Problem 1)
Express the following delta function in terms of delta functions of the variable x:

$$\delta\Big(\frac{\sin x}{x}\Big). \tag{1}$$

Recall that we can write

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|}, \tag{2}$$

where $f(x)$ has simple roots x_i[1]. Observe that $\sin x/x$ has zeros when $\sin x = 0$ or $x = n\pi$ for $n = \pm1, \pm2, \ldots$ and $\frac{\mathrm{d}}{\mathrm{d}x} \sin x/x|_{x=n\pi} = [\cos x/x - \sin x/x^2]_{x=n\pi} = (-1)^n/n\pi$. Note that there is not a zero when $n = 0$ (i.e. $x = 0$) since the $1/x$ factor makes the function indeterminate there and $\sin x/x \to 1$ as $x \to 0$.

Thus,

$$\boxed{\delta\Big(\frac{\sin x}{x}\Big) = \sum_{n=1}^{\infty}(-1)^n n\pi\Big[\delta(x - n\pi) - \delta(x + n\pi)\Big]}. \tag{3}$$

Problem 2)
Calculate

$$I(z) = \Gamma(1 + z)\Gamma(1 - z) \tag{4}$$

at $z = 1/4$.

We can write

$$\boxed{\begin{aligned} \Gamma(1+z)\Gamma(1-z) = z\Gamma(z)\Gamma(1-z) &= \frac{\pi z}{\sin \pi z} \\ \Rightarrow \Gamma(1+z)\Gamma(1-z)|_{z=1/4} = \frac{\pi/4}{\sin \pi/4} &= \frac{\pi}{2\sqrt{2}} \end{aligned}}. \tag{5}$$

Problem 3)
Using the definition of the complete elliptical integrals $E(m)$ and $K(m)$, express the derivative $\partial E(m)/\partial m$ in terms of $K(m)$ and $E(m)$.

[1] Otherwise we should expand to higher orders in the Taylor series of $f(x)$ around its root x_i

Problem 4)
Find the values of $e^{\pm i\pi/2}$, $e^{i\pi n}$, $\ln(-1)$ where $n = 0, \pm 1, \pm 2, \ldots$.

Problem 5)
Calculate the following series:

$$I_1 = \sum_{n=0}^{\infty} p^n \sin(qn) \quad \text{and} \quad I_2 = \sum_{n=0}^{\infty} p^n \cos(qn), \tag{6}$$

where p and q are real parameters.
HINT: Use the sum of geometric series with complex r.

Observe the following:

$$I = \sum_{n=0}^{\infty} p^n e^{iqn} = I_2 + iI_1. \tag{7}$$

That is $I_2 = \text{Re}(I)$ and $I_1 = \text{Im}(I)$. We can use the geometric series formula with complex $r = pe^{iq}$, giving

$$I = \frac{1}{1 - pe^{iq}} = \frac{1}{(1 - p\cos q) - ip\sin q} = \frac{(1 - p\cos q) + ip\sin q}{(1 - p\cos q)^2 + p^2\sin^2 q}. \tag{8}$$

Taking real and imaginary parts of I, we have

$$\boxed{I_1 = \frac{p\sin q}{1 + p^2 - 2p\cos q}} \tag{9}$$

$$\boxed{I_2 = \frac{1 - p\cos q}{1 + p^2 - 2p\cos q}}. \tag{10}$$