Problem 1)
What are the lines of constant $u(x,y)$ and $v(x,y)$ for the function $f(z) = u(x,y) + iv(x,y) = \ln z$?

Problem 2)
Calculate the integral

$$I(z) = \oint_{|z|=1} z^\alpha \, dz \tag{1}$$

along the circle of radius $|z| = 1$, where $\alpha \in \mathbb{R}$.

Problem 3)
Calculate the integral

$$I = \oint_C \frac{z+1}{z^2 + 4} \, dz \text{ if :} \tag{2}$$

(a) The point $2i$ is inside the contour C, and the point $-2i$ is outside the contour C.
(b) The point $-2i$ is inside the contour C, and the point $2i$ is outside the contour C.
(c) Both points $2i$ and $-2i$ are inside the contour C.

Problem 4)
The function defined by the series

$$I_4(z) = \frac{1}{1 + z^2} = 1 - z^2 + z^4 - z^6 + \ldots = \sum_{n=0}^{\infty} (-1)^n z^{2n} \tag{3}$$

is convergent at $|z| < 1$. Find the convergent series which analytically continues this function into the region $|z| > 1$.

Problem 5)
Expand the following function in the Laurent series in the neighborhood of $z = \infty$:

$$I_5(z) = \ln \frac{z^2 - a^2}{z^2 - b^2}. \tag{4}$$

Problem 6)
Expand the following function in the Laurent series at $z = 0$:

$$I_6(z) = z^3 \exp\left(-\frac{1}{z^2}\right). \tag{5}$$