

# **04-630**

# **Data Structures and Algorithms for Engineers**

## Lecture 7: Constrainers and Dictionaries I

# Agenda

- Containers and Dictionaries
  - Arrays
  - Lists
  - List ADT
    - Array implementation
    - Linked list implementation

# CONTAINERS AND DICTIONARIES

# Containers and Dictionaries

- Containers
- Dictionaries
- List ADT
  - Array implementation
  - Linked list implementation

# Containers and Dictionaries

- **Containers** are data structures that permit storage and retrieval of data items **independent of content**
- **Dictionaries** are data structures that retrieve data based on **key** values (i.e., content)

# Containers and Dictionaries

- **Containers** are distinguished by the particular retrieval order they support
- In the case of **stacks** and **queues**, the retrieval order depends on the insertion order

# Containers and Dictionaries

**Stack:** supports retrieval by last-in, first-out (LIFO) order

- $\text{Push}(x, S)$                       Insert item  $x$  at the **top** of a stack  $S$
- $\text{Pop}(S)$                               Return (and remove) the **top** item of a stack  $S$

# Containers and Dictionaries

**Queue:** support retrieval by first-in, first-out (FIFO) order

- Enqueue( $x, Q$ )      Insert item  $x$  at the **back** of a queue  $Q$
- Dequeue ( $Q$ )      Return (and remove) the the **front** item from a queue  $Q$



# Containers and Dictionaries

**Dictionaries** permits access to data items by content/key

- You put an item into a dictionary so that you can find it when you need it

# Containers and Dictionaries

Main dictionary operations are

- $\text{Search}(D, k)$     Given a search key  $k$ , return a pointer to the element in dictionary  $D$  whose key value is  $k$ , if one exists
- $\text{Insert}(D, x)$     Given a data item  $x$ , add it to the dictionary  $D$
- $\text{Delete}(D, x)$     Given a pointer to a given data item  $x$  in the dictionary  $D$ , remove it from  $D$

# Containers and Dictionaries

Some dictionary data structures also **efficiently** support other useful operations

- $\text{Max}(D)$                       Retrieve the item with the largest key from  $D$
- $\text{Min}(D)$                       Retrieve the item with the smallest key from  $D$

These operations allows the dictionary to serve as a **priority queue**; more on this later.

# Containers and Dictionaries

Some dictionary data structures also **efficiently** support other useful operations

- $\text{Predecessor}(D, x)$       Retrieve the item from  $D$  whose key is immediately before  $x$  in sorted order
- $\text{Successor}(D, x)$       Retrieve the item from  $D$  whose key is immediately after  $x$  in sorted order

These operations enable us to iterate through the elements of the data structure

# Containers and Dictionaries

- We have defined these container and dictionary operations in an **abstract** manner,  
  
without reference to their implementation or the implementation of the structure itself
- There are many implementation options
  - Unsorted arrays
  - Sorted arrays
  - Singly-linked lists
  - Doubly-linked lists
  - Binary search trees
  - Balanced binary search trees
  - Hash tables
  - Heaps
  - ...

# Lists

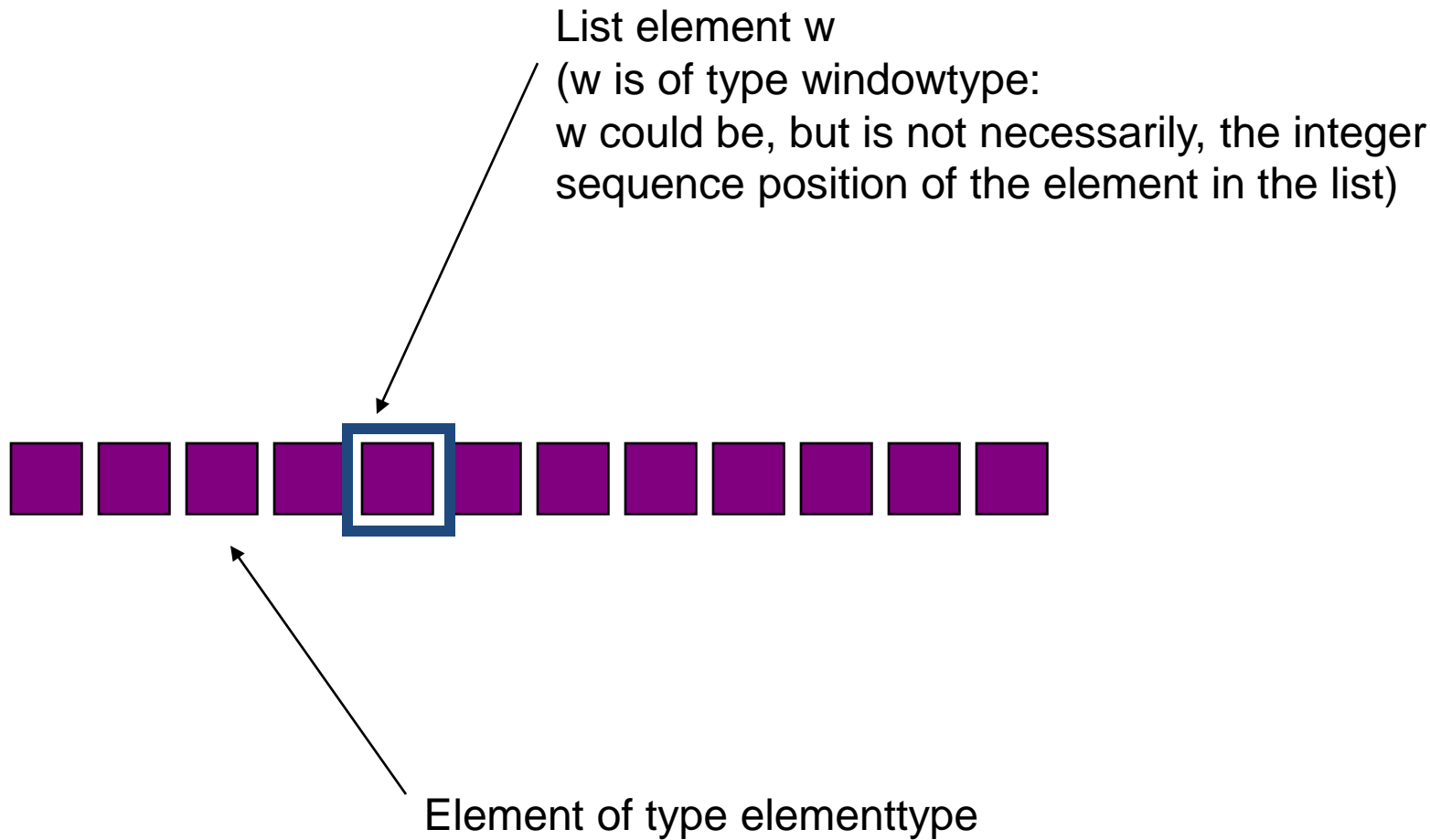
# Lists

A list is an ordered sequence of zero or more elements of a given type

$a_1, a_2, a_3, \dots a_n$

- $a_i$  is of type **elementtype**
- $a_i$  precedes  $a_{i+1}$
- $a_{i+1}$  succeeds or follows  $a_i$
- If  $n=0$  the list is empty: a null list
- The position of  $a_i$  is  $i$

# Lists





# LIST: An ADT specification of a list type

- Let **L** denote all possible values of type LIST (*i.e.* lists of elements of type *elementtype*)
- Let **E** denote all possible values of type *elementtype*
- Let **B** denote the set of Boolean values *true* and *false*
- Let **W** denote the set of values of type *windowtype*

# LIST Operations

|               |                     |
|---------------|---------------------|
| Declare(L)    | returns listtype    |
| End(L)        | returns windowtype  |
| Empty(L)      | returns windowtype  |
| IsEmpty(L)    | returns Boolean     |
| First(L)      | returns windowtype  |
| Next(w,L)     | returns windowtype  |
| Previous(w,L) | returns windowtype  |
| Last(L)       | returns windowtype  |
| Insert(e,w,L) | returns listtype    |
| Delete(w,L)   | returns listtype    |
| Examine(w,L)  | returns elementtype |

# LIST Operations

Syntax of ADT Definition:

Operation:

**What\_You\_Pass\_It** → **What\_It\_Returns** :

# LIST Operations

Declare:  $\rightarrow \mathbf{L}$  :

The function value of **Declare(L)** is an empty list

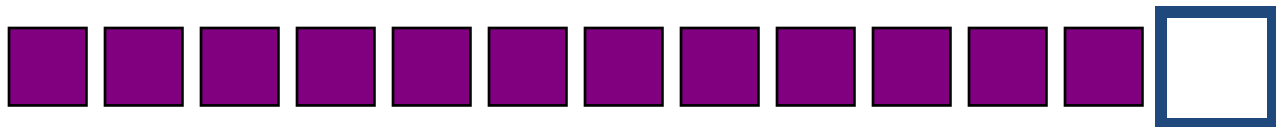
- alternative syntax: **LIST L**

# LIST Operations

End:  $L \rightarrow W$  :

The function **End(L)** returns the position after the last element in the list

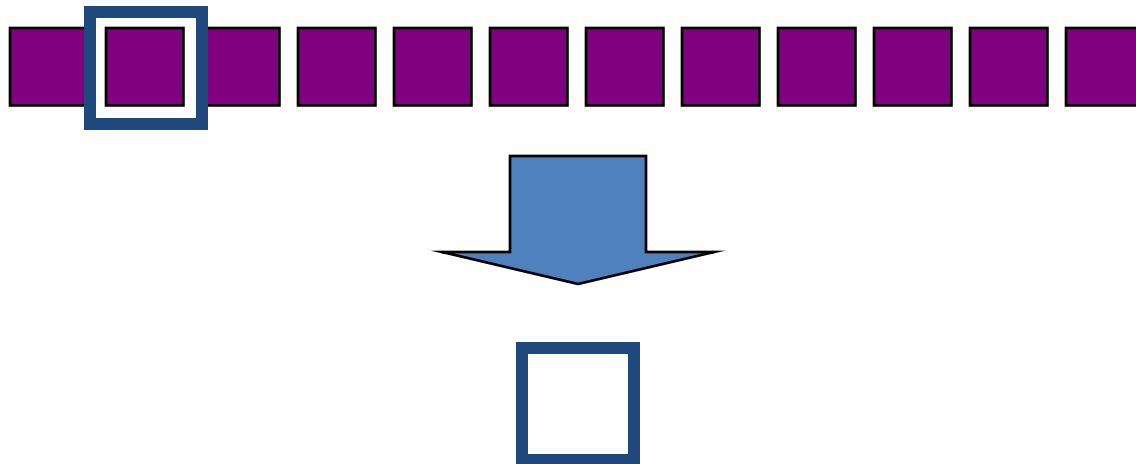
(i.e. the value of the function is the window position after the last element in the list)



# LIST Operations

Empty:  $L \rightarrow L \times W$  :

The function **Empty** causes the list to be emptied and it returns position **End(L)**



# LIST Operations

IsEmpty:  $L \rightarrow B$  :

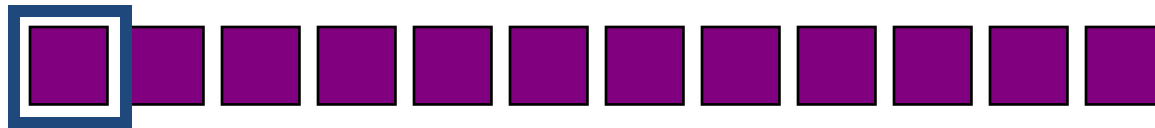
The function value `IsEmpty(L)` is `true` if  $L$  is empty; otherwise it is `false`

# LIST Operations

First:  $L \rightarrow W$  :

The function value  $\text{First}(L)$  is the window position of the first element in the list;

if the list is empty, it has the value  $\text{End}(L)$





# LIST Operations

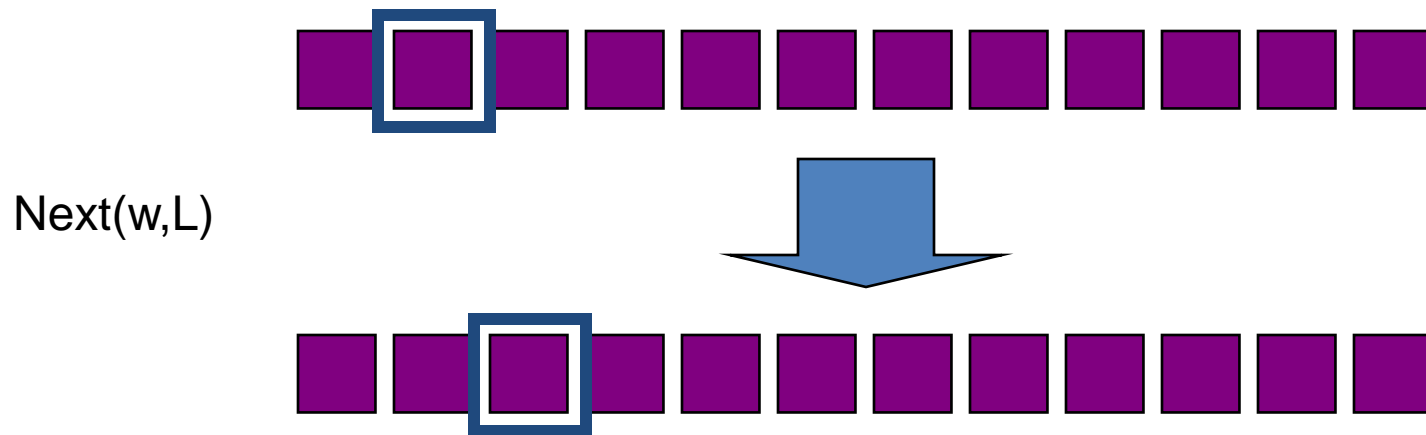
Next:  $L \times W \rightarrow W$  :

The function value  $\text{Next}(w, L)$  is the window position of the next successive element in the list;

if we are already at the last element of the list then the value of  $\text{Next}(w, L)$  is  $\text{End}(L)$ ;

if the value of  $w$  is  $\text{End}(L)$ , then the operation is undefined

# LIST Operations



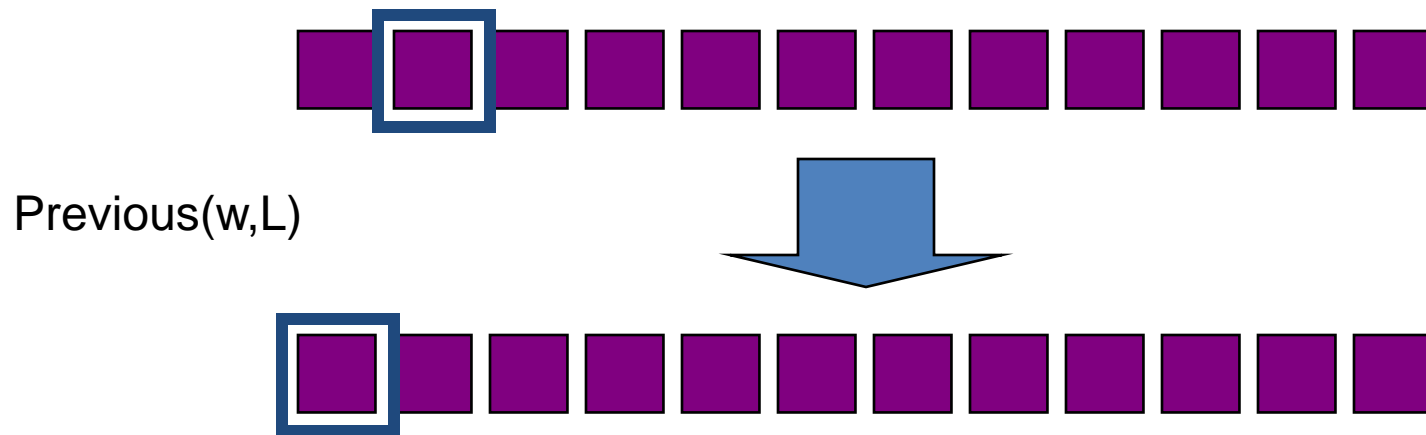
# LIST Operations

Previous:  $L \times W \rightarrow W$  :

The function value  $\text{Previous}(w, L)$  is the window position of the previous element in the list;

if we are already at the beginning of the list ( $w = \text{First}(L)$ ), then the value is undefined

# LIST Operations

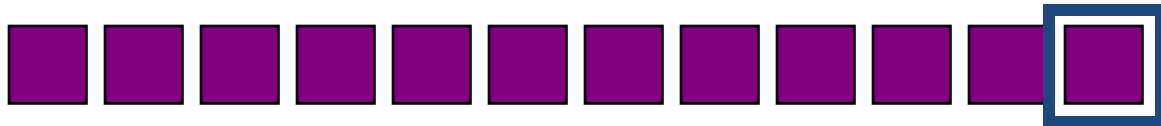


# LIST Operations

Last:  $L \rightarrow W$  :

The function value  $\text{Last}(L)$  is the window position of the last element in the list;

if the list is empty, it has the value  $\text{End}(L)$



# LIST Operations

Insert:  $E \times L \times W \rightarrow L \times W$  :

Insert( $e, w, L$ )

Insert an element  $e$  at position  $w$  in the list  $L$ , moving elements at  $w$  and following positions to the next higher position

$$a_1, a_2, \dots, a_n \rightarrow a_1, a_2, \dots, a_{w-1}, e, a_w, \dots, a_n$$

# LIST Operations

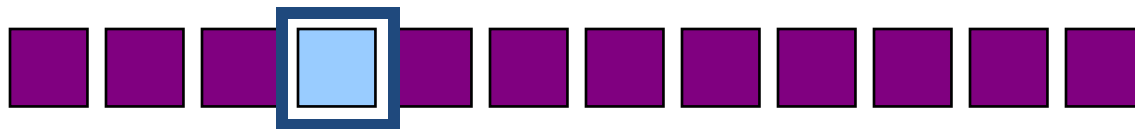
If  $w = \text{End}(L)$  then

$a_1, a_2, \dots, a_n \rightarrow a_1, a_2, \dots, a_n, e$

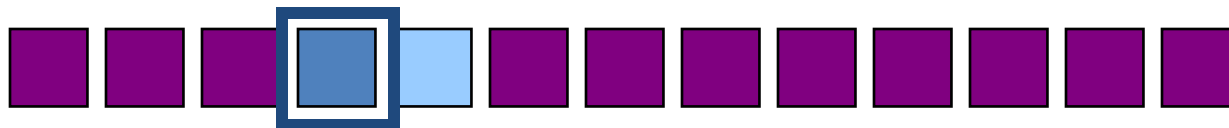
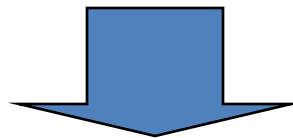
The window w is moved over the new element e

The function's value is the list with the element inserted

# LIST Operations

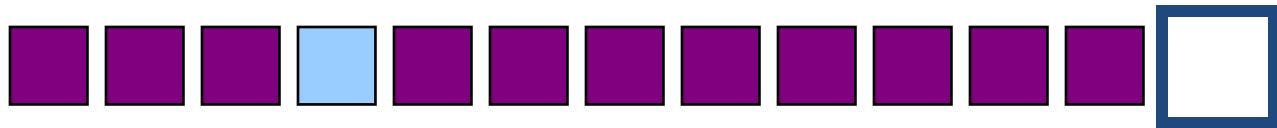


Insert(e,w,L)

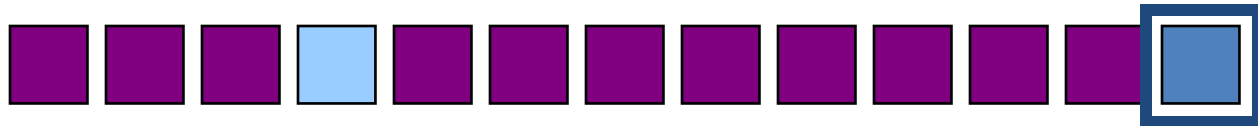
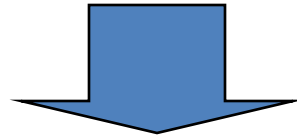




# LIST Operations ( $w = \text{End}(L)$ )



Insert( $e, w, L$ )



# LIST Operations

Delete:  $L \times W \rightarrow L \times W$  :

Delete( $w, L$ )

Delete the element at position  $w$  in the list  $L$

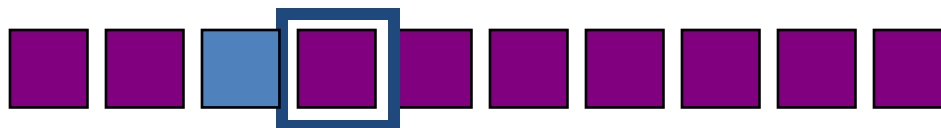
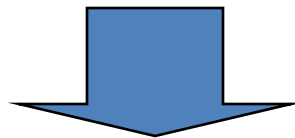
$a_1, a_2, \dots, a_n \rightarrow a_1, a_2, \dots, a_{w-1}, a_{w+1}, \dots, a_n$

- If  $w = \text{End}(L)$  then the operation is undefined
- The function value is the list with the element deleted

# LIST Operations



Delete(w,L)



# LIST Operations

Examine:  $L \times W \rightarrow E$ :

The function value  $\text{Examine}(w, L)$  is the value of the element at position  $w$  in the list;

if we are already at the end of the list (*i.e.*  $w = \text{End}(L)$ ), then the value is undefined

# LIST Operations

Example of List manipulation

Declare(L)  
w = End(L)



empty list

# LIST Operations

Example of List manipulation

$w = \text{End}(L)$



$\text{Insert}(e, w, L)$



# LIST Operations

Example of List manipulation

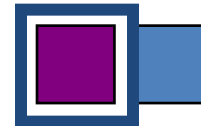
$w = \text{End}(L)$



$\text{Insert}(e, w, L)$



$\text{Insert}(e, w, L)$



# LIST Operations

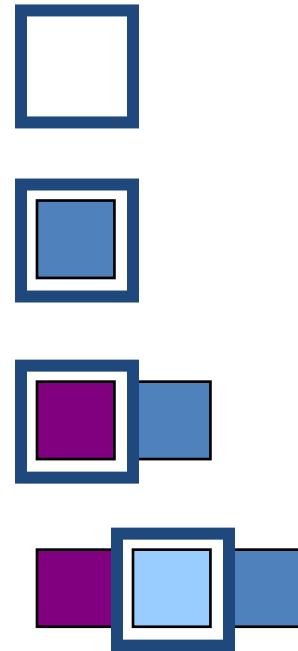
Example of List manipulation

$w = \text{End}(L)$

$\text{Insert}(e, w, L)$

$\text{Insert}(e, w, L)$

$\text{Insert}(e, \text{Last}(L), L)$

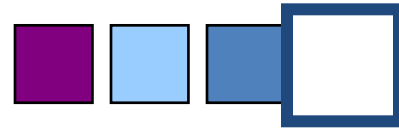




# LIST Operations

Example of List manipulation

$w = \text{Next}(\text{Last}(L), L)$

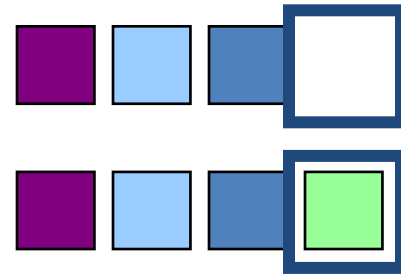


# LIST Operations

Example of List manipulation

$w = \text{Next}(\text{Last}(L), L)$

$\text{Insert}(e, w, L)$



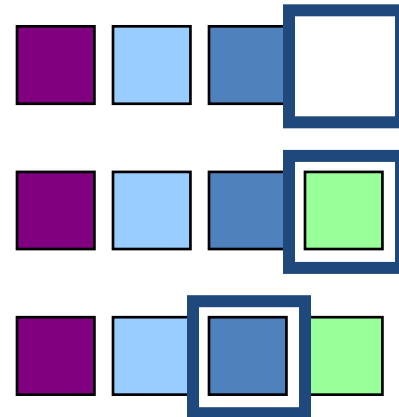
# LIST Operations

Example of List manipulation

$w = \text{Next}(\text{Last}(L), L)$

$\text{Insert}(e, w, L)$

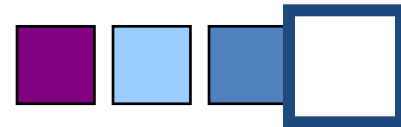
$w = \text{Previous}(w, L)$



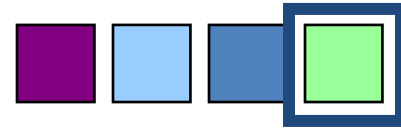
# LIST Operations

Example of List manipulation

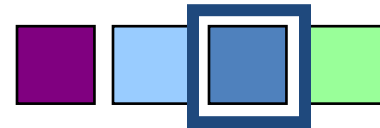
$w = \text{Next}(\text{Last}(L), L)$



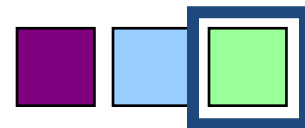
$\text{Insert}(e, w, L)$



$w = \text{Previous}(w, L)$



$\text{Delete}(w, L)$



# ADT Specification

- The key idea is that we have not specified how the lists are to be implemented, merely their values and the operations of which they can be operands
- This 'old' idea of data abstraction is one of the key features of object-oriented programming
- C++ is a particular implementation of this object-oriented methodology

# ADT Implementation

- Of course, we still have to implement this ADT specification
- The choice of implementation will depend on the requirements of the application

# ADT Implementation

We will look at two implementations

- Array implementation
  - uses a static data-structure
  - reasonable if we know in advance the maximum number of elements in the list
- Pointer implementation
  - Also known as a linked-list implementation
  - uses dynamic data-structure
  - best if we don't know in advance the number of elements in the list (or if it varies significantly)
  - overhead in space: the pointer fields

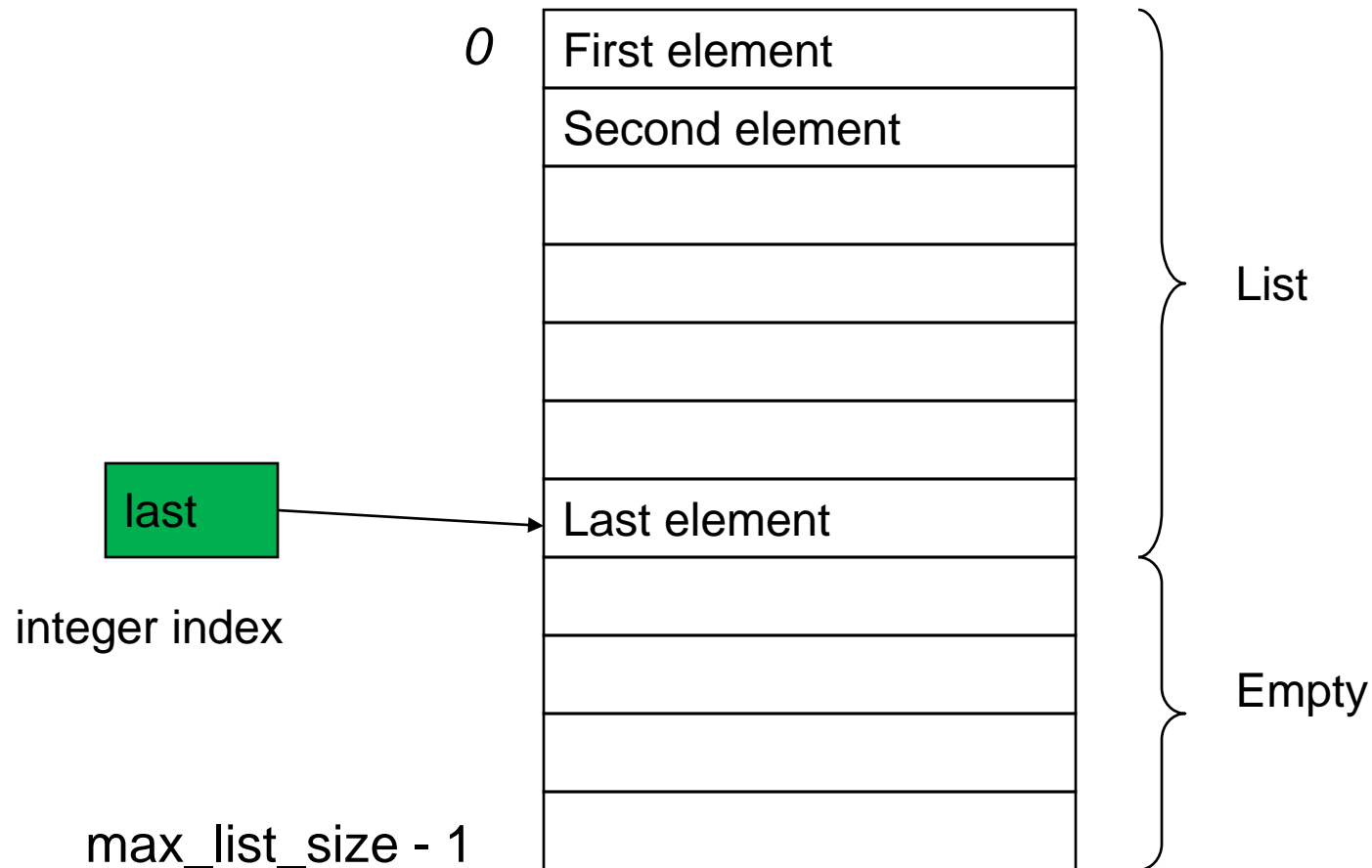
# LIST: Array Implementation

We will do this in two steps:

- the implementation (or representation) of the four constituents datatypes of the ADT:
  - list
  - elementtype
  - Boolean
  - Windowtype
- the implementation of each of the ADT operations



# LIST: Array Implementation



# LIST: Array Implementation

type elementtype

type LIST

type Boolean

type windowtype

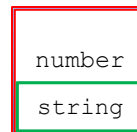
# LIST: Array Implementation

```
/* array implementation of LIST ADT */

#include <stdio.h>
#include <math.h>
#include <string.h>

#define MAX_LIST_SIZE 100
#define FALSE 0
#define TRUE 1

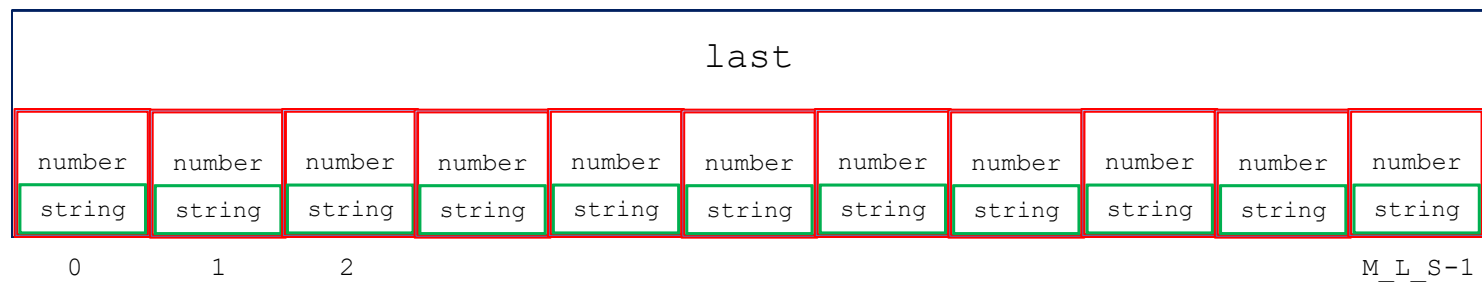
typedef struct {
    int number;
    char *string;
} ELEMENT_TYPE;
```



# LIST: Array Implementation

```
typedef struct {  
    int last;  
    ELEMENT_TYPE a[MAX_LIST_SIZE];  
} LIST_TYPE;
```

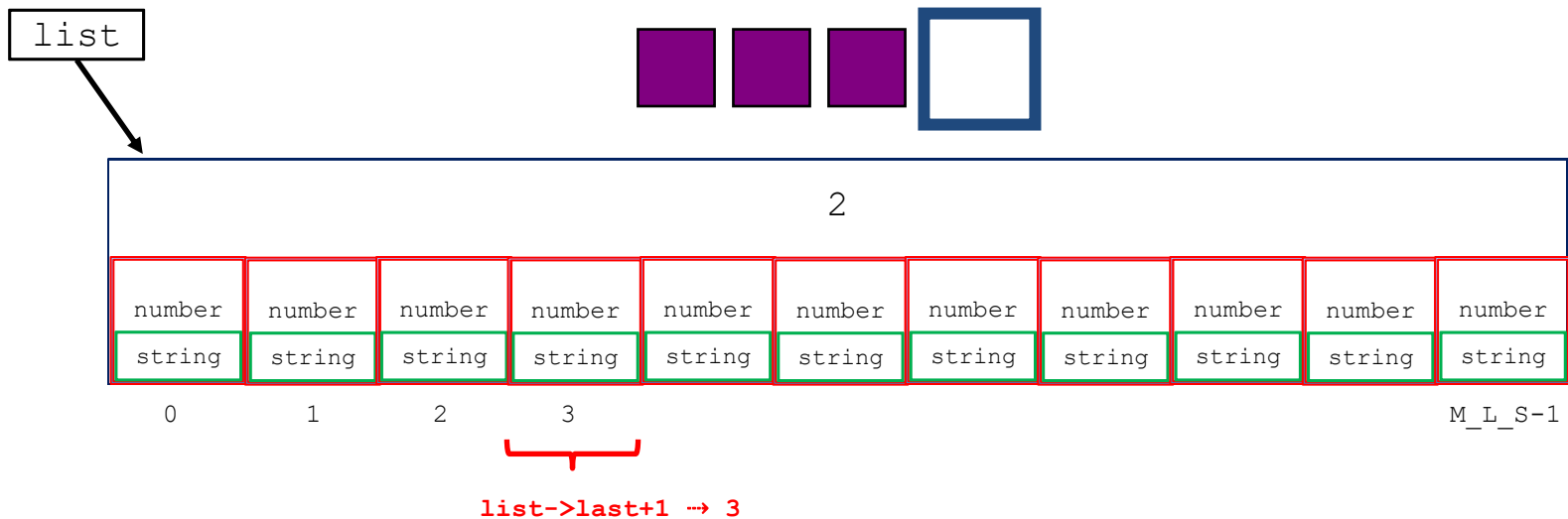
```
typedef int WINDOW_TYPE;
```



# LIST: Array Implementation

```
/** position following last element in a list **/
```

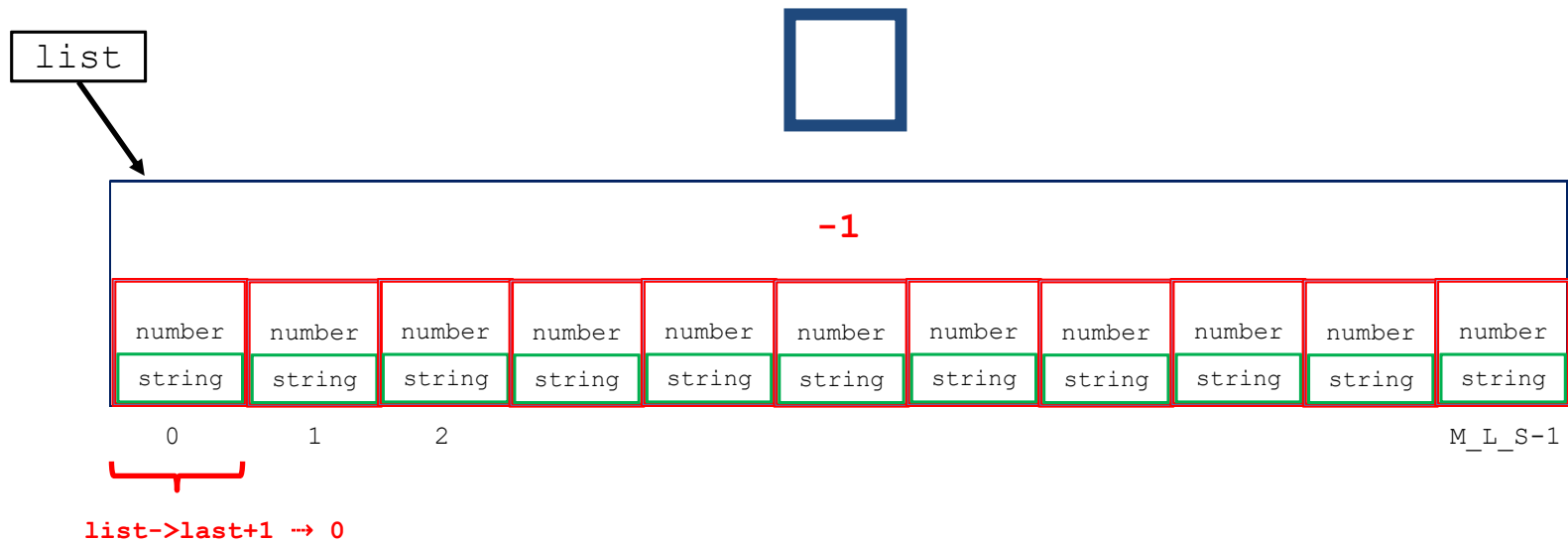
```
WINDOW_TYPE end(LIST_TYPE *list) {  
    return(list->last+1);  
}
```



# LIST: Array Implementation

```
/** empty a list */
```

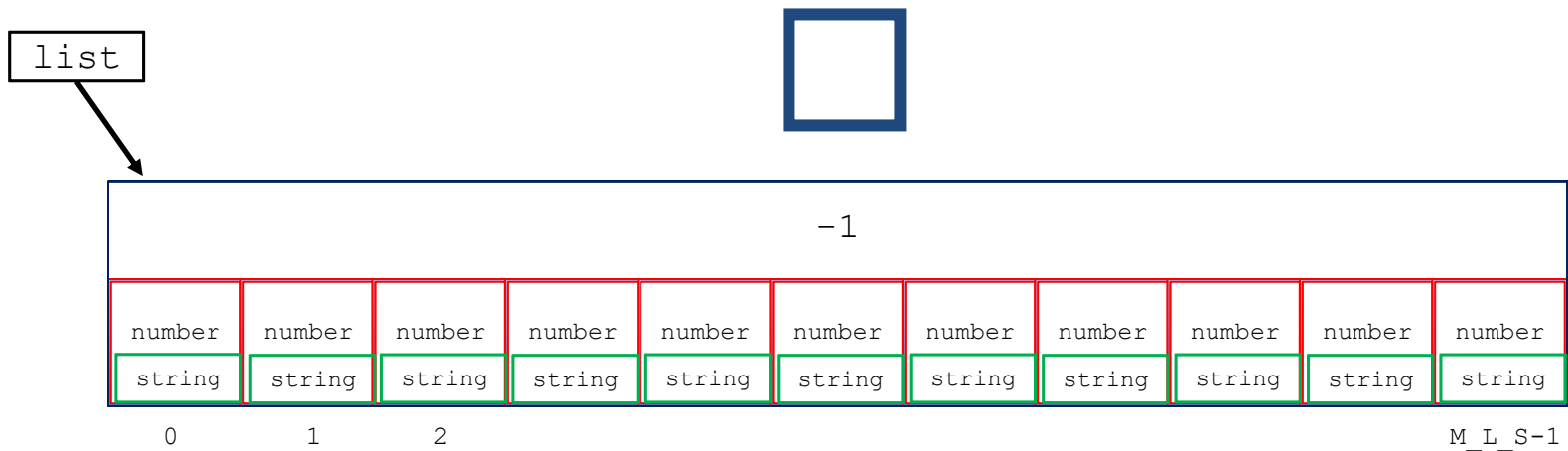
```
WINDOW_TYPE empty(LIST_TYPE *list) {  
    list->last = -1;  
    return(end(list));  
}
```



# LIST: Array Implementation

```
/** test to see if a list is empty */
```

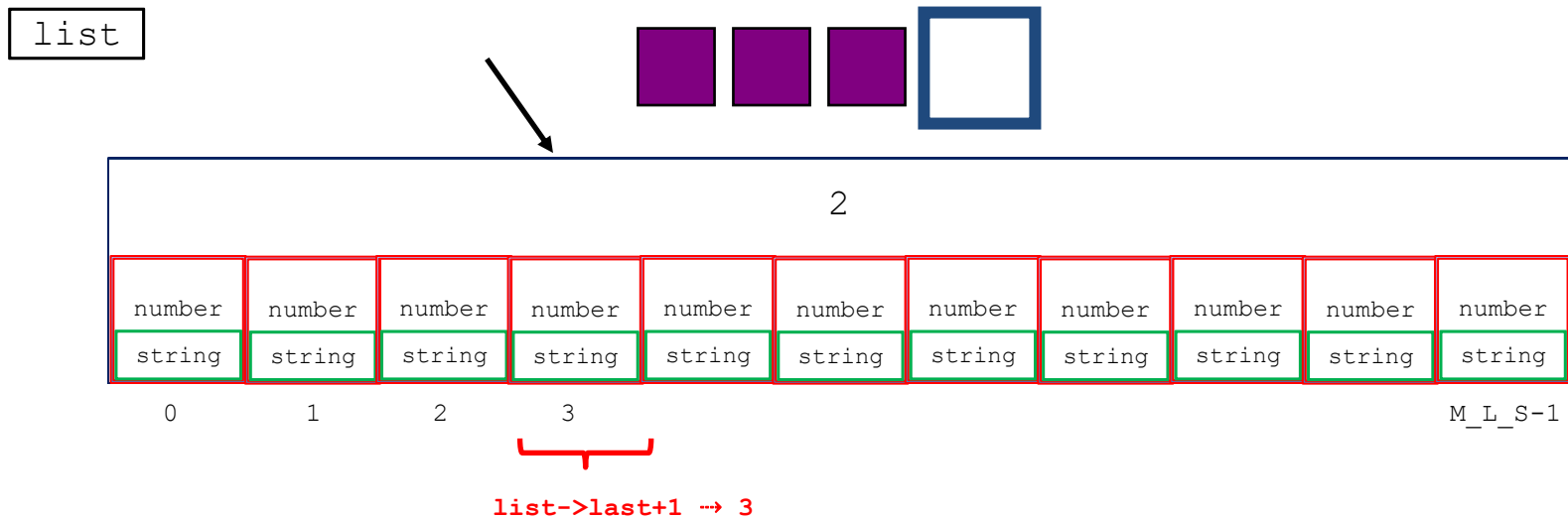
```
int is_empty(LIST_TYPE *list) {  
    if (list->last == -1)  
        return(TRUE);  
    else  
        return(FALSE)  
}
```



# LIST: Array Implementation

```
/** position at first element in a list */
```

```
WINDOW_TYPE first(LIST_TYPE *list) {  
    if (is_empty(list) == FALSE) {  
        return(0);  
    }  
    else  
        return(end(list));  
}
```

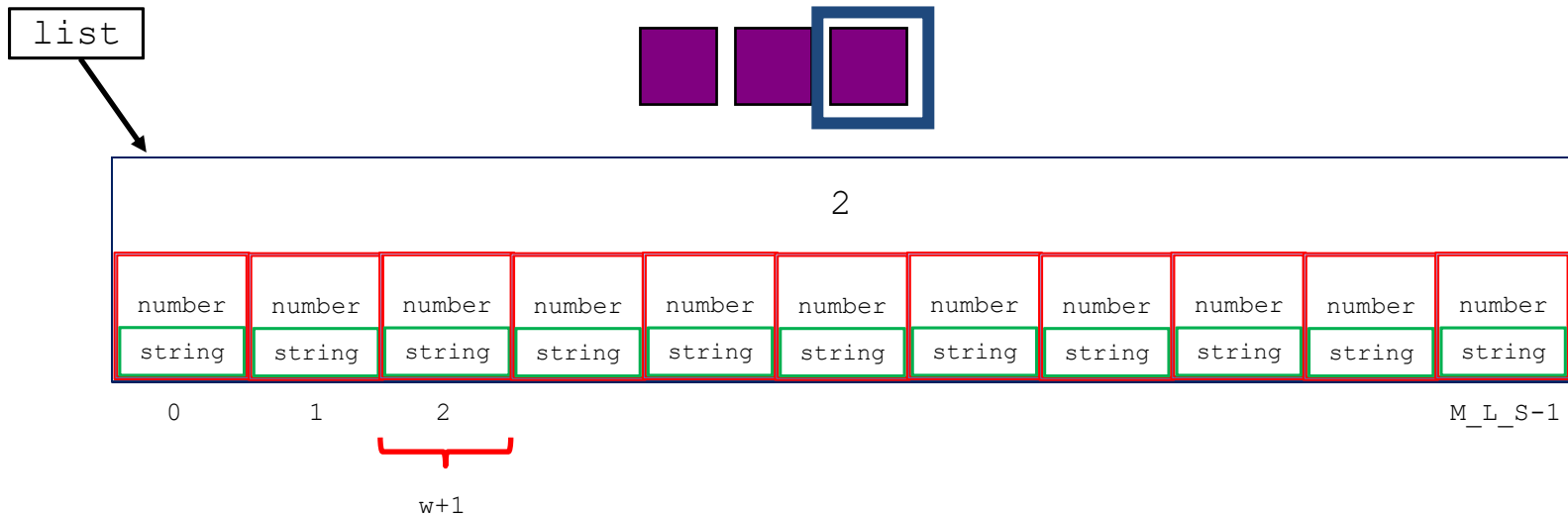
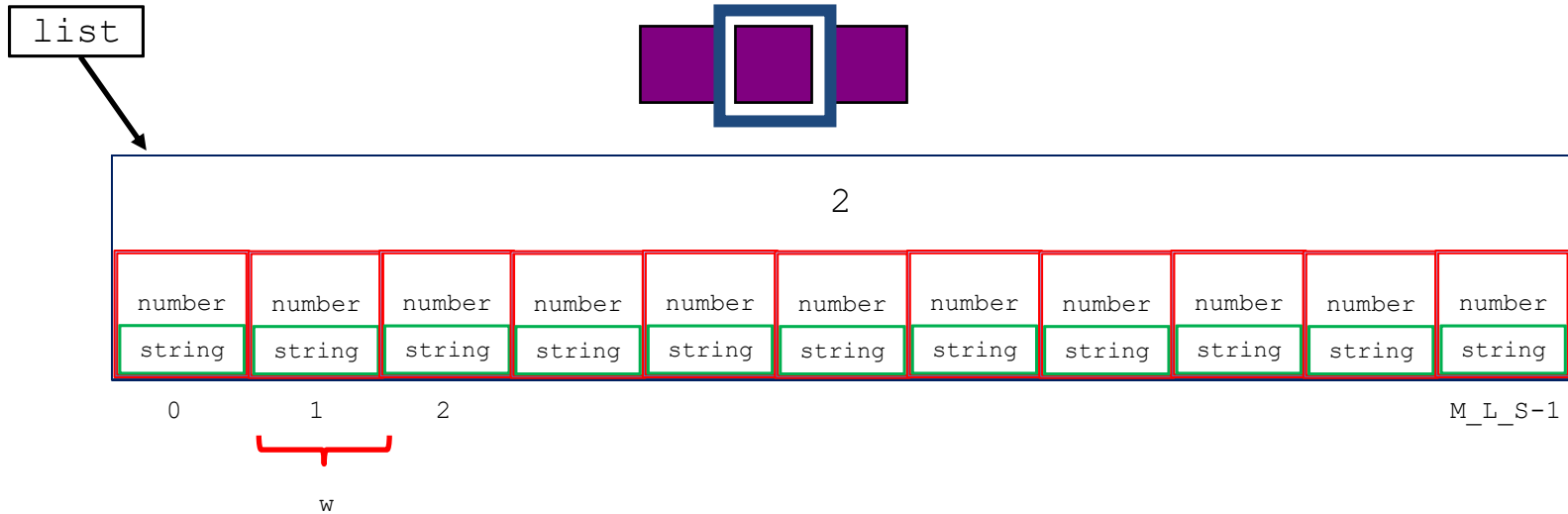




# LIST: Array Implementation

```
/** position at next element in a list */  
  
WINDOW_TYPE next(WINDOW_TYPE w, LIST_TYPE *list) {  
    if (w == last(list)) {  
        return(end(list));  
    }  
    else if (w == end(list)) {  
        error("can't find next after end of list");  
    }  
    else {  
        return(w+1);  
    }  
}
```

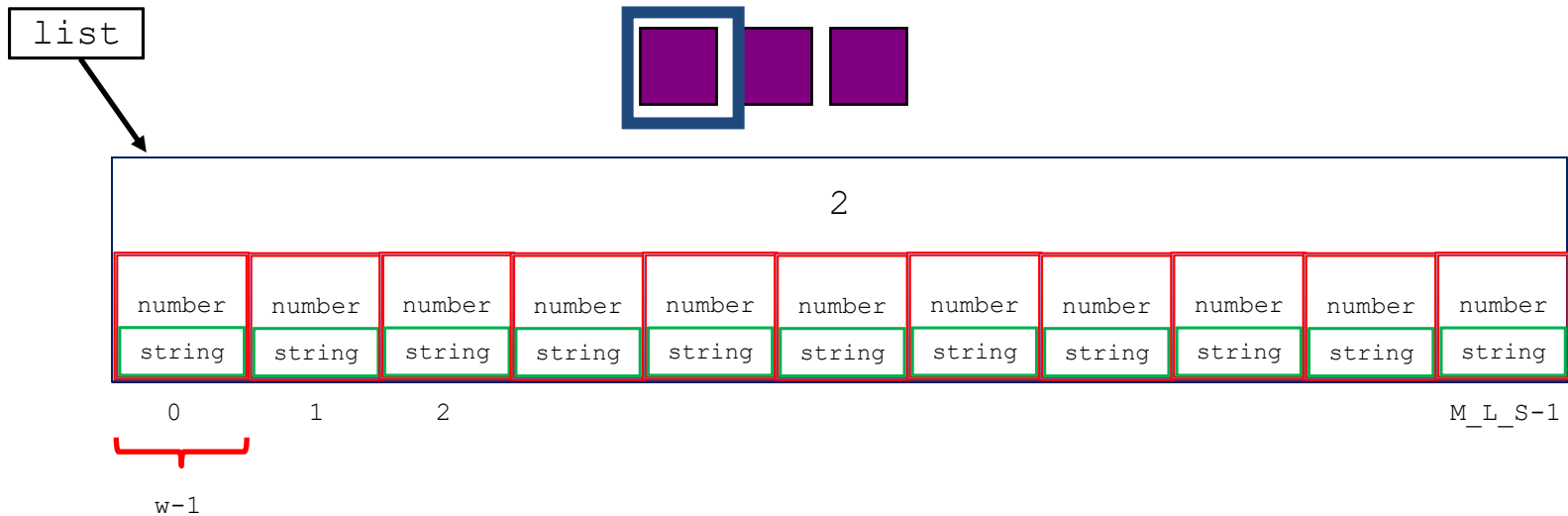
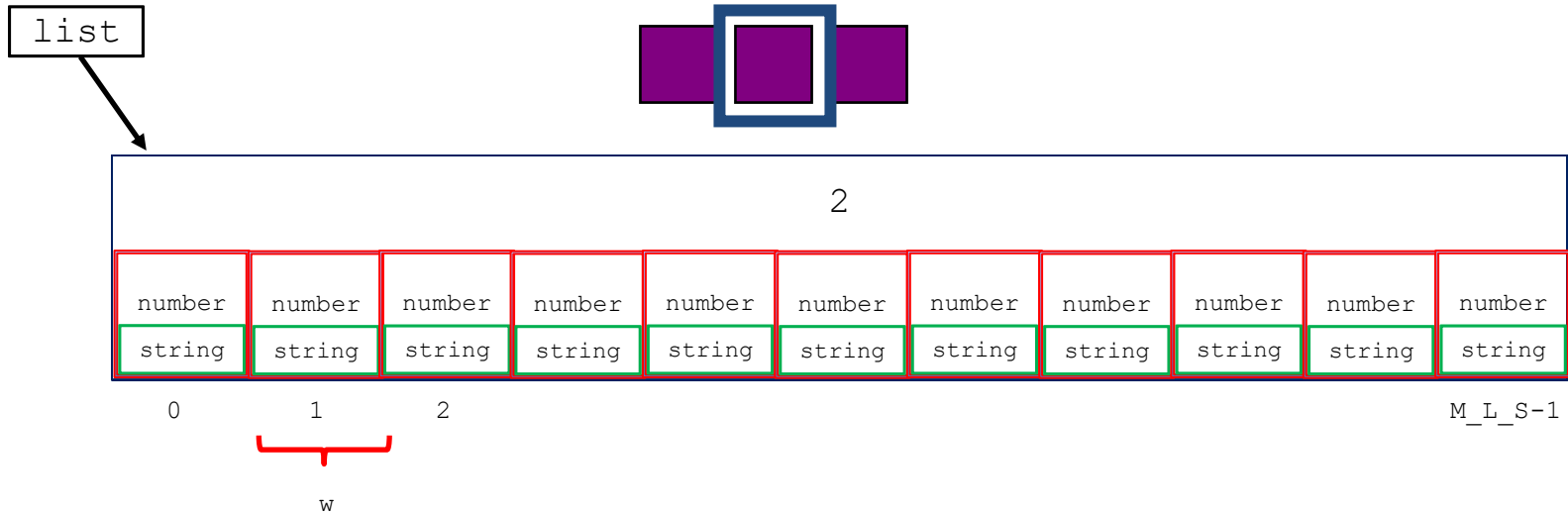
# LIST: Array Implementation



# LIST: Array Implementation

```
/** position at previous element in a list */  
  
WINDOW_TYPE previous(WINDOW_TYPE w, LIST_TYPE *list) {  
    if (w != first(list)) {  
        return(w-1);  
    }  
    else {  
        error("can't find previous before first element of list");  
        return(w);  
    }  
}
```

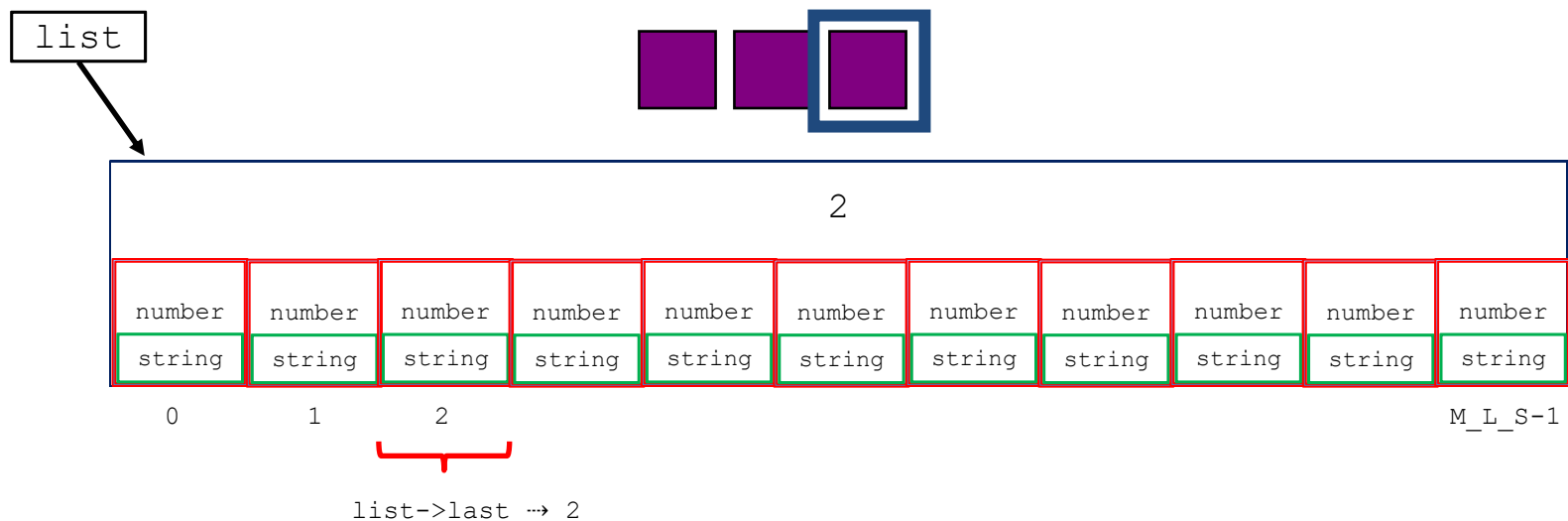
# LIST: Array Implementation



# LIST: Array Implementation

```
/** position at last element in a list */
```

```
WINDOW_TYPE last(LIST_TYPE *list) {  
    return(list->last);  
}
```



# LIST: Array Implementation

```
/** insert an element in a list */  
  
LIST_TYPE *insert(ELEMENT_TYPE e, WINDOW_TYPE w,  
                  LIST_TYPE *list) {  
    int i;  
    if (list->last >= MAX_LIST_SIZE-1) {  
        error("Can't insert - list is full");  
    }  
    else if ((w > list->last + 1) || (w < 0)) {  
        error("Position does not exist");  
    }  
    else {  
        /* insert it ... shift all after w to the right */
```

# LIST: Array Implementation

```
for (i=list->last; i >= w; i--) {  
    list->a[i+1] = list->a[i];  
}
```

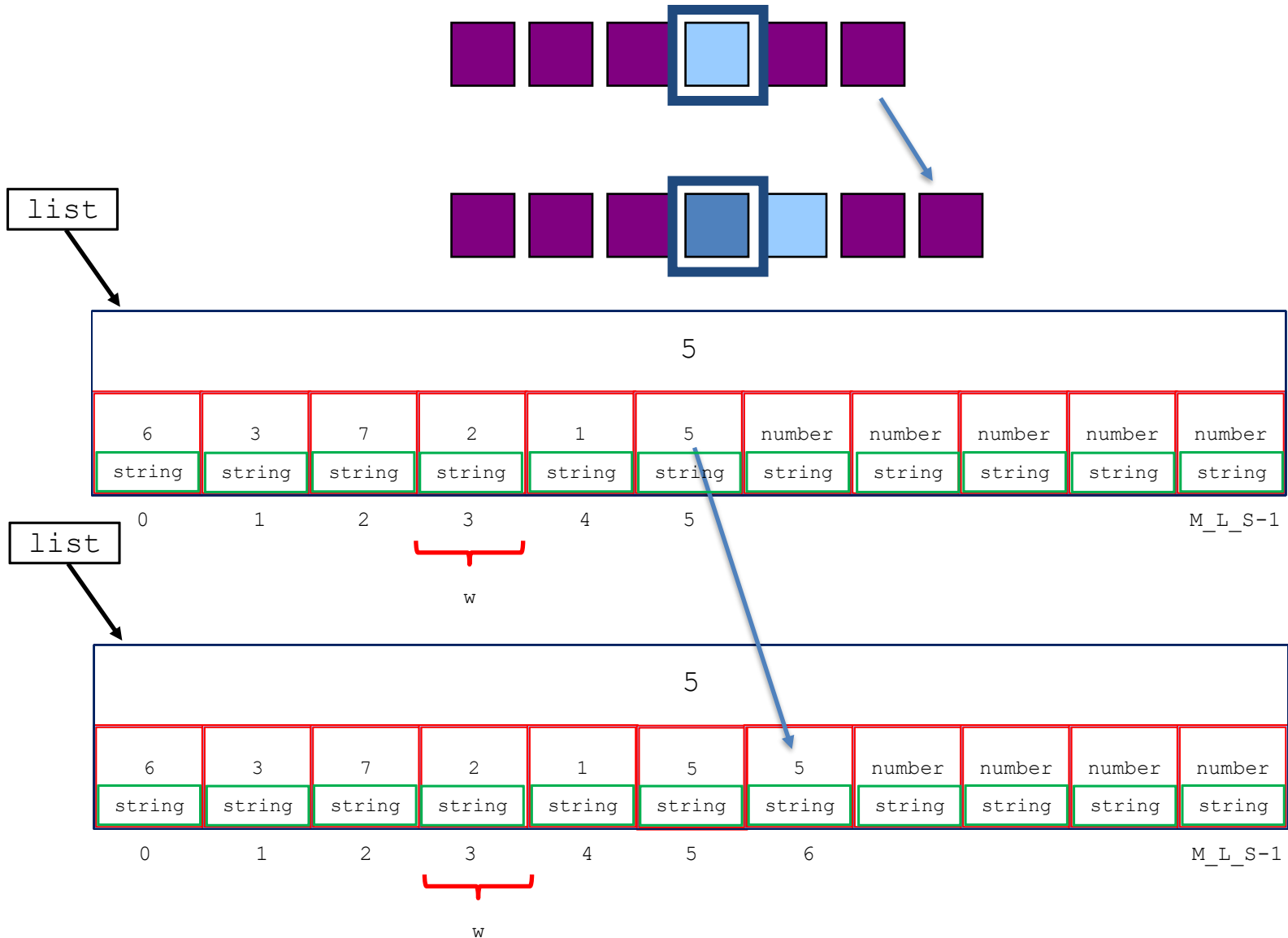
```
list->a[w] = e;  
list->last = list->last + 1;
```

```
return(list);
```

```
}
```

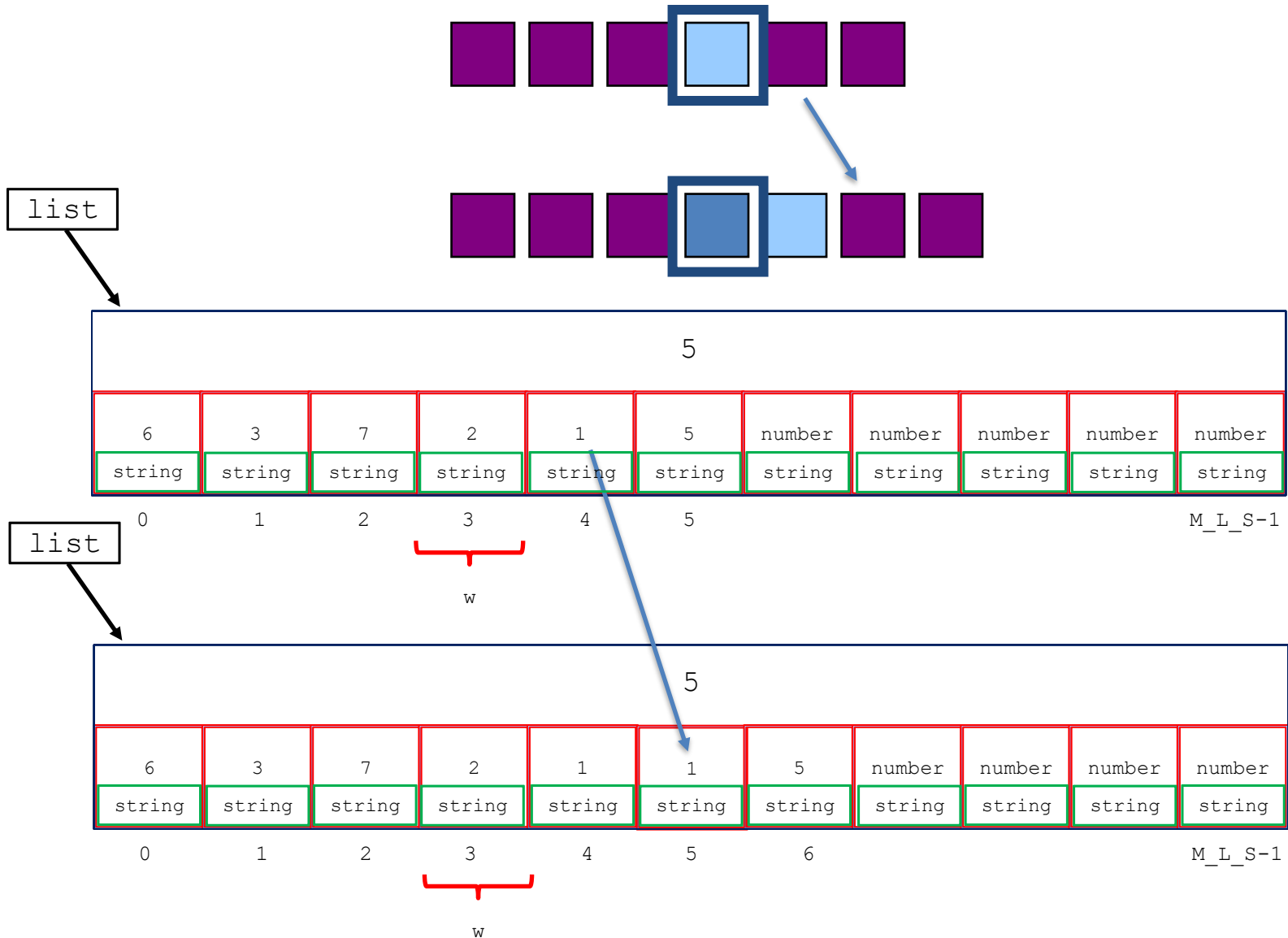
```
}
```

# LIST: Array Implementation

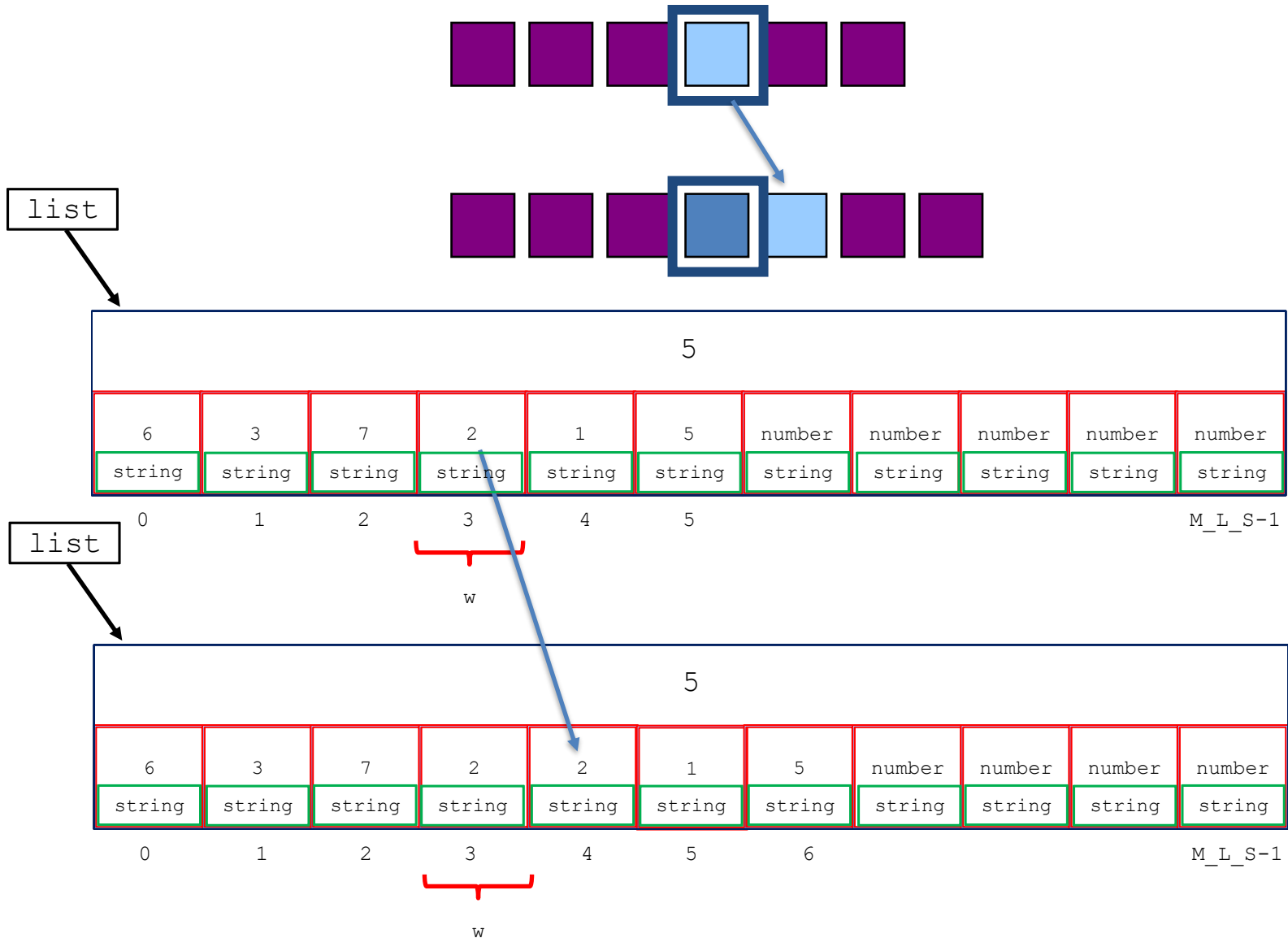




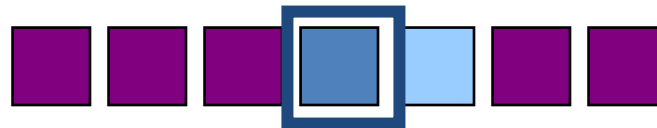
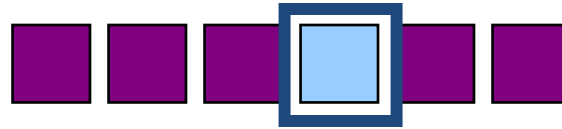
# LIST: Array Implementation



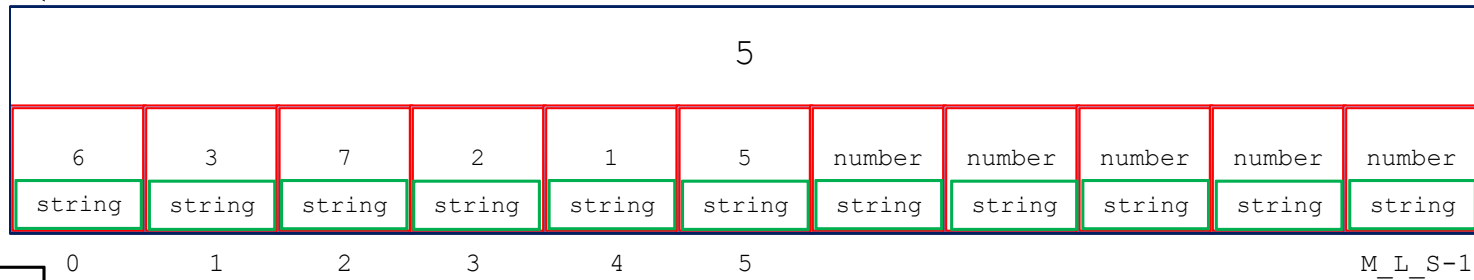
# LIST: Array Implementation



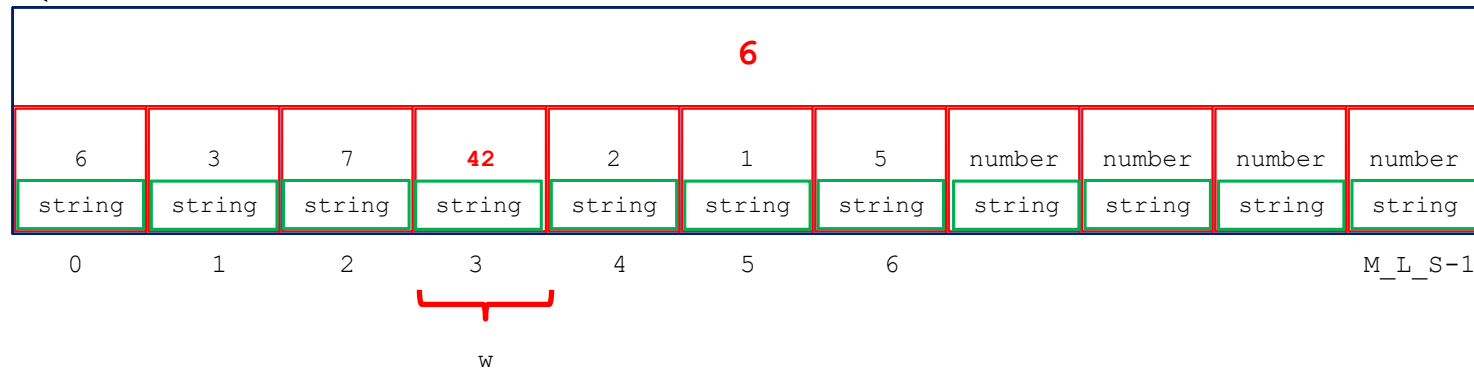
# LIST: Array Implementation



list



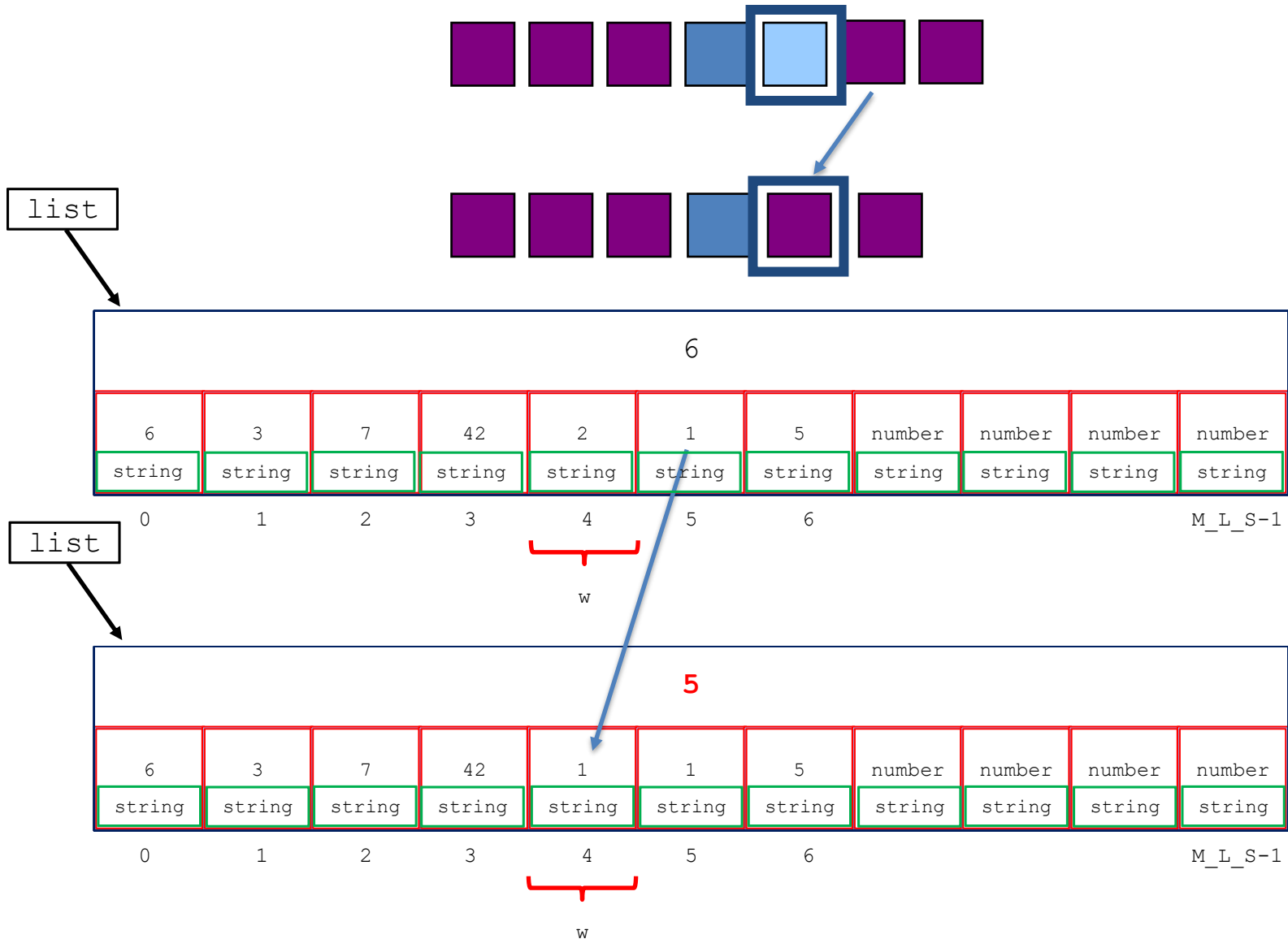
list



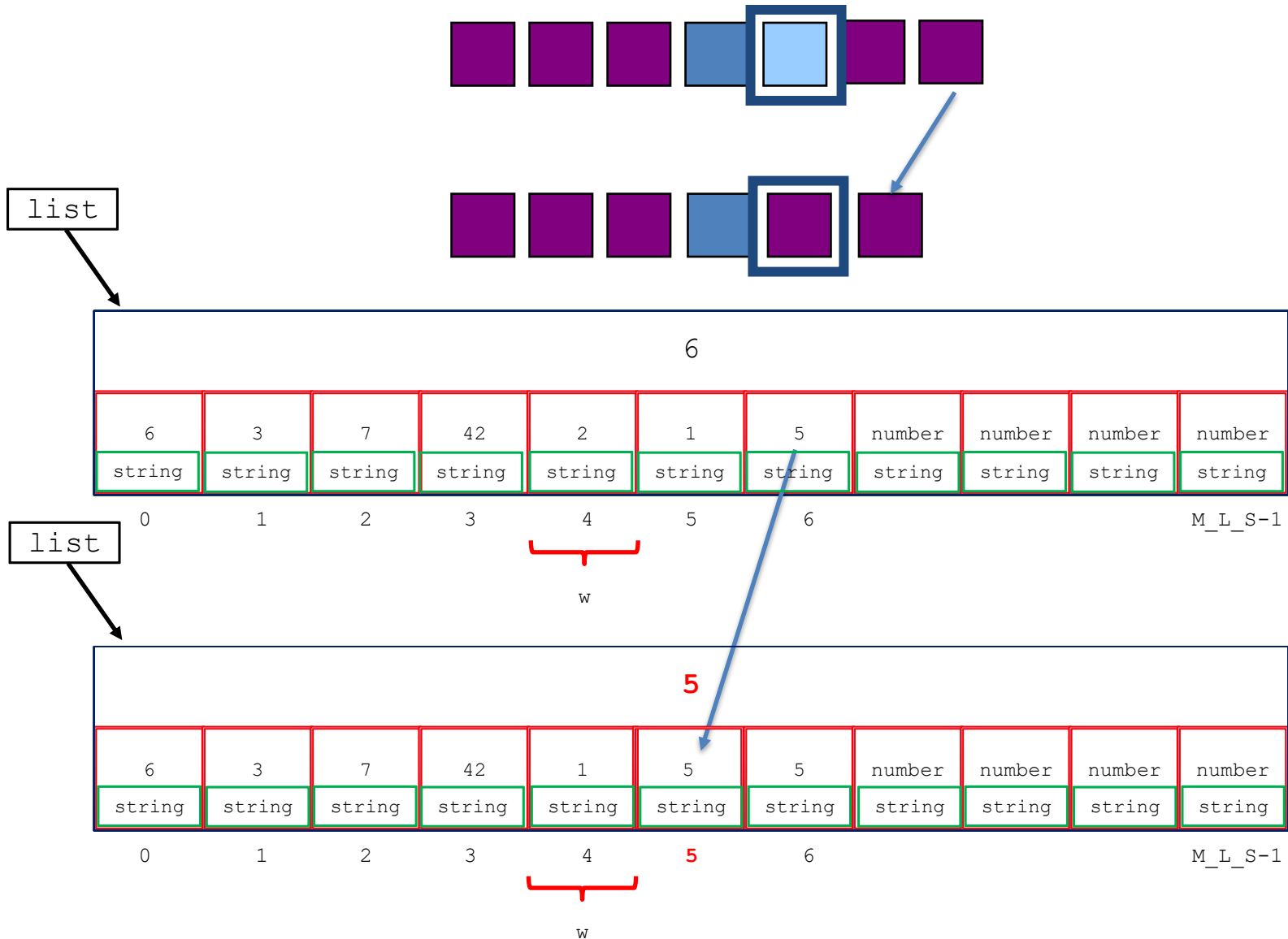
# LIST: Array Implementation

```
/** delete an element from a list */  
  
LIST_TYPE *delete(WINDOW_TYPE w, LIST_TYPE *list) {  
    int i;  
    if ((w > list->last) || (w < 0)) {  
        error("Position does not exist");  
    }  
    else {  
        /* delete it ... shift all after w to the left */  
        list->last = list->last - 1;  
        for (i=w; i < list->last; i++) {  
            list->a[i] = list->a[i+1];  
        }  
        return(list);  
    }  
}
```

# LIST: Array Implementation



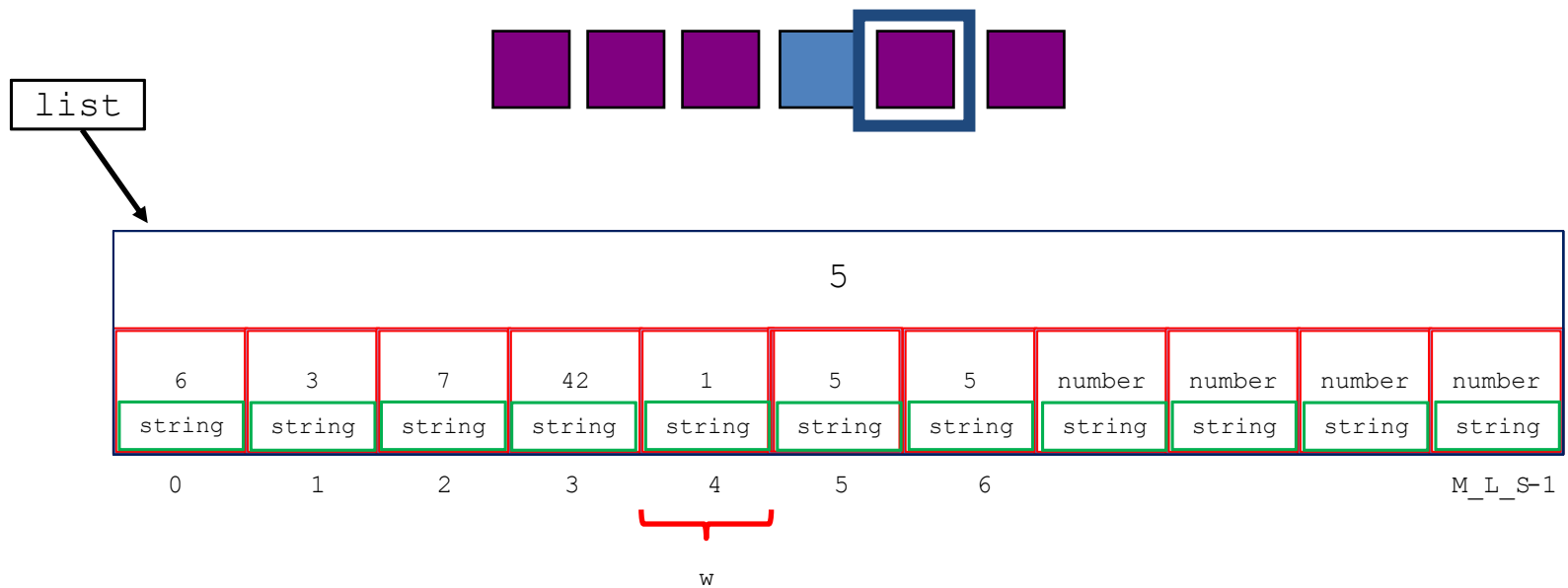
# LIST: Array Implementation



# LIST: Array Implementation

```
/** retrieve an element from a list */  
  
ELEMENT_TYPE retrieve(WINDOW_TYPE w, LIST_TYPE *list) {  
    if ( (w < 0) || (w > list->last) ) {  
  
        /* list is empty */  
  
        error("Position does not exist");  
    }  
    else {  
        return(list->a[w]);  
    }  
}
```

# LIST: Array Implementation



`list->a[4].number ... 1`



# LIST: Array Implementation

```
/** assign values to an element */  
  
int assign_element_values(ELEMENT_TYPE *e, int number, char s[]){  
    e->string = (char *) malloc(sizeof(char)* (strlen(s)+1));  
    strcpy(e->string, s);  
    e->number = number;  
}
```

# LIST: Array Implementation

```
/** print all elements in a list */

int print(LIST_TYPE *list) { // rewrite as application code
    WINDOW_TYPE w;
    ELEMENT_TYPE e;
    printf("Contents of list: \n");
    w = first(list);
    while (w != end(list)) {
        e = retrieve(w, list);
        printf("%d %s\n", e.number, e.string);
        w = next(w, list);
    }
    printf("---\n");
    return(0);
}
```

# LIST: Array Implementation

```
/** error handler: print message passed as argument and  
    take appropriate action                                ***/  
  
int error(char *s) {  
    printf("Error: %s\n", s);  
    exit(0);  
}
```

# LIST: Array Implementation

```
/** main application routine */  
  
WINDOW_TYPE w;  
ELEMENT_TYPE e;  
LIST_TYPE list;  
int i;  
  
empty(&list);  
print(&list);  
  
assign_element_values(&e, 1, "String A");  
w = first(&list);  
insert(e, w, &list);  
print(&list);
```

# LIST: Array Implementation

```
assign_element_values(&e, 2, "String B");  
insert(e, w, &list);  
print(&list);
```

```
assign_element_values(&e, 3, "String C");  
insert(e, last(&list), &list);  
print(&list);
```

```
assign_element_values(&e, 4, "String D");  
w = next(last(&list), &list);  
insert(e, w, &list);  
print(&list);
```

# LIST: Array Implementation

```
w = previous(w, &list);  
delete(w, &list);  
print(&list);
```

```
}
```

# LIST: Array Implementation

Key points:

- we have implemented all list manipulation operations with dedicated access functions
- we never directly access the data-structure when using it but we always use the access functions
- Why?

# LIST: Array Implementation

## Key points:

- greater security: localized control and more resilient software maintenance
- data hiding: the implementation of the data-structure is hidden from the user and so we can change the implementation and the user will never know



# LIST: Array Implementation

Possible problems with the implementation:

- have to shift elements when inserting and deleting (i.e. insert and delete are  $O(n)$ )
- have to specify the maximum size of the list at compile time

# Acknowledgement

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