Problem 1)
Find the branch points of this function:

$$f(z) = \sqrt{z^2 + 2z - 1}. \tag{1}$$

What branch cuts can make this function single-valued?

Problem 2)
Find the residues and all isolated singularities of the function

$$I_2(z) = \tan z. \tag{2}$$

Problem 3)
Calculate the following real integral using the Cauchy theorem:

$$I_3 = \int_0^{2\pi} \frac{\mathrm{d}x}{2 + \cos^2 x}. \tag{3}$$

Problem 4)
Calculate the following real integral using the Cauchy theorem:

$$I_4 = \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{1 + x^4}. \tag{4}$$

Problem 5)
Calculate the following real integral using teh Cauchy theorem:

$$I_5(a) = \int_0^{\infty} \frac{x \sin ax}{b^2 + x^2} \, \mathrm{d}x \,, \tag{5}$$

where $a > 0$.