1) Which operators are linear?

a) $Au = -\partial^2 u$, $\mathcal{D} = \{u \in C^2[0,\pi] \bigm| u(0) = 0 \text{ and } u'(\pi) = 0\}$

b) $Au = -\partial^2 u$, $\mathcal{D} = \{u \in C^2[0,\pi] \bigm| u(0) = 0 \text{ and } u'(\pi) = \pi\}$

c) $Au = u_{xx} + x^2 u$, $\mathcal{D} = C^2(\mathbb{R})$

d) $Au = u_{xx} + u^2$, $\mathcal{D} = C^2(\mathbb{R})$

2) Solve $u_t = 3u_x + 5u$, $u(x,0) = f(x)$.

3) Solve $u_{tt} - u_{xx} = 0$, $x \in \mathbb{R}$, $t > 0$ for $u(x,0) = 3\sin 5x - \cos 3x$, $u_t(x,0) = 2x + 3x^2$. Find only $u(0,t)$.

4) Find $u(x,t)$ for the equation $u_t - ku_{xx} = 0$, $x \in \mathbb{R}$, $t > 0$ subject to $u(x,0) = e^{-(x+1)^2}$.

5) Find the Fourier transform $\hat{f}(\omega) = \displaystyle\int_{-\infty}^{\infty} f(x)e^{-i\omega x}$ for $f(x) = xe^{-x^2}$.

6*) Find the Fourier transform for $f(x) = -\mathrm{H}(x+1) + 2\mathrm{H}(x) - \mathrm{H}(x-1)$, where
$\mathrm{H}(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$.

7*) Solve $u_t = 5u_x - 3xu + u$, $x \in \mathbb{R}$, $t > 0$ with $u(x,0) = f(x)$. Display the graph of $f(x)$.