1) Assume x is an m-dimensional vector, prove $||x||_\infty \leq ||x||_2$ and $||x||_2 \leq \sqrt{m}||x||_\infty$.

Recall that

$$||x||_2 = \sqrt{\sum_{i=1}^{m} |x_i|^2} \tag{1}$$

$$||x||_\infty = \max\{|x_1|, |x_2|, \ldots, |x_m|\}. \tag{2}$$

Thus, for the first inequality we have the following. Suppose that $|x_k| = ||x||_\infty$, then

$$\boxed{||x||_2 = \sqrt{||x||_\infty^2 + \sum_{i \neq k} |x_i|^2} \leq \sqrt{||x||_\infty^2} \leq ||x||_\infty}, \tag{3}$$

since $|x_i| \geq 0$ for all i.

For the second inequality, we know that $|x_i| \leq ||x||_\infty$ for all i. Hence,

$$\boxed{||x||_2 \leq \sqrt{\sum_{i=1}^{m} ||x||_\infty^2} = \sqrt{m}||x||_\infty}. \tag{4}$$

2) Show that if a matrix A is both triangular and unitary, then it is diagonal.

3) Prove that matrix ∞-norm is

$$||A||_\infty = \max_i \sum_{j=1}^{n} |a_{ij}|. \tag{5}$$

4) Let $||\cdot||$ denote any norm on $\mathbb{C}$ and also the induced matrix norm on $\mathbb{C}^{m\times m}$. Show that $\rho(A) \leq ||A||$, where ρ is the spectral radius of A, which is the largest absolute value of an eigenvalue of A.

5) Find l_1, l_2, and l_∞ norms of the following vectors and matrices, also, please verify your results by MATLAB.

(a) $x = (3, -4, 0, \frac{3}{2})^{\mathrm{T}}$

(b) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

6) Determine SVDs of the following matrices by hand calculation and MATLAB.

(a) $\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$