6.5) Do the integral

$$\oint_C \frac{\mathrm{d}z}{z^2-1} \tag{1}$$

in which the contour C is counterclockwise about the circle $|z| = 2$

6.9) Use Cauchy's integral formula

$$f^{(n)}(z) = \frac{n!}{2\pi i}\oint \mathrm{d}z' \, \frac{f(z')}{(z'-z)^{n+1}} \tag{2}$$

and Rodrigues's expression

$$P_n(x) = \frac{1}{2^n n!}\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^n (x^2-1)^n \tag{3}$$

for Legendre's polynomial $P_n(x)$ to derive Schlaefli's formula

$$P_n(z) = \frac{1}{2^n 2\pi i}\oint \frac{(z'^2-1)^n}{(z'-z)^{n+1}} \, \mathrm{d}z' \, . \tag{4}$$

6.20) Use a contour integral to evaluate the integral

$$I_a = \int_0^\pi \frac{\mathrm{d}\theta}{a+\cos\theta}, \quad a > 1. \tag{5}$$

6.26) Show that

$$\int_0^\infty \cos ax e^{-x^2} \, \mathrm{d}x = \frac{1}{2}\sqrt{\pi}e^{-a^2/4}. \tag{6}$$

6.33) The Bessel function J_n is given by the integral

$$J_n(x) = \frac{1}{2\pi i}\oint_C e^{(x/2)(z-1/z)} \frac{\mathrm{d}z}{z^{n+1}} \tag{7}$$

along a counterclockwise about the origin. Find the generating function for these Bessel functions, that is, the function $G(x, z)$ whose Laurent series has the $J_n(x)$'s as coefficients

$$G(x,z) = \sum_{n=-\infty}^{\infty} J_n(x) z^n. \tag{8}$$

6.34) Show that the Heaviside function $\theta(y) = (y + |y|)/(2|y|)$ is given by the integral

$$\theta(y) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} e^{iyx} \frac{\mathrm{d}x}{x - i\epsilon} \tag{9}$$

in which ϵ is an infinitesimal positive number.