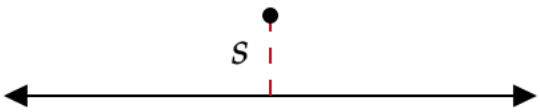
2.22) Find the potential a distance s from an infinitely long straight wire that carries a uniform line charge λ. Compute the gradient of your potential, and check that it yields the correct field.



We see that

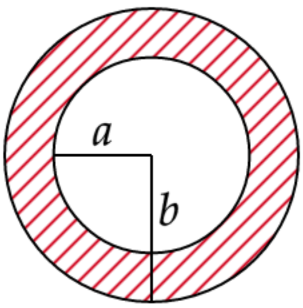
$$V = -\int \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{\ell}} = -\frac{\lambda}{2\pi\epsilon_0}\int_{s_0}^{s} \frac{1}{s'}\,\mathrm{d}s$$

$$\boxed{V = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{s_0}{s}\right)}$$

where s_0 is an arbitrary reference point not on the wire or at infinity (e.g. $s_0 = 1$ in units of length). We can check this result as follows:

$$\vec{\boldsymbol{E}} = -\vec{\boldsymbol{\nabla}}V = -\frac{\partial V}{\partial s}\hat{\boldsymbol{s}} = \frac{\lambda}{2\pi\epsilon_0}\frac{\partial}{\partial s}\ln\left(\frac{s}{s_0}\right) = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}$$

2.23) For the charge configuration of Prob. 2.15, find the potential at the center, using infinity as your reference point.



Notice that the electric field, using spheres as our Gaussian surfaces, is

$$E = \frac{k}{\epsilon_0}\begin{cases} 0, & r < a \\ \frac{r-a}{r^2}, & a < r < b \\ \frac{b-a}{r^2}, & r > b \end{cases}$$

Then, the potential at the center, using infinity as our reference, is

$$\begin{aligned} V &= -\int_0^\infty E\,\mathrm{d}r = -\frac{k}{\epsilon_0}\left[\int_0^a 0\,\mathrm{d}r + \int_a^b \frac{r-a}{r^2}\,\mathrm{d}r + \int_b^\infty \frac{b-a}{r^2}\,\mathrm{d}r\right] \\ &= -\left[\ln(b) + \frac{a}{b} - \ln(a) - 1 + \frac{b-a}{b}\right] \end{aligned}$$

$$\boxed{V = -\frac{k}{\epsilon_0}\ln\left(\frac{b}{a}\right)}$$