

# Accessing Gluon Polarization in SIDIS

Richard Whitehill



# Roadmap

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- Introduction
- Background
- Motivation
- Theoretical Framework
- Numerical Simulations
- Outlook

# Introduction

About me:

- Senior at WSU
- Summer 2021: REU at ODU (partnered with JLab)
- Summer 2022: SULI program at JLab

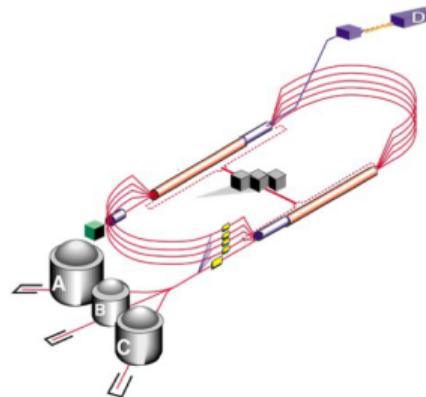


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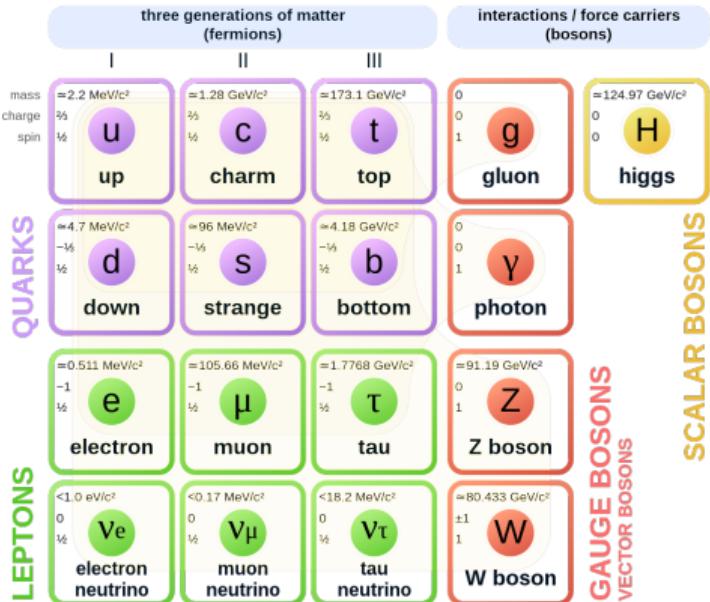
About Jefferson Lab (JLab):

- Department of Energy National Lab
- Location: Newport News, VA
- Mission: Understand nuclear matter
  - CEBAF + 4 experimental halls
  - Theory & Computing



# Standard Model of Physics

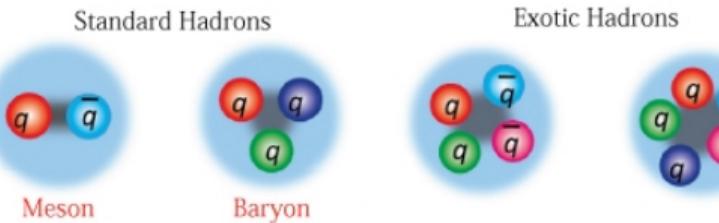
## Standard Model of Elementary Particles



Modern theory of particle physics:

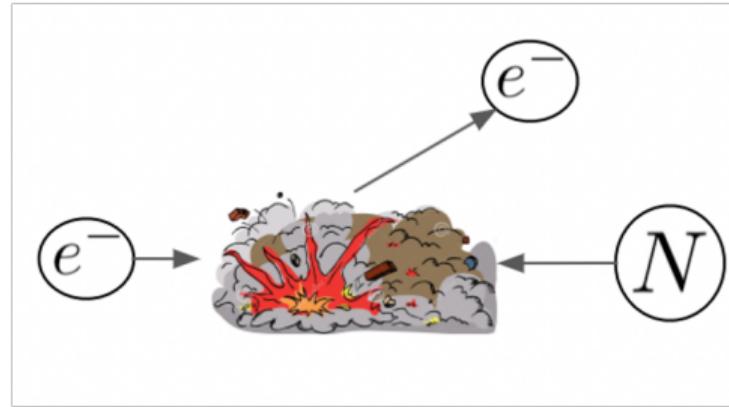
- Describes fundamental particles and their interactions

JLab focuses on nuclear and hadronic structure of matter



# Scattering

1. Cause “Collision” between particles
2. Detect/measure what comes out
3. Infer what happened during the scattering process



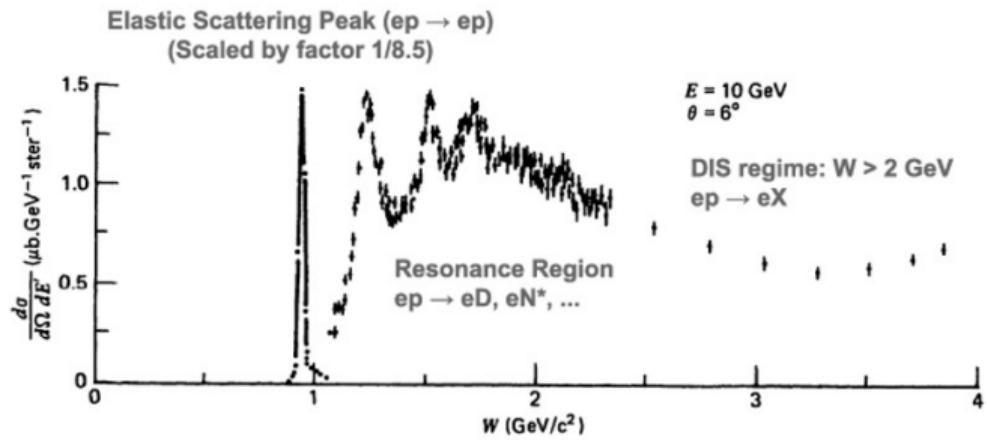
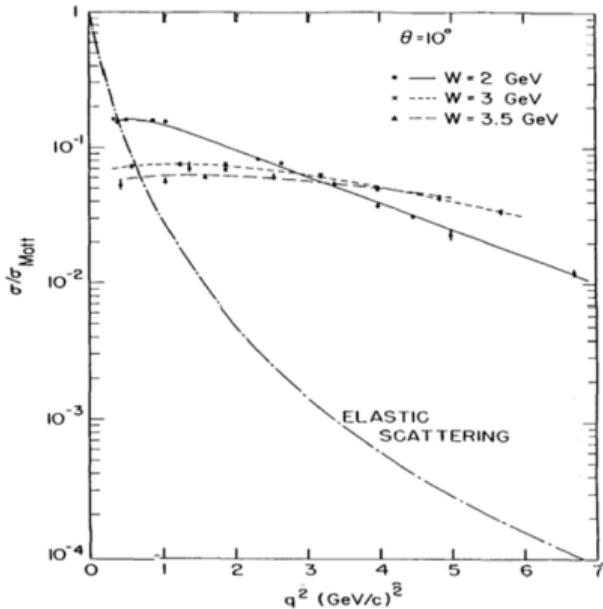
$$\frac{dN}{d\Omega} = \mathcal{L} \frac{d\sigma}{d\Omega}$$

Experimentally Measurable →

Theoretically Calculable ←

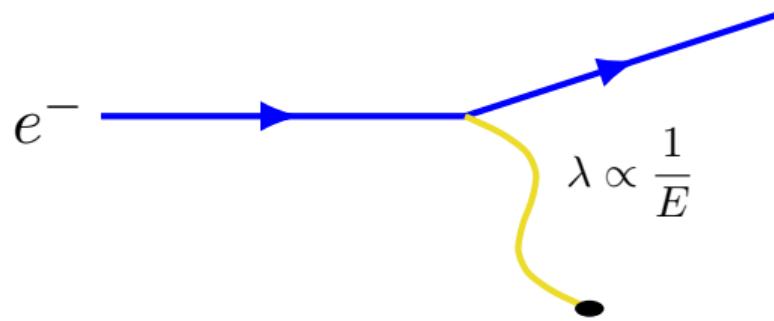
# What has scattering taught us?

A lot! The proton is a composite particle not fundamental



# Probing the proton substructure

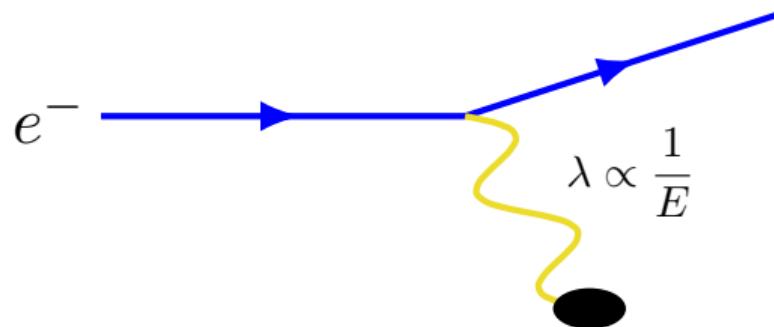
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Fundamental fermion:  $s = \frac{1}{2}$

# Probing the proton substructure

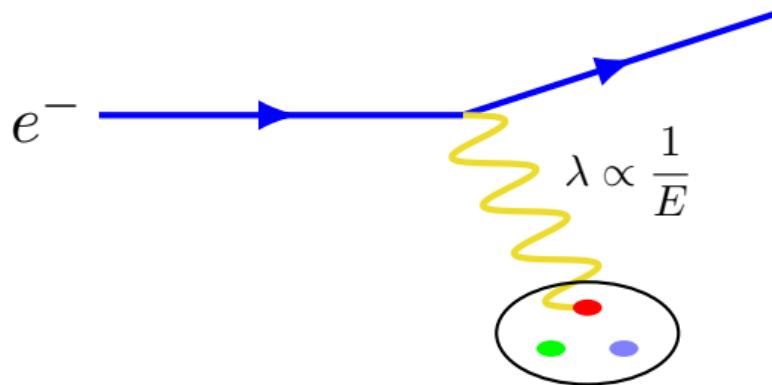
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Fundamental fermion:  $s = \frac{1}{2}$

# Probing the proton substructure

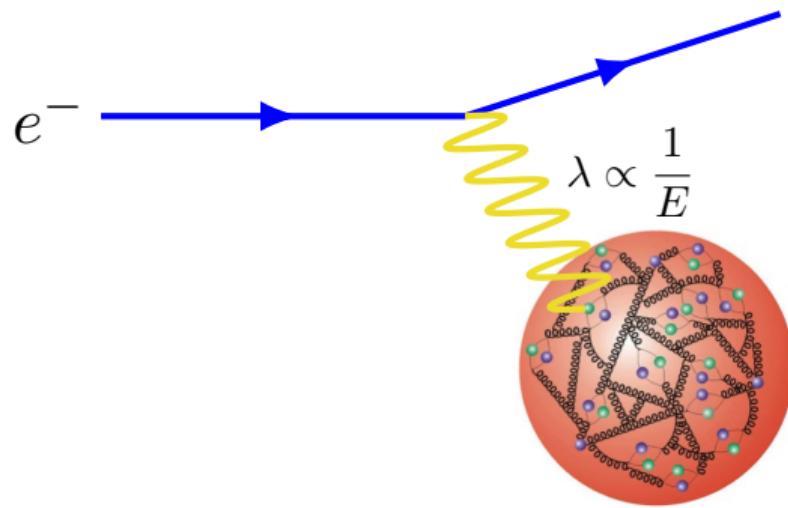
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$$\begin{aligned} |p_+\rangle = \frac{1}{\sqrt{18}} & \left[ 2|u_+d_-u_+\rangle + 2|u_+u_+d_-\rangle + 2|d_-u_+u_+\rangle \right. \\ & - |u_+u_-d_+\rangle - |u_-u_+d_+\rangle - |u_+d_+u_-\rangle \quad \Rightarrow \quad \langle p_+|\hat{S}|p_+\rangle = \frac{1}{2} \\ & \left. - |u_-d_+u_+\rangle - |d_+u_+u_-\rangle - |d_+u_-u_+\rangle \right] \end{aligned}$$

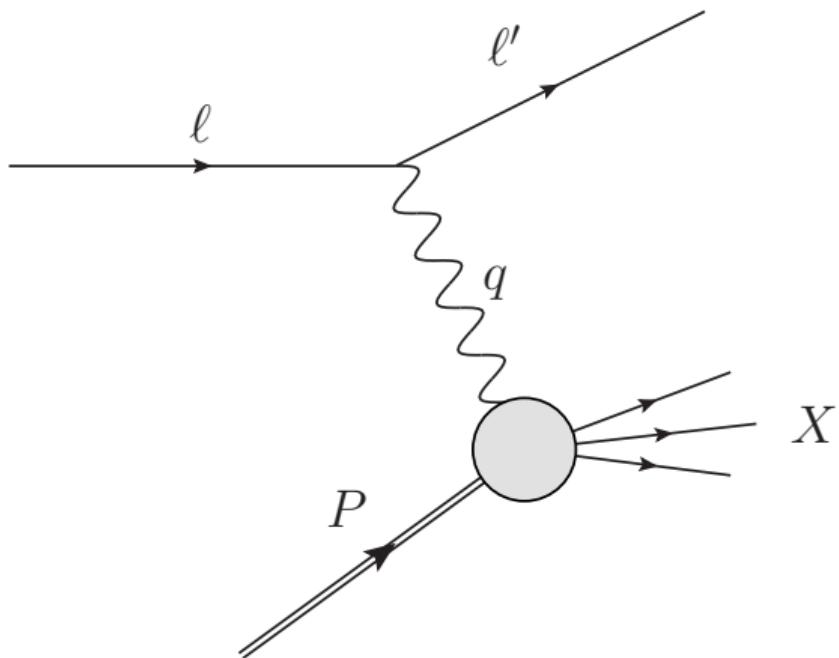
# Probing the proton substructure

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$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L$$

# Deep Inelastic Scattering (DIS)



DIS:  $\ell P \rightarrow \ell X$

$$\frac{d\sigma}{dx dQ^2} \propto L_{\mu\nu} W^{\mu\nu}$$

$$Q^2 = -q^2 \quad x = \frac{Q^2}{2P \cdot q}$$

# Leptonic Tensor

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Exactly calculable in QED

$$L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu} \ell \cdot \ell' + i\epsilon_{\mu\nu\alpha\beta} s^\alpha q^\beta)$$

Assuming longitudinal polarization (i.e.  $s^\mu = \lambda_\ell \ell^\mu$ )

$$L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu} \ell \cdot \ell' - i\lambda_\ell \epsilon_{\mu\nu\alpha\beta} \ell^\alpha \ell'^\beta)$$

# Hadronic Tensor

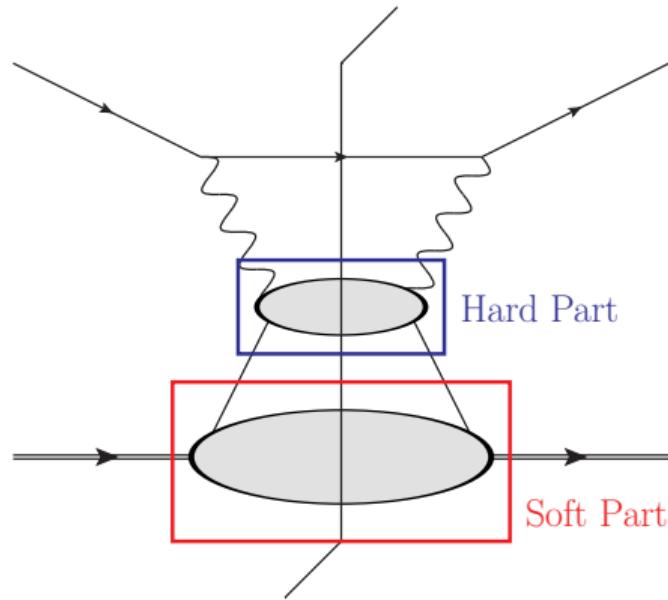
Not exactly calculable

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{(2\pi)^4} \sum_X \int d^4z e^{iq \cdot z} \langle P, S | J^\mu(z) | X \rangle \langle X | J^\nu(0) | P, S \rangle \\ &= \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x, Q^2) + \left( P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) \left( P^\nu - \frac{P \cdot q}{q^2} q^\nu \right) F_2(x, Q^2) \\ &\quad + \frac{i}{P \cdot q} \epsilon^{\mu\nu\alpha\beta} q_\alpha \left[ S_\beta g_1(x, Q^2) + (S_\beta - \frac{S \cdot q}{P \cdot q} P_\beta) g_2(x, Q^2) \right] \end{aligned}$$

- Current conservation ( $q_\mu W^{\mu\nu} = 0$ ), covariant expression
- Symmetric and antisymmetric pieces (parity/spin dependence)
- Structure functions  $F_{1,2}$  and  $g_{1,2}$  parameterize “ignorance”

# Collinear factorization

$$L_{\mu\nu} W^{\mu\nu} = L_{\mu\nu} \left[ \int_x^1 \frac{d\xi}{\xi} \widehat{W}^{\mu\nu} f_{i/P}(\xi) + \mathcal{O}\left(\frac{m}{Q}\right) \right]$$



## Naïve parton model

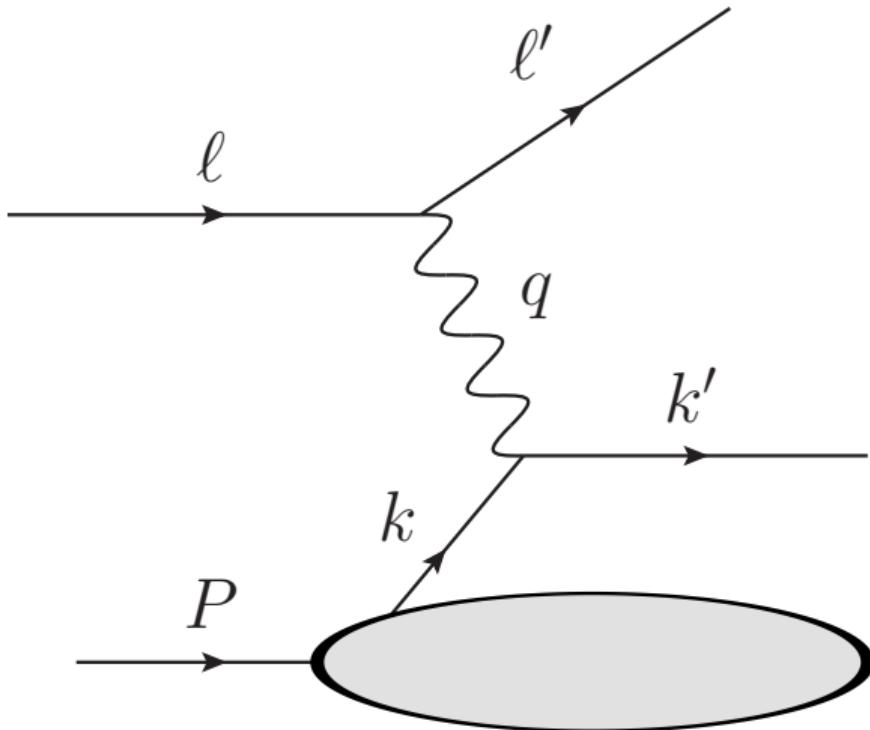
Leading order (LO):  $\gamma + q \rightarrow q$

$$F_1 = \frac{1}{2} \sum_q e_q^2 f_q(x, Q^2)$$

$$F_2 = 2x F_1$$

$$g_1 = \frac{1}{2} \sum_q e_q^2 \Delta f_q(x, Q^2)$$

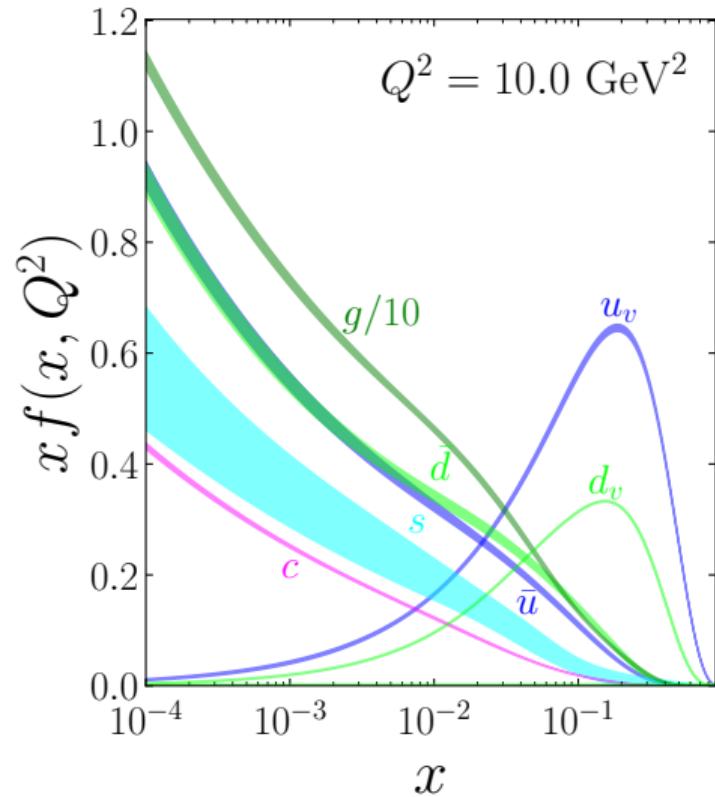
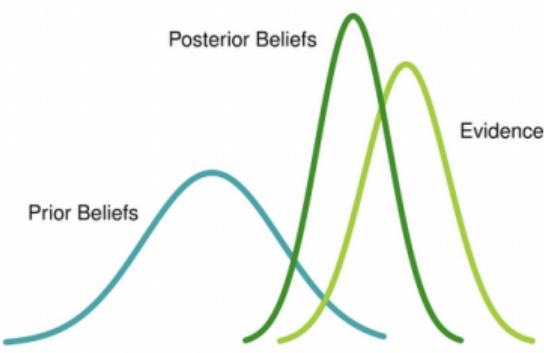
$$g_2 = 0$$



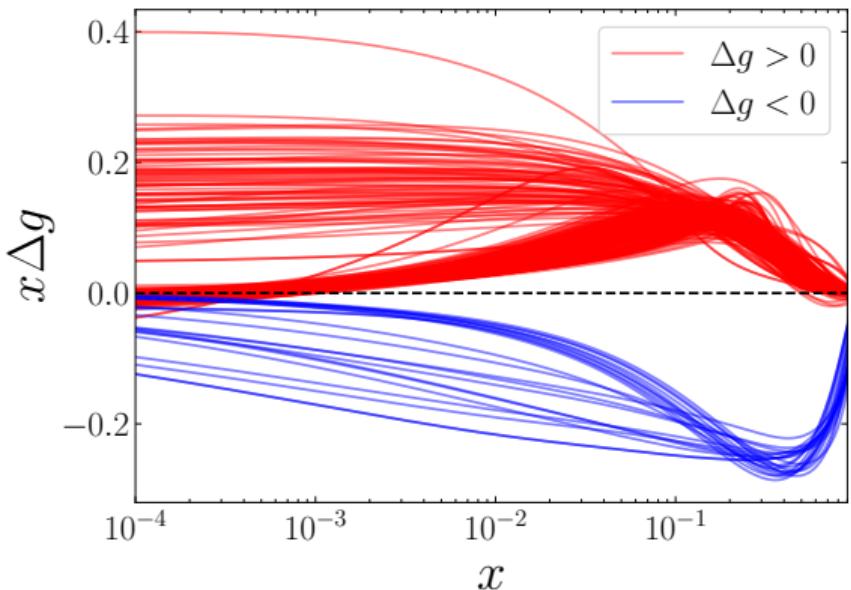
# Parton distribution functions

JLab Angular Momentum Collaboration (JAM):

- Global QCD analysis – extract PDFs from experimental data
- Parameterization:  $f(x) = x^\alpha(1-x)^\beta\eta(x)$
- Blend of bayesian statistics and monte carlo methods:  $\rho(\vec{a}|\text{data}) = \mathcal{L}(\vec{a}, \text{data})\pi(\vec{a})$



# Tension in $\Delta g$ fit



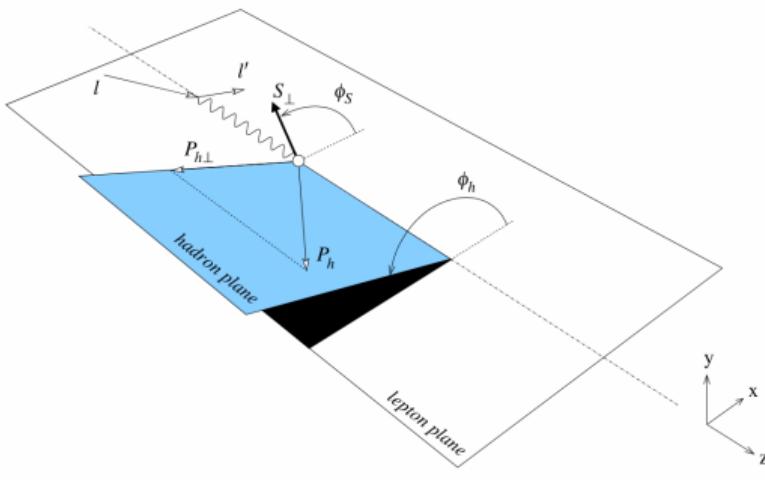
How: Current data sets allow this

Why is this a problem?

- Proton Spin Crisis:  $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$
- $\Delta G = \int_{x_{\min}}^1 \Delta g(x, Q^2) dx$

Goal: Find new data to distinguish  $\Delta g > 0$  and  $\Delta g < 0$

# Semi-inclusive DIS (SIDIS)



SIDIS:  $\ell P \rightarrow \ell' H X$

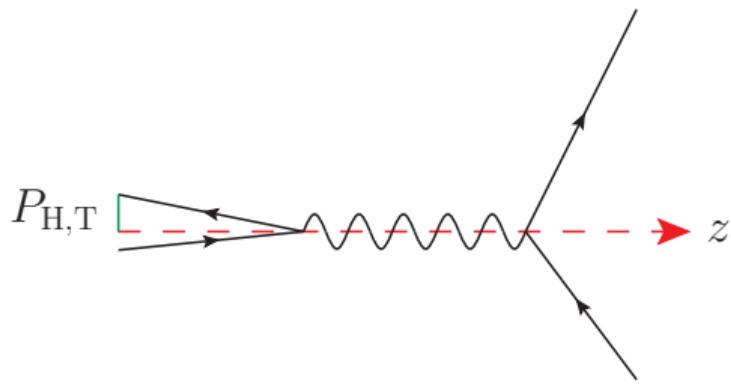
$$\frac{d\sigma}{dx dQ^2 dz dP_{H,T}^2} = \frac{\pi^2 \alpha^2 y}{2z Q^4} L_{\mu\nu} W^{\mu\nu}$$

$$z = \frac{P \cdot P_H}{P \cdot q} \quad q_T = \frac{P_{H,T}}{z}$$

# Transverse momentum

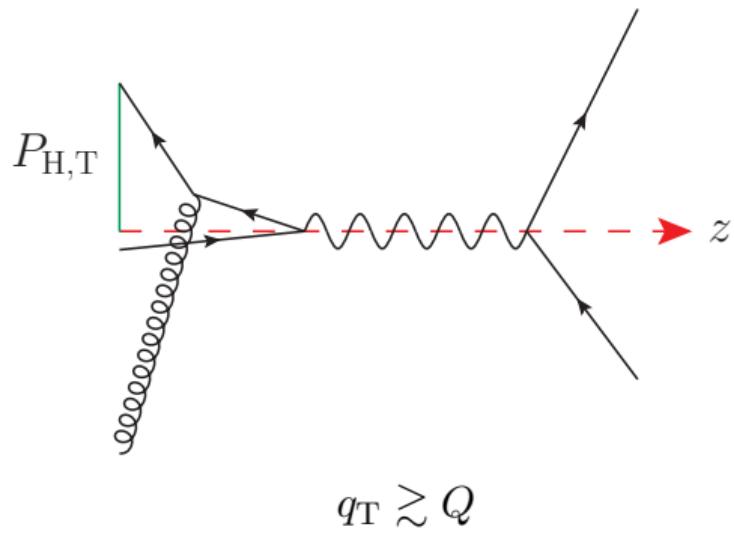
Small intrinsic transverse momentum

- TMD description more appropriate



Large transverse momentum

- Hard gluon radiations

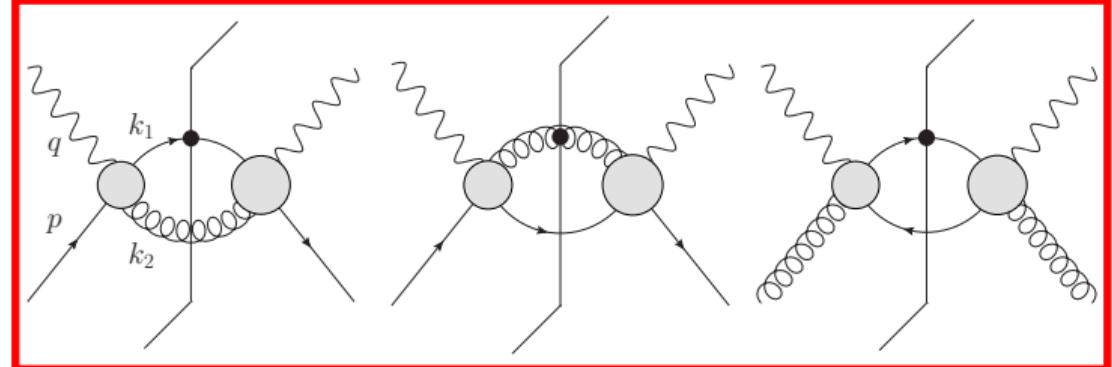


# Collinear factorization and hard scattering amplitudes

$$W^{\mu\nu} = \int_x^1 \frac{d\xi}{\xi} \int_z^1 \frac{d\zeta}{\zeta^2} \widehat{W}^{\mu\nu} f_{i/P}(\xi) D_{H/j}(\zeta)$$

$\mathcal{JAM}$

$$\widehat{W}^{\mu\nu} = \sum_{i,j} \mathcal{M}^\mu \mathcal{M}^{\dagger\nu}$$



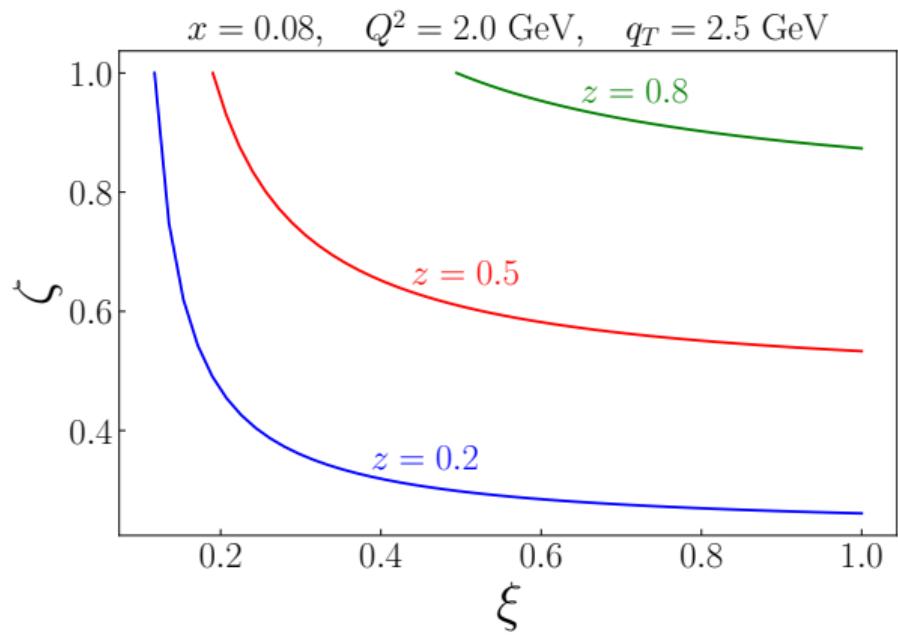
# Momentum Fractions

Conservation of momentum:  $q + p = k_1 + k_2$

$$p = \xi P \quad k_{1,T} = P_{H,T}/\zeta$$

$$\zeta = z \left[ 1 + \frac{x}{\xi - x} \frac{q_T^2}{Q^2} \right]$$

$$\xi_{\min} = x \left[ 1 + \frac{z}{1 - z} \frac{q_T^2}{Q^2} \right]$$



# Double Spin Asymmetry

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Definition:

$$A_{LL} = \frac{d\Delta\sigma}{d\sigma}$$

Uncertainties:

1. Statistical:  $\delta A_{LL} = \sqrt{\frac{1 + A_{LL}^2}{N}} \approx \frac{1}{\sqrt{N}}$  when  $A_{LL} \ll 1$

→ Experiments are Poisson processes:  $\delta N = \sqrt{N} = \sqrt{\mathcal{L}\sigma}$

→  $\sigma = \int_{\text{bin}} dx dQ^2 dz dP_{H,T}^2 \frac{d\sigma}{dx dQ^2 dz dP_{H,T}^2} \approx \Delta x \Delta Q^2 dz \Delta P_{H,T}^2 \frac{d\sigma(\text{center})}{dx dQ^2 dz dP_{H,T}^2}$

2. PDF replicas: one standard deviation ( $1\sigma$ ) of values across replicas at a kinematic point

# Kinematic Definitions

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Center of mass energy squared:  $s = (\ell + P)^2$

- JLab: beam energy  $E$  ( $s = 2ME + M^2$ )  
→ Available energies: 12 GeV ( $\sqrt{s} = 4.8$  GeV) and 22 GeV ( $\sqrt{s} = 6.5$  GeV)
- EIC: projected  $\sqrt{s} = 140$  GeV

Inelasticity variable:  $y = \frac{P \cdot q}{P \cdot \ell}$

- Useful relation:  $Q^2 = (s - M^2)xy$

Squared final state masses:

- $W^2 = (P + q)^2$
- $W_{\text{SIDIS}}^2 = (P + q - P_H)^2$

# Phase Space Constraints

$$Q_{\max}^2 = (s - M^2)x$$

$$Q_{\min}^2 = \max\{Q_1^2, Q_2^2, Q_3^2\}:$$

1.  $Q_1^2 = m_c^2 = (1.28 \text{ GeV})^2$

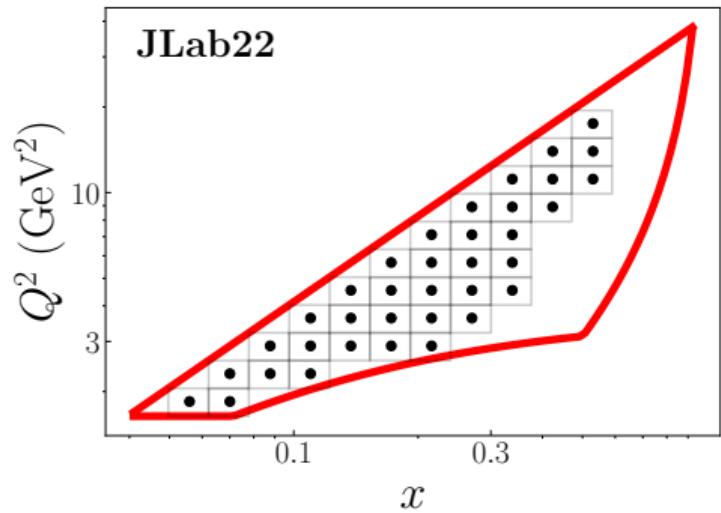
2.  $W^2 > W_{\min}^2: Q_2^2 = \frac{x}{1-x}(W_{\min}^2 - M^2)$

3.  $\theta > \theta_{\min}: Q_3^2 = \frac{4E^2 \sin^2(\theta_{\min}/2)}{1 + (2E/Mx) \sin^2(\theta_{\min}/2)}$

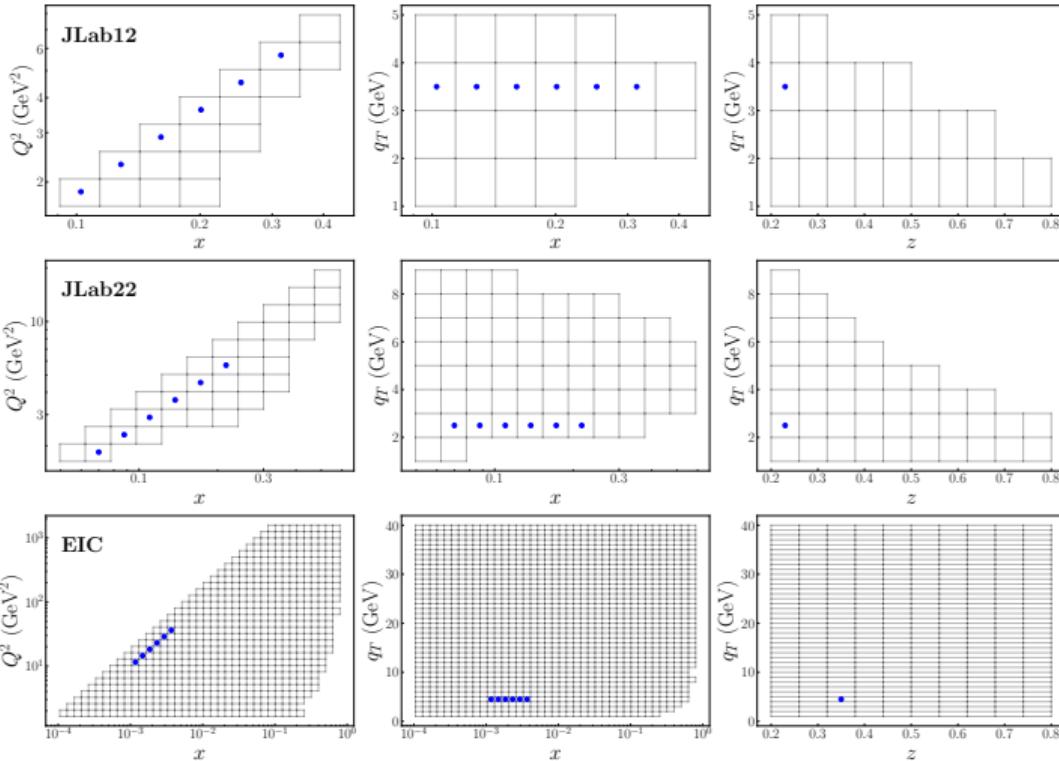
$$x_{\min} = \frac{m_c^2}{s - M^2} \quad x_{\max} = \frac{s - W^2}{s - M^2}$$

$$0.2 < z < 0.8$$

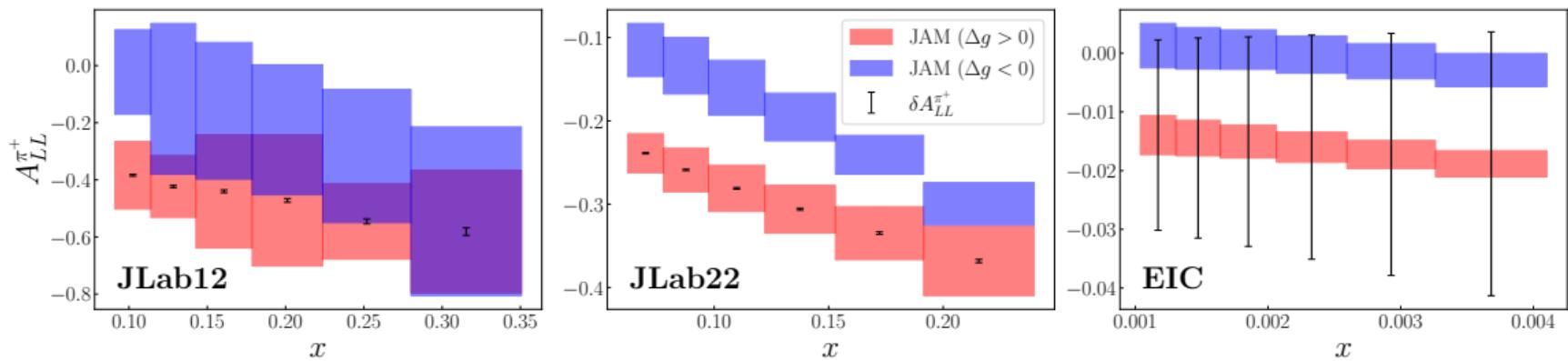
$$q_{T,\min} = Q, \quad q_{T,\max}: W_{\text{SIDIS}}^2 > M^2$$



# Results



# Results



$$\mathcal{L}_{\text{JLab}} = 86 \text{ fb}^{-1}, \quad \mathcal{L}_{\text{EIC}} = 10 \text{ fb}^{-1}$$

## Outlook

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- Significant sensitivity to the gluon channel
- JLab with a 22 GeV beam is well positioned to discriminate  $\Delta g$
- Strong scaling behavior with  $\sqrt{s}$  may make further constraint of  $\Delta g$  difficult at small- $x$  from this process
- Continue search for observable/process that is better positioned for constraint of  $\Delta g$  from EIC data in future global QCD analyses

# Backup Slides

# Spinor and polarization vector products

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$$u(p)\bar{u}(p) = \frac{1}{2}(1 + \lambda\gamma_5)\not{p}$$
$$\epsilon^\mu(p)\epsilon^{*\nu}(p) = \frac{1}{2}\left(-g^{\mu\nu} + \frac{i\lambda}{p \cdot X}\epsilon^{\mu\nu\alpha\beta}p_\alpha X_\beta\right)$$

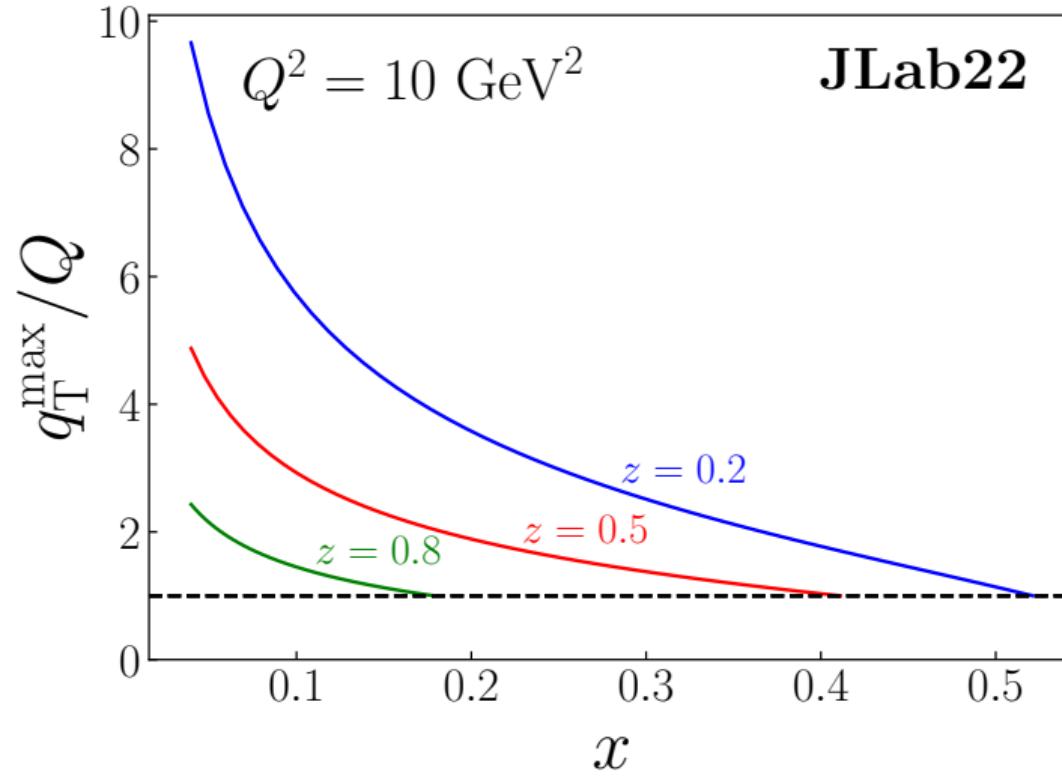
## Hidden Expressions – Kinematics

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$$W^2 = M^2 - \left(1 - \frac{1}{x}\right) Q^2$$

$$\begin{aligned} W_{\text{SIDIS}}^2 &= M^2 + M_h^2 + Q^2 \frac{1-x-z}{x} \\ &\quad + \frac{Q^4 z}{2M^2 x^2} \left( \sqrt{1 + \frac{4M^2 x^2}{Q^2}} \sqrt{1 - \frac{4M^2 x^2 (M_h^2 + q_T^2 z^2)}{Q^4 z^2}} - 1 \right) \end{aligned}$$

# Maximum $q_T$



# Hidden Expressions – Unpolarized Amplitudes

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$$\begin{aligned}\frac{d\mathcal{H}_{qq}^U}{d\hat{x} dy d\hat{z} dP_{H,T}^2} &= \frac{64\pi\alpha_s^2}{3\hat{x}(1-\hat{x})\textcolor{red}{y^2}\mathcal{Q}_1^2} \left[ (1 + \hat{x}^2 \hat{z}^2)(1 + \bar{y}^2) Q^4 + 8\hat{x}^2 \hat{z}^2 \bar{y} Q^2 q_T^2 \right. \\ &\quad \left. + \hat{x}^2 \hat{z}^2 (1 + \bar{y}^2) q_T^4 \right] \\ \frac{d\mathcal{H}_{qg}^U}{d\hat{x} dy d\hat{z} dP_{H,T}^2} &= \frac{64\pi\alpha_s^2}{3(1-\hat{x})\textcolor{red}{y^2}\mathcal{Q}_2^2} \left[ \left( (2 + \hat{x}^2 \hat{z}^2)(1 + \bar{y}^2) - 4\hat{x}\hat{z}\bar{y} - 2\hat{x}y^2(1 - \hat{x}(1 - \hat{z})) \right) Q^4 \right. \\ &\quad \left. + 2\hat{x}\hat{z}(4\hat{x}\hat{z}\bar{y} + \hat{x}y^2 - 1 - \bar{y}^2) Q^2 q_T^2 + \hat{x}^2 \hat{z}^2 (1 + \bar{y}^2) q_T^4 \right] \\ \frac{d\mathcal{H}_{qg}^U}{d\hat{x} dy d\hat{z} dP_{H,T}^2} &= \frac{64\pi\alpha_s^2}{3(1-\hat{x})\textcolor{red}{y^2}\mathcal{Q}_2^2} \left[ \left( (2 + \hat{x}^2 \hat{z}^2)(1 + \bar{y}^2) - 4\hat{x}\hat{z}\bar{y} - 2\hat{x}y^2(1 - \hat{x}(1 - \hat{z})) \right) Q^4 \right. \\ &\quad \left. + 2\hat{x}\hat{z}(4\hat{x}\hat{z}\bar{y} + \hat{x}y^2 - 1 - \bar{y}^2) Q^2 q_T^2 + \hat{x}^2 \hat{z}^2 (1 + \bar{y}^2) q_T^4 \right]\end{aligned}$$

# Hidden Expressions – Polarized Amplitudes

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$$\begin{aligned}\frac{d\mathcal{H}_{qq}^P}{d\hat{x} dy d\hat{z} dP_{H,T}^2} &= -\frac{64\pi\alpha_s^2 (2-y)}{3\hat{x}(1-\hat{x})y\mathcal{Q}_1^2} \left[ (1 + \hat{x}^2 \hat{z}^2) Q^4 - \hat{x}^2 \hat{z}^2 q_T^4 \right] \\ \frac{d\mathcal{H}_{qg}^P}{d\hat{x} dy d\hat{z} dP_{H,T}^2} &= -\frac{64\pi\alpha_s^2 \hat{x}(2-y)}{3(1-\hat{x})y\mathcal{Q}_2^2} \left[ (2 + \hat{x} \hat{z}^2 - 2\hat{x} \hat{z}) Q^4 + 2\hat{z}(1-\hat{x}) Q^2 q_T^2 - \hat{x} \hat{z}^2 q_T^4 \right] \\ \frac{d\mathcal{H}_{gq}^P}{d\hat{x} dy d\hat{z} dP_{H,T}^2} &= \frac{8\pi\alpha_s^2 (2-y) Q^2}{\hat{x}y\mathcal{Q}_1^2\mathcal{Q}_2^2} \left[ (2\hat{x}^2 \hat{z}^2 - 2\hat{x}^2 \hat{z} + 2\hat{x} - 1) Q^4 \right. \\ &\quad \left. + 2\hat{x} \hat{z}(1-\hat{x}) Q^2 q_T^2 - 2\hat{x}^2 \hat{z}^2 q_T^4 \right]\end{aligned}$$

# Scaling with $\sqrt{s}$

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