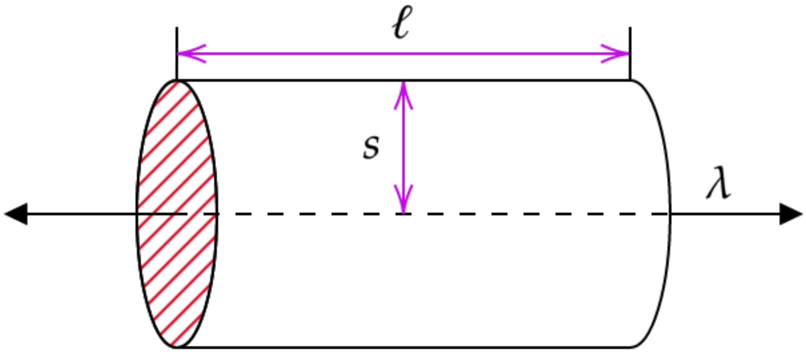
2.13) Find the electric field a distance s from an infinitely long straight wire that carries a uniform line charge λ. Compare Eq. 2.9.

For this problem, the setup is rotationally symmetric about the wire, so we will use Gauss's Law with a cylindrical Gaussian surface of radius s.



Gauss's law tells us that

$$\oint \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{a}} = \frac{Q_{\text{enc}}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$

It is observed that contribution to the flux through the ends of the cylinder is zero, so only obtain a radial component, pointing normal to the curved surface of the cylinder.

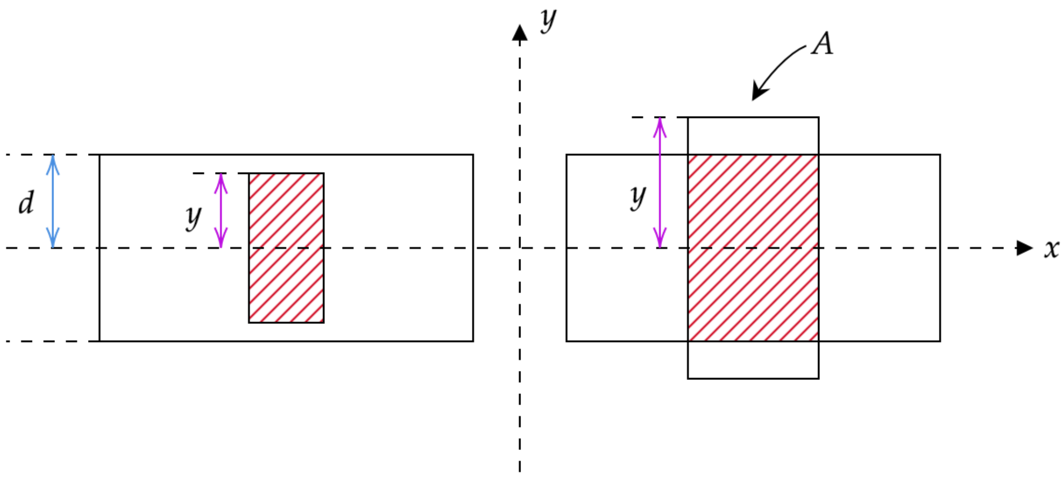
$$E(2\pi s \ell) = \frac{\lambda \ell}{\epsilon_0}$$

$$\boxed{\vec{\boldsymbol{E}} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}\hat{\boldsymbol{s}} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{s}\hat{\boldsymbol{s}}}$$

Notice that this is the same as Eq. 2.9, since the z axis in that example is not uniquely defined since we could rotate about the wire and end up with an equivalent physical setup.

2.17) An infinite plane slab, of thickness $2d$, carries a uniform volume charge density ρ. Find the electric field, as a function of y, where $y = 0$ at the center. Plot E versus y, calling E positive when it points in the $+y$ direction and negative when it points in the $-y$ direction.

In the figure below, we see the slab set up. Note that it extends infinitely in the x and z directions. For this problem, we can set up a Gaussian surface, which is a cylinder with a height $2y$. Note that there are two cases: $|y| < d$ or $|y| \geq d$ shown in the left and right figures.



Since there is rotational symmetry in the xz-plane, the only component of the electric field is in the y direction. From Gauss's law, we have

$$\oint \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{a}} = 2A|E| = \frac{Q_{\text{enc}}}{\epsilon_0} = \frac{A}{\epsilon_0} \int \rho(y)\,\mathrm{d}y$$
$$\Rightarrow |E| = \frac{1}{2\epsilon_0} \int \rho(y)\,\mathrm{d}y$$

Note that ρ is constant inside the slab and $\rho = 0$ outside the slab, so the integral over y gives

$$\boxed{|E| = \frac{\rho}{\epsilon_0} \begin{cases} y, & |y| < d \\ d, & |y| \geq d \end{cases}}$$

Notice that the electric field linearly increases as a function of y and is constant outside of the slab, pointing away from the xz-plane. A plot of this is shown below.