6.7) An infinitely long circular cylinder carries a uniform magnetization $\vec{\mathbf{M}}$ parallel to its axis. Find the magnetic field (due to $\vec{\mathbf{M}}$ inside and outside the cylinder.

The magnetization is $\vec{\mathbf{M}} = M\hat{\boldsymbol{z}}$, which gives the following bound current densities:

$$\vec{\mathbf{J}}_b = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{M}} = 0$$
$$\vec{\mathbf{K}}_b = \vec{\mathbf{M}} \times \hat{\boldsymbol{s}} = M\hat{\boldsymbol{\phi}}.$$

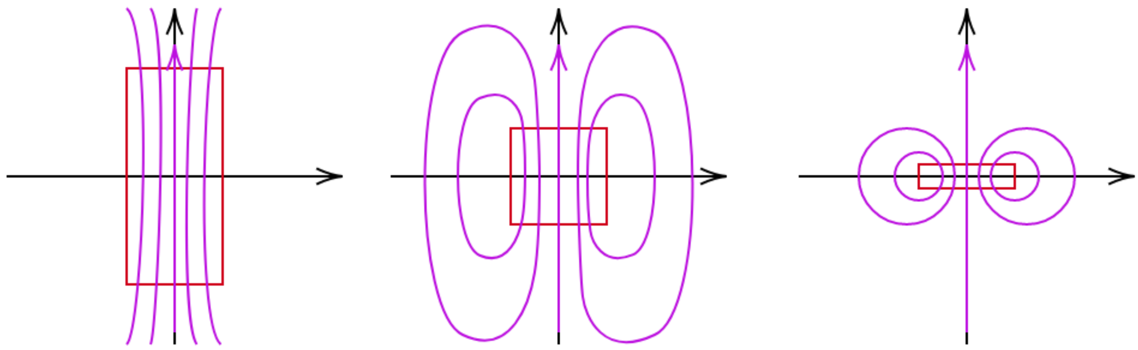
This is essentially a solenoid, and we can write down the magnetic field simply using $nI = M$, giving

$$\boxed{\vec{\mathbf{B}} = \mu_0\vec{\mathbf{M}}}$$

Notice that the field is constant along the axis of the cylinder inside and zero outside the cylinder.

6.9) A short circular cylinder of radius a and length L carries a "frozen-in" uniform magnetization $\vec{\mathbf{M}}$ parallel to its axis. Find the bound current, and sketch the magnetic field of the cylinder. (Make three sketches: one for $L \gg a$, one for $L \ll a$, and one for $L \approx a$.) Compare this **bar magnet** with the bar electret of Prob. 4.11.

In this case, the current densities are the same. Notice that there is none on the caps since the normal vector and magnetization are parallel.



The left figure is the field when $L \gg a$, the center is for $L \approx a$, and the right is for $L \ll a$. Notice that for $L \gg a$, we essentially recover the constant field of a solenoid with some fringe field behavior at the caps of the solenoid. For $L \approx a$, the field is somewhat uniform in the center but circles around the cylinder with a nonvanishing (non-negligible) field outside the cylinder, and lastly, for $L \ll a$ we essentially have the field of a current loop.

Recall that for the bar electret we have the following cases.

1. $L \gg a$: The electric field is that of a physical dipole.

2. $L \approx a$: The electric field and magnetic fields are similar outside the cylinder, but inside they point opposite each other if the magnetization and polarization point in the same direction.

3. $L \ll a$: The electric field is essentially that of a parallel plate capacitor.