1.15) Calculate the divergence of the following vector functions.

(a) $\vec{v}_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$

$$\vec{\nabla} \cdot \vec{v}_a = \frac{\partial}{\partial x}\left(x^2\right) + \frac{\partial}{\partial y}\left(3xz^2\right) + \frac{\partial}{\partial z}(-2xz) = 2x - 2x$$

$$\boxed{\vec{\nabla} \cdot \vec{v}_a = 0}$$

(b) $\vec{v}_b = xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}$

$$\boxed{\vec{\nabla} \cdot \vec{v}_b = y + 2z + 3x}$$

(c) $\vec{v}_c = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}$

$$\boxed{\vec{\nabla} \cdot \vec{v}_c = 2(x + y)}$$

1.18) Calculate the curl of the vector functions in Prob. 1.15.

(a) $\vec{v}_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$

$$\begin{aligned}\vec{\nabla} \times \vec{v}_a = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 3xz^2 & -2xz \end{vmatrix} &= \left[\frac{\partial}{\partial y}(-2xz) - \frac{\partial}{\partial z}\left(3xz^2\right)\right]\hat{x} \\ &+ \left[\frac{\partial}{\partial z}\left(x^2\right) - \frac{\partial}{\partial x}(-2xz)\right]\hat{y} \\ &+ \left[\frac{\partial}{\partial x}\left(3xz^2\right) - \frac{\partial}{\partial y}\left(x^2\right)\right]\hat{z}\end{aligned}$$

$$\boxed{\vec{\nabla} \times \vec{v}_a = -6xz\hat{x} + 2z\hat{y} + 3z^2\hat{z}}$$

(b) $\vec{v}_b = xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}$

$$\begin{aligned}\vec{\nabla} \times \vec{v}_b = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & -3zx \end{vmatrix} &= \left[\frac{\partial}{\partial y}(3zx) - \frac{\partial}{\partial z}(2yz)\right]\hat{x} \\ &+ \left[\frac{\partial}{\partial z}(xy) - \frac{\partial}{\partial x}(3zx)\right]\hat{y} \\ &+ \left[\frac{\partial}{\partial x}(2yz) - \frac{\partial}{\partial y}(xy)\right]\hat{z}\end{aligned}$$

$$\boxed{\vec{\nabla} \cdot \vec{v}_b = -2y\hat{x} - 3z\hat{y} - x\hat{z}}$$

(c) $\vec{v}_c = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}$

$$\vec{\nabla} \times \vec{v}_c = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & (2xy+z^2) & 2yz \end{vmatrix} = \left[\frac{\partial}{\partial y}(2yz) - \frac{\partial}{\partial z}\left(2xy+z^2\right)\right]\hat{x}$$
$$+ \left[\frac{\partial}{\partial z}\left(y^2\right) - \frac{\partial}{\partial x}(2yz)\right]\hat{y}$$
$$+ \left[\frac{\partial}{\partial x}\left(2xy+z^2\right) - \frac{\partial}{\partial y}\left(y^2\right)\right]\hat{z}$$

$$\boxed{\vec{\nabla} \cdot \vec{v}_c = 0}$$

1.26) Calculate the Laplacian of the following functions:

(a) $T_a = x^2 + 2xy + 3z + 4$

$$\boxed{\vec{\nabla}^2 T_a = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) T_a = (2) + (0) + (0) = 2}$$

(b) $T_b = \sin x \sin y \sin z$

$$\boxed{\vec{\nabla}^2 T_b = -\sin x \sin y \sin z - \sin x \sin y \sin z - \sin x \sin y \sin z = -3T_b}$$

(c) $T_c = e^{-5x} \sin 4y \cos 3z$

$$\boxed{\vec{\nabla}^2 T_c = 25T_c - 16T_c - 9T_c = 0}$$

(d) $\vec{v} = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$

$$\boxed{\vec{\nabla}^2 \vec{v} = \left(\vec{\nabla}^2 x^2\right)\hat{x} + \left(\vec{\nabla}^2 3xz^2\right)\hat{y} - \left(\vec{\nabla}^2 2xz\right)\hat{z} = 2\hat{x} + 6x\hat{y}}$$