

1) For each of the following pairs of integers a and b , determine their greatest common divisor, their least common multiple, and write their greatest common divisor in the form $ax + by$ for some integers x and y

(a) $a = 60$, $b = 17$

Observe that,

$$60 = 3(17) + 9$$

$$17 = 1(9) + 8$$

$$9 = 1(8) + 1$$

$$8 = 8(1),$$

so $\gcd(60, 17) = 1$, and

$$\begin{aligned}\gcd(60, 17) = 1 &= 9 - (1 * 8) = 9 - (17 - 9) = 17 + 2(9) = -17 + 2(60 - 3 * 17) \\ &= 60(2) + 17(-7).\end{aligned}$$

Finally, we have $\text{lcm}(60, 17) \gcd(60, 17) = \text{lcm}(60, 17) = 60(17) = 1020$.

(b) $a = 11391$, $b = 5673$

Notice that

$$11391 = 2(5673) + 45$$

$$5673 = 126(45) + 3$$

$$45 = 15(3),$$

so $\gcd(11391, 5673) = 3$, and

$$\begin{aligned}\gcd(11391, 5673) &= 5673 - 126(45) = 5673 - 126(11391 - 2 * 5673) \\ &= 11391(-126) + 5763(253).\end{aligned}$$