PHYS 621 – Quantum Mechanics Study Notes

Richard Whitehill

October 9, 2023

Contents

CHAPTER 1

The Failure of Classical Physics

The following is a quick summary of some of the phenomena which classical mechanics and electromagnetic theory could not properly model and explain.

1.1 Black-body radiation

The primary quantity of interest here is the energy density of some box which is kept at temperature T, denoted as $u(\nu, T)$. The total energy that strikes an area A of the wall of the box in time t is

$$\int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi/2} \mathrm{d}\theta \sin\theta \int_0^{ct} r^2 \frac{A\cos\theta}{4\pi r^2} u(\nu, T)\,\mathrm{d}\nu = \frac{ctA}{4} u(\nu, T)\,\mathrm{d}\nu\,, \tag{1.1}$$

implying that we can write the total energy absorbed by the box as

$$E(\nu, T) = \frac{c}{4} f(\nu, T) u(\nu, T). \tag{1.2}$$

A body is called "black" if $f \equiv 1$, meaning that all light is perfectly absorbed.

1.1.1 Classical treatment

We can model the electromagnetic radiation by an infinite set of uncoupled harmonic oscillators. Solving Maxwell's equations in the box and using the equipartition theorem (which states that $U = \frac{1}{2}k_B T$ is the energy contribution from each quadratic degree of freedom in the Hamiltonian – of which there are two for a given harmonic oscillator), we recover the Rayleigh-Jeans law for the energy density of a blackbody:

$$\boxed{u(\nu, T) = 8\pi \frac{k_B T}{c^3} \nu^2}\,, \tag{1.3}$$

which clearly diverges to ∞ for more energetic light (which is where the term "ultraviolet catastrophe" originates).

1.1.2 Quantum treatment

The quantum nature comes from using Einstein's formula for the energy of a photon of light at frequency ν: $E = nh\nu$ ($n = 0, 1, 2, \ldots$). We can use the partition function to derive the fact that the average energy of a harmonic oscillator is

$$\langle E \rangle = \frac{h\nu}{e^{\beta h\nu} - 1}, \tag{1.4}$$

where $\beta = 1/k_B T$. Using this fact instead of the equipartition theorem, we find

$$\boxed{u(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\beta h\nu} - 1}}. \tag{1.5}$$

Notice that this resolves the ultraviolet catastrophe since the energy density now is bounded at all frequencies of light, peaking at some ν_0 (is there a well-known formula for this?).

1.2 Photo-electric effect