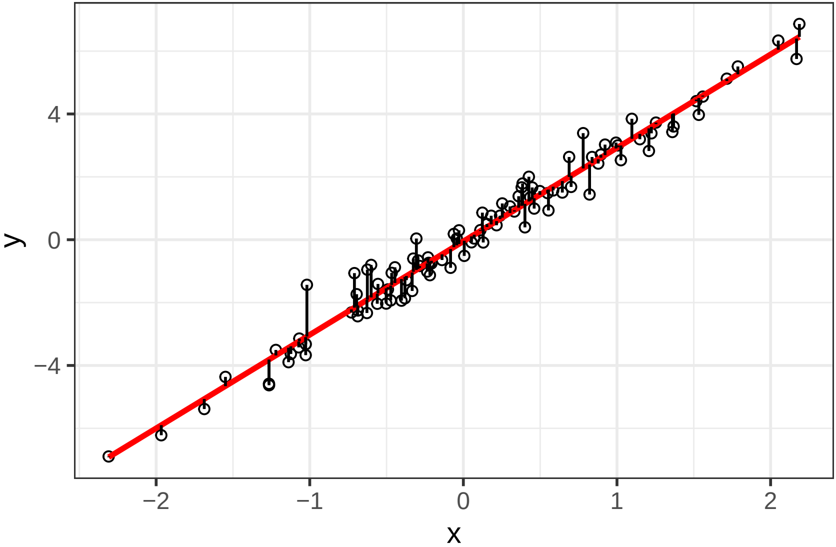
Fake data, better fit



Fake data, better fit

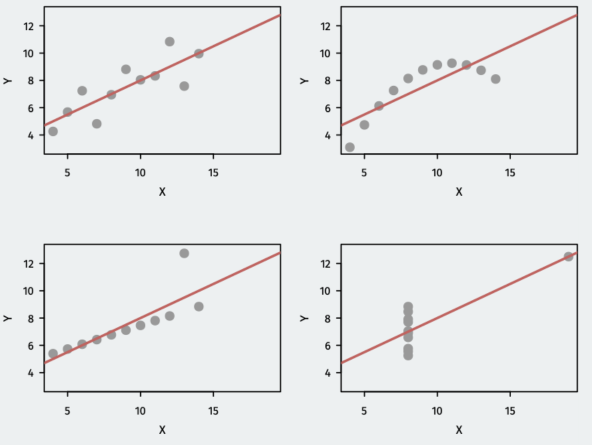


Fake data, better fit



Is R-squared useful?

- Can be very misleading. Each of these samples have the same R^2 even though they are vastly different:



Overfitting

- **In-sample fit**: how well your model predicts the data used to estimate it.
  - R^2 is a measure of in-sample fit.
- **Out-of-sample fit**: how well your model predicts new data.
- **Overfitting**: OLS optimizes in-sample fit; may do poorly out of sample.
  - Example: predicting winner of Democratic presidential primary with gender of the candidate.
  - Until 2016, gender was a **perfect** predictor of who wins the primary.
  - Prediction for 2016 based on this: Bernie Sanders as Dem. nominee.
  - Bad out-of-sample prediction due to overfitting!

Multiple Predictors (Multivariate Regression)

- What if we want to predict Y as a function of many variables?

$$\texttt{seat.change}_i = \alpha + \beta_1 \texttt{approval}_i + \beta_2 \texttt{rdi.change}_i + \epsilon_i$$

- Better predictions (at least in-sample).
    - Better interpretation as ceteris paribus relationships:
        - β_1 is the relationship between `approval` and `seat.change` holding `rdi.change` constant.

Multiple regression in R

```r
mult.fit <- lm(seat.change ~ approval + rdi.change, data = midterms)
mult.fit
```

```
##
## Call:
## lm(formula = seat.change ~ approval + rdi.change, data = midterms)
##
## Coefficients:
## (Intercept)     approval   rdi.change
##    -120.436        1.572        3.334
```

- $\hat{\alpha} = -120.4$: average seat change president has 0% approval and no change in income levels.
- $\hat{\beta}_1 = 1.57$: average increase in seat change for additional percentage point of approval, **holding RDI change fixed**
- $\hat{\beta}_1 = 3.334$: average increase in seat change for each additional percentage point increase of RDI, **holding approval fixed**

Least squares with multiple regression

- How do we estimate the coefficients?
- The same exact way as before: minimize prediction error!
- Residuals (aka prediction error) with multiple predictors:

$$\hat{\epsilon}_i = \texttt{seat.change}_i - \hat{\alpha} - \hat{\beta}_1 \texttt{approval}_i - \hat{\beta}_2 \texttt{rdi.change}_i$$

- Find the coefficients that minimizes the **sum of the squared residuals**:

$$\text{SSR} = \sum_{i=1}^{n} \hat{\epsilon}_i^2 = (Y_i - \hat{\alpha} - \hat{\beta}_1 X_{i1} - \hat{\beta}_2 X_{i2})^2$$

Model fit with multiple predictors

- R^2 mechanically increases when you add a variables to the regression.
  - But this could be overfitting!!
- Solution: penalize regression models with more variables.
  - Occam's razor: **simpler models are preferred**
- Adjusted R^2: lowers regular R^2 for each additional covariate.
  - If the added covariates don't help predict, adjusted R^2 goes down!

Comparing Model Fits

```r
summary(fit.app)$r.squared
```

```
## [1] 0.4307133
```

```r
summary(mult.fit)$r.squared
```

```
## [1] 0.4448387
```

```r
summary(mult.fit)$adj.r.squared
```

```
## [1] 0.3655299
```

Binary and Categorical Predictors

- Political effects of government programs
  - *Progresa*: Mexican conditional cash transfer program (CCT) from ~2000
    - Welfare $ given if kids enrolled in schools, get regular check-ups, etc.
  - Do these programs have political effects?
    - Program had support from most parties.
    - Was implemented in a nonpartisan fashion.
    - Would the incumbent presidential party be rewarded?

The Data

- Randomized roll-out of the CCT program:
  - treatment: receive CCT 21 months before 2000 election
  - control: receive CCT 6 months before 2000 election
  - Does having CCT longer mobilize voters for incumbent PRI party?

Name	Description
`treatment`	early Progresa (1) or late Progresa (0)
`pri2000s`	PRI votes in the 2000 election as a share of adults in precinct
`t2000`	turnout in the 2000 election as share of adults in precinct

```r
cct <- read.csv("../data/progresa.csv")
```

Difference in Means Estimates

- Does CCT affect turnout?

```
cct.turn.ate <- cct %>% group_by(treatment) %>%
  summarize(t2000_mean = mean(t2000)) %>%
  pivot_wider(names_from = treatment, values_from = t2000_mean) %>%
  mutate(turnout_ate = `1` - `0`)
cct.turn.ate$turnout_ate
```

```
## [1] 4.269676
```

- Does CCT affect PRI (incumbent) votes?

```
cct.pri.ate <- cct %>% group_by(treatment) %>%
  summarize(pri2000s_mean = mean(pri2000s)) %>%
  pivot_wider(names_from = treatment, values_from = pri2000s_mean) %>%
  mutate(pri_ate = `1` - `0`)
cct.pri.ate$pri_ate
```

```
## [1] 3.622496
```

Binary independent variables

$$Y_i = \alpha + \beta X_i + \epsilon_i$$

- When independent variable X_i is binary:
- Intercept α is the average outcome in the $X = 0$ group.
- Slope β is the difference-in-means of Y between $X = 1$ group and $X = 0$ group.

$$\hat{\beta} = \bar{Y}_{treated} - \bar{Y}_{control}$$

- If there are other independent variables, this becomes the difference-in-means controlling for those covariates.

Linear regression for experiments

- Under **randomization**, we can estimate the ATE with regression:

```
cct.pri.ate <- cct %>%
  group_by(treatment) %>%
  summarize(pri2000s_mean = mean(pri2000s)) %>%
  pivot_wider(names_from = treatment, values_from = pri2000s_mean) %>%
  mutate(pri_ate = `1` - `0`)
cct.pri.ate$pri_ate
```

```
## [1] 3.622496
```

```
lm(pri2000s ~ treatment, data = cct)
```

```
##
## Call:
## lm(formula = pri2000s ~ treatment, data = cct)
##
## Coefficients:
## (Intercept)    treatment
##      34.489        3.622
```

Categorical variables in regression

- We often have **categorical variables**:
  - Race/ethnicity: white, Black, Latino, Asian.
  - Partisanship: Democrat, Republican, Independent
  - Strategy for including in a regression: create a **series of binary variables**

Unit	Party	Democrat	Republican	Independent
1	Democrat	1	0	0
2	Democrat	1	0	0
3	Independent	0	0	1
4	Republican	0	1	0
⋮	⋮	⋮	⋮	⋮

- Then include all but one of these binary variables:

$$turnout_i = \alpha + \beta_1 \text{Republican}_i + \beta_2 \text{Independent}_i + \epsilon_i$$

Interpreting categorical variables

$$turnout_i = \alpha + \beta_1 \text{Republican}_i + \beta_2 \text{Independent}_i + \epsilon_i$$

- $\hat{\alpha}$: average outcome in the **omitted group/baseline** (Democrats).
- $\hat{\beta}$ coefficients: average difference between each group and the baseline.
    - $\hat{\beta}_1$: average difference in turnout between Republicans and Democrats
    - $\hat{\beta}_2$: average difference in turnout between Independents and Democrats