Problem 1)

Consider the classical Lagrangian densities for the following relativistic quantum field theories,

$$\mathcal{L}_Y = \frac{i}{2}\bar{\psi}\overleftrightarrow{\partial}\psi - M\bar{\psi}\psi + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_s^2\phi^2 - g\bar{\psi}\psi\phi - \frac{\lambda_1}{3!}\phi^3 - \frac{\lambda_2}{4!}\phi^4 \tag{1}$$

$$\mathcal{L}_V = \frac{i}{2}\bar{\psi}\overleftrightarrow{\partial}\psi - M\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{m_V^2}{2}A^\mu A_\mu - g\bar{\psi}\not{A}\psi. \tag{2}$$

We could envision each of these being proposed as moels of the interactions between spinor "nucleons" of mass M represented by the Fermi field ψ. In the first case, the interaction is then mediated by a scalar "pion" field ϕ with mass m_s, and in the second it is mediated by a vector field A^μ with mass m_V. As usual, the field strength tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. (Note that to be realistic we should really have <u>pseudo</u>scalar and vector interactions.) In the second theory, we would get the Maxwell field if we set $m_V = 0$.

- **(a)** Use the fast mnemonic that we developed in class for translating a classical Lagrangian density into QFT Feynman rules to write down all the Feynman rules for the two theories above. Make a comment about where each factor of "i" comes from. Use straight lines for ψ, dashed lines for ϕ, and wavy lines for A^μ.
- **(b)** Draw a two-loop diagram for the vector field case. Draw an example of a diagram that would give problems if you have not worried about the "reduction" of external leg states.
- **(c)** What happens if we then place $m_V = 0$ in the vector field? A standard way to deal with the problem is to replace $\frac{m_V^2}{2}A_\mu A^\mu \to -\frac{1}{2\xi}(\partial_\mu A^\mu)^2$. This effectively uses the Lagrange multiplier technique to fix the Lorenz gauge condition $\partial_\mu A^\mu = 0$. What are the Feynman rules if I make this replacement? What if I further specify the gauge by fixing $\xi = 1$, where ξ is a real constant that will be chosen later?
- **(d)** Using Eq. (1), draw all the Feynman diagrams that would contribute to the $2 \to 2$ cross section, $\psi\bar{\psi} \to \psi\bar{\psi}$ <u>through</u> order g^2 and $g^2\lambda_1^2$.
- **(e)** Still using Eq. (1), consider the cross section for the $2 \to 2$ scattering process $\psi\psi \to \psi\psi$. Start with the general cross section expression derived in class,

$$\mathrm{d}\sigma = \frac{|M|^2}{2\sqrt{\lambda(s, m_A^2, m_B^2)}} \frac{\mathrm{d}^3\boldsymbol{p}_1}{(2\pi)^3 2E_1} \cdots \frac{\mathrm{d}^3\boldsymbol{p}_N}{(2\pi)^3 2E_N} (2\pi)^4 \delta\Big(k_A + k_B - \sum_{i=1}^{N} p_i\Big), \tag{3}$$

with

$$\lambda(s, m_A^2, m_B^2) = s^2 + m_A^4 + m_B^4 - 2sm_A^2 - 2sm_B^2 - 2m_A^2 m_B^2, \tag{4}$$

and derive the order g^2 expression for the unpolarized differential cross section $\mathrm{d}\sigma/\mathrm{d}\Omega\,|_{\mathrm{CM}}$ in the center-of-mass system. Since it is an unpolarized cross section, you should sum over the final and average over initial nucleon spins. Let p_A and p_B label the initial four-momenta and p_C and p_D label the final four-momenta and express your result in terms of Mandelstam variables.