1) Prove that

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}). \tag{1}$$

In the following work, we will use Einstein's summation convention, where repeated indices are summed over implicitly unless otherwise stated (which for this problem will not happen). This will make the tedious accounting of sums and indices out front of the expressions happen automatically, making the work much cleaner and straightforward. Note that the indices here are roman letters, so the implicit sums will run over the set $\{1, 2, 3\}$.

Without further ado, we know that the ith component of a cross product can be written as

$$(\vec{A} \times \vec{B})_i = \epsilon_{ijk} A_j B_k, \tag{2}$$

and the dot product between two vectors is simply

$$\vec{A} \cdot \vec{B} = \delta_{ij} A_i B_j = A_i B_i. \tag{3}$$

Hence, we can write

$$[\vec{A} \times (\vec{B} \times \vec{C})]_i = \epsilon_{ijk} A_j (\vec{B} \times \vec{C})_k = \epsilon_{ijk} A_j \epsilon_{k\ell m} B_\ell C_m = \epsilon_{kij} \epsilon_{k\ell m} A_j B_\ell C_m, \tag{4}$$

noting that we can permute the indices of the Levi-Civita tensor without picking up any sign difference. We can then simplify Eq. (4) by making use of the identity $\epsilon_{kij} \epsilon_{k\ell m} = \delta_{i\ell}\delta_{jm} - \delta_{im}\delta_{j\ell}$, which gives us that

$$\begin{aligned}
[\vec{A} \times (\vec{B} \times \vec{C})]_i &= [\delta_{i\ell}\delta_{jm} - \delta_{im}\delta_{j\ell}] A_j B_\ell C_m \\
&= B_i A_j C_j - C_i A_j B_j = B_i(\vec{A} \cdot \vec{C}) - C_i(\vec{A} \cdot \vec{B}).
\end{aligned} \tag{5}$$

Thus, we can write the whole vector equivalence as

$$\boxed{\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})}, \tag{6}$$

which is what we were after.