

Notes on PVDIS

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I. OVERVIEW OF DEEP-INELASTIC SCATTERING (DIS)

The deep-inelastic scattering process refers generally to the lepton-nucleon collision $\ell N \rightarrow \ell' X$, where X is the unobserved hadronic final state particle(s). Since the original experiments at SLAC, DIS has been one of the most fruitful methods by which we have learned about the structures of the proton and neutron, and it will continue to be so for the foreseeable future, especially given the imminent operation of the Electron-Ion Collider (EIC) at Brookhaven national lab, which will give unprecedented access to a larger kinematic phase space.

For these notes, we restrict our attention to neutral-current processes, and in particular, we only consider contributions from processes that enter at first-order in boson exchanges between the lepton and nucleon (as shown in Fig. 1). In doing so, we have the following kinematic definitions:

- $\nu = \frac{P \cdot q}{M} = E - E'$ is the energy lost by the lepton in the nucleon rest frame.
- $Q^2 = -q^2 = -(\ell - \ell')^2$ is the virtuality of the virtual photon.
- $x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M\nu}$ is the typical Bjorken scaling variable.
- $y = \frac{P \cdot q}{P \cdot \ell} = \frac{\nu}{E}$ is the inelasticity variable.
- $W^2 = (P + q)^2 = M^2 + Q^2 \left(1 - \frac{1}{x}\right)$ is the invariant mass of the hadronic final state.
- $s = (P + \ell)^2 = M^2 + \frac{Q^2}{xy}$ is the squared center of mass energy of the lepton-nucleon system.

Note that DIS is characterized by the following kinematic constraints: $Q^2, W^2 \gg M^2$.

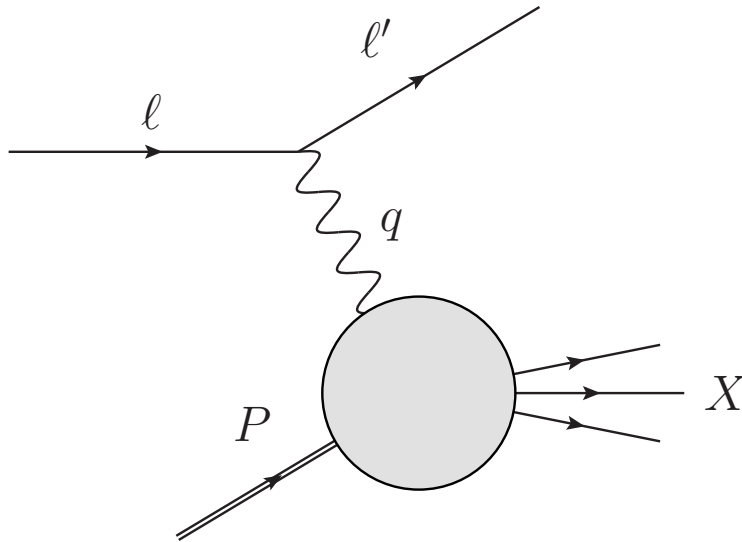


FIG. 1. Generic feynman diagram representing the DIS process in the one-“photon” exchange approximation.

Using Fermi's Golden rule, we can write the cross section for this process (summing and integrating over the hadronic final states) as

$$d\sigma = \frac{1}{4P \cdot \ell} \frac{d^3 \ell'}{(2\pi)^3 2E'} \sum_X \int \left(\prod_{i \in X} \frac{d^3 \mathbf{P}_i}{(2\pi)^3 2E_i} \right) (2\pi)^4 \delta^{(4)}(P + q - P_X) |\mathcal{M}|^2, \quad (1)$$

Rewriting, we find

$$\frac{d\sigma}{dE' d\Omega} = \frac{1}{2(s - M^2)} \frac{E'}{2(2\pi)^3} \sum_X \int d\Phi_X (2\pi)^4 \delta^{(4)}(P + q - P_X) |\mathcal{M}|^2. \quad (2)$$

At this juncture, we analyze the form of the scattering amplitude. The full amplitude is given as a sum of contributions from the processes mediated by a virtual photon and Z -boson, respectively,

$$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z, \quad (3)$$

where $\mathcal{M}_{\gamma(Z)} = \langle \ell', s' | j_\mu^{\gamma(Z)} | \ell, s \rangle \frac{g^{\mu\nu}}{Q^2} \langle X | J_\nu^{\gamma(Z)} | P, S \rangle = \frac{1}{Q^2} \langle \ell', s' | j_\mu^{\gamma(Z)} | \ell, s \rangle \langle X | J_{\gamma(Z)}^\mu | P, S \rangle$, where $j_\mu^{\gamma(Z)}$ and $J_\mu^{\gamma(Z)}$ are electromagnetic (weak) lepton and nucleon currents. Note that the factor $g^{\mu\nu}/Q^2$ is the virtual boson propagator, which is exact when the boson is a photon and approximate when the virtual particle is a Z -boson ($Q^2 \ll M_Z^2$). The squared amplitude is then

$$|\mathcal{M}|^2 = |\mathcal{M}_\gamma|^2 + \mathcal{M}_\gamma^* \mathcal{M}_Z + \mathcal{M}_Z^* \mathcal{M}_\gamma + |\mathcal{M}_Z|^2. \quad (4)$$

Using the current matrix elements above, we can separate the leptonic and hadronic contributions to the cross section as a tensor product:

$$\frac{d\sigma}{dE' d\Omega} = \frac{1}{2(s - M^2)} \frac{E'}{2(2\pi)^3} \frac{e^4}{Q^4} \sum_i [\eta_i L_{\mu\nu}^i (4\pi W_i^{\mu\nu})], \quad (5)$$

where $i \in \{\gamma, \gamma Z, Z\}$, $L_{\mu\nu}^i$ is the leptonic tensor, $W_i^{\mu\nu}$ is the hadronic tensor (where the factor of 4π is a normalization factor introduced by convention), and η_i collects kinematic factors from the squared matrix elements. We write down the kinematic factors η_i with the corresponding lepton tensor $L_{\mu\nu}^i$:

- $\eta_\gamma = 1$ and $L_{\mu\nu} = 2(\ell^\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu} \ell \cdot \ell' - i\lambda_\ell \epsilon_{\mu\nu\alpha\beta} \ell^\alpha \ell'^\beta)$
- $\eta_{\gamma Z} = \left(\frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \right) \left(\frac{Q^2}{Q^2 + M_Z^2} \right)$ and $L_{\mu\nu}^Z = (g_V^\ell - \lambda g_A^\ell) L_{\mu\nu}^\gamma$
- $\eta_Z = \eta_{\gamma Z}^2$ and $L_{\mu\nu}^Z = (g_V^\ell - \lambda g_A^\ell)^2 L_{\mu\nu}^\gamma$

Note that $g_V^\ell = g_A^\ell - 2Q_\ell \sin^2 \theta_W$ and $g_A^\ell = \frac{1}{2} \frac{Q_\ell}{|Q_\ell|}$ are the vector and axial weak couplings (θ_W is the weak mixing angle) for lepton ℓ (with charge Q_ℓ), respectively, and λ_ℓ is the helicity of the lepton (assuming longitudinal polarization). Finally, we write down the definition of the hadronic tensor as

$$W_\gamma^{\mu\nu} = \frac{1}{4\pi} \sum_X \int d\Phi_X \delta_{\text{PS}}^{(4)} \langle P, S | J_\gamma^{\mu\dagger} | X \rangle \langle X | J_\gamma^\nu | P, S \rangle \quad (6a)$$

$$W_{\gamma Z}^{\mu\nu} = \frac{1}{4\pi} \sum_X \int d\Phi_X \delta_{\text{PS}}^{(4)} [\langle P, S | J_\gamma^{\mu\dagger} | X \rangle \langle X | J_Z^\nu | P, S \rangle + \langle P, S | J_Z^{\mu\dagger} | X \rangle \langle X | J_\gamma^\nu | P, S \rangle] \quad (6b)$$

$$W_Z^{\mu\nu} = \frac{1}{4\pi} \sum_X \int d\Phi_X \delta_{\text{PS}}^{(4)} \langle P, S | J_Z^{\mu\dagger} | X \rangle \langle X | J_Z^\nu | P, S \rangle, \quad (6c)$$

where $\delta_{\text{PS}}^{(4)} = (2\pi)^4 \delta^{(4)}(P+q-P_X)$. Generally, we can parameterize the hadronic tensor in terms of structure functions, ensuring that it obeys current conservation (i.e. $\partial_\mu J^\mu = 0 \Leftrightarrow q_\mu W^{\mu\nu} = 0$) and Lorentz covariance, as

$$\begin{aligned} W^{\mu\nu} = & -\tilde{g}^{\mu\nu} F_1(x, Q^2) + \tilde{P}^\mu \tilde{P}^\nu \frac{F_2(x, Q^2)}{P \cdot q} - i\epsilon^{\mu\nu\alpha\beta} \frac{q^\alpha P^\beta}{2P \cdot q} F_3(x, Q^2) \\ & + i\epsilon^{\mu\nu\alpha\beta} \frac{q^\alpha}{P \cdot q} \left[S^\beta g_1(x, Q^2) + \left(S^\beta - \frac{S \cdot q}{P \cdot q} \right) g_2(x, Q^2) \right] \\ & + \frac{1}{P \cdot q} \left[\frac{1}{2} (\tilde{P}^\mu \tilde{S}^\nu + \tilde{S}^\mu \tilde{P}^\nu) - \frac{S \cdot q}{P \cdot q} \tilde{P}^\mu \tilde{P}^\nu \right] g_3(x, Q^2) \\ & + \frac{S \cdot q}{P \cdot q} \left[\tilde{P}^\mu \tilde{P}^\nu \frac{g_4(x, Q^2)}{P \cdot q} - \tilde{g}^{\mu\nu} g_5(x, Q^2) \right], \end{aligned} \quad (7)$$

where $\tilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$ and $\tilde{P}^\mu = \tilde{g}^{\mu\nu} P_\nu$. Furthermore, we note that S is the nucleon 4-vector with $S^2 = -M^2$ and $S \cdot P = 0$, and lastly, we note that $F_j(x, Q^2)$ and $g_j(x, Q^2)$ are the unpolarized and polarized¹ dimensionless Bjorken-scaling structure functions, respectively.

Now, we can change variables from (E', Ω) to (x, y) , which gives us the conventional DIS cross section as

$$\frac{d\sigma}{dx dy} = \frac{Q^2}{x} \frac{\pi}{E'} \frac{d\sigma}{dE' d\Omega} = \frac{2\pi\alpha^2 y}{Q^4} \sum_i \eta_i L_{\mu\nu}^i W_i^{\mu\nu}, \quad (8)$$

where we have used the relation $e^2 = 4\pi\alpha$. Note that $d\sigma = d\sigma(\lambda_\ell, S)$ depends on the helicity of the lepton (in the initial state) and spin of the nucleon. We can therefore define cross sections that depend on exclusively on the spin of either the lepton or the nucleon. When we refer to the polarization of a cross section, we explicitly refer to the polarization of the nucleon. Therefore, the unpolarized cross section is given by

$$\begin{aligned} \frac{d\sigma}{dx dy} &= \frac{1}{2} \left[\frac{d\sigma(\lambda_\ell, -S)}{dx dy} + \frac{d\sigma(\lambda_\ell, S)}{dx dy} \right] = \frac{2\pi\alpha^2 y}{Q^4} \sum_i \eta_i L_{\mu\nu}^i W_{U,i}^{\mu\nu} \\ &= \frac{4\pi\alpha^2}{xyQ^2} \left[xy^2 F_1^{\text{NC}}(x, Q^2) + \left(1 - y - \frac{x^2 y^2 M^2}{Q^2} \right) F_2^{\text{NC}}(x, Q^2) - \lambda_\ell x \left(y - \frac{y^2}{2} \right) F_3^{\text{NC}}(x, Q^2) \right]. \end{aligned} \quad (9)$$

where $W_U^{\mu\nu} = \frac{1}{2} [W^{\mu\nu}(P, q, S) + W^{\mu\nu}(P, q, -S)]$ and

$$F_j^{\text{NC}} = F_j^\gamma - \eta_{\gamma Z} (g_V^\ell - \lambda_\ell g_A^\ell) F_j^{\gamma Z} + \eta_Z (g_V^\ell - \lambda_\ell g_A^\ell)^2 F_j^Z. \quad (10)$$

Finally, the polarized cross section is given by²

$$\begin{aligned} \frac{d\Delta\sigma}{dx dy} &= \frac{1}{2} \left[\frac{d\sigma(\lambda_\ell, -S)}{dx dy} - \frac{d\sigma(\lambda_\ell, S)}{dx dy} \right] = \frac{2\pi\alpha^2 y}{Q^4} \sum_i \eta_i L_{\mu\nu}^i W_{P,i}^{\mu\nu} \\ &= \frac{8\pi\alpha^2}{xyQ^2} \left[-\lambda_\ell xy \left(2 - y - 2x^2 y^2 \frac{M^2}{Q^2} \right) g_1^{\text{NC}} + 4\lambda_\ell x^3 y^2 \frac{M^2}{Q^2} g_2^{\text{NC}} \right. \\ &\quad \left. + 2x^2 y \frac{M^2}{Q^2} \left(1 - y - x^2 y^2 \frac{M^2}{Q^2} \right) g_3^{\text{NC}} \right. \\ &\quad \left. - \left(1 + 2x^2 y \frac{M^2}{Q^2} \right) \left(\left[1 - y - x^2 y^2 \frac{M^2}{Q^2} \right] g_4^{\text{NC}} \right) + xy^2 g_5^{\text{NC}} \right], \end{aligned} \quad (11)$$

where $W_P^{\mu\nu} = \frac{1}{2} [W^{\mu\nu}(P, q, S) - W^{\mu\nu}(P, q, -S)]$ and g_j^{NC} are defined similarly as for the unpolarized neutral current structure functions in Eq. (10). Finally, before moving on, we define

$$R^i = \frac{\sigma_L}{\sigma_T} = \frac{F_2^i}{2xF_1^i} r^2 - 1, \quad (12)$$

where $r^2 = 1 + Q^2/\nu^2$ and σ_L and σ_T are the longitudinal and transverse photoabsorption cross sections.

¹ It should be noted that $W^{\mu\nu} = W^{\mu\nu}(P, q, S)$, and unpolarized structure functions appear in the sum $W^{\mu\nu}(P, q, S) + W^{\mu\nu}(P, q, -S)$ while polarized structure functions appear in the difference $W^{\mu\nu}(P, q, S) - W^{\mu\nu}(P, q, -S)$.

² We assume longitudinal polarization such that $S^\mu = M(0, 0, 0, \lambda_n)$, where λ_n is the nucleon's helicity, in the proton's rest frame, which gives $S \cdot \ell = -\lambda_n M E$ and $S \cdot \ell' = -\lambda_n M E' \cos \theta$, where θ is the angle of deflection of the lepton. Note that $E = Q^2/[2Mxy]$, $E' = E(1 - y)$, and $\cos \theta = 1 - Q^2/[2E^2(1 - y)]$

II. THE PARTON MODEL

In the previous section, we outlined the calculation of the hadronic tensor and differential cross sections at the structure function level. In this section, we outline the calculation of these quantities at the parton level, using a factorization framework. At large Q^2 , we can write the hadronic tensor as a convolution of hard scattering amplitudes and parton distribution functions (PDFs)³:

$$W_U^{\mu\nu} = \sum_i \int_0^1 \frac{d\xi}{\xi} \widehat{W}_U^{\mu\nu}(\hat{x}, Q^2) f_{i/P}(\xi, \mu^2) \quad (13a)$$

and

$$W_P^{\mu\nu} = \sum_i \int_0^1 \frac{d\xi}{\xi} \widehat{W}_P^{\mu\nu}(\hat{x}, Q^2) \Delta f_{i/P}(\xi, \mu^2), \quad (13b)$$

where $\widehat{W}^{\mu\nu}$ is the partonic tensor (defined as the partonic analogue to the leptonic tensor), $(\Delta)f_{i/N}$ is the (polarized) unpolarized PDF for a parton of type i in nucleon N , ξ is the parton momentum fraction defined by $p = \xi P$, $\hat{x} = x/\xi$ is the partonic analogue to Bjorken- x , and μ is the renormalization scale for the PDFs (usually set such that $\mu^2 = Q^2$). In the following text, we are interested in computing the hadronic structure functions in this factorization framework. It is noted that several components go into the computation of the factorization integrand and the extraction of the structure functions. Since these notes are focused on the phenomenology of PVDIS, we make no attempt to discuss the determination of the PDFs. We simply utilize those PDFs, which were obtained from previous global QCD analyses, assuming that these studies were competent. In the next sections, we outline the calculation of the partonic tensor (Sec. II A) and the determination of the hadronic structure functions (Sec. II B).

A. Calculation of the partonic tensor

The partonic structure tensor is calculated as the squared amplitude for the virtual compton scattering process, shown in Fig. 2, as

$$\widehat{W}_i^{\mu\nu} = \sum_{\text{graphs}} \mathcal{M}_i^\mu{}^\dagger \mathcal{M}_i^\nu. \quad (14)$$

In this work, we compute the partonic tensor at leading order, where the leading order process is simply $\gamma^*(q) + q(p) \rightarrow q(p')$ (shown in Fig. 3), which gives

$$\widehat{W}_\gamma^{\mu\nu} = \int d\Pi \left\{ \sum_{s'} e_q^2 [\bar{u}(p', s') \gamma^\mu u(p, s)]^* [\bar{u}(p', s') \gamma^\nu u(p, s)] \right\} \quad (15a)$$

$$\begin{aligned} \widehat{W}_{\gamma Z}^{\mu\nu} = \int d\Pi \left\{ \sum_{s'} e_q ([\bar{u}(p', s') (g_V^i - g_A^i \gamma^5) \gamma^\mu u(p, s)]^* [\bar{u}(p', s') \gamma^\nu u(p, s)] \right. \\ \left. + [\bar{u}(p', s') \gamma^\mu u(p, s)]^* [\bar{u}(p', s') \gamma^\nu (g_V^i - g_A^i \gamma^5) u(p, s)] \right\} \end{aligned} \quad (15b)$$

$$\widehat{W}_Z^{\mu\nu} = \int d\Pi \left\{ \sum_{s'} [\bar{u}(p', s') \gamma^\mu (g_V^i - g_A^i \gamma^5) u(p, s)]^* [\bar{u}(p', s') \gamma^\nu (g_V^i - g_A^i \gamma^5) u(p, s)] \right\}. \quad (15c)$$

Note that e_q is the fractional charge of the quarks. For the above tensors, note that

$$\int d\Pi = \int \frac{d^3 \mathbf{p}'}{(2\pi)^3 2p'^0} \delta^{(3)}(p + q - p') = \int d^4 p' \delta^{(4)}(p + q - p') \delta_+(p'^2), \quad (16)$$

³ Neglecting higher order m_q/Q corrections

where $\delta_+(p'^2) = \delta_+([p+q]^2) = \frac{x}{Q^2} \delta(\xi - x)$ is the on-shell condition for the scattered parton, and⁴

$$\begin{aligned}
& \sum_{s'} [\bar{u}(p', s') \gamma^\mu (v_1 - a_1 \gamma^5) u(p, s)]^* [\bar{u}(p', s') \gamma^\nu (v_2 - a_2 \gamma^5) u(p, s)] \\
&= \sum_{s'} [\bar{u}(p, s) (v_1 + a_1 \gamma^5) \gamma^\mu u(p', s') \bar{u}(p', s') \gamma^\nu (v_2 - a_2 \gamma^5) u(p, s)] \\
&= \frac{1}{2} \text{Tr} \left[\left(1 + \frac{\gamma^5 \not{p}}{m_q} \right) (\not{p} + m_q) (v_1 + a_1 \gamma^5) \gamma^\mu (\not{p}' + m_q) \gamma^\nu (v_2 - a_2 \gamma^5) \right] \\
&= 2(a_1 a_2 + v_1 v_2) [2p^\mu p^\nu + p^\mu q^\nu + q^\mu p^\nu - g^{\mu\nu} p \cdot q] \\
&\quad - 2a_1 v_2 [2s^\mu p^\nu - g^{\mu\nu} s \cdot q] - 2a_2 v_1 [2p^\mu s^\nu - g^{\mu\nu} s \cdot q] - 2(a_1 v_2 + a_2 v_1) [s^\mu q^\nu + q^\mu s^\nu] \\
&\quad + 2i\epsilon^{\mu\nu\alpha\beta} [(a_1 v_2 + a_2 v_1) p_\alpha q_\beta - 2a_1 a_2 s_\alpha p_\beta - (a_1 a_2 + v_1 v_2) s_\alpha q_\beta].
\end{aligned} \tag{17}$$

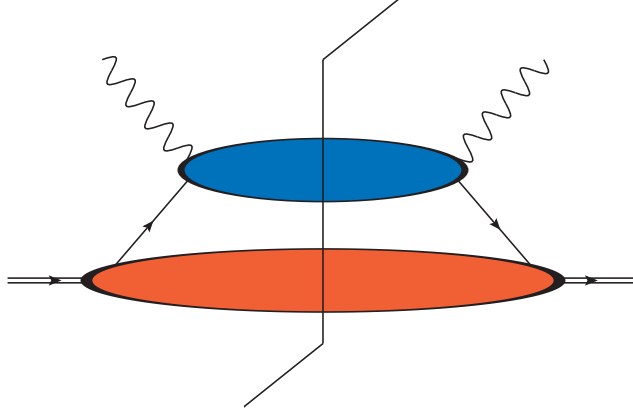


FIG. 2. Generic feynman diagram for virtual compton scattering of partons from the exchanged photon in the DIS process. Note that the blue blob represents all the possible hard scattering (virtual compton scattering) diagrams that can be drawn, which contribute to the scattering process, and the red blob represents the soft part of the collinear factorization, which is captured by the PDFs.

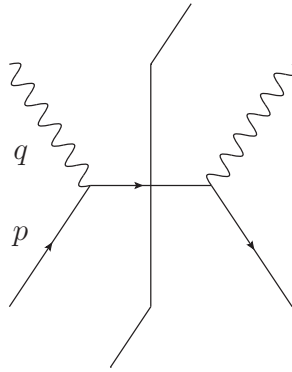


FIG. 3. Leading order virtual compton scattering feynman diagram for the hard scattering part of Fig. 2: $\gamma^* q \rightarrow q$.

⁴ Recall the dirac adjoint: $\bar{\psi} = \psi^\dagger \gamma^0$

B. Determination of the hadronic structure functions

From the last section, we have the form of the electromagnetic, weak, and interference partonic tensors, and the connection between the hadronic and partonic tensors is captured in Eq. (13). The structure functions enter at the hadronic level as shown in Eq. (6). Generically, we can express this form as a linear combination of structure functions⁵

$$W_U^{\mu\nu} = \sum_j a_j^{\mu\nu} F_j. \quad (18)$$

As with regular vectors, we can use 2nd-order projection tensors with the defining property that $\mathcal{P}_{\mu\nu}^k W_U^{\mu\nu} = F_k$. For the unpolarized projectors, we can write a general projector as

$$\mathcal{P}_{\mu\nu}^k = c_g^k \mathcal{P}_{\mu\nu}^g + c_{PP}^k \mathcal{P}_{\mu\nu}^{PP} + c_{\epsilon Pq}^k \mathcal{P}_{\mu\nu}^{\epsilon Pq}, \quad (19)$$

where $\mathcal{P}_g^{\mu\nu} = g^{\mu\nu}$, $\mathcal{P}_{PP}^{\mu\nu} = P^\mu P^\nu$, and $\mathcal{P}_{\epsilon Pq} = i\epsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta$. It is clear then from this setup that we can solve for the coefficients $c_{g,PP,\epsilon Pq}^k$ by imposing $\mathcal{P}_{\mu\nu}^k a_j^{\mu\nu} = \delta_{jk}$. Doing so, gives the following projectors for the unpolarized structure functions

$$\mathcal{P}_1^{\mu\nu} = -\frac{1}{2}\mathcal{P}_g^{\mu\nu} + \frac{2x^2}{4M^2x^2 + Q^2}\mathcal{P}_{PP}^{\mu\nu} \quad (20a)$$

$$\mathcal{P}_2^{\mu\nu} = -\frac{Q^2x}{4M^2x^2 + Q^2}\left[\mathcal{P}_g - \frac{12x^2}{4M^2x^2 + Q^2}\mathcal{P}_{PP}^{\mu\nu}\right] \quad (20b)$$

$$\mathcal{P}_3^{\mu\nu} = -\frac{2x}{4M^2x^2 + Q^2}\mathcal{P}_{\epsilon Pq}. \quad (20c)$$

For the calculations below, we will assume that $M^2 \ll Q^2$ such that the nucleon mass can be neglected. Using this fact, putting everything into the factorization, and extracting the structure functions using the projectors above, we find the leading order expressions for the unpolarized structure functions:

$$F_1^{[\gamma, \gamma^Z, Z]}(x, Q^2) = \frac{1}{2} \sum_q \left[e_q^2, 2e_q g_V^q, (g_V^q)^2 + (g_A^q)^2 \right] f_q^+(x, Q^2) \quad (21)$$

$$F_3^{[\gamma, \gamma^Z, Z]}(x, Q^2) = \sum_q \left[0, 2e_q g_A^q, 2g_V^q g_A^q \right] f_q^-(x, Q^2), \quad (22)$$

where we have defined $f_q^\pm = f_q \pm f_{\bar{q}}$. Also, note that $F_2 = 2xF_1$ (which is the Callan-Gross relation) and the sum over q is a sum over quark flavors (u, d, s, c, b).⁶

III. PARITY-VIOLATING ASYMMETRY

Now that we have the expressions for the hadronic cross sections and structure functions, we may write down and compute the parity violating asymmetry A_{PV} over a wide-range of kinematics. In particular, we are interested in the dependence of A_{PV} on the strange sea. Current experimental scattering data do not allow global QCD analyses to constrain the strange sea well, but because A_{PV} depends directly on combinations of the strange and anti-strange PDFs (as will be shown below), it may be possible that this asymmetry serves as a good candidate for data input to future global QCD analyses.

The conventional asymmetry is defined as the ratio below⁷:

$$A_{PV} = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-} = \frac{G_F Q^2}{8\pi\alpha} \left[a_1 Y_1 + a_3 Y_3 \right], \quad (23)$$

⁵ The argument is given here for the unpolarized and can be repeated for the polarized sector.

⁶ The top (anti-)quark does not contribute at the \sqrt{s} we consider since it is so massive ($m_t \approx 173 \text{ GeV}$)!.

⁷ We have defined $d\sigma_\pm = d\sigma(\lambda_\ell = \pm 1)$, where $d\sigma$ is the unpolarized cross section of Eq. (9).

where

$$a_1 = 2g_A^\ell \frac{F_1^{\gamma Z}}{F_1^\gamma} \quad \text{and} \quad a_3 = g_V^\ell \frac{F_3^{\gamma Z}}{F_3^\gamma}. \quad (24)$$

Additionally, the factors

$$Y_1 = \left(\frac{1 + R^{\gamma Z}}{1 + R^\gamma} \right) \frac{1 + (1 - y^2) - \frac{y^2}{2} \left[1 + r^2 - \frac{2r^2}{1 + R^{\gamma Z}} \right]}{1 + (1 - y)^2 - \frac{y^2}{2} \left[1 + r^2 - \frac{2r^2}{1 + R^\gamma} \right]} \quad (25)$$

$$Y_3 = \left(\frac{r^2}{1 + R^\gamma} \right) \frac{1 - (1 - y^2)}{1 + (1 - y)^2 - \frac{y^2}{2} \left[1 + r^2 - \frac{2r^2}{1 + R^\gamma} \right]}. \quad (26)$$

In the leading order calculation we completed above, we had $F_2 = 2xF_1$, implying that $R = r^2 - 1$. This gives $Y_1 = 1$ and $Y_3 = [1 - (1 - y)^2]/[1 + (1 - y)^2]$. It should be noted that the expression recorded for A_{PV} in Eq. (23) is the leading-order term in the power series expansion of the general A_{PV} in G_F/α and M_Z/Q .