1) Prove that the following equations have at least one solution in the given intervals.

(a) $x - (\ln x)^3 = 0, \quad [5, 7]$

(b) $5x \cos(\pi x) - 2x^2 + 3 = 0, \quad [0, 2]$

2) Verify that the function $|| \cdot ||_1$ defined on $\mathbb{R}^n$ by

$$||x|| = \sum_{i=1}^{n} |x_i|$$

is a norm on $\mathbb{R}^n$.

3) Find l_1, l_2, and l_∞ norms of the following vectors or matrices.

(a) $x = (2, 1, -3, 4)^T$

(b) $x = (\sin k, \cos k, 2^k)^T$

(c)
$$\begin{bmatrix} 10 & 15 \\ 0 & 1 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

4) Taylor expand the following function.

(a) e^x around $x = 0$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + O\left(x^8\right)$$

(b) $\log(x + 1)$ around $x = 0$

$$\log(x + 1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} + O\left(x^8\right)$$

```python
#!/usr/bin/env python3

import sympy as sp

x = sp.Symbol('x')
f1 = sp.exp(x)
f2 = sp.log(x + 1)

print(sp.latex(f1.series(x,0,8)))
print()
print(sp.latex(f2.series(x,0,8)))
```
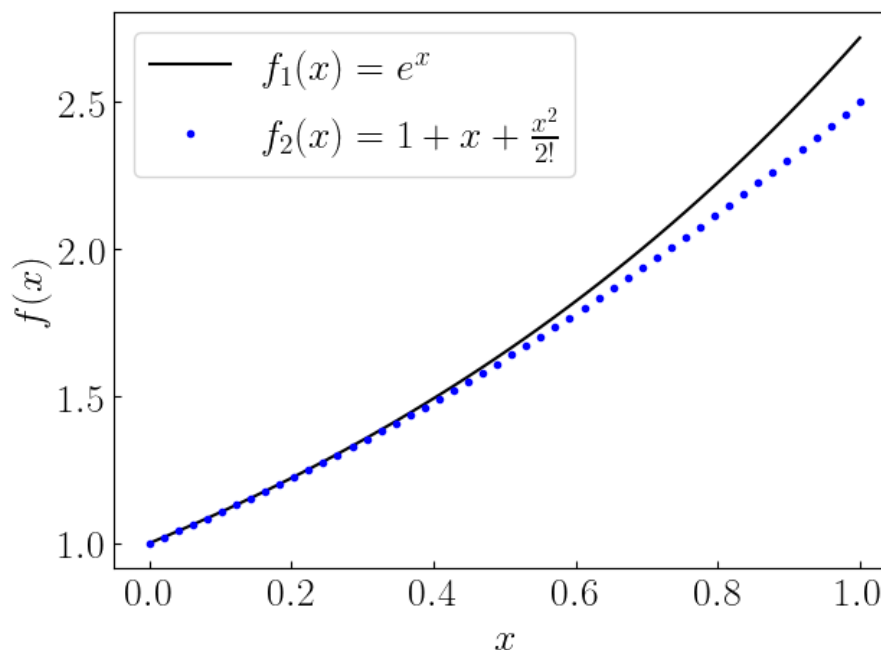
5) Plot the function e^x on $[0, 1]$ in a black solid line. On the same graph, plot the function $1 + x + \frac{x^2}{2!}$ in blue circle.


```
#!/usr/bin/env python3

import numpy as np
import matplotlib.pyplot as plt

from matplotlib import rcParams
rcParams['text.latex.preamble'] = r'\usepackage{amsmath}'
rcParams['text.usetex']          = True
rcParams['font.family']          = 'sans-serif'
rcParams['font.sans-serif']      = ['Helvetica']

x = np.linspace(0,1)
f1 = np.exp(x)
f2 = 1 + x + x**2/np.math.factorial(2)

fig, ax = plt.subplots(nrows=1,ncols=1,figsize=(7,5))
ax.plot(x,f1,'k-',label=r'$f_1(x) = e^x$')
ax.plot(x,f2,'b.',label=r'$f_2(x) = 1+x+\frac{x^2}{2!}$')
ax.legend(loc=2,fontsize=20)

ax.set_xlabel(r'$x$',size=20)
ax.set_ylabel(r'$f(x)$',size=20)
ax.tick_params(axis='both',which='major',labelsize=20,direction='in')

plt.savefig('prob5fig.png',bbox_inches='tight')
```