Problem 1)
Find the Laplace transform of the following function:

$$I(t) = t^n e^{-at}, \quad a > 0 \text{ and even } n. \tag{1}$$

Problem 2)
Solve the following equation by the Laplace transform

$$\ddot{y} + 2\lambda\dot{y} + \omega_0^2 y = 0, \tag{2}$$

where $y(0) = 0$ and $\dot{y}(0) = v$.

Problem 3)
A unit vector $\hat{\boldsymbol{n}}$ makes angles θ and α with the Cartesian axes z and x, respectively, and a unit vector $\hat{\boldsymbol{n}}'$ makes angles θ' and α' with z and x, respectively. Find $\cos\varphi$, where φ is the angle between $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{n}}'$.

Problem 4)
Find a scalar function $\varphi(r)$ of $r = |\vec{r}|$ which satisfies the equation

$$\vec{\nabla} \cdot [\varphi(r)\vec{r}] = 0. \tag{3}$$

Problem 5)
Calculate the following: (1) $\vec{\nabla}\cdot[(\vec{a}\cdot\vec{r})\vec{b}]$, (2) $\vec{\nabla}\times[(\vec{a}\cdot\vec{r})\vec{b}]$, (3) $\vec{\nabla}\cdot\vec{a}\times\vec{r}$, (4) $\vec{\nabla}\times(\vec{a}\times\vec{r})$, (5) $\vec{\nabla}\cdot[\vec{r}\times(\vec{a}\times\vec{r})]$, where $\vec{a}$ and $\vec{b}$ are constant vectors.