

5.1) A particle of charge q enters a region of uniform magnetic field \vec{B} (pointing *into* the page). The field deflects the particle a distance d above the original line of flight, as shown in Fig. 5.8. Is the charge positive or negative? In terms of a , d , B , and q , find the momentum of the particle.

The magnetic force is given as a cross product of the velocity of a charge and the magnetic field which it is immersed in:

$$\vec{F} = q\vec{v} \times \vec{B}$$

We can determine the sign of the charge on q by seeing if the direction of the force is parallel or anti-parallel to the direction of $\vec{v} \times \vec{B}$. Using the right hand rule, we see that the force points up, implying that the charge must be positive.

Now, we know that a charge moving in a constant magnetic field with velocity perpendicular to the field undergoes circular motion:

$$\begin{cases} x = r \cos\left(\frac{qB}{m}t\right) \\ y = r \sin\left(\frac{qB}{m}t\right) - r \end{cases}$$

We can solve for the radius of motion using the point (d, a) , giving

$$r = \frac{a^2 + d^2}{2d}$$

Hence, using basic facts about centripetal motion, we have

$$\begin{aligned} F &= \frac{mv^2}{r} = qvB \\ \Rightarrow p &= qBr \end{aligned}$$

giving

$$\boxed{p = \frac{qB(a^2 + d^2)}{2d}}$$

5.5) A current I flows down a wire of radius a .

a) If it is uniformly distributed over the surface, what is the surface current density K ?

The length of the strip perpendicular to the flow of current is $2\pi a$, and since the current is uniform over the surface, then

$$\boxed{K = \frac{I}{2\pi a}}$$

b) If it is distributed in such a way that the volume current density is inversely proportional to the distance from the axis, what is $J(s)$?

We can write

$$J = \frac{k}{s}$$

find k in terms of the total current as follows:

$$I = \int \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}} = \int \frac{k}{s} s d\phi ds = k(2\pi)a$$
$$\Rightarrow k = \frac{I}{2\pi a}$$

Hence,

$$\boxed{J(s) = \frac{I}{2\pi a s}}$$