1) Investigate the local truncation error for the difference formula

$$f''(x) \approx \frac{f(x+3h) - 4f(x) + 3f(x-h)}{6h^2}.$$

2) Consider a quadrature of the form

$$\int_{-1}^{1} |x|\, f(x)\, \mathrm{d}x = \frac{1}{4}\left[f(-1) + 2f(0) + f(1)\right].$$

Show that it is exact for any polynomial $f(x)$ of degree at most 3.

3) For function $f(x) = \sin x$, use the forward-difference formula and backward-difference formula to determine $f'(a)$ at $a = 0.5, \,, 0.6, 0.7$ for $h = 0.1, 0.05, 0.025, 0.0125, 0.00625$. Calculate the exact derivatives exactly by directly calculating the derivative function. We define the order to be

$$\text{order}(h) = \log_2 \frac{\text{error}\,(2h)}{\text{error}\,(h)}.$$

Table 1: Table for 0.5, where $f'(0.5) = 0.87758$

h	Forward Difference	Forward Error	Forward Order	Backward Difference	Backward Error	Backward Order
0.1	0.85217	0.02541	-	0.90007	0.02249	-
0.05	0.86523	0.01235	1.04121	0.88920	0.01162	0.95294
0.025	0.87150	0.00608	1.02129	0.88348	0.00590	0.97725
0.0125	0.87456	0.00302	1.01082	0.88056	0.00297	0.98881
0.00625	0.87608	0.00150	1.00546	0.87908	0.00149	0.99445

Table 2: Table for 0.6, where $f'(0.6) = 0.82534$

h	Forward Difference	Forward Error	Forward Order	Backward Difference	Backward Error	Backward Order
0.1	0.79575	0.02958	-	0.85217	0.02683	-
0.05	0.81088	0.01446	1.03303	0.83910	0.01377	0.96260
0.025	0.81819	0.00714	1.01704	0.83231	0.00697	0.98187
0.0125	0.82179	0.00355	1.00865	0.82884	0.00351	0.99108
0.00625	0.82357	0.00177	1.00436	0.82709	0.00176	0.99557

Table 3: Table for 0.7, where $f'(0.7) = 0.76484$

h	Forward Difference	Forward Error	Forward Order	Backward Difference	Backward Error	Backward Order
0.1	0.73138	0.03346	-	0.79575	0.03091	-
0.05	0.74842	0.01642	1.02684	0.78063	0.01578	0.96966
0.025	0.75671	0.00813	1.01384	0.77281	0.00797	0.98528
0.0125	0.76080	0.00405	1.00703	0.76885	0.00401	0.99275
0.00625	0.76282	0.00202	1.00354	0.76685	0.00201	0.99640