Accessing Gluon Polarization in SIDIS

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Roadmap

- Introduction
- Background
- Motivation
- Theoretical Framework
- Numerical Simulations
- Outlook

Introduction

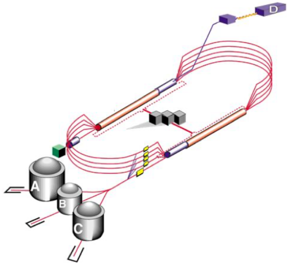
About me:

- Senior at WSU
- Summer 2021: REU at ODU (partnered with JLab)
- Summer 2022: SULI program at JLab

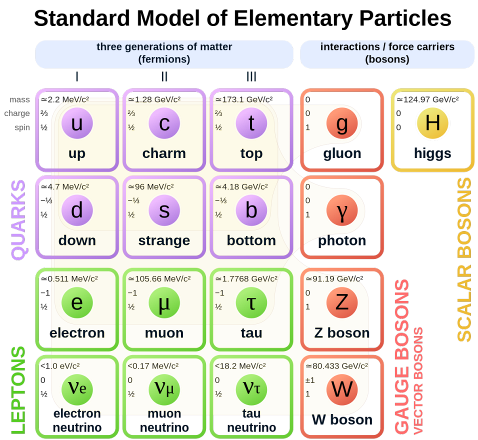
About Jefferson Lab (JLab):

- Department of Energy National Lab
- Location: Newport News, VA
- Mission: Understand nuclear matter

$\rightarrow$ CEBAF + 4 experimental halls
$\rightarrow$ Theory & Computing



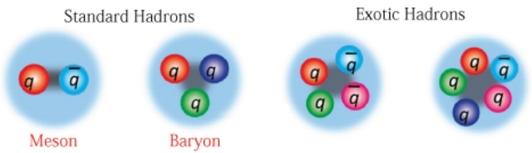
Standard Model of Physics



Modern theory of particle physics:

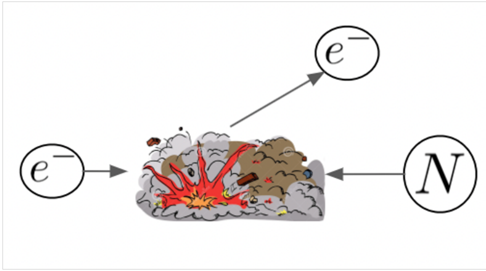
- Describes fundamental particles and their interactions

JLab focuses on nuclear and hadronic structure of matter



Scattering

1. Cause "Collision" between particles
2. Detect/measure what comes out
3. Infer what happened during the scattering process

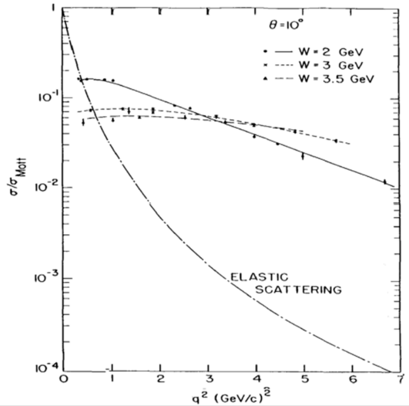


$$\frac{\mathrm{d}N}{\mathrm{d}\Omega} = \mathcal{L}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

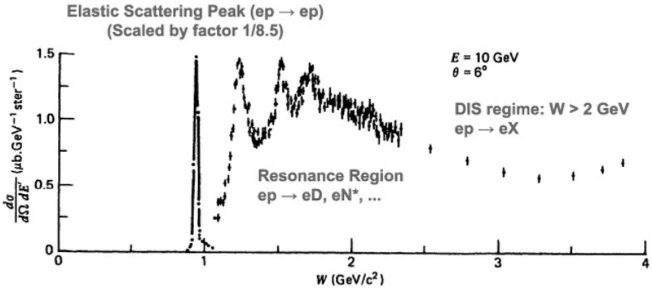
Experimentally Measurable

Theoretically Calculable

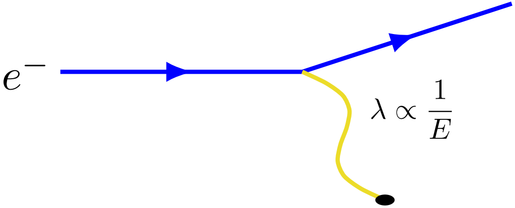
What has scattering taught us?



A lot! The proton is a composite particle not fundamental

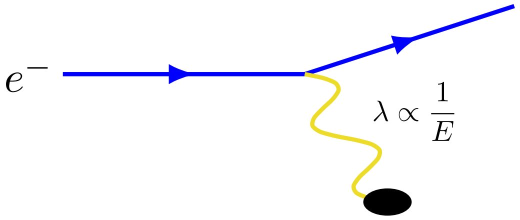


Probing the proton substructure



Fundamental fermion: $s = \frac{1}{2}$

Probing the proton substructure



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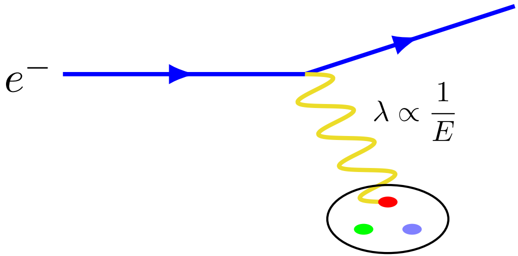
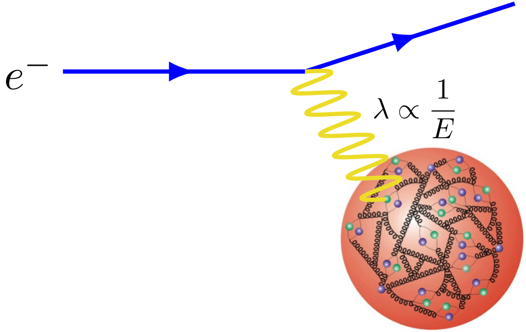
Probing the proton substructure



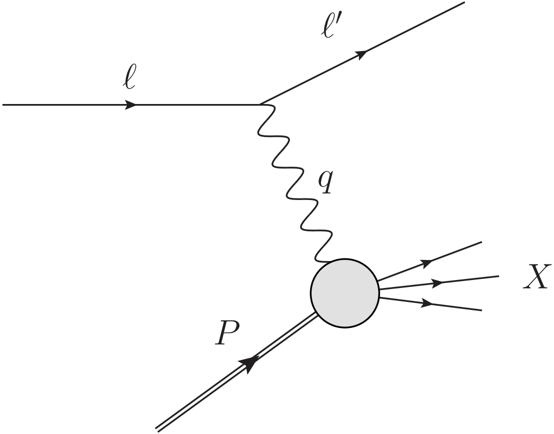
$$
\begin{aligned}
|p_+\rangle = \frac{1}{\sqrt{18}}\Big[&2|u_+d_-u_+\rangle + 2|u_+u_+d_-\rangle + 2|d_-u_+u_+\rangle \\
&- |u_+u_-d_+\rangle - |u_-u_+d_+\rangle - |u_+d_+u_-\rangle \\
&- |u_-d_+u_+\rangle - |d_+u_+u_-\rangle - |d_+u_-u_+\rangle\Big]
\end{aligned}
\quad \Rightarrow \quad \langle p_+|\hat{S}|p_+\rangle = \frac{1}{2}
$$

Probing the proton substructure



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

Deep Inelastic Scattering (DIS)



$$\text{DIS: } \ell P \to \ell X$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}Q^2} \propto L_{\mu\nu} W^{\mu\nu}$$

$$Q^2 = -q^2 \qquad x = \frac{Q^2}{2P \cdot q}$$

Leptonic Tensor

Exactly calculable in QED

$$L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu} \ell \cdot \ell' + i\epsilon_{\mu\nu\alpha\beta} s^\alpha q^\beta)$$

Assuming longitudinal polarization (i.e. $s^\mu = \lambda_\ell \ell^\mu$)

$$L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu} \ell \cdot \ell' - i\lambda_\ell \epsilon_{\mu\nu\alpha\beta} \ell^\alpha \ell'^\beta)$$

Hadronic Tensor

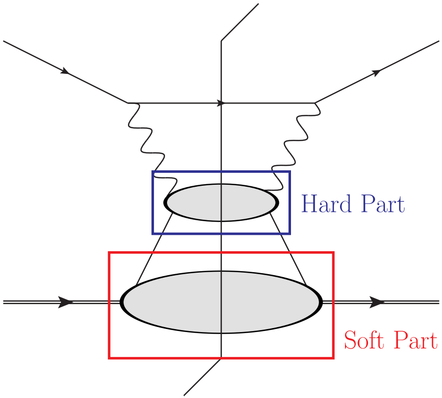
Not exactly calculable

$$
\begin{aligned}
W^{\mu\nu} &= \frac{1}{(2\pi)^4} \sum_X \int \mathrm{d}^4 z \, e^{iq\cdot z} \langle P, S | J^\mu(z) | X \rangle \langle X | J^\nu(0) | P, S \rangle \\
&= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x, Q^2) + \left(P^\mu - \frac{P\cdot q}{q^2} q^\mu \right) \left(P^\nu - \frac{P\cdot q}{q^2} q^\nu \right) F_2(x, Q^2) \\
&+ \frac{i}{P\cdot q} \epsilon^{\mu\nu\alpha\beta} q_\alpha \left[S_\beta g_1(x, Q^2) + (S_\beta - \frac{S\cdot q}{P\cdot q} P_\beta) g_2(x, Q^2) \right]
\end{aligned}
$$

- Current conservation ($q_\mu W^{\mu\nu} = 0$), covariant expression
- Symmetric and antisymmetric pieces (parity/spin dependence)
- Structure functions $F_{1,2}$ and $g_{1,2}$ parameterize “ignorance”

Collinear factorization

$$L_{\mu\nu}W^{\mu\nu} = L_{\mu\nu}\left[\int_x^1 \frac{\mathrm{d}\xi}{\xi}\,\widehat{W}^{\mu\nu} f_{i/P}(\xi) + \mathcal{O}\left(\frac{m}{Q}\right)\right]$$



Naïve parton model

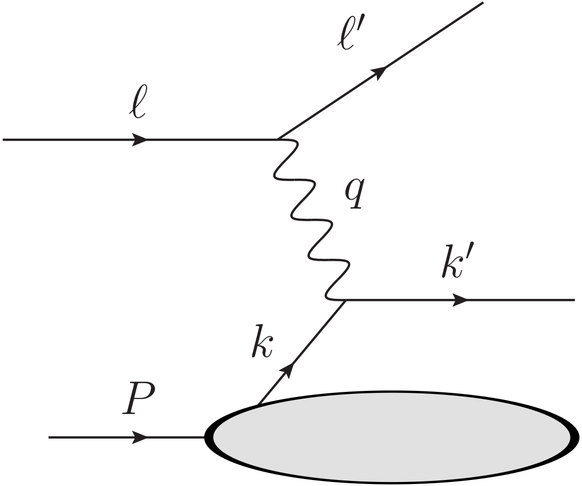
Leading order (LO): $\gamma + q \to q$

$$F_1 = \frac{1}{2}\sum_q e_q^2 f_q(x, Q^2)$$

$$F_2 = 2xF_1$$

$$g_1 = \frac{1}{2}\sum_q e_q^2 \Delta f_q(x, Q^2)$$

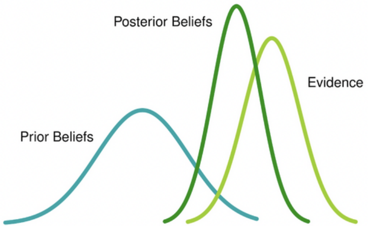
$$g_2 = 0$$

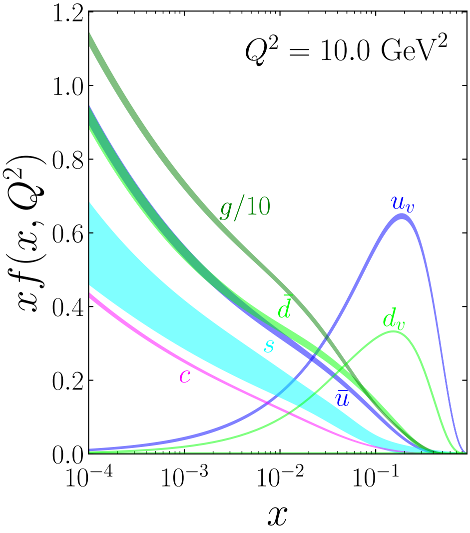


Parton distribution functions

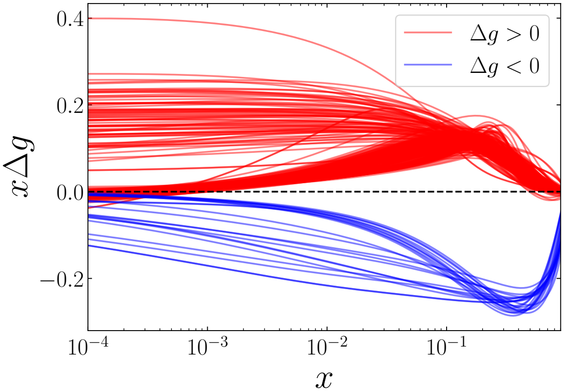
JLab Angular Momentum Collaboration (JAM):

- Global QCD analysis – extract PDFs from experimental data
- Parameterization: $f(x) = x^{\alpha}(1-x)^{\beta}\eta(x)$
- Blend of bayesian statistics and monte carlo methods: $\rho(\vec{a}|\text{data}) = \mathcal{L}(\vec{a}, \text{data})\pi(\vec{a})$





Tension in Δg fit



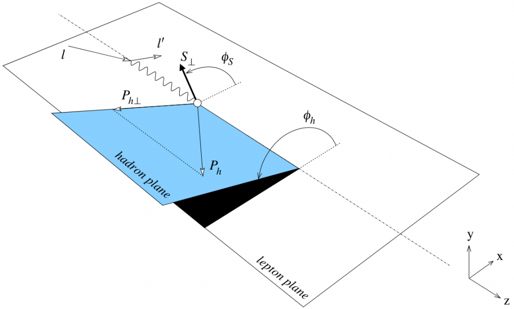
How: Current data sets allow this

Why is this a problem?

- Proton Spin Crisis: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$
- $\Delta G = \int_{x_{\min}}^{1} \Delta g(x, Q^2)\mathrm{d}x$

Goal: Find new data to distinguish $\Delta g > 0$ and $\Delta g < 0$

Semi-inclusive DIS (SIDIS)



SIDIS: $\ell P \to \ell' H X$

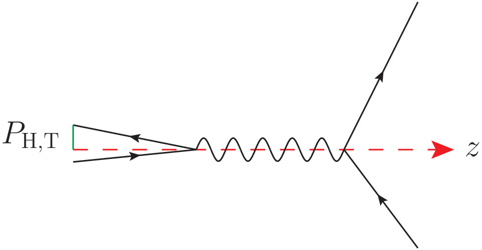
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}Q^2\,\mathrm{d}z\,\mathrm{d}P_{\mathrm{H,T}}^2} = \frac{\pi^2\alpha^2 y}{2zQ^4} L_{\mu\nu} W^{\mu\nu}$$

$$z = \frac{P \cdot P_{\mathrm{H}}}{P \cdot q} \qquad q_{\mathrm{T}} = \frac{P_{\mathrm{H,T}}}{z}$$

Transverse momentum

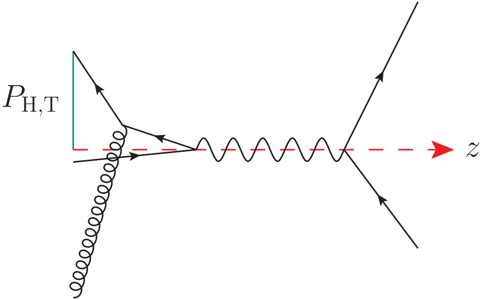
Small intrinsic transverse momentum

- TMD description more appropriate



Large transverse momentum

- Hard gluon radiations

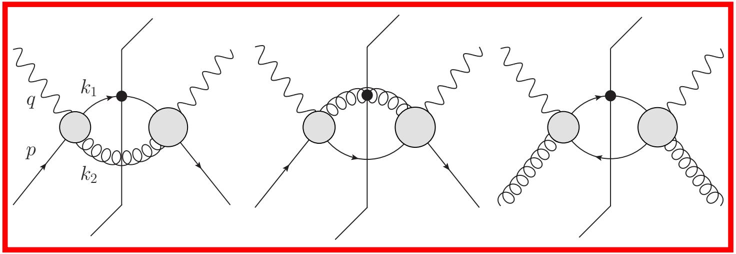


$$q_{\mathrm{T}} \gtrsim Q$$

Collinear factorization and hard scattering amplitudes

$$W^{\mu\nu} = \int_x^1 \frac{\mathrm{d}\xi}{\xi} \int_z^1 \frac{\mathrm{d}\zeta}{\zeta^2} \boxed{\widehat{W}^{\mu\nu}} \boxed{f_{i/P}(\xi) D_{H/j}(\zeta)} \longleftarrow \boxed{\mathcal{JAM}}$$

$$\boxed{\widehat{W}^{\mu\nu} = \sum_{i,j} \underbrace{\mathcal{M}^{\mu}\mathcal{M}^{\dagger\nu}}}$$



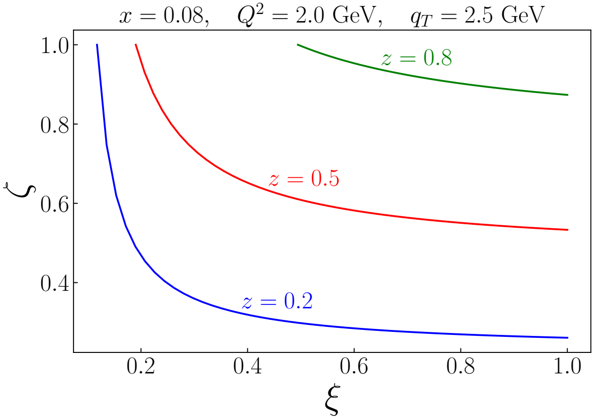
Momentum Fractions

Conservation of momentum: $q + p = k_1 + k_2$

$$p = \xi P \quad k_{1,T} = P_{\mathrm{H,T}}/\zeta$$

$$\zeta = z\left[1 + \frac{x}{\xi - x}\frac{q_T^2}{Q^2}\right]$$

$$\xi_{\min} = x\left[1 + \frac{z}{1-z}\frac{q_T^2}{Q^2}\right]$$



Double Spin Asymmetry

Definition:

$$A_{LL} = \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}\sigma}$$

Uncertainties:

1. Statistical: $\delta A_{LL} = \sqrt{\frac{1 + A_{LL}^2}{N}} \approx \frac{1}{\sqrt{N}}$ when $A_{LL} \ll 1$

$\rightarrow$ Experiments are Poisson processes: $\delta N = \sqrt{N} = \sqrt{\mathcal{L}\sigma}$

$\rightarrow$ $\sigma = \int_{\mathrm{bin}} \mathrm{d}x\, \mathrm{d}Q^2\, \mathrm{d}z\, \mathrm{d}P_{\mathrm{H,T}}^2 \frac{\mathrm{d}\sigma}{\mathrm{d}x\, \mathrm{d}Q^2\, \mathrm{d}z\, \mathrm{d}P_{\mathrm{H,T}}^2} \approx \Delta x\, \Delta Q^2\, \mathrm{d}z\, \Delta P_{\mathrm{H,T}}^2 \frac{\mathrm{d}\sigma(\mathrm{center})}{\mathrm{d}x\, \mathrm{d}Q^2\, \mathrm{d}z\, \mathrm{d}P_{\mathrm{H,T}}^2}$

2. PDF replicas: one standard deviation (1σ) of values across replicas at a kinematic point

Kinematic Definitions

Center of mass energy squared: $s = (\ell + P)^2$

- JLab: beam energy E ($s = 2ME + M^2$)

$\rightarrow$ Available energies: 12 GeV ($\sqrt{s} = 4.8$ GeV) and 22 GeV ($\sqrt{s} = 6.5$ GeV)

- EIC: projected $\sqrt{s} = 140$ GeV

Inelasticity variable: $y = \dfrac{P \cdot q}{P \cdot \ell}$

- Useful relation: $Q^2 = (s - M^2)xy$

Squared final state masses:

- $W^2 = (P + q)^2$
- $W^2_{\text{SIDIS}} = (P + q - P_{\text{H}})^2$

Phase Space Constraints

$$Q^2_{\max} = (s - M^2)x$$

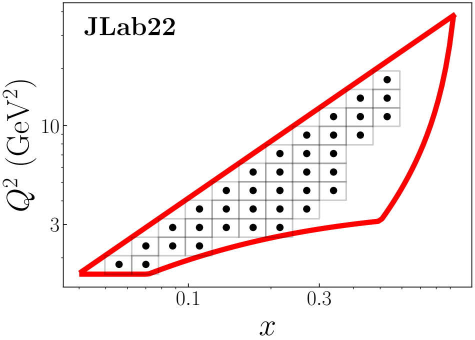
$$Q^2_{\min} = \max\{Q_1^2, Q_2^2, Q_3^2\}:$$

1. $Q_1^2 = m_c^2 = (1.28\,\mathrm{GeV})^2$
2. $W^2 > W^2_{\min}$: $Q_2^2 = \dfrac{x}{1-x}(W^2_{\min} - M^2)$
3. $\theta > \theta_{\min}$: $Q_3^2 = \dfrac{4E^2 \sin^2(\theta_{\min}/2)}{1 + (2E/Mx)\sin^2(\theta_{\min}/2)}$

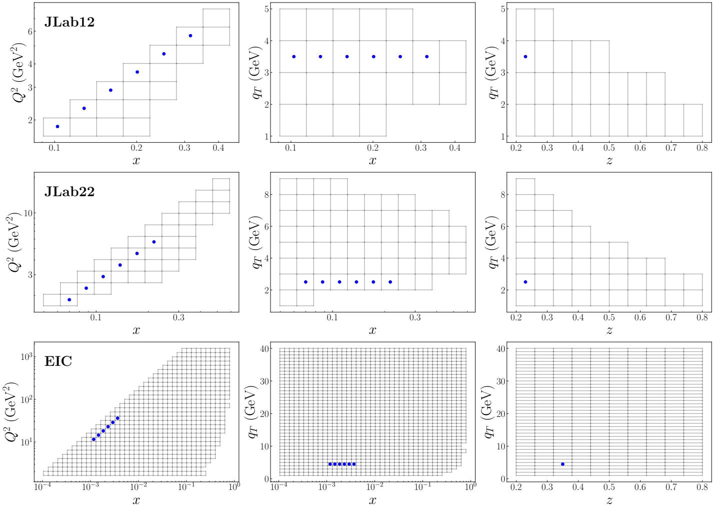
$$x_{\min} = \frac{m_c^2}{s - M^2} \qquad x_{\max} = \frac{s - W^2}{s - M^2}$$

$$0.2 < z < 0.8$$

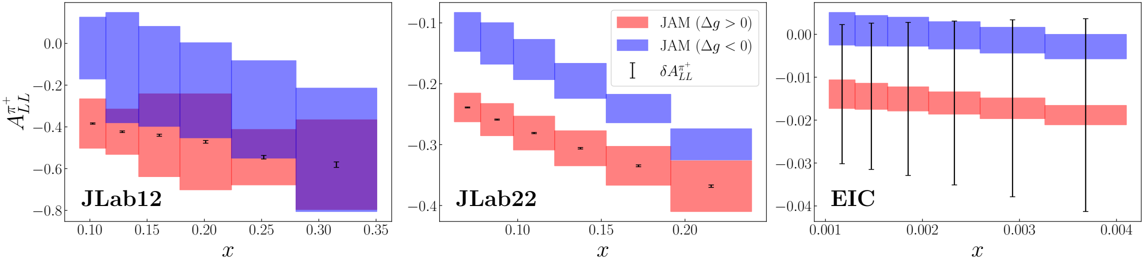
$$q_{\mathrm{T},\min} = Q, \quad q_{\mathrm{T},\max}:\ W^2_{\mathrm{SIDIS}} > M^2$$



Results



Results



$$\mathcal{L}_{\text{JLab}} = 86\,\text{fb}^{-1}, \quad \mathcal{L}_{\text{EIC}} = 10\,\text{fb}^{-1}$$

Outlook

- Significant sensitivity to the gluon channel
- JLab with a 22 GeV beam is well positioned to discriminate Δg
- Strong scaling behavior with $\sqrt{s}$ may make further constraint of Δg difficult at small-x from this process
- Continue search for observable/process that is better positioned for constraint of Δg from EIC data in future global QCD analyses

Backup Slides

Spinor and polarization vector products

$$u(p)\bar{u}(p) = \frac{1}{2}(1+\lambda\gamma_5)\not{p}$$

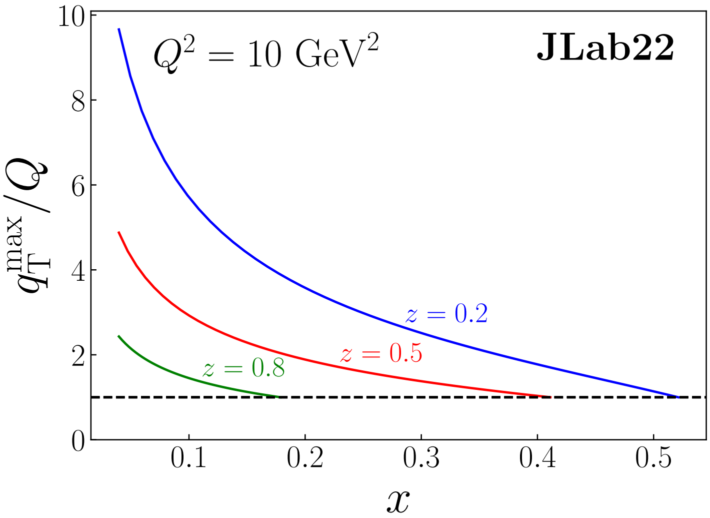
$$\epsilon^{\mu}(p)\epsilon^{*\nu}(p) = \frac{1}{2}\left(-g^{\mu\nu} + \frac{i\lambda}{p\cdot X}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}X_{\beta}\right)$$

Hidden Expressions – Kinematics

$$W^2 = M^2 - \left(1 - \frac{1}{x}\right)Q^2$$

$$W_{\text{SIDIS}}^2 = M^2 + M_h^2 + Q^2\frac{1-x-z}{x} + \frac{Q^4 z}{2M^2x^2}\left(\sqrt{1+\frac{4M^2x^2}{Q^2}}\sqrt{1-\frac{4M^2x^2(M_h^2+q_T^2z^2)}{Q^4z^2}} - 1\right)$$

Maximum q_T



$Q^2 = 10\ \mathrm{GeV}^2$
JLab22
$z = 0.2$
$z = 0.5$
$z = 0.8$
$q_\mathrm{T}^\mathrm{max}/Q$
x
0
2
4
6
8
10
0.1
0.2
0.3
0.4
0.5

Hidden Expressions – Unpolarized Amplitudes

$$\frac{\mathrm{d}\mathcal{H}^{\mathrm{U}}_{qq}}{\mathrm{d}\hat{x}\,\mathrm{d}y\,\mathrm{d}\hat{z}\,\mathrm{d}P^2_{\mathrm{H,T}}} = \frac{64\pi\alpha_s^2}{3\hat{x}(1-\hat{x})y^2\,\mathcal{Q}_1^2}\Big[(1+\hat{x}^2\hat{z}^2)(1+\bar{y}^2)\,Q^4 + 8\hat{x}^2\hat{z}^2\bar{y}\,Q^2 q_T^2 + \hat{x}^2\hat{z}^2(1+\bar{y}^2)\,q_T^4\Big]$$

$$\frac{\mathrm{d}\mathcal{H}^{\mathrm{U}}_{qg}}{\mathrm{d}\hat{x}\,\mathrm{d}y\,\mathrm{d}\hat{z}\,\mathrm{d}P^2_{\mathrm{H,T}}} = \frac{64\pi\alpha_s^2}{3(1-\hat{x})y^2\,\mathcal{Q}_2^2}\Big[\Big((2+\hat{x}^2\hat{z}^2)(1+\bar{y}^2) - 4\hat{x}\hat{z}\bar{y} - 2\hat{x}y^2\big(1-\hat{x}(1-\hat{z})\big)\Big)Q^4 + 2\hat{x}\hat{z}\big(4\hat{x}\hat{z}\bar{y} + \hat{x}y^2 - 1 - \bar{y}^2\big)\,Q^2 q_T^2 + \hat{x}^2\hat{z}^2(1+\bar{y}^2)\,q_T^4\Big]$$

$$\frac{\mathrm{d}\mathcal{H}^{\mathrm{U}}_{qg}}{\mathrm{d}\hat{x}\,\mathrm{d}y\,\mathrm{d}\hat{z}\,\mathrm{d}P^2_{\mathrm{H,T}}} = \frac{64\pi\alpha_s^2}{3(1-\hat{x})y^2\,\mathcal{Q}_2^2}\Big[\Big((2+\hat{x}^2\hat{z}^2)(1+\bar{y}^2) - 4\hat{x}\hat{z}\bar{y} - 2\hat{x}y^2\big(1-\hat{x}(1-\hat{z})\big)\Big)Q^4 + 2\hat{x}\hat{z}\big(4\hat{x}\hat{z}\bar{y} + \hat{x}y^2 - 1 - \bar{y}^2\big)\,Q^2 q_T^2 + \hat{x}^2\hat{z}^2(1+\bar{y}^2)\,q_T^4\Big]$$

Hidden Expressions – Polarized Amplitudes

$$
\begin{aligned}
\frac{\mathrm{d}\mathcal{H}^{\mathrm{P}}_{qq}}{\mathrm{d}\hat{x}\,\mathrm{d}y\,\mathrm{d}\hat{z}\,\mathrm{d}P^2_{\mathrm{H,T}}} &= -\frac{64\pi\alpha_s^2\,(2-y)}{3\hat{x}(1-\hat{x})y\mathcal{Q}_1^2}\Big[(1+\hat{x}^2\hat{z}^2)Q^4-\hat{x}^2\hat{z}^2q_T^4\Big] \\
\frac{\mathrm{d}\mathcal{H}^{\mathrm{P}}_{qg}}{\mathrm{d}\hat{x}\,\mathrm{d}y\,\mathrm{d}\hat{z}\,\mathrm{d}P^2_{\mathrm{H,T}}} &= -\frac{64\pi\alpha_s^2\,\hat{x}(2-y)}{3(1-\hat{x})y\mathcal{Q}_2^2}\Big[\big(2+\hat{x}\hat{z}^2-2\hat{x}\hat{z}\big)Q^4+2\hat{z}(1-\hat{x})Q^2q_T^2-\hat{x}\hat{z}^2q_T^4\Big] \\
\frac{\mathrm{d}\mathcal{H}^{\mathrm{P}}_{gq}}{\mathrm{d}\hat{x}\,\mathrm{d}y\,\mathrm{d}\hat{z}\,\mathrm{d}P^2_{\mathrm{H,T}}} &= \frac{8\pi\alpha_s^2\,(2-y)Q^2}{\hat{x}y\mathcal{Q}_1^2\mathcal{Q}_2^2}\Big[(2\hat{x}^2\hat{z}^2-2\hat{x}^2\hat{z}+2\hat{x}-1)\,Q^4 \\
&\qquad +2\hat{x}\hat{z}(1-\hat{x})\,Q^2q_T^2-2\hat{x}^2\hat{z}^2\,q_T^4\Big]
\end{aligned}
$$

Scaling with $\sqrt{s}$