

# Impact of PVDIS with 22 GeV electrons at JLab on the nucleon strangeness

Richard Whitehill

Collaborators: M. M. Dalton, T. Liu, W. Melnitchouk, J. Qiu, and N. Sato



# JAM Collaboration

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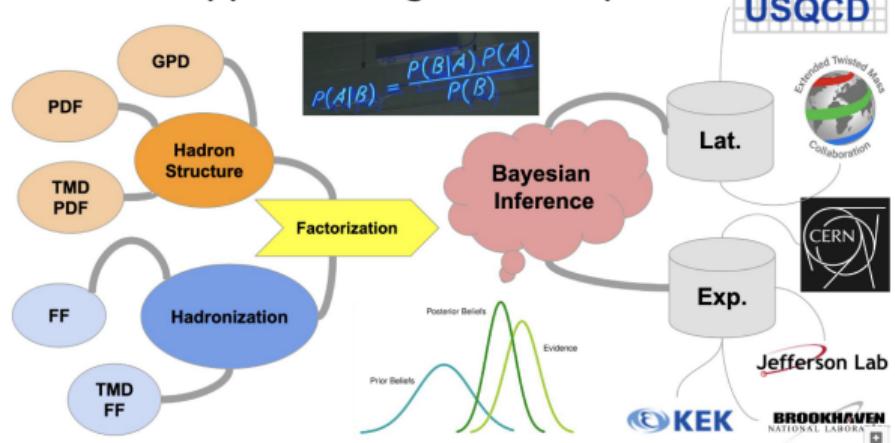
The Jefferson Lab Angular Momentum (JAM) Collaboration is an enterprise involving theorists, experimentalists, and computer scientists from the Jefferson Lab community using QCD to study the internal quark and gluon structure of hadrons and nuclei. Experimental data from high-energy scattering processes are analyzed using modern Monte Carlo techniques and state-of-the-art uncertainty quantification to simultaneously extract various quantum correlation functions, such as parton distribution functions (PDFs), fragmentation functions (FFs), transverse momentum dependent (TMD) distributions, and generalized parton distributions (GPDs). Inclusion of lattice QCD data and machine learning algorithms are being explored to potentially expand the reach and efficacy of JAM analyses and our understanding of hadron structure in QCD.

**TL;DR:** Group interested in understanding partonic structure of nucleons/hadrons through determination of relevant QCFs via global QCD analysis

# JAM Global Analysis Paradigm

Factorization:  $\mathcal{O} = \mathcal{H} \otimes \text{QCFs}$

## A holistic approach to global analysis



$$f(x) = \mathcal{N}x^\alpha(1-x)^\beta(1 + \gamma\sqrt{x} + \delta x)$$

# JAM Global Analysis Paradigm

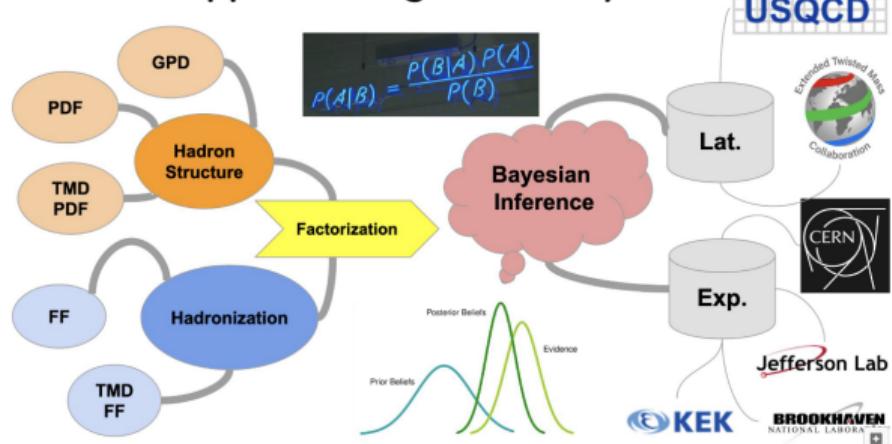
Factorization:  $\mathcal{O} = \mathcal{H} \otimes \text{QCFs}$

Bayes' theorem:

$$\rightarrow \mathcal{P}(\mathbf{a}|\text{data}) \sim \mathcal{L}(\mathbf{a}, \text{data})\pi(\mathbf{a})$$

$$\rightarrow \mathcal{L}(\mathbf{a}, \text{data}) = \exp(-\chi^2(\mathbf{a}, \text{data})/2)$$

## A holistic approach to global analysis



$$\chi^2(\mathbf{a}, \text{data}) = \sum_{e,i} \left( \frac{d_{e,i} - \sum_k r_{e,k} \beta_{e,i}^k - T_{e,i}(\mathbf{a})/N_e}{\alpha_{e,i}} \right)^2 + \sum_{e,k} r_{e,k}^2 + \sum_e \left( \frac{1 - N_e}{\delta N_e} \right)^2$$

# JAM Global Analysis Paradigm

Factorization:  $\mathcal{O} = \mathcal{H} \otimes \text{QCFs}$

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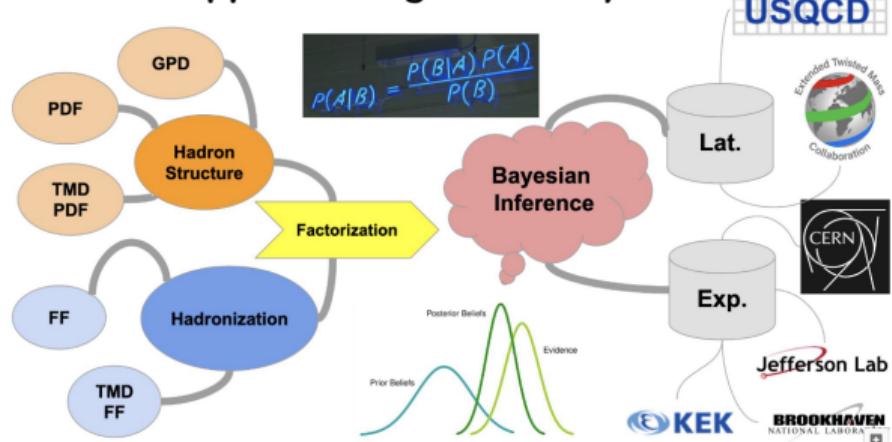
$$\rightarrow \mathcal{P}(\mathbf{a}|\text{data}) \sim \mathcal{L}(\mathbf{a}, \text{data})\pi(\mathbf{a})$$

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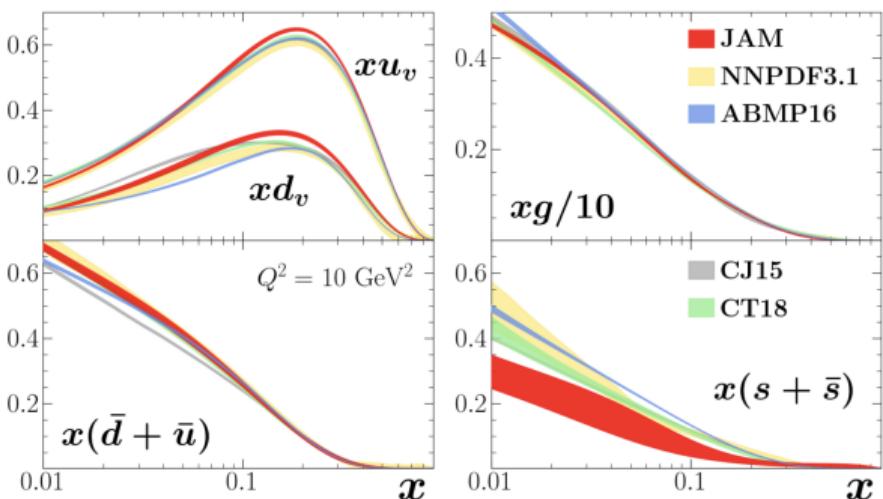
Monte Carlo Replicas:  $\tilde{\mathbf{d}} \sim \mathcal{N}(\mathbf{d}, \Sigma)$

$$\text{E}[\mathcal{O}] = \frac{1}{N_{\text{reps}}} \sum_{i=1}^{N_{\text{reps}}} \mathcal{O}(\mathbf{a}_i) \quad \text{V}[\mathcal{O}] = \frac{1}{N_{\text{reps}}} \sum_{i=1}^{N_{\text{reps}}} \left( \mathcal{O}(\mathbf{a}_i) - \text{E}[\mathcal{O}] \right)^2$$

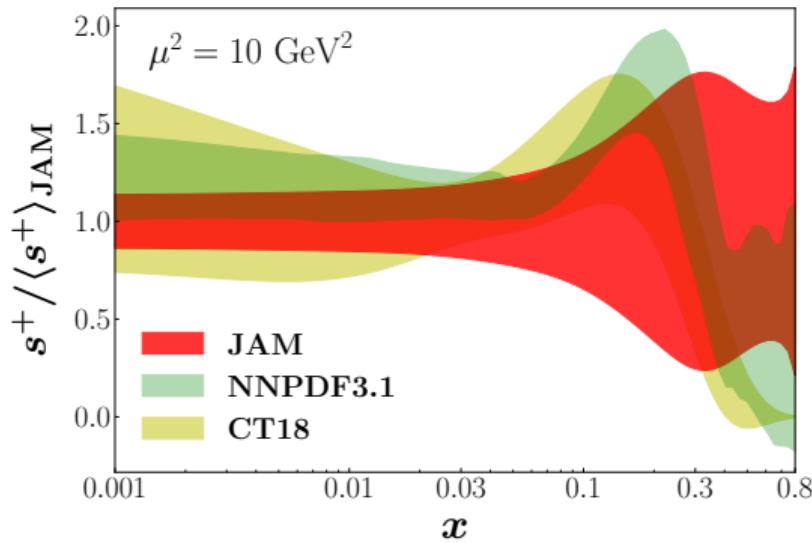
## A holistic approach to global analysis



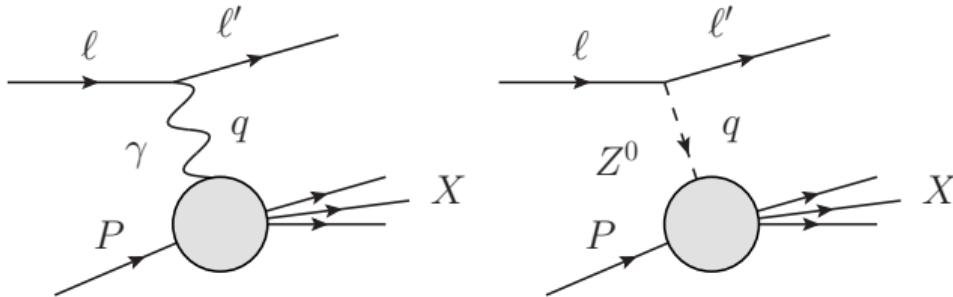
# Current Status – PDFs



C. Cocuzza, W. Melnitchouk, A. Metz, N. Sato  
Phys. Rev. D 104 (2021)



# Parity-Violating Deep-Inelastic Scattering (PVDIS)



$$Q^2 = -q^2 = -(\ell - \ell')^2$$

$$x_B = \frac{Q^2}{2P \cdot q} \quad s = (P + \ell)^2$$

$$y = \frac{P \cdot q}{P \cdot \ell} \quad W^2 = (P + q)^2$$

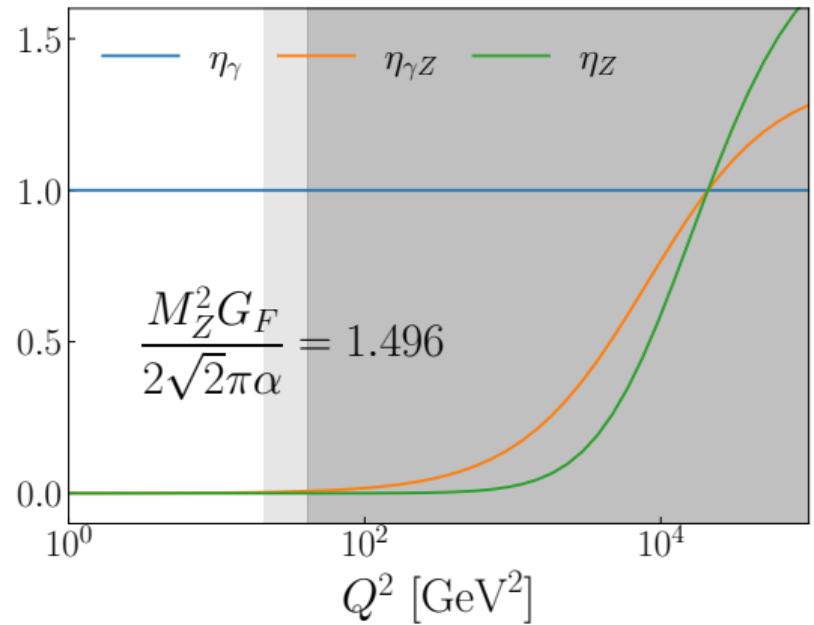
$$\frac{d\sigma}{dE' d\Omega} = \frac{1}{2(s - M^2)} \frac{E'}{2(2\pi)^3} \sum_X \int d\Phi_X (2\pi)^4 \delta^{(4)}(P + q - P_X) \underbrace{|\mathcal{M}_\gamma + \mathcal{M}_Z|^2}_{|\mathcal{M}_\gamma|^2 + 2 \operatorname{Re}(\mathcal{M}_\gamma \mathcal{M}_Z^*) + |\mathcal{M}_Z|^2}$$

## PVDIS cross section

$$\frac{d\sigma_{\lambda_\ell}}{dx_B dy} = \frac{2\pi\alpha^2 y}{Q^4} \sum_i \eta_i C_i L_{\mu\nu}^\gamma W_{i,U}^{\mu\nu}$$

$$\begin{aligned} W_{i,U}^{\mu\nu} &= \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1^i(x_B, Q^2) \\ &+ \left( P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) \left( P^\nu - \frac{P \cdot q}{q^2} q^\nu \right) \frac{F_2^i(x_B, Q^2)}{P \cdot q} \\ &+ i\epsilon^{\mu\nu\alpha\beta} \frac{P_\alpha q_\beta}{2P \cdot q} F_3^i(x_B, Q^2) \end{aligned}$$

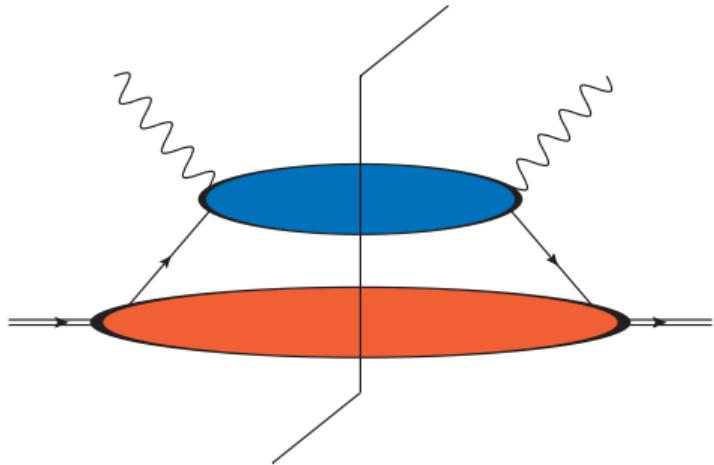
$$\eta_{\gamma Z} = \frac{M_Z^2 G_F}{2\sqrt{2}\pi\alpha} \frac{Q^2}{Q^2 + M_Z^2}, \quad \eta_Z = \eta_{\gamma Z}^2$$



# Collinear factorization and the parton model

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$$W_{i,U}^{\mu\nu} = \sum_i \int_{\xi_{\min}}^1 \frac{d\xi}{\xi} \widehat{W}_{i,U}^{\mu\nu}(\hat{x}, Q^2) f_{i/P}(\xi, \mu^2)$$



At LO in  $\alpha_S$ :

$$\rightarrow F_1^{\{\gamma, \gamma Z, Z\}}(x_B, Q^2) = \frac{1}{2} \sum_q \left\{ e_q^2, 2e_q g_V^q, (g_V^q)^2 + (g_A^q)^2 \right\} \underbrace{[q(x_B, Q^2) + \bar{q}(x_B, Q^2)]}_{q^+(x_B, Q^2)}$$

$$\rightarrow F_3^{\{\gamma, \gamma Z, Z\}}(x_B, Q^2) = \sum_q \left\{ 0, 2e_q g_A^q, 2g_V^q g_A^q \right\} \underbrace{[q(x_B, Q^2) - \bar{q}(x_B, Q^2)]}_{q^-(x_B, Q^2)}$$

# Parity-Violating Asymmetry

$$A_{\text{PV}} = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-} \approx \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \left[ 2g_A^e \frac{F_1^{\gamma Z}}{F_1^\gamma} Y_1 + g_V^e \frac{F_3^{\gamma Z}}{F_1^\gamma} Y_3 \right]$$

$$Y_1 = \left( \frac{1 + R^{\gamma Z}}{1 + R^\gamma} \right) \frac{1 + (1 - y)^2 - \frac{y^2}{2} \left[ 1 + r^2 - \frac{2r^2}{1+R^{\gamma Z}} \right]}{1 + (1 - y)^2 - \frac{y^2}{2} \left[ 1 + r^2 - \frac{2r^2}{1+R^\gamma} \right]}, \quad r^2 = 1 + 4M^2 x_B^2 / Q^2$$

$$Y_3 = \left( \frac{1 + R^{\gamma Z}}{1 + R^\gamma} \right) \frac{1 - (1 - y)^2}{1 + (1 - y)^2 - \frac{y^2}{2} \left[ 1 + r^2 - \frac{2r^2}{1+R^{\gamma Z}} \right]}, \quad R^i = \frac{F_2^i}{2x_B F_1^i} r^2 - 1$$

# Parity-Violating Asymmetry

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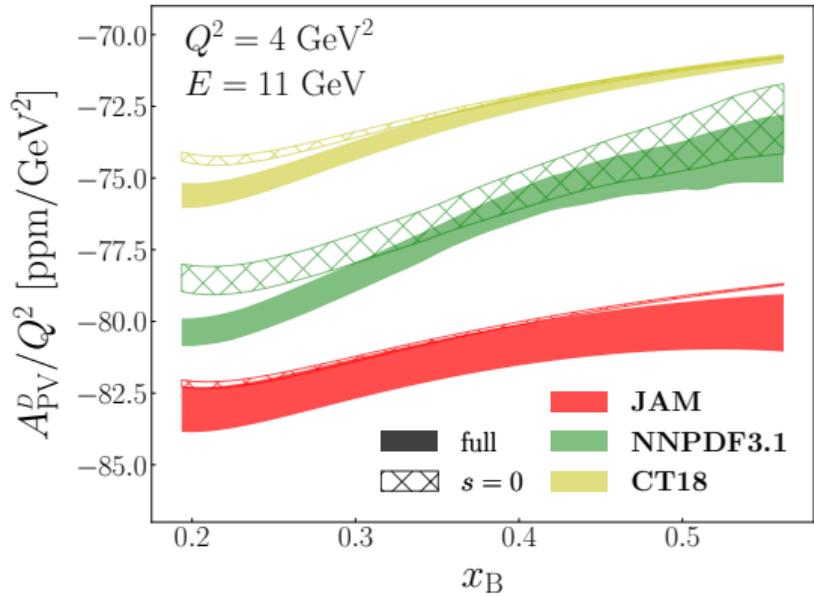
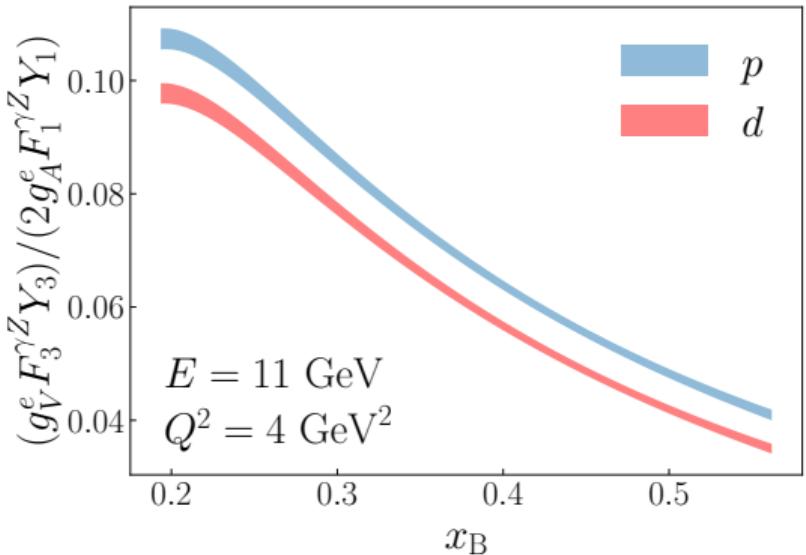
# Parity-Violating Asymmetry

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$$A_{\text{PV}} = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-} \approx \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \left[ 2g_A^e \frac{F_1^{\gamma Z}}{F_1^\gamma} Y_1 + g_V^e \frac{F_3^{\gamma Z}}{F_1^\gamma} Y_3 \right]$$

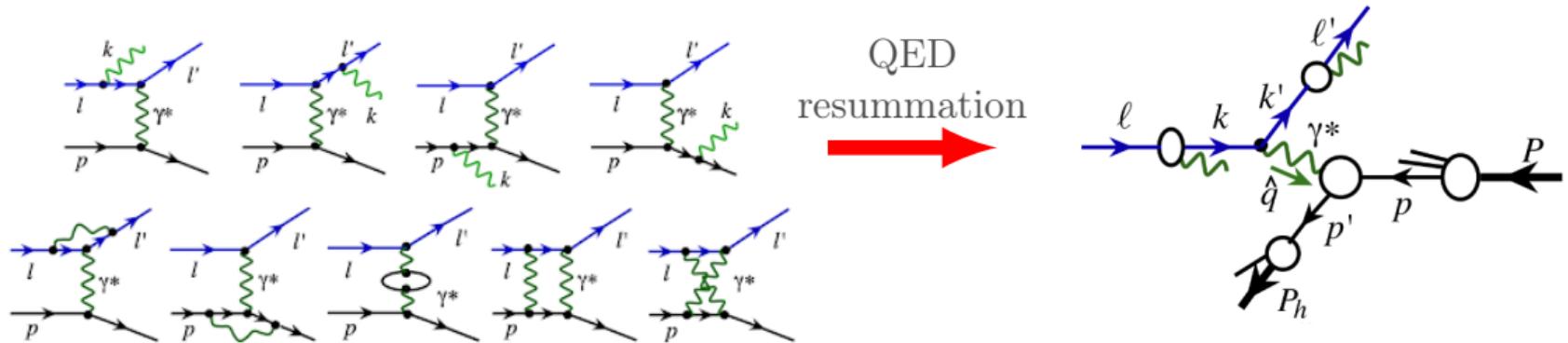
$$Y_1 = 1, \quad Y_3 = \frac{1 - (1 - y)^2}{1 + (1 - y)^2}$$

# $A_{\text{PV}}$ on a deuterium target



$$A_{\text{PV}}^D \approx -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \left[ \left( \frac{9}{5} - 4 \sin^2 \theta_W \right) + \frac{2}{25} \frac{s^+}{u^+ + d^+} \right]$$

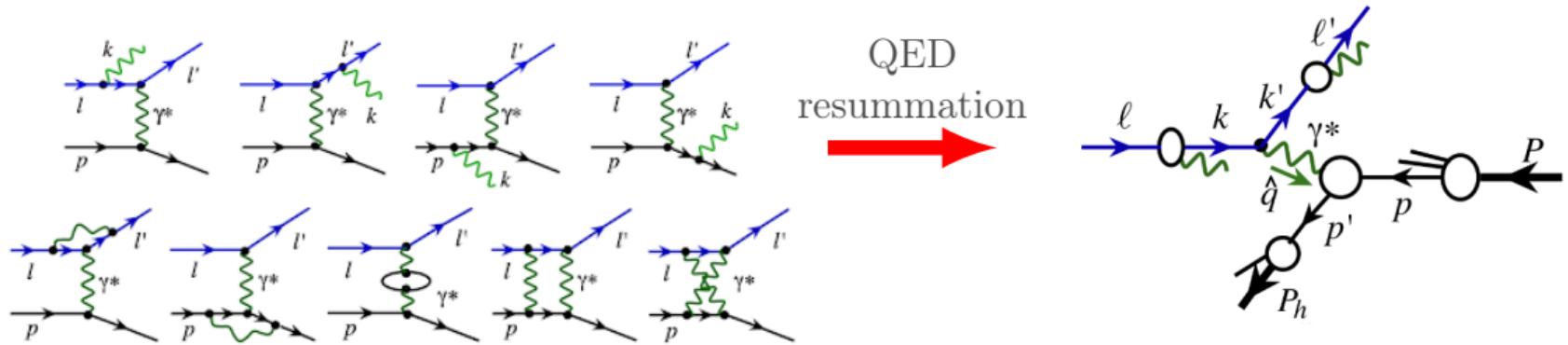
# Hybrid QED+QCD factorization



*T. Liu, W. Melnitchouk, J. Qiu, N. Sato  
JHEP 11 (2021)*

$$\frac{d\sigma_{(U/L)U}}{dx_B dy} = \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \underbrace{D_{e/e}(\zeta, \mu^2)}_{\text{LFF}} \int_{\xi_{\min}}^1 d\xi \underbrace{f_{e/e}(\xi, \mu^2)}_{\text{LDF}} \left[ \frac{Q^2}{x_B} \frac{\hat{x}_B}{\hat{Q}^2} \right] \frac{d\hat{\sigma}_{(U/L)U}}{d\hat{x}_B d\hat{y}}$$

# Hybrid QED+QCD factorization

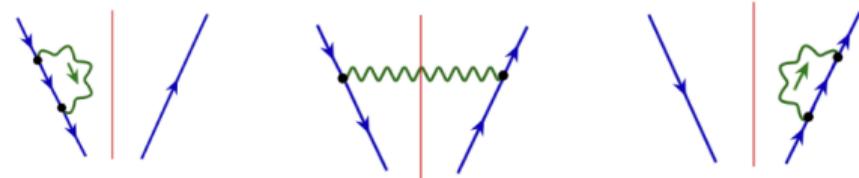


T. Liu, W. Melnitchouk, J. Qiu, N. Sato  
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$$f_{i/e}(\xi) = \int \frac{dz^-}{4\pi} e^{i\xi \ell^+ z^-} \langle e | \bar{\psi}_i(0) \gamma^+ \Phi_{[0,z^-]} \psi_i(z^-) | e \rangle$$

$$D_{e/j}(\zeta) = \frac{\zeta}{2} \sum_X \int \frac{dz^-}{4\pi} e^{i\ell^+ z^- / \zeta} \text{Tr} \left[ \gamma^+ \langle 0 | \bar{\psi}_j(0) \Phi_{[0,\infty]} | e, X \rangle \langle e, X | \psi_j(z^-) \Phi_{[z^-, \infty]} | 0 \rangle \right]$$

## LDF and LFF RGE



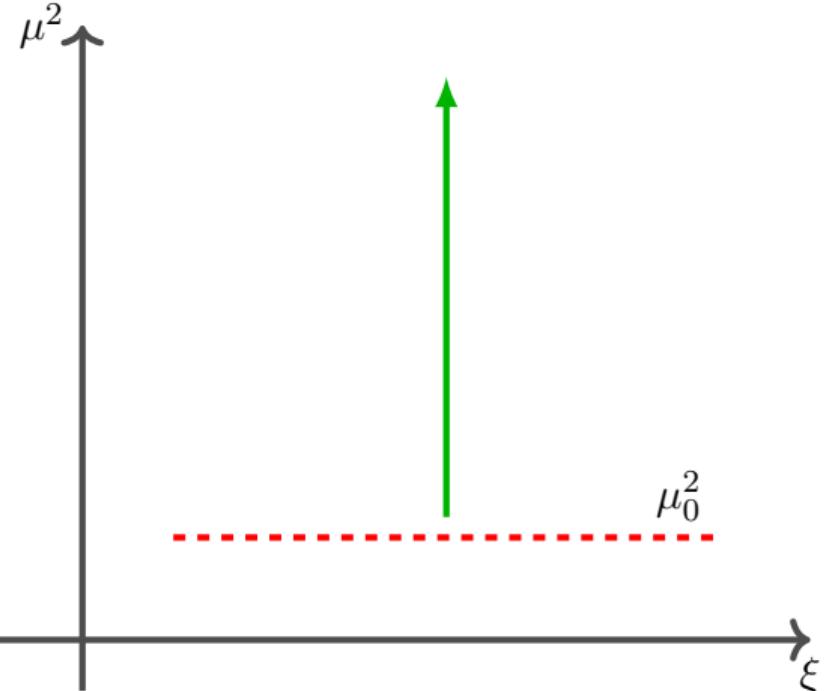
$$f_{e/e}^{(0)}(\xi, \mu_0^2) = \delta(\xi - 1)$$

$$f_{e/e}^{(1)}(\xi, \mu_0^2) = \frac{\alpha}{2\pi} \left[ \frac{1 + \xi^2}{1 - \xi} \ln \frac{\mu_0^2}{(1 - \xi)^2 m_e^2} \right]_+$$

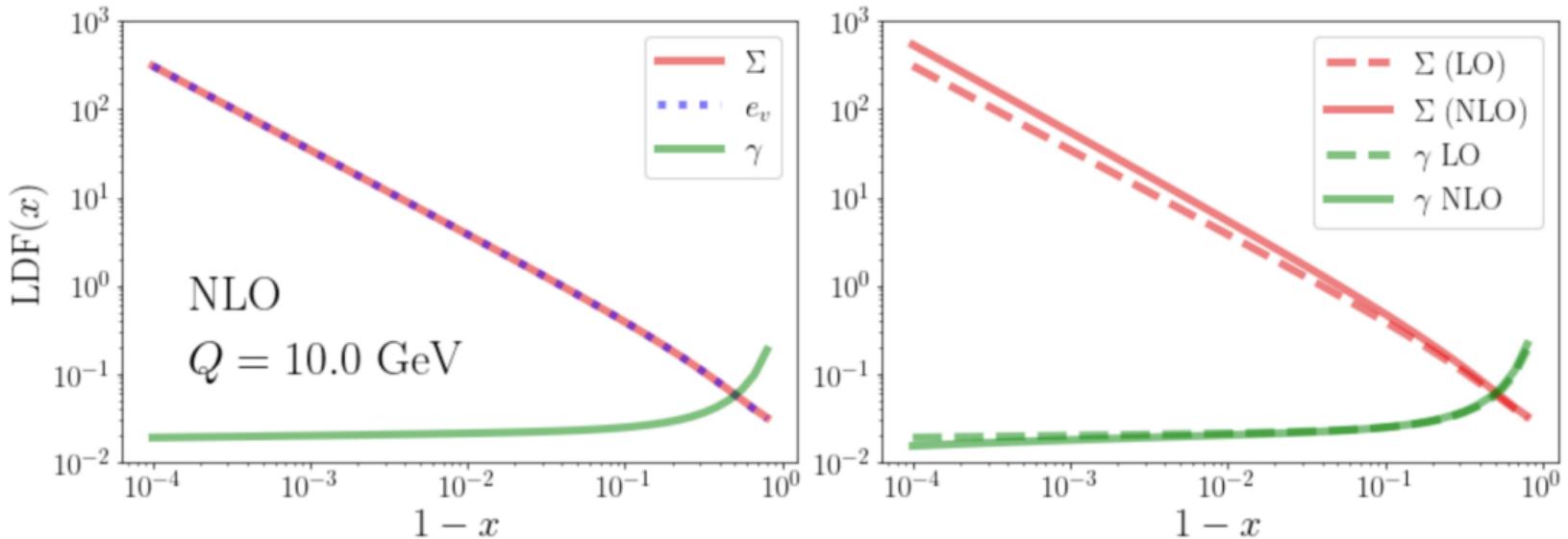
$$D_{e/e}^{(0)}(\zeta, \mu_0^2) = \delta(\zeta - 1)$$

$$D_{e/e}^{(1)}(\zeta, \mu_0^2) = \frac{\alpha}{2\pi} \left[ \frac{1 + \zeta^2}{1 - \zeta} \ln \frac{\mu_0^2}{(1 - \zeta)^2 m_e^2} \right]_+$$

$$\frac{\partial f_{i/\ell}(\xi, \mu^2)}{\partial \ln \mu^2} = \sum_j \int_\xi^1 \frac{dz}{z} P_{ij}(z) f_{j/\ell}\left(\frac{z}{\xi}, \mu^2\right)$$



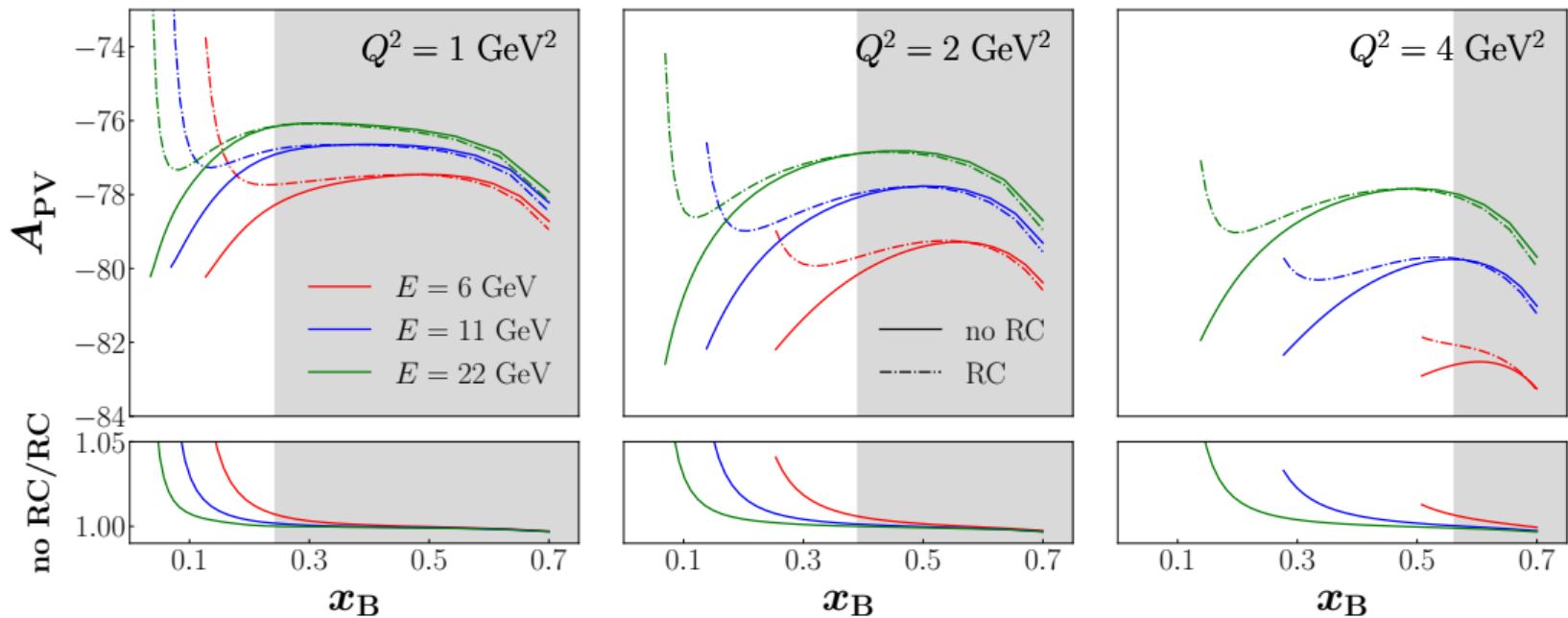
# LDFs and LFFs



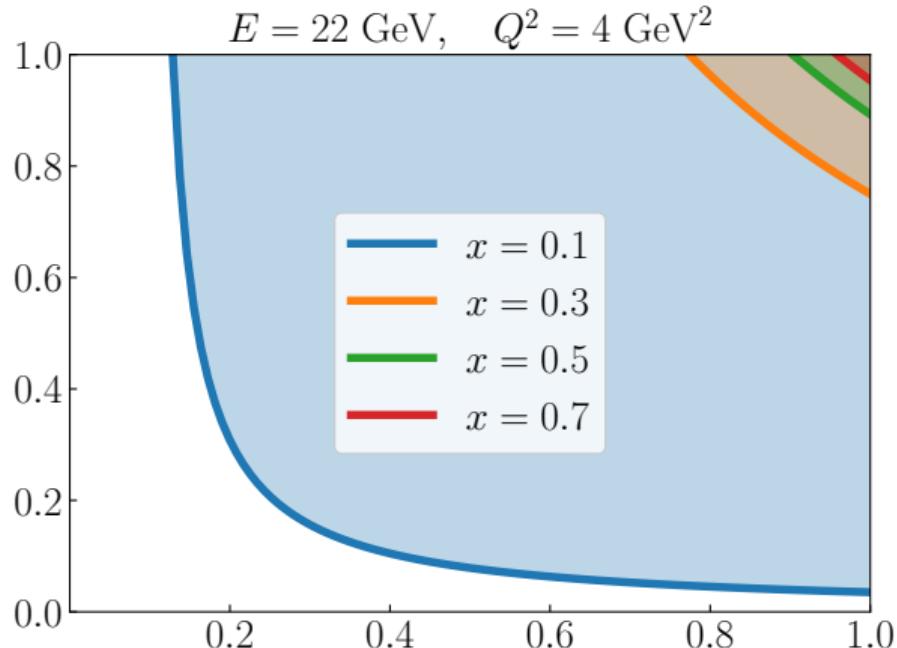
Subtraction “trick”:

$$\rightarrow \int_{\xi_{\min}}^1 d\xi \mathcal{H}(\xi) f(\xi) = \int_{\xi_{\min}}^1 d\xi [\mathcal{H}(\xi) - \mathcal{H}(1)] f(\xi) + \mathcal{H}(1) \frac{\xi_{\min}}{2\pi i} \int dN \xi_{\min}^{-N} \frac{F_N}{N-1}$$

# Size of the radiative corrections



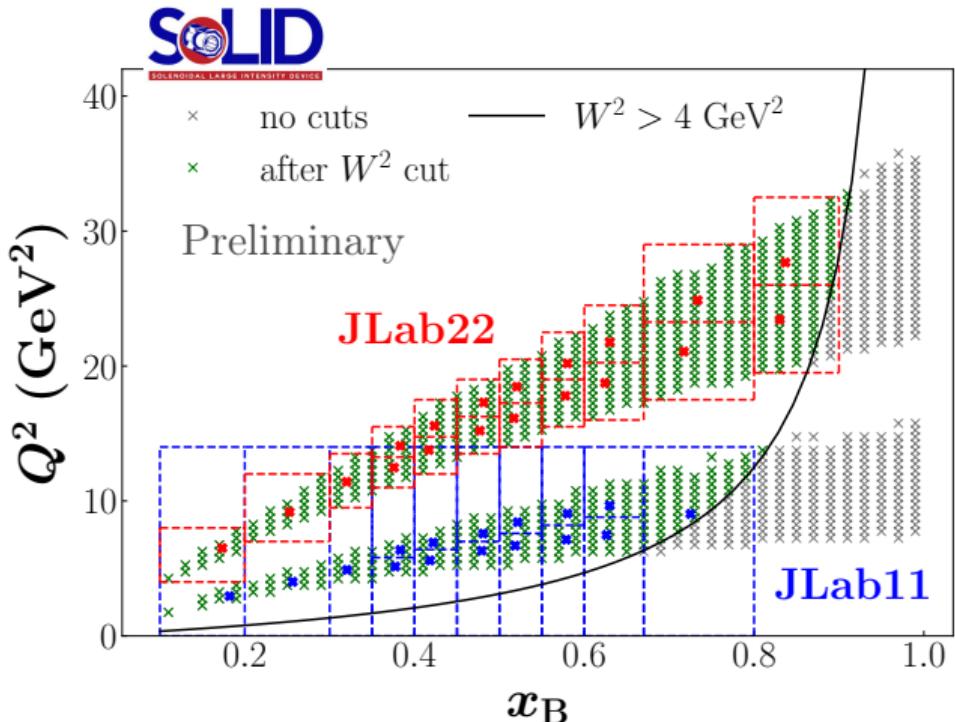
# Why not fit all QCFs within hybrid factorization?



$$d\sigma \sim \text{LFF} \otimes \text{LDF} \otimes \text{PDF} \otimes d\hat{\sigma}$$

- additional convolution integrals
- large radiative phase spaces
- end point instabilities
- how to model LFF, LDFs?

# Simulating pseudo-data



$$\langle A_{\text{PV}} \rangle_{\text{bin}} = \sum_{i \in \text{replicas}} A_{\text{PV},i} / N_{\text{replicas}}$$

Error budget:

$$\rightarrow \delta^{\text{stat}} A_{\text{PV}} = \left( P \sqrt{\mathcal{L} \sigma_{\text{bin}}} \right)^{-1}$$

$$\rightarrow \delta^{\text{syst}} A_{\text{PV}} / A_{\text{PV}} = 0.5\%$$

$$\rightarrow \delta^{\text{thy}} A_{\text{PV}} = |A_{\text{PV}}^{(\text{RC})} - A_{\text{PV}}|$$

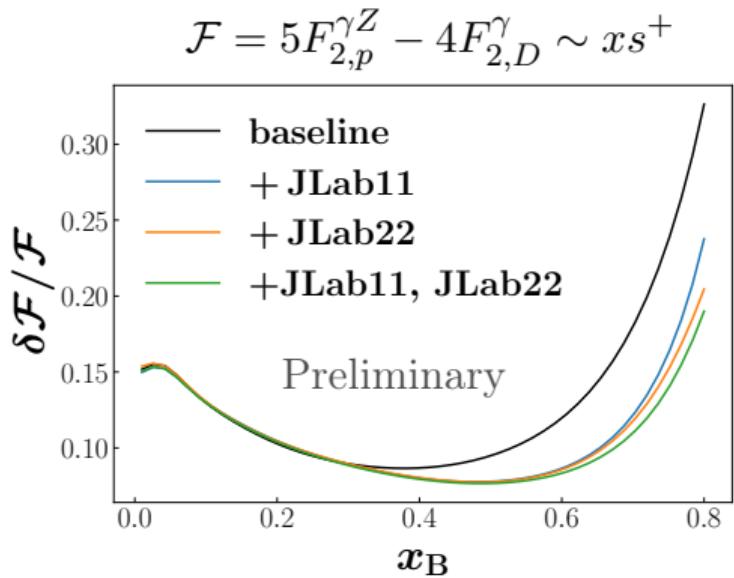
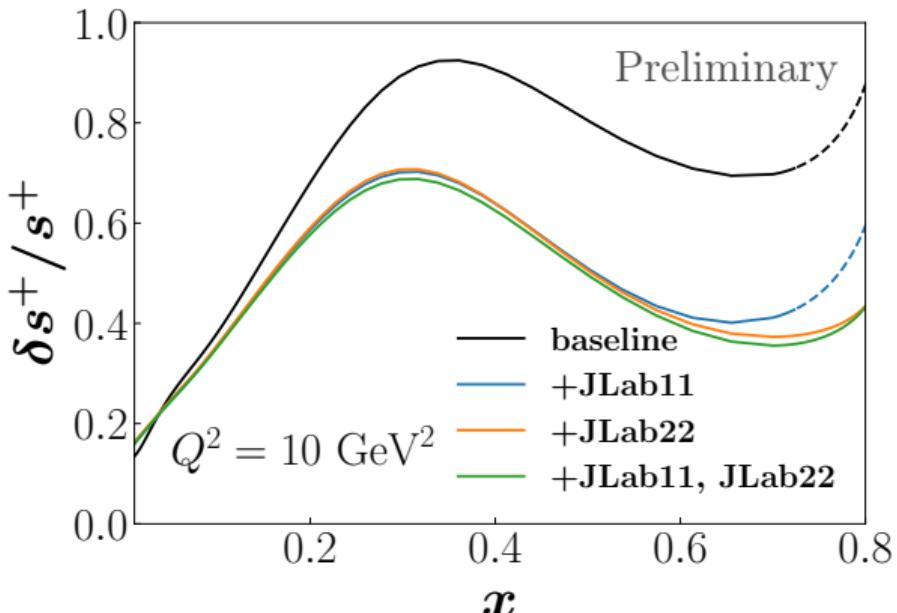
Note:

$$\rightarrow P = 85\%$$

$$\rightarrow d\mathcal{L}/dt = 4.85 \times 10^{38} \text{ cm}^{-2} \text{ s}^{-1}$$

→ run time: 50 days/target

## Impact on $s^+$



# Summary and Outlook

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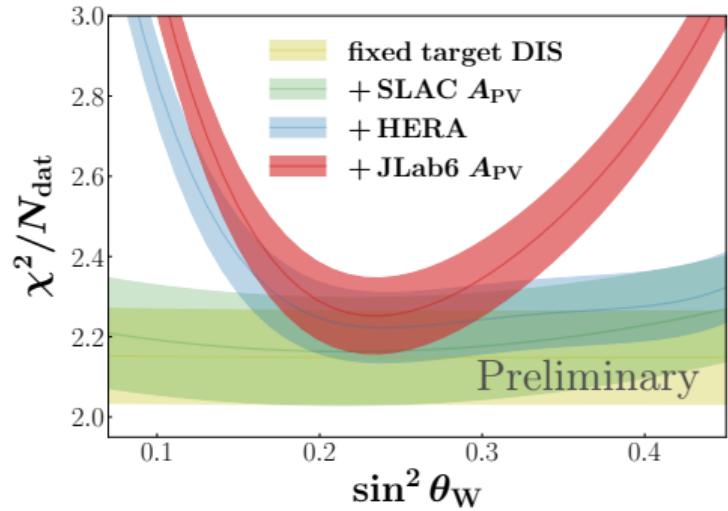
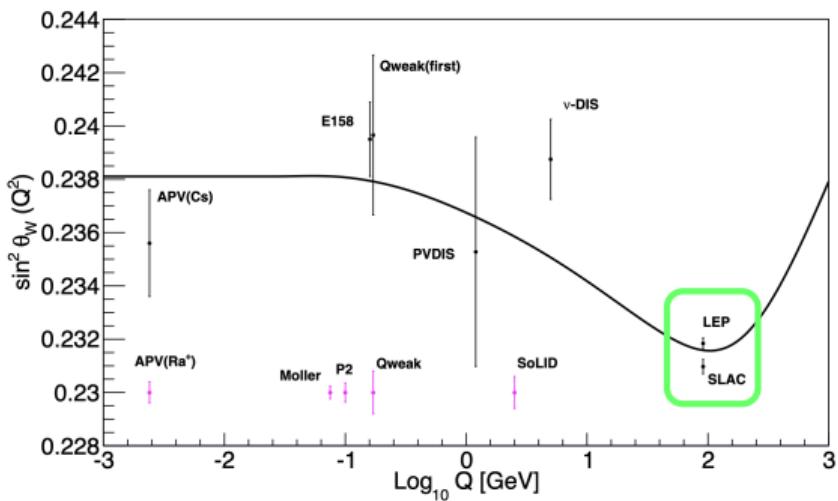
- Strange sea inside the nucleon needs to be constrained to achieve a better understanding of the longitudinal structure of the nucleon
  - also a primary source of uncertainty that contaminates BSM physics searches through  $A_{PV}$  measurements
- $A_{PV}$  is a unique and clean observable that can be used in future global analyses to make progress toward this goal
- Future work:
  - Quantification of higher twist effects/uncertainties
  - electron/positron PVDIS for constraint of sea quark asymmetries
  - Charge symmetry violation
  - Polarized  $A_{PV}$ ?

# Backup Slides

# Current Status – $\sin^2 \theta_W$

Fundamental Standard Model parameter

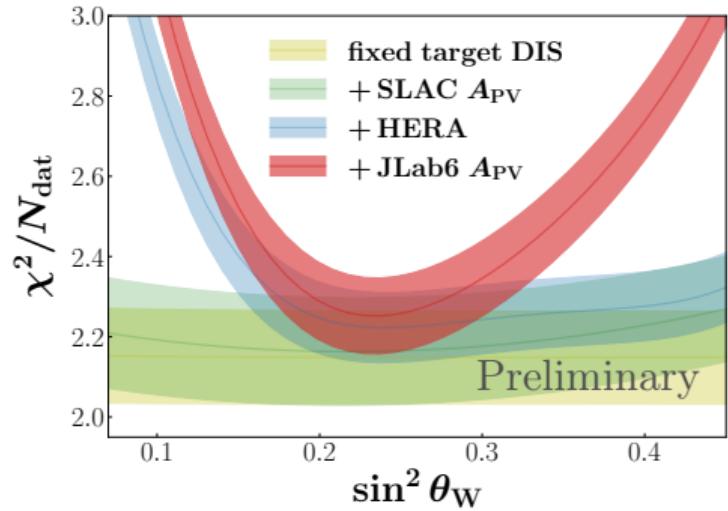
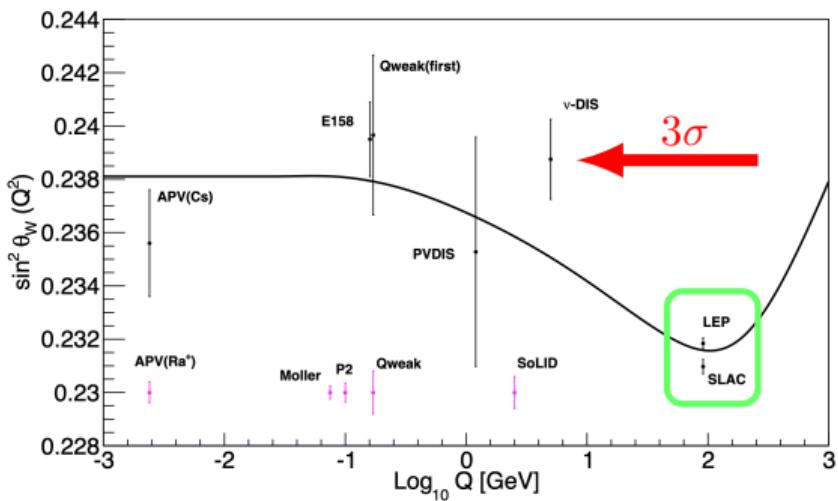
$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}, \quad \cos \theta_W = \frac{m_W}{m_Z}$$



# Current Status – $\sin^2 \theta_W$

Fundamental Standard Model parameter

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}, \quad \cos \theta_W = \frac{m_W}{m_Z}$$



# Combined impact on $\sin^2 \theta_W$ and $s^+$

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