1.2) Derive the cyclicity (1.24) of the trace from Eq. (1.23).

Using the definition of the trace, we can write

$$\mathrm{Tr}(AB) = \sum_{k=1}^{n} (AB)_{kk} = \sum_{k=1}^{n} \sum_{\ell=1}^{n} A_{k\ell} B_{\ell k}. \tag{1}$$

Note that this sum is symmetric in the indices $\{k, \ell\}$, and since $A_{k\ell}$ and $B_{\ell k}$ are just scalars

$$\boxed{\mathrm{Tr}(AB) = \sum_{\ell=1}^{n} \sum_{k=1}^{n} B_{\ell k} A_{k\ell} = \mathrm{Tr}(BA)}. \tag{2}$$

1.3) Show that $(AB)^{\mathrm{T}} = B^{\mathrm{T}} A^{\mathrm{T}}$, which is Eq. (1.26).

Recall the definition of a matrix transpose: $(A^{\mathrm{T}})_{ij} = A_{ji}$, so

$$(AB)^{\mathrm{T}}_{ij} = (AB)_{ji} = \sum_{k=1}^{n} A_{jk} B_{ki} = \sum_{k=1} B^{\mathrm{T}}_{ik} A^{\mathrm{T}}_{kj} = (B^{\mathrm{T}} A^{\mathrm{T}})_{ij}. \tag{3}$$

Hence,

$$\boxed{(AB)^{\mathrm{T}} = B^{\mathrm{T}} A^{\mathrm{T}}}. \tag{4}$$

1.5) Show that $(AB)^{\dagger} = B^{\dagger} A^{\dagger}$, which is Eq. (1.29).

Recall that the hermitian adjoint of a matrix is just $A^{\dagger} = (A^{\mathrm{T}})^{*}$, so

$$(AB)^{\dagger} = ((AB)^{\mathrm{T}})^{*} = (B^{\mathrm{T}} A^{\mathrm{T}})^{*}. \tag{5}$$

Observe that

$$(AB)^{*}_{ij} = \left(\sum_{k=1}^{n} A_{ik} B_{kj} \right)^{*} = \sum_{k=1}^{n} A^{*}_{ik} B^{*}_{kj} = (A^{*} B^{*})_{ij}, \tag{6}$$

so Eq. (5) becomes

$$\boxed{(AB)^{\dagger} = (B^{\mathrm{T}} A^{\mathrm{T}})^{*} = (B^{\mathrm{T}})^{*} (A^{\mathrm{T}})^{*} = B^{\dagger} A^{\dagger}}. \tag{7}$$

1.7) Show that the two 4×4 matrices (1.46) satisfy Grassman's algebra (1.11) for $n = 2$.

1.12) Show that the Minkowski product $(x, y) = \vec{\mathbf{x}} \cdot \vec{\mathbf{y}} - x^0 y^0$ of two 4-vectors x and y is an inner product obeying the rules (1.78,1.79,1.84).

1.18) Use the Gram-Schmidt method to find orthonormal linear combinations of the three vectors

$$\vec{\mathbf{s}}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{\mathbf{s}}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{\mathbf{s}}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{8}$$

1.21) Show that a linear operator A that is represented by a hermitian matrix (1.167) in an orthonormal basis satisfies $(g, Af) = (Ag, f)$.