Slide 1:

* Introduction

Slide 2:

* Rutherford did the famous gold foil experiments where we learned that the atom is not a delicious plum pudding but rather mostly empty
* DIS experiments gave us verification of the proton substructure 🡪 gave rise to a whole field of nuclear physics
* Higgs discovery gave us a large missing piece of the standard model
* Lots of activity devoted to scattering theory and experiments including large facilities such as LHC, RHIC, EIC

Slide 3:

* Our picture of scattering: send in particle beam, particles interact with target via potential and are deflected
* The probability for scattering into solid angle from small circular cross section is the differential cross section
* Classical picture: concrete objects
* Quantum: wave mechanics
  + Need to solve Schrodinger equation to extract information about states

Slide 4:

* Consider potentials have effectively finite range (go to zero far from center)
* Basic setup: scattering experiments prepare particles not affected by potential and measure outgoing particles free from potential
* Write wavefunction outside potential as superposition of incoming and outgoing state
  + Normalization: factor out front and the 1/r for the spherical wave
  + f(theta,phi) modulates the scattered amplitude in different directions
  + Same wavenumbers for complex exponentials: no recoil so incoming particle is essentially just deflected (elastic scattering)

Slide 5:

* Calculate the differential cross section by equating the flux into d(sigma) and the flux into d^3(r)
* Definition of flux (comes from differentiating the probability density)
* Rearranging factors we find a simple relation between the cross section and scattering amplitude
* Now we just need to know how to calculate the amplitude

Slide 6:

* Rearrange Schrodinger equation to put potential on right side
* Multiply by inverse of left operator on right to get equation for psi in terms of psi and solution to homogeneous equation phi (with energy E)

Slide 7:

* The previous slide was quite abstract – what do the states and operators actually represent
* Insert position states and obtain simple position space integral representation of Lippman-Schwinger equation
* Now need to determine function G

Slide 8:

* Step 1: insert complete set of momentum states
  + Motivated by fact that H\_0 is free particle Hamiltonian so can use fact that f(H\_0) phi = f(E) \phi
* Use relation between momentum and wavenumber to simplify, recall form of momentum eigenfunctions in position space, eliminate one integral using normalization

Slide 9:

* Step 2: do the integration in momentum space (spherical coordinates)
  + Note that we extended our integration range from half the real line to the full real line because the function is even (difference of complex exponential is sin)
* Now we have two integrals over the real line to do with poles

Slide 10:

* Step 3: this looks like a job for the Cauchy integral formula
* We have to choose form of pole shift in order to choose proper integration contour
* Choice also influences whether we are looking at outgoing or incoming spherical waves
  + We care about outgoing so with hindsight we choose the “+” shift

Slide 11:

* First integral
  + Contour choice: complex exponential factor goes to zero
  + Use the Cauchy integral formula to obtain result

Slide 12:

* Second integral basically the same, just different contour and different pole but same result as first integral with minus sign

Slide 13:

* We now have the function G
  + You may recognize that this is the Green’s function for the Helmholtz differential equation
  + Can rewrite the Schrodinger equation to look like the Helmholtz equation with dependence on psi on the right side

Slide 14:

* Now we have integral form of Schrodinger equation
* Still not incredibly useful unless we already know the form of the wavefunction (which is preposterous since this is what we want to calculate)

Slide 15:

* First set of approximations: arguments based on relevant distance from scattering region
  + Then find form similar to our original wavefunction far from scattering center but scattering amplitude still depends on psi
* Born approximation: the incoming particle has sufficiently high energy and/or interaction potential is sufficiently weak so that psi (on the right side) is approximately phi
  + This is basically an argument of successive approximations
* This gets us somewhere: we now have the scattering amplitude as essentially the Fourier transform of the potential in transferred momentum space
  + This is kind of an interesting fact to keep in mind when looking at the relativistic version of the theory

Slide 16:

* q is the momentum tranfer
  + notice that incoming and outgoing momenta have the same magnitudes so q is really just telling us by how much the particle is deflected
  + q has magnitude of ell modulated by some angular factor

Slide 17:

* Let’s use this to look at the scattering of two charged objects (Rutherford scattering)
* This potential is spherically symmetric
  + So scattering amplitude does not depend on polar angle
  + Align integration axis along y and do angular integrations
* Introduce coulomb potential explicitly
  + We have a problem now: this integral does not converge

Slide 18:

* But we can play a clever mathematical trick to do the integral
  + Introduce decaying exponential factor
  + This is essentially making our potential of the Yukawa form which is interesting in its own right
  + Take the limit of the exponential parameter to zero in our final answer to obtain scattering amplitude for coulomb potential
* Take norm squared and retrieve sin^-4 dependence on azimuthal angle
  + What does this mean?
  + Probability of scattering suppressed quickly at angles away from beam axis
  + Plum pudding model qualitatively expects slow fall off near central angle with steep fall off for backscattering
    - Here the fall off is steep everywhere (monotonically decreasing)
  + Interpretation: most particles will pass through undeflected while a few will dramatically scatter backward
    - Quote from Rutherford: “It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.”
  + New model of the atom born with very small positive nucleus
* Also note that this result is found in classical and relativistic quantum scattering
  + As Griffiths says: this is a robust result

Slide 19:

* Back to the Born approximations
* Continue method of successive approximations to obtain a series representation of psi
  + Interpretation in feynman diagrams: propagators weighting propagation of particle from between interactions and vertex factors representing those interactions
* Convergence of series: sufficient to have small multiplicative factor out front that damps/suppresses graphs with vertex factors
  + Can go through motions of showing that Coulomb potential depends on small parameter (fine-structure constant)
  + Scattering amplitude related to overlap between phi and psi with operator series sandwiched in between (not proven here but a common result)
* Looks an awful lot like modern calculations and it is – has many of the same features as in the relativistic theories except that governing equations are different
  + This is a perturbative series

Slide 20:

* Final remarks: read off slide