

4.4) A point charge q is situated a large distance r from a neutral atom of polarizability α . Find the force of attraction between them.

The charge q serves as the external field which polarizes the neutral atom. The dipole moment of the atom from this field is given as

$$p = \alpha \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \right)$$

Thus, we can calculate the field produced by the induced dipole

$$E = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} \left(\frac{2\alpha q}{4\pi\epsilon_0 r^2} \right)$$

and the force on q from this field

$$F = qE_{\text{atom}} = \frac{2\alpha q^2}{4\pi\epsilon_0 r^5}$$

4.6) A (perfect) dipole \vec{p} is situated a distance z above an infinite grounded conducting plane. The dipole makes an angle θ with the perpendicular to the plane. Find the torque on \vec{p} . If the dipole is free to rotate, in what orientation will it come to rest?

The torque on the dipole is given as

$$\vec{N} = \vec{p} \times \vec{E}$$

We can produce an equivalent set up by placing a mirror dipole on the opposite side of the infinite conducting plane, which gives us the electric field from the plane. That is,

$$\begin{aligned} \vec{p} &= p [\cos \theta \hat{r} + \sin \theta \hat{\theta}] \\ \vec{E} &= \frac{p}{4\pi\epsilon_0 (2z)^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta}) \end{aligned}$$

Hence, carrying out the cross product and simplifying the result,

$$\vec{N} = -\frac{p^2 \sin(2\theta)}{4\pi\epsilon_0 (16z^3)} \hat{\phi}$$

From this we can see that the dipole “attempts” to orient itself parallel to the field. If $-\pi/2 < \theta < \pi/2$ the dipole will point away from the plane. Otherwise the dipole will rotate such that it points towards the plane.