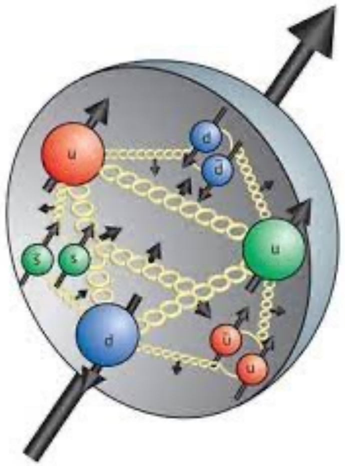
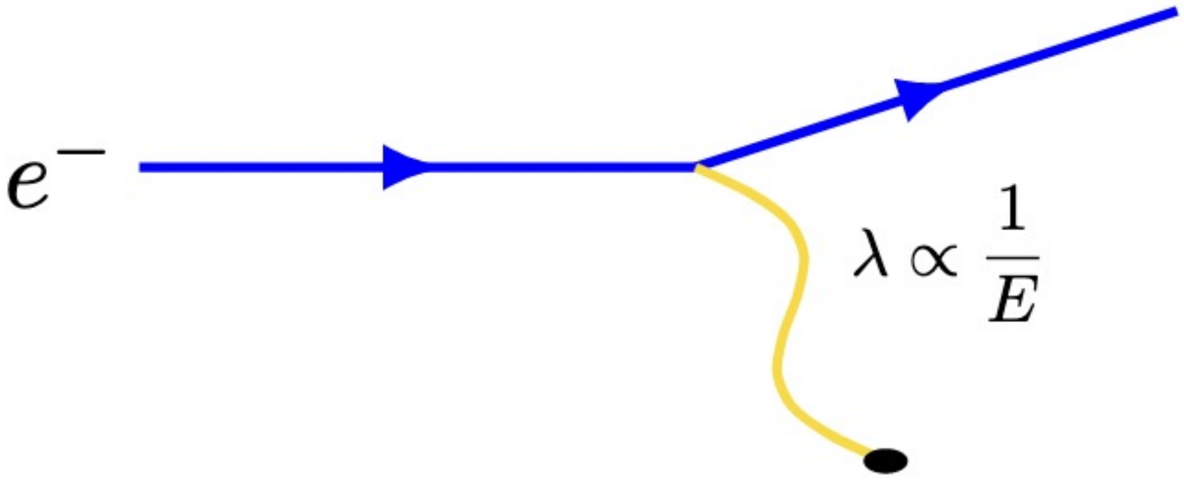
The proton spin puzzle: a surprisingly difficult problem

Richard Whitehill



Motivations: development of quark model



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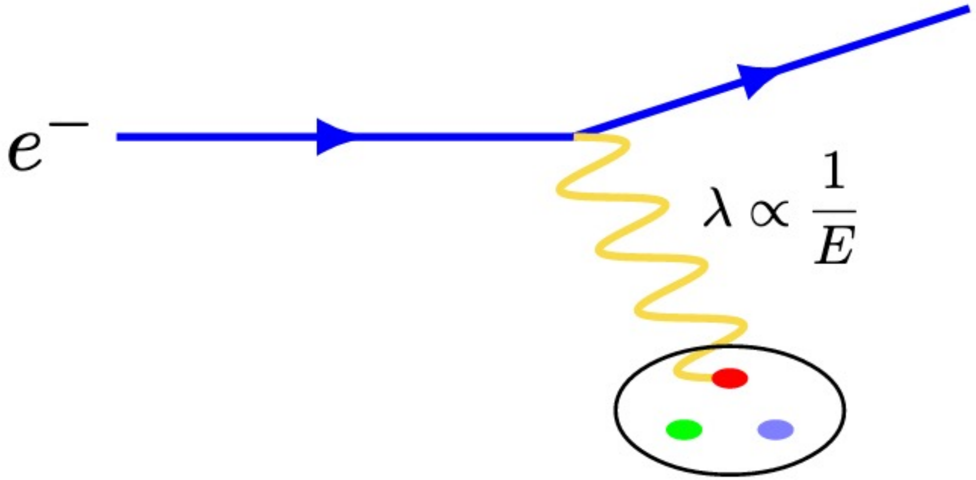
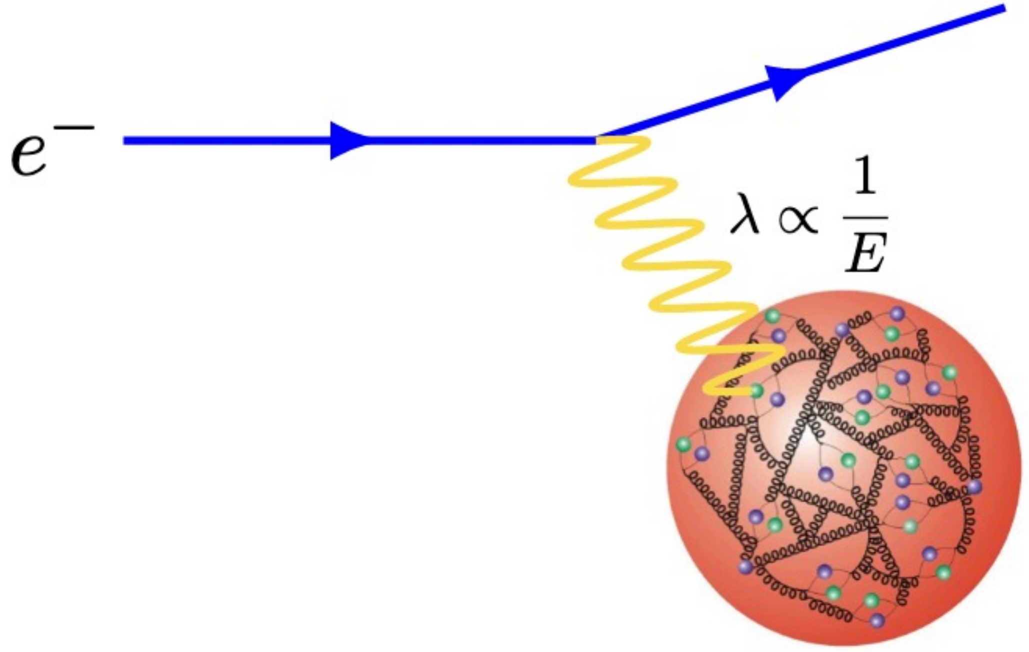


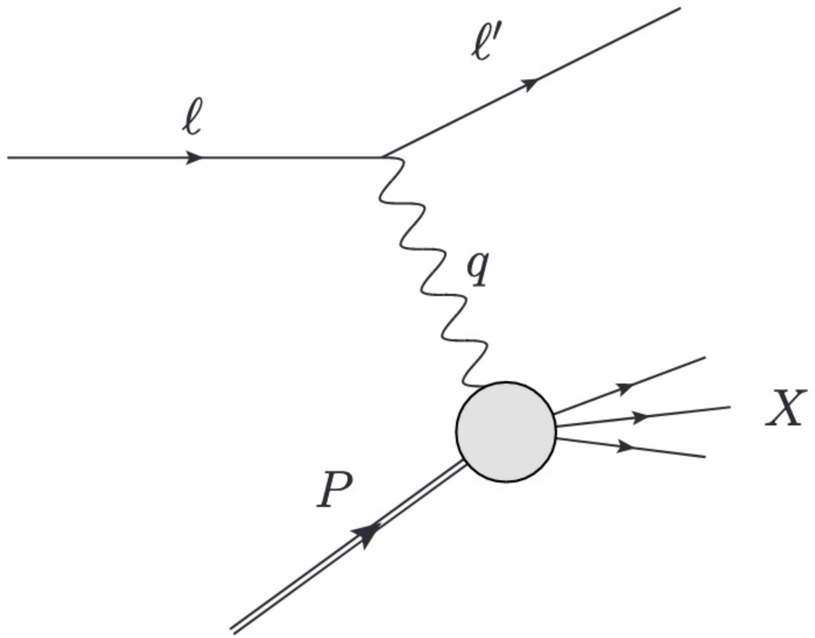
$$|p_+\rangle = \frac{1}{\sqrt{18}}\Big[2|u_+d_-u_+\rangle + 2|u_+u_+d_-\rangle + 2|d_-u_+u_+\rangle - |u_+u_-d_+\rangle - |u_-u_+d_+\rangle - |u_+d_+u_-\rangle - |u_-d_+u_+\rangle - |d_+u_+u_-\rangle - |d_+u_-u_+\rangle\Big] \quad \Rightarrow \quad \langle p_+|\hat{S}|p_+\rangle = \frac{1}{2}$$

Motivations: development of quark model



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

Formalism: DIS and structure functions



$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}Q^2} \propto L_{\mu\nu}W^{\mu\nu}$$

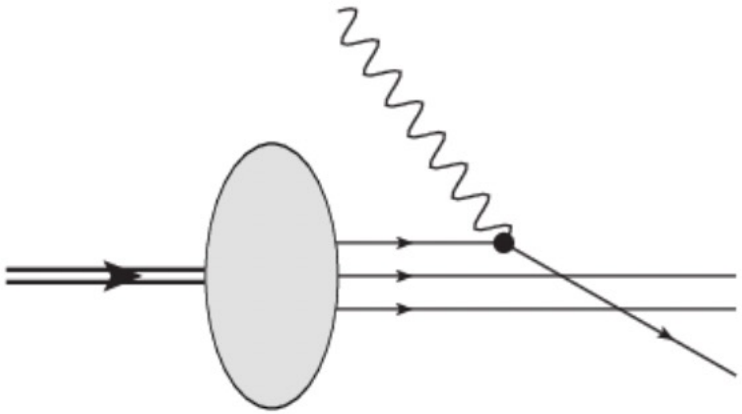
$$L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu}\ell\cdot\ell' - i\lambda_\ell \epsilon_{\mu\nu\alpha\beta}\ell^\alpha \ell'^\beta)$$

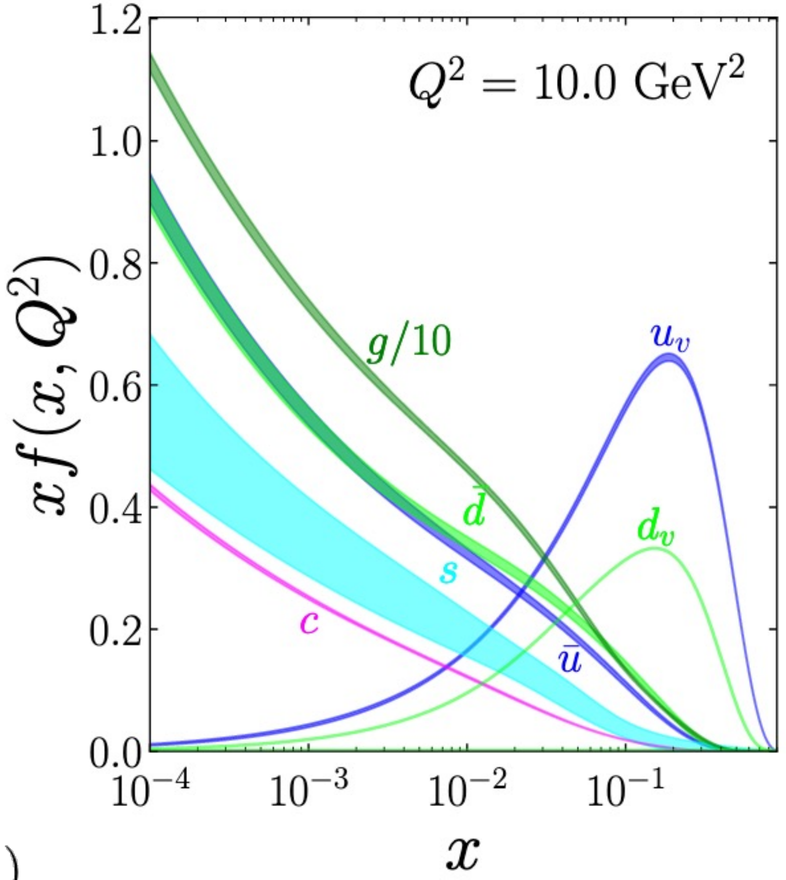
$$\begin{aligned} W^{\mu\nu} &= \frac{1}{(2\pi)^4}\sum_X \int \mathrm{d}^4 z\, e^{iq\cdot z}\langle P,S|J^\mu(z)|X\rangle\langle X|J^\nu(0)|P,S\rangle \\ &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1(x,Q^2) + \left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)F_2(x,Q^2) \\ &+ \frac{i}{P\cdot q}\epsilon^{\mu\nu\alpha\beta}q_\alpha\left[S_\beta g_1(x,Q^2) + (S_\beta - \frac{S\cdot q}{P\cdot q}P_\beta)g_2(x,Q^2)\right] \end{aligned}$$

$$Q^2 = -q^2 = (\ell' - \ell)^2 \quad x = \frac{Q^2}{2P\cdot q}$$

Formalism: parton distribution functions

Factorize DIS cross section





$$F_1(x,Q^2) = \int_x^1 \frac{\mathrm{d}\xi}{\xi}\hat{H}\Big(\frac{x}{\xi},\frac{\mu}{Q}\Big)f(\xi,\mu) = \frac{1}{2}\sum_i e_i^2 f_i(x,Q^2)$$

$$g_1(x,Q^2) = \int_x^1 \frac{\mathrm{d}\xi}{\xi}\Delta\hat{H}\Big(\frac{x}{\xi},\frac{\mu}{Q}\Big)\Delta f(\xi,\mu) = \frac{1}{2}\sum_i e_i^2 \Delta f_i(x,Q^2)$$

Sum rules

Some "facts" to be recovered from PDFs (model-independent)

1) Sum over parton momentum fractions is 1
2) Proton = uud
3) Proton has charge +1

$$\sum_i \int_0^1 x f_i(x, Q^2)\,\mathrm{d}x = 1$$

$$\int_0^1 \left[u(x) - \bar{u}(x)\right] \mathrm{d}x = 2$$

$$\int_0^1 \left[d(x) - \bar{d}(x)\right] \mathrm{d}x = 1$$

$$\int_0^1 \left[s(x) - \bar{s}(x)\right] \mathrm{d}x = 0$$

Ellis-Jaffe (EJ) sum rule

Other sum rules are derived assuming a particular parton model

- Note: this is the QCD corrected sum rule

$$\Gamma_1 = \int_0^1 \mathrm{d}x \; g_1(x, Q^2) = \frac{1}{12} g_A^{(3)} \left[1 - \frac{\alpha_s}{\pi}\right] + \frac{5}{36} g_A^{(8)} \left[1 - \frac{7}{15} \frac{\alpha_s}{\pi}\right] = 0.171 \pm 0.006$$

EMC determination of g1

EMC = European Muon Collaboration

Experimental check of EJ sum rule to determine validity of quark model

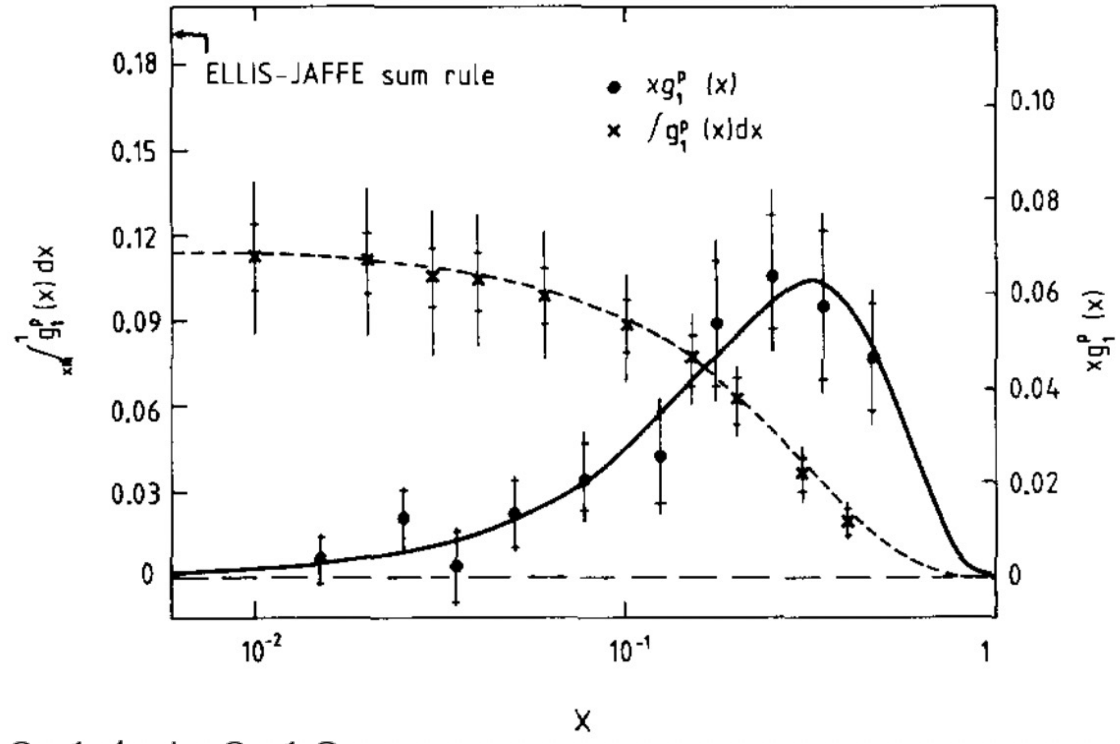
$$A_{\parallel}(x, Q^2) = \frac{\mathrm{d}\sigma^{++} - \mathrm{d}\sigma^{+-}}{\mathrm{d}\sigma^{++} + \mathrm{d}\sigma^{+-}} \sim \frac{g_1(x, Q^2)}{F_1(x, Q^2)}$$

Measure asymmetry – extract g1, integrate over x, and compare …

EMC results

Overall findings:

- Poor agreement with EJ sum rule
- Total quark spin contribution consistent with zero
- Large negative strange polarization



$$\Delta u = 0.77(6)$$
$$\Delta d = -0.49(6) \Rightarrow \Delta\Sigma(10.0\,\mathrm{GeV}^2) = 0.14 \pm 0.18$$
$$\Delta s = -0.15(6)$$

Verification of the EMC result

Other experiments (CERN, HERMES, SLAC) have confirmed the EMC result of the small quark spin contribution at precise confidence level

Experiment	Target	$Q^2(\text{GeV}^2)$ range	x range	$\Gamma_1^{\text{target}}(Q^2)$
E80/E130 [13, 14]	p	$1 < Q^2 < 10$	$0.1 < x < 0.7$	$\Gamma_1^p(10) = 0.17 \pm 0.05^*$
E142 [15]	n	$1 < Q^2 < 10$	$0.03 < x < 0.6$	$\Gamma_1^n(2) = -0.031 \pm 0.006 \pm 0.009$
E143 [16]	p,d	$1 < Q^2 < 10$	$0.03 < x < 0.8$	$\Gamma_1^p(3) = 0.132 \pm 0.003 \pm 0.009$
				$\Gamma_1^d(3) = 0.047 \pm 0.003 \pm 0.006$
E154 [17]	n	$1 < Q^2 < 17$	$0.014 < x < 0.7$	$\Gamma_1^n(5) = -0.041 \pm 0.004 \pm 0.006$
E155 [18]	p,d	$1 < Q^2 < 17$	$0.01 < x < 0.9$	$\Gamma_1^d(5) = 0.0266 \pm 0.0025 \pm 0.0071$
EMC [4]	p	$1 < Q^2 < 200$	$0.01 < x < 0.7$	$\Gamma_1^p(10.7) = 0.126 \pm 0.010 \pm 0.015^{**}$
SMC [19]	p,d	$1 < Q^2 < 60$	$0.003 < x < 0.7$	$\Gamma_1^p(10) = 0.120 \pm 0.005 \pm 0.006 \pm 0.014$
				$\Gamma_1^n(10) = -0.078 \pm 0.013 \pm 0.008 \pm 0.014$
				$\Gamma_1^d(10) = 0.019 \pm 0.006 \pm 0.003 \pm 0.013$
HERMES [20]	p,n,d	$1 < Q^2 < 10$	$0.023 < x < 0.6$	$\Gamma_1^n(3) = -0.037 \pm 0.013 \pm 0.008^\dagger$

What does this mean?

Proton quark spin content is well established now

- Only roughly 30% of the total proton angular momentum
- Still not fully established how individual quarks contribute

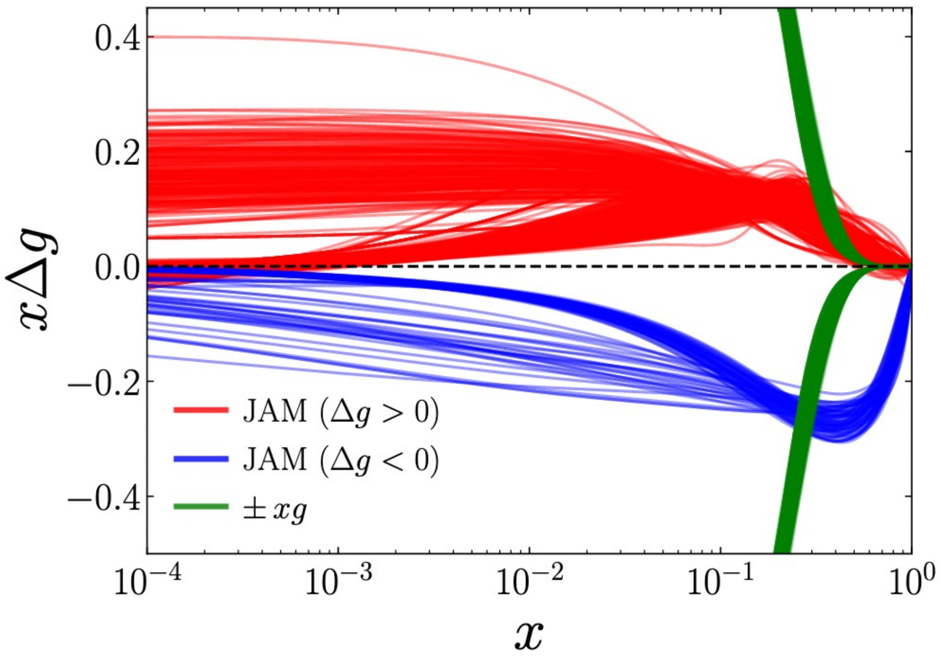
Rest of angular momentum from gluon spin or in quark/gluon orbital angular momentum

- Gluon spin: $\Delta G(Q^2) = \int_{x_{\mathrm{min}}}^{1} \Delta g(x, Q^2)\,\mathrm{d}x$
- Angular momentum of quarks and gluons contained in generalized parton densities (GPDs)

Gluon polarization – theory

This is a hot area right now

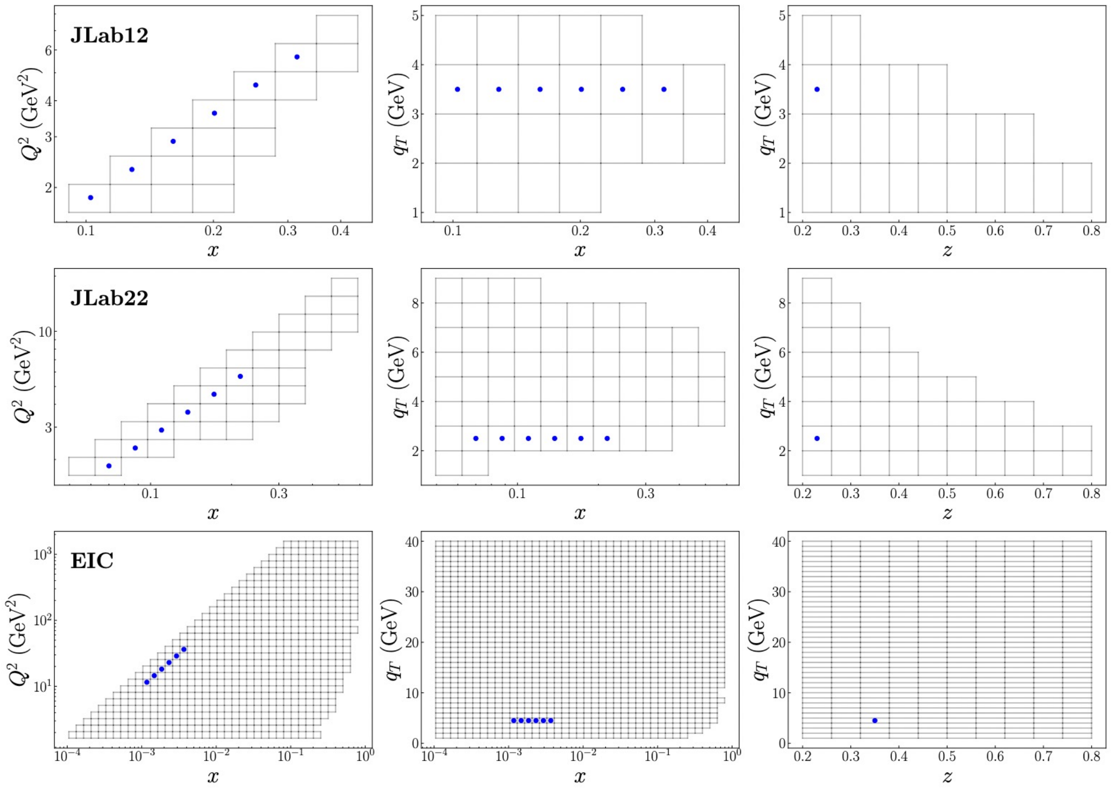
- Lattice work indicates about 50% of spin budget allocated to gluons
- Perturbative work has hit a bit of a snag recently

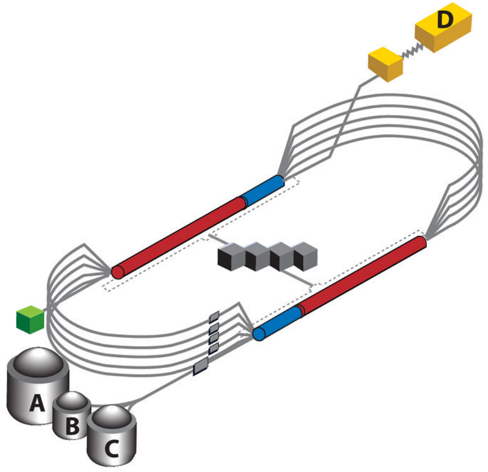


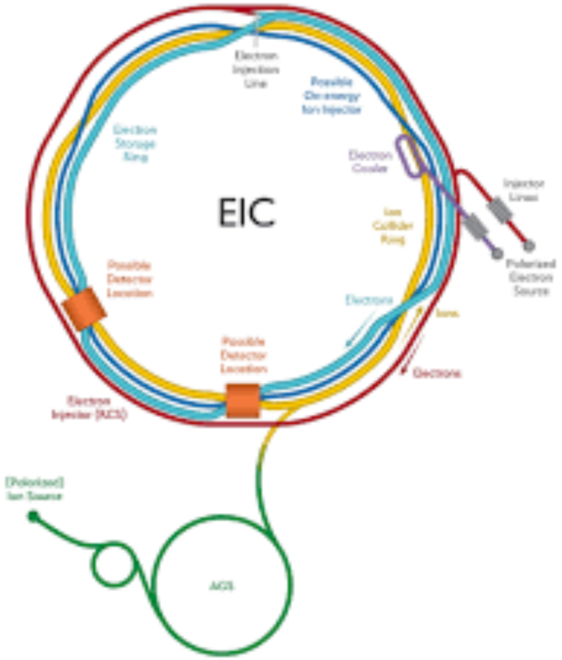
$\int\Delta f$	SU(2)		SU(3)		SU(3)+pos
Δu^+	0.8(1)		0.80(1)		0.81(1)
Δd^+	−0.4(1)		−0.37(1)		−0.38(2)
Δs^+	0.1(7)		−0.08(3)		−0.07(2)
Δg	0.0(6)		0.3(5)		0.39(9)
	$\Delta g > 0$	$\Delta g < 0$	$\Delta g > 0$	$\Delta g < 0$	
	0.4(2)	−0.8(2)	0.4(1)	−0.9(2)	

Gluon polarization – experiment

Future experiments will pin down gluon spin contribution precisely in the next couple decades







Summary

- Modern accelerators/colliders give insight into proton substructure
- EMC collaboration shattered expectation that quark spin dominates proton spin
- Current work looking into gluon polarization and GPDs
- Exciting progress to be made at EIC and JLab for DIS experiments in next few decades