Gluon Polarization in Large-PT SIDIS

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~ March 2, 2022 ~

Outline

1. REU and Collaborators
2. Background
3. Research Problem
4. Results

Research Experience for Undergraduates

- Applied to ~10 REUs – 4 offers

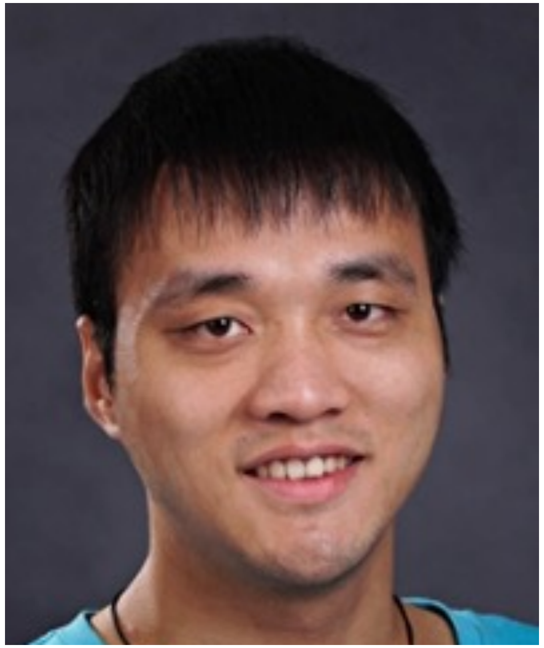
- 10-week program over summer 2021 (remote)
  - Funded by National Science Foundation (NSF)
- Run in conjunction with Jefferson Lab (JLab)

Collaborators

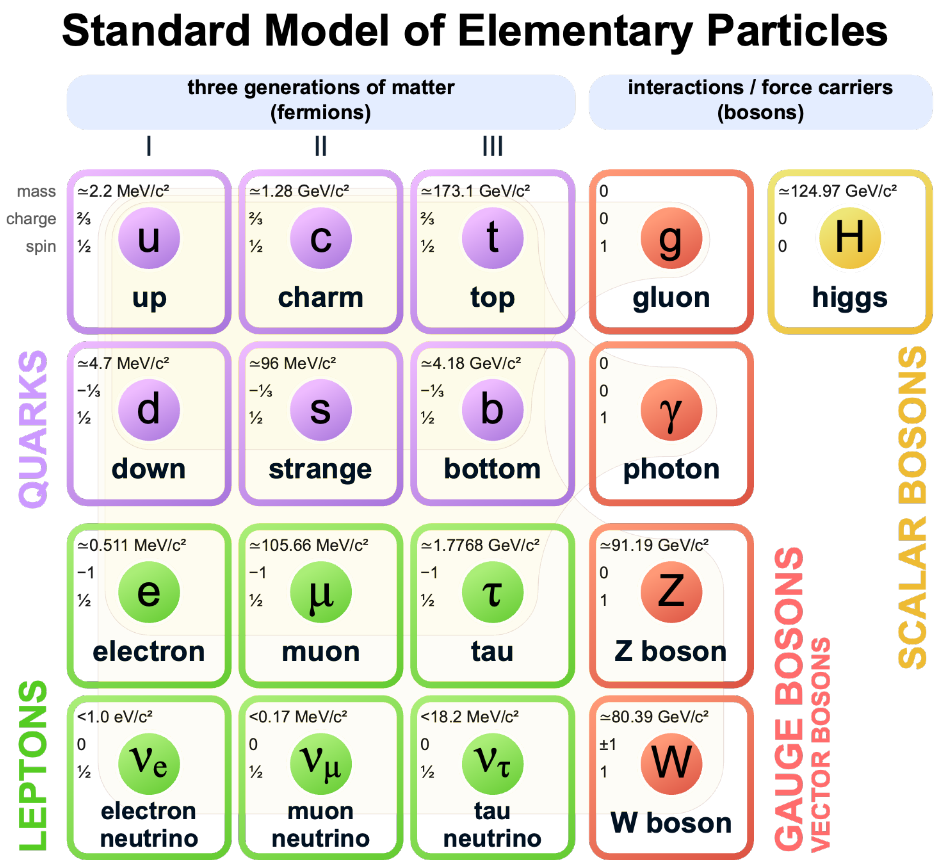
Wally Melnitchouk

Nobuo Sato

Yiyu Zhou



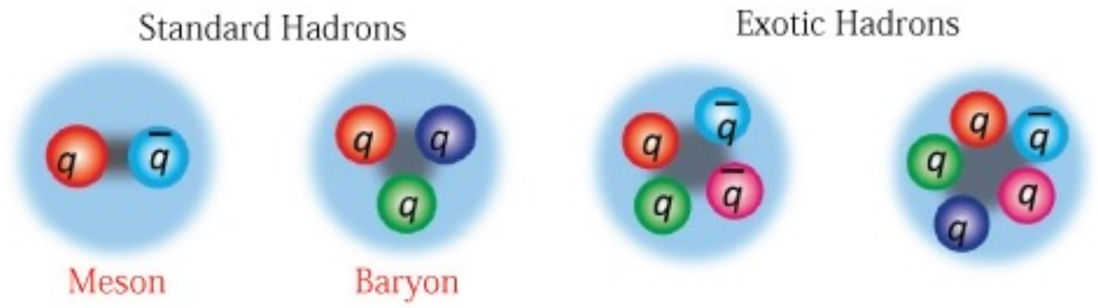
Standard Model



Modern theory for particle physics

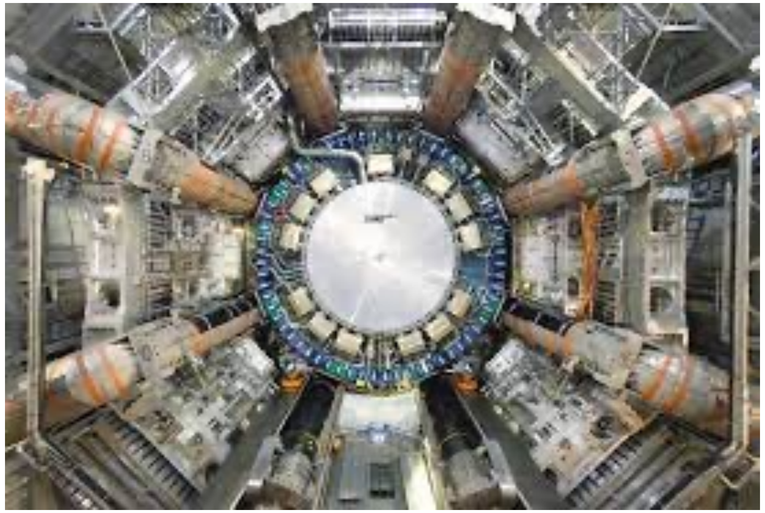
- Describes fundamental particles and their interactions

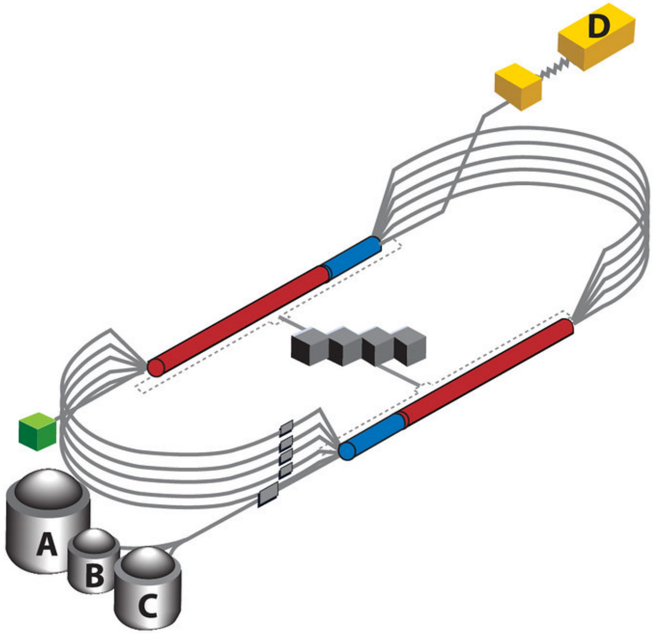
Jefferson Lab focuses on nuclear and hadronic structure of matter

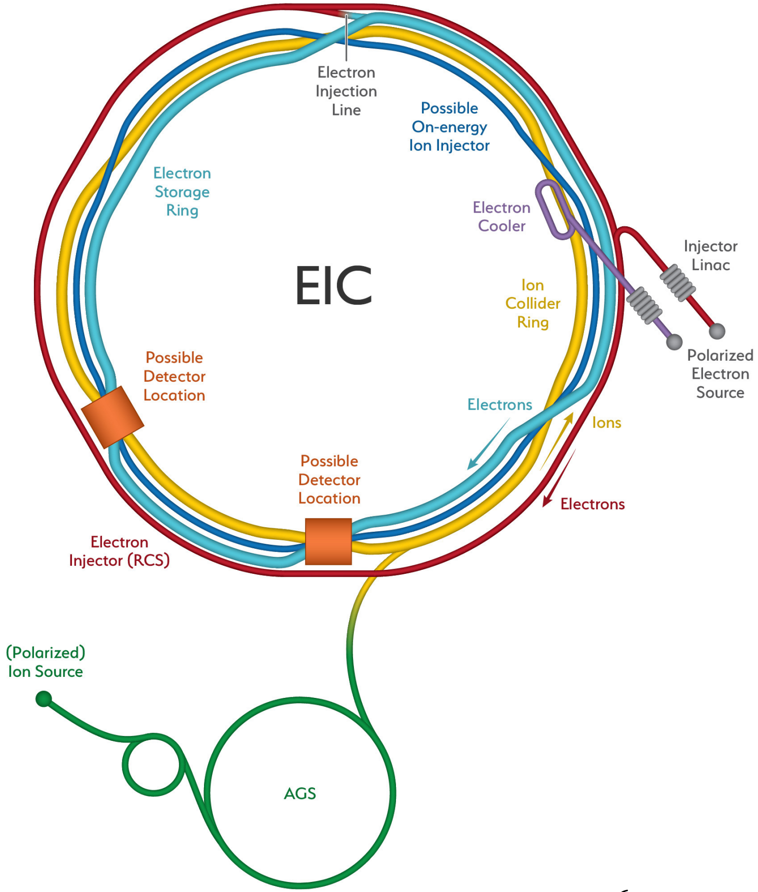


How do we learn about these particles?

Scattering Experiments!

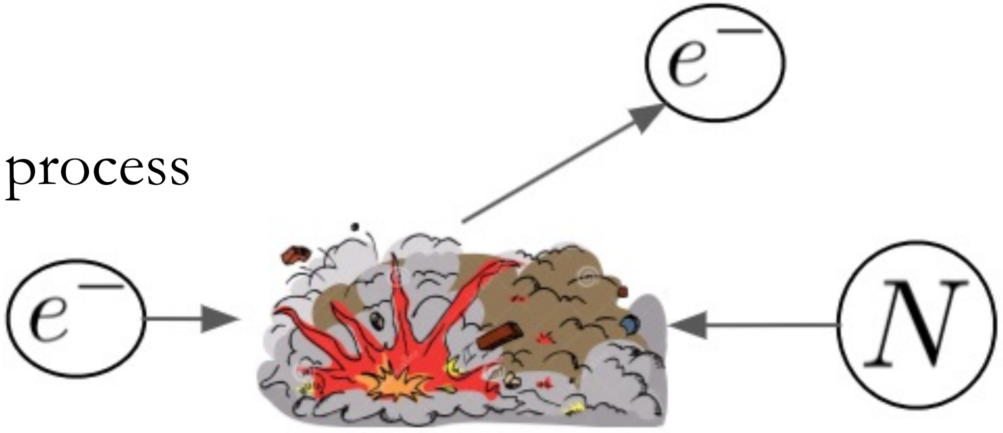






Scattering

1. Cause ”collision” between particles
2. Detect/measure what comes out
3. Infer what happened during the scattering process



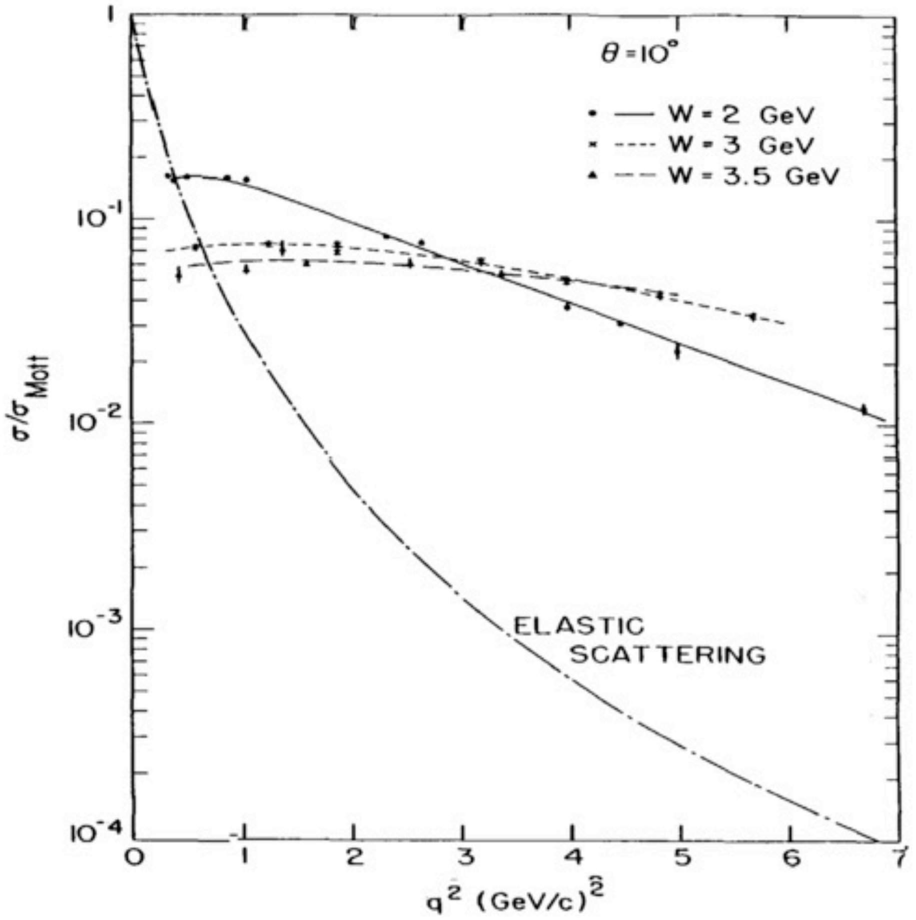
Cross section: main measurable observable

$$\frac{dN}{d\Omega} = \mathcal{L}\frac{d\sigma}{d\Omega}$$

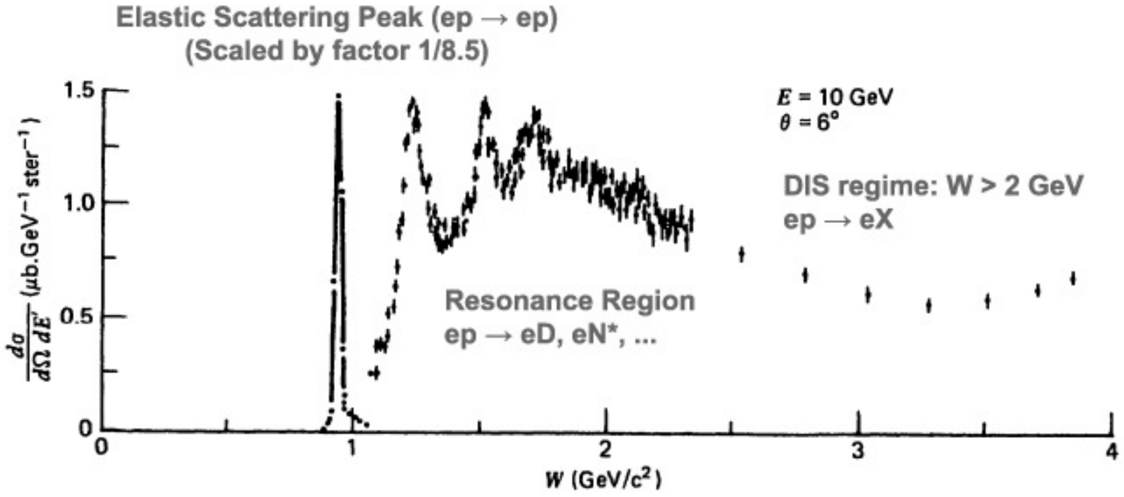
Experimentally Measurable

Theoretically Calculable

What does scattering tell us?



A lot – proton is composite particle not fundamental

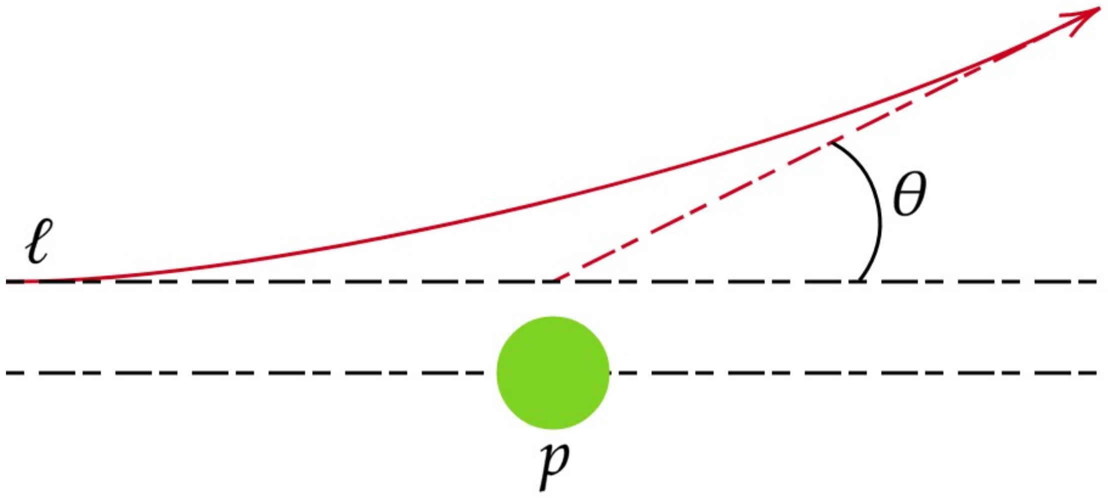


Rutherford Scattering

Nonrelativistic scattering between point-like spin-1/2 Dirac particles

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2 \sin^4 \frac{\theta}{2}}$$

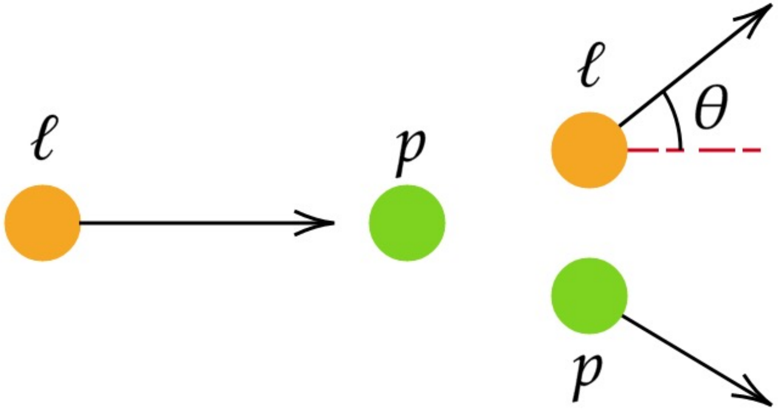
- Assumes $E \sim m \ll M$ (no proton recoil)

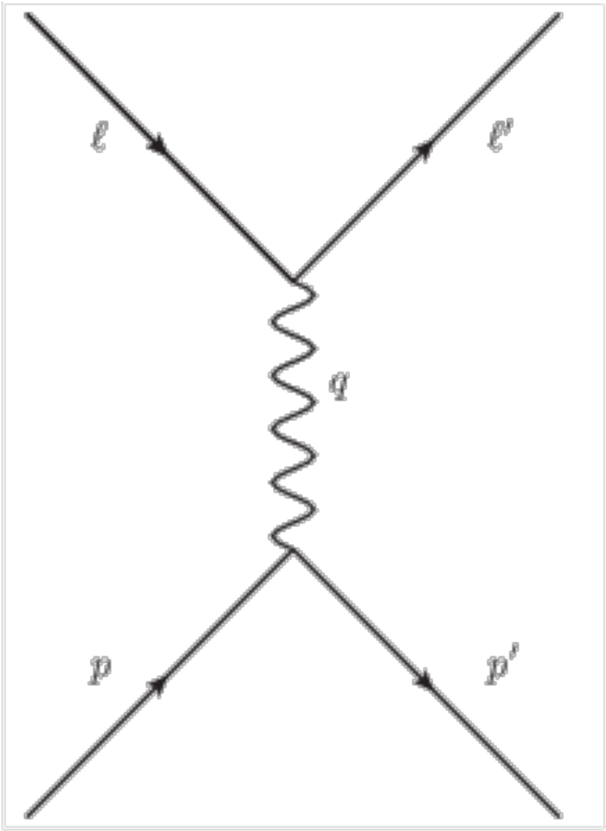


Mott Scattering

Relativistic correction to Rutherford Scattering ($\ell p \to \ell p$)

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}}}_{\text{Rutherford}} \underbrace{\frac{E'}{E}}_{\text{Recoil}} \left(\underbrace{\cos^2 \frac{\theta}{2}}_{\text{Electric/Magnetic Interactions}} + \underbrace{\frac{Q^2}{2M^2} \sin^2 \frac{\theta}{2}}_{\text{Spin interaction}} \right)$$





“Mott Scattering” with finite size

With finite size – first order perturbative correction

$$\mathcal{M} = \langle\psi_f|\, V(\vec{r})\, |\psi_i\rangle = \int d^3\vec{r} e^{-i\vec{\ell}\cdot\vec{r}} V(\vec{r}) e^{i\vec{\ell}'\cdot\vec{r}}$$

$$= \int\int d^3\vec{r}\, d^3\vec{r}'\; e^{i\vec{q}\cdot\vec{r}} \frac{Q\rho(\vec{r}')}{4\pi\epsilon_0|\vec{r}-\vec{r}'|} = \mathcal{M}_{\mathrm{point}} F(q^2)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} |F(q^2)|^2$$

Proton with extended structure

Finite size proton – Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right)$$

Note: $\tau = Q^2/4M^2$

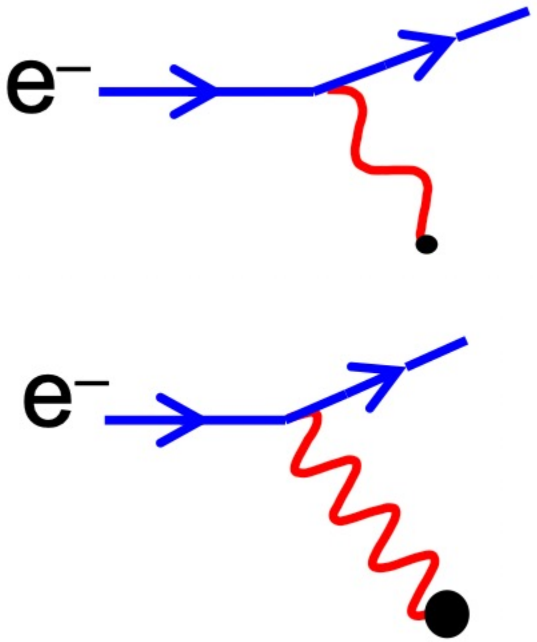
- $G_{E/M}(q^2)$: form factors
  - Like Fourier transform of charge/moment distributions (not quite though!)
  - Interpretation not as clean as with Mott scattering

Probing composite proton structure

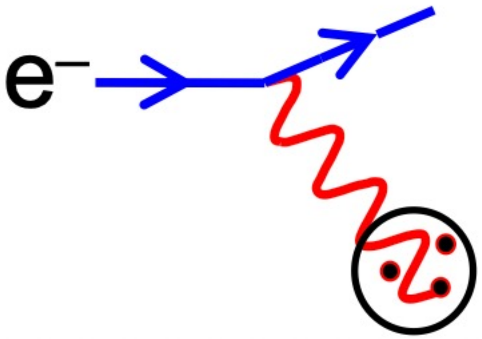
Experiments support finite size of proton

- But does not necessarily imply that proton has substructure

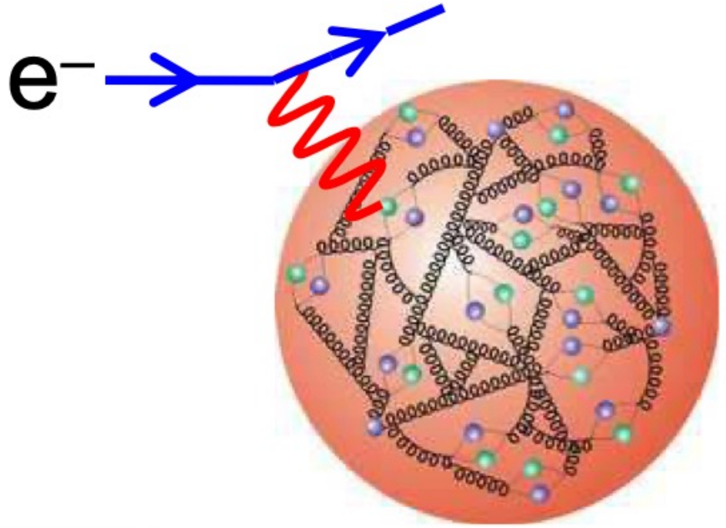
Elastic scattering



Deep Inelastic Scattering



$$\lambda \propto \frac{1}{E}$$

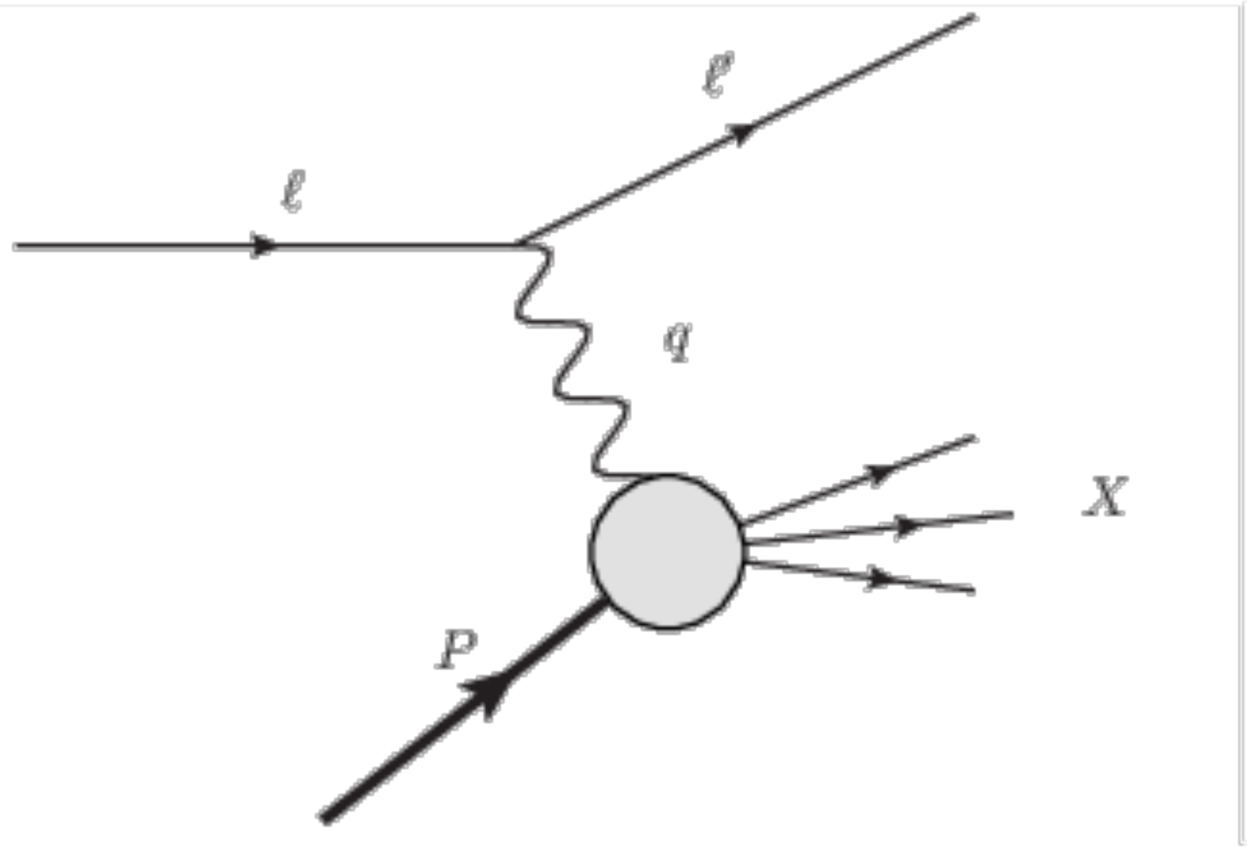


Deep Inelastic Scattering

Deep: probes internal proton structure

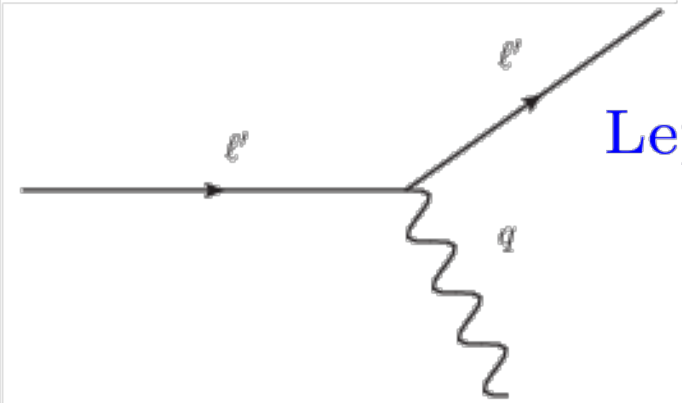
Inelastic: other particles produced in scattering

$$lp \to lX$$

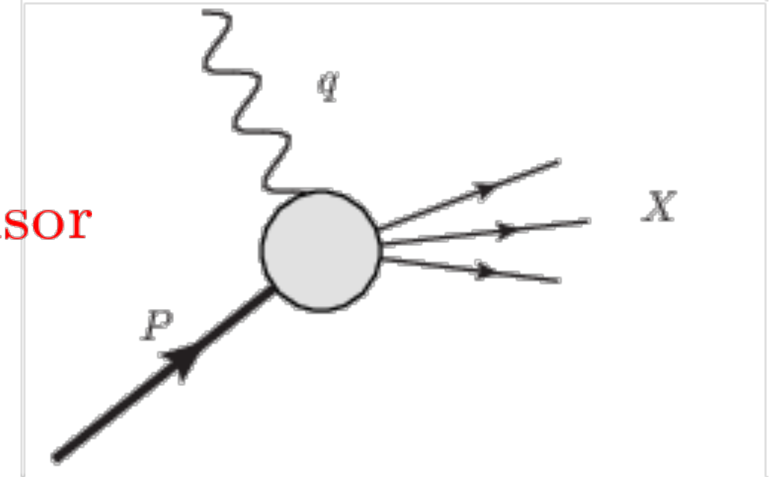


Scattering Process

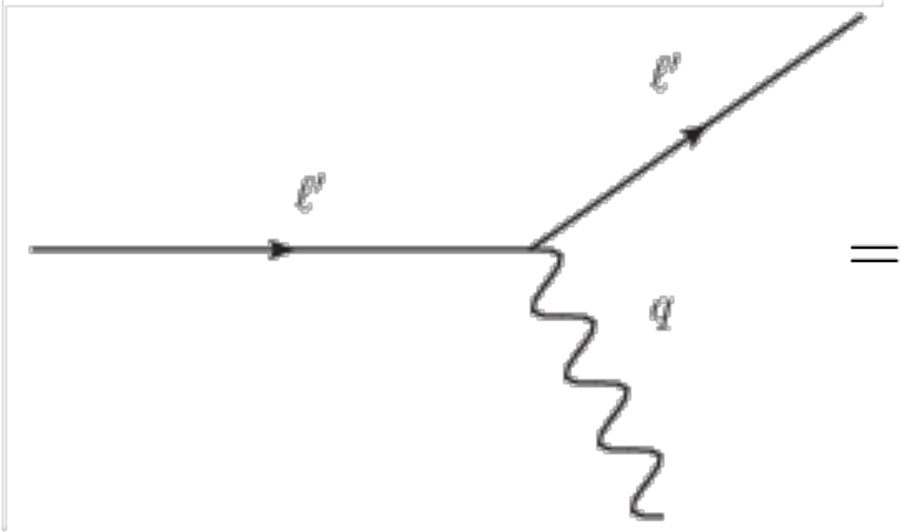
Separate calculation over photon line into leptonic and hadronic parts



$$\frac{d\sigma}{d^3\ell'} \sim \underbrace{L_{\mu\nu}}_{\text{Leptonic Tensor}} \overbrace{W^{\mu\nu}}^{\text{Hadronic Tensor}}$$



Lepton Tensor



$$= i\mathcal{M}_\mu = u_s(\ell)\frac{i(\not{\ell} + m)}{\ell^2 - m^2} ie\gamma^\mu \frac{i(\not{\ell} - m)}{\ell^2 - m^2}\bar{u}_{s'}(\ell')$$

$$L_{\mu\nu} = \frac{1}{2}\sum_s \sum_{s'} (i\mathcal{M}_\mu)(-i\mathcal{M}^*_\nu) = 2[\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu}(\ell \cdot \ell' - m^2)]$$

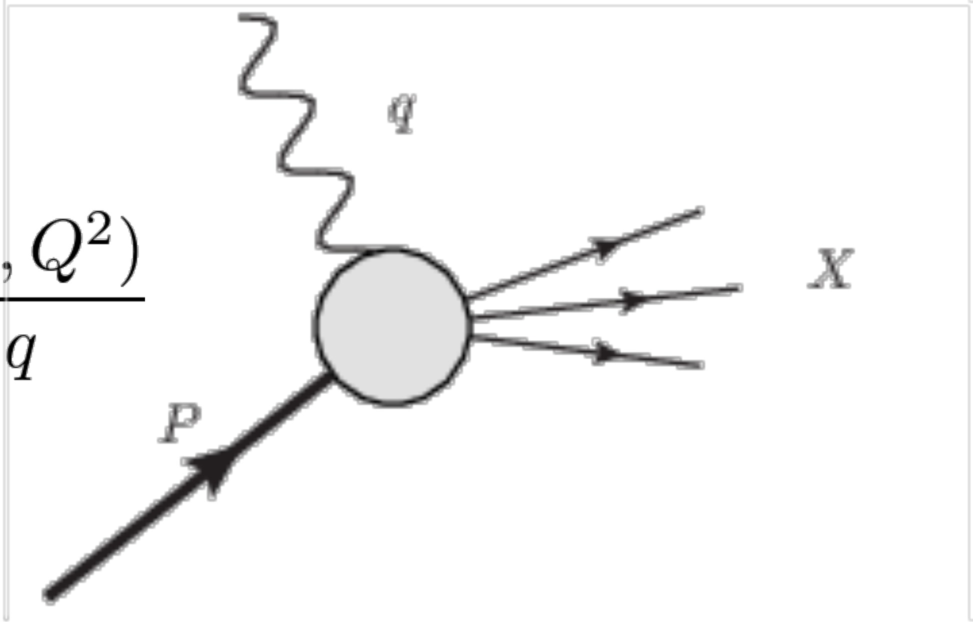
Hadron Tensor

$$x_{\mathrm{Bj}} = \frac{Q^2}{2P\cdot q}$$

$$W^{\mu\nu} \equiv \frac{1}{4\pi}\sum_X \langle P,S|\, j^\mu(0)\,|X\rangle\,\langle X|\, j^\nu(0)\,|P,S\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1(x_{\mathrm{Bj}},Q^2)$$

$$+ \left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)\frac{F_2(x_{\mathrm{Bj}},Q^2)}{P\cdot q}$$



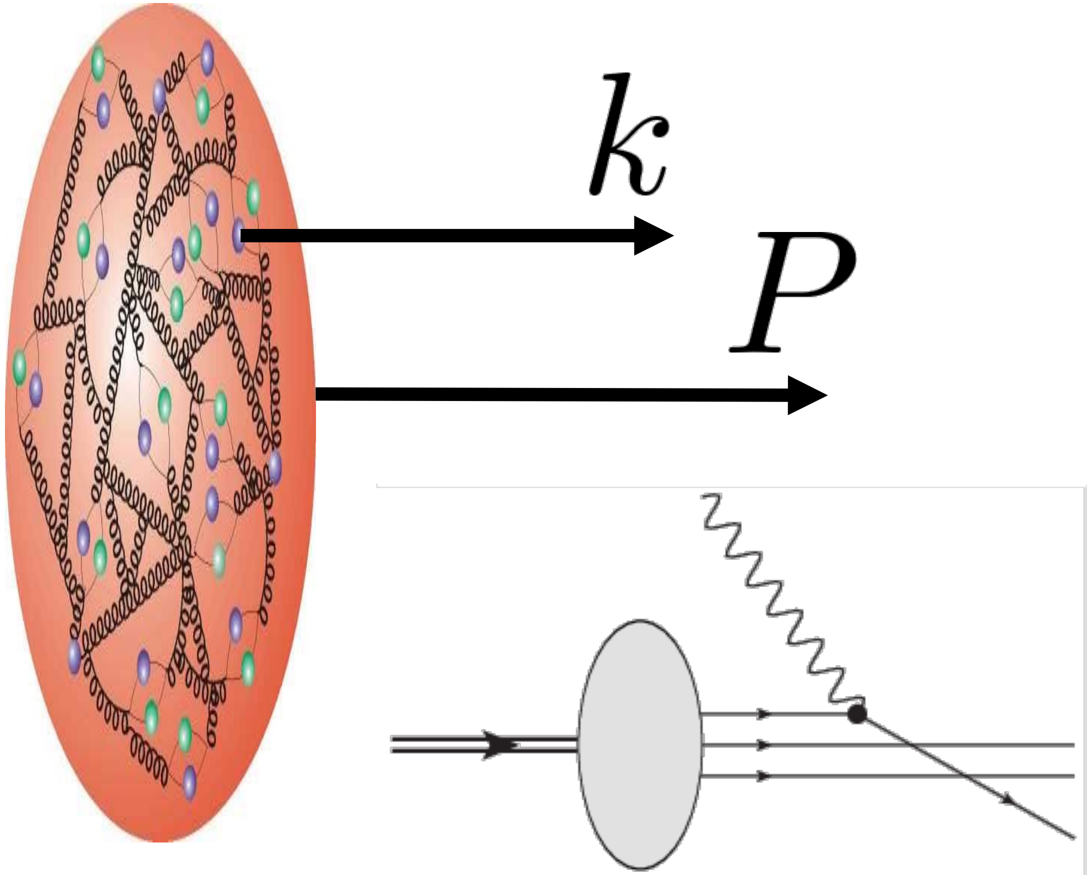
Collinear Factorization

Scattering in Breit (infinite momentum) frame

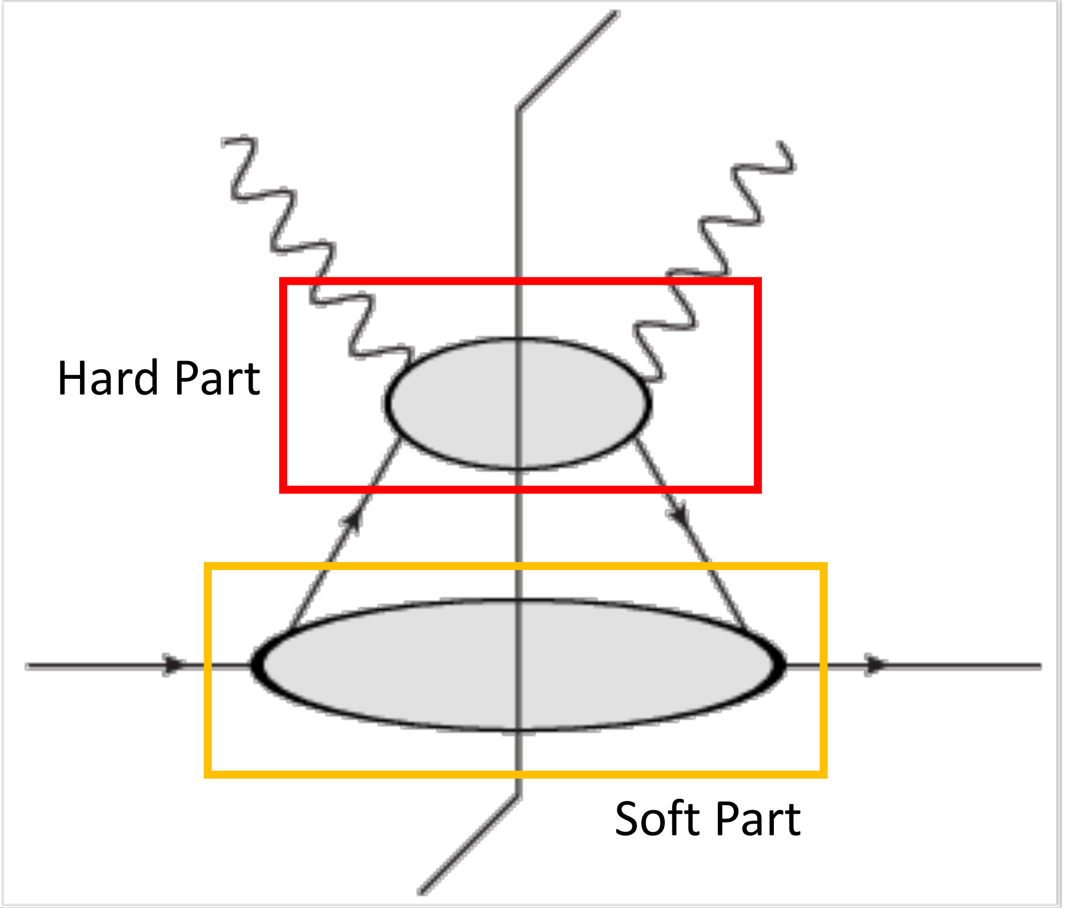
- Parton momentum parallel to proton momentum

$$\xi = \frac{k^+}{P^+}$$

- Partons are essentially non-interacting
  - Imagine virtual photon interacts with one parton (hard part)
  - Other partons (soft part)



Collinear Factorization



$$F(x,Q^2) = \int_x^1 \frac{d\xi}{\xi} \overbrace{\hat{H}\left(\frac{x}{\xi},\frac{\mu}{Q}\right)}^{\text{Hard scattering amplitude}} \underbrace{f(\xi,\mu)}_{\text{PDF}} + O\left(\frac{m}{Q}\right)$$

What is a PDF?

Parton distribution function

$$f_i(\xi) = \int \frac{\mathrm{d}w^-}{4\pi} e^{-i\xi p^+ w^-} \langle N|\bar{\psi}_i(0, w^-, \mathbf{0}_\mathrm{T})\gamma^+\psi_i(0)|N\rangle$$

Non-interacting QCD:

$$\psi_i(x) = \sum_{k,\alpha} b_{k,\alpha}(x^+) u_{k,\alpha} e^{-ik^+x^- + ik_\mathrm{T}\cdot x_\mathrm{T}} + d^\dagger_{k,\alpha}(x^+) u_{k,-\alpha} e^{ik^+x^- - ik_\mathrm{T}\cdot x_\mathrm{T}}$$

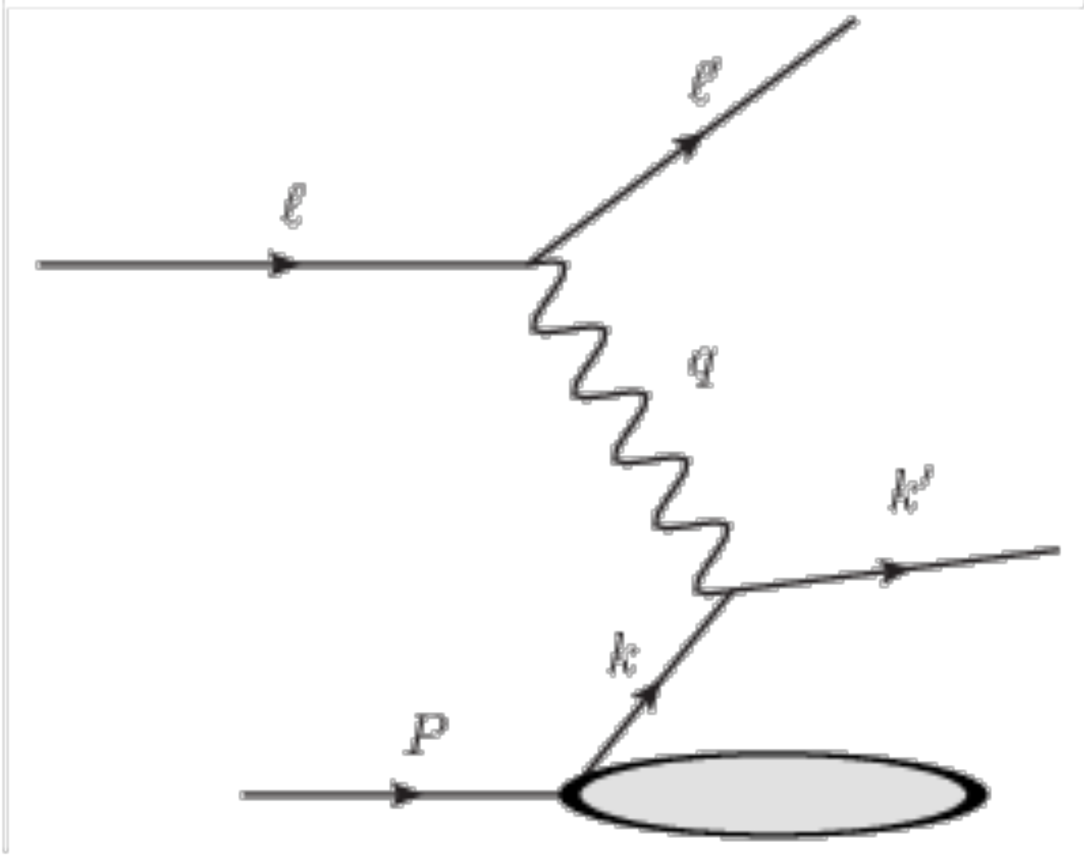
$$f_i(\xi) \sim \sum_\alpha \int \mathrm{d}^2 k_\mathrm{T} \, \langle N| \underbrace{b^\dagger_{k,\alpha} b_{k,\alpha}(\xi p^+, k_\mathrm{T}, \alpha)}_{\text{number operator}} |N\rangle$$

Naïve Parton Model

At leading order:

$$F_1 = \frac{1}{2}\sum_q e_q^2 f_q(x, Q^2)$$

$$F_2 = 2xF_1$$



Agreement with Experiment

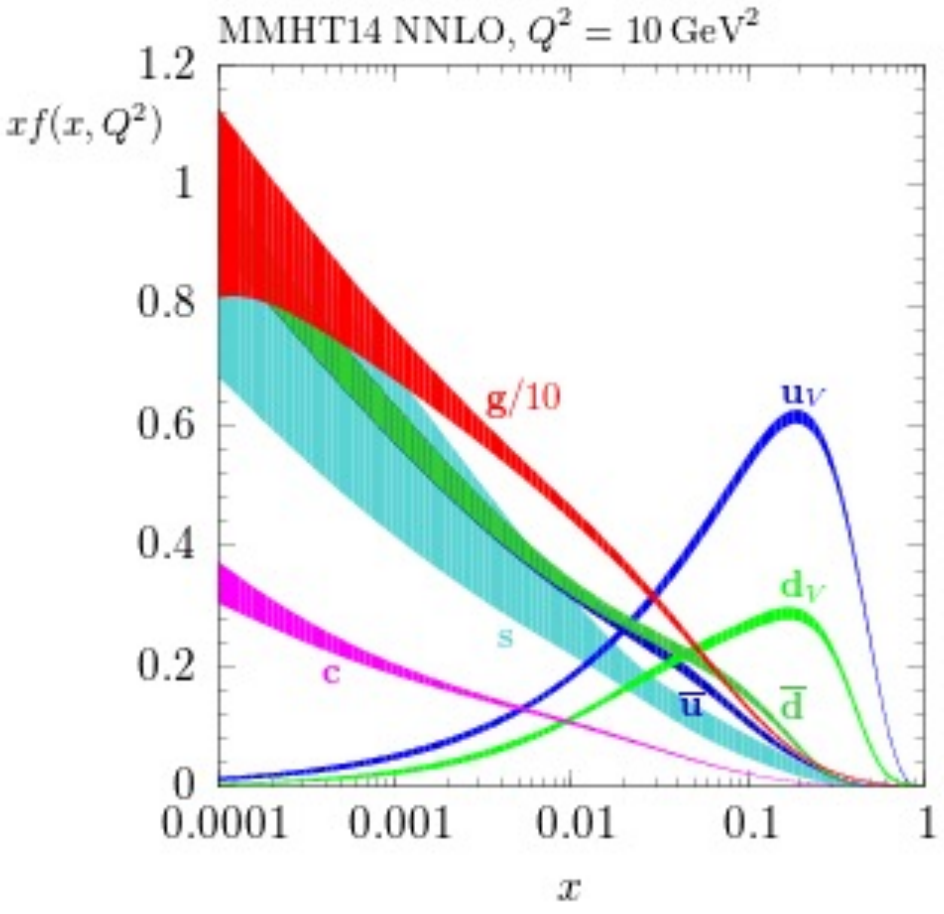
Experimental results allow theorists to work backwards to extract PDFs

- Show that proton is indeed composite

Many questions

- JAM: why is the proton a “spin-1/2” particle?

$$\boxed{\frac{1}{2} = \Delta\Sigma + \Delta G + L}$$



My Work

Polarized PDFs give insight into spin structure of proton

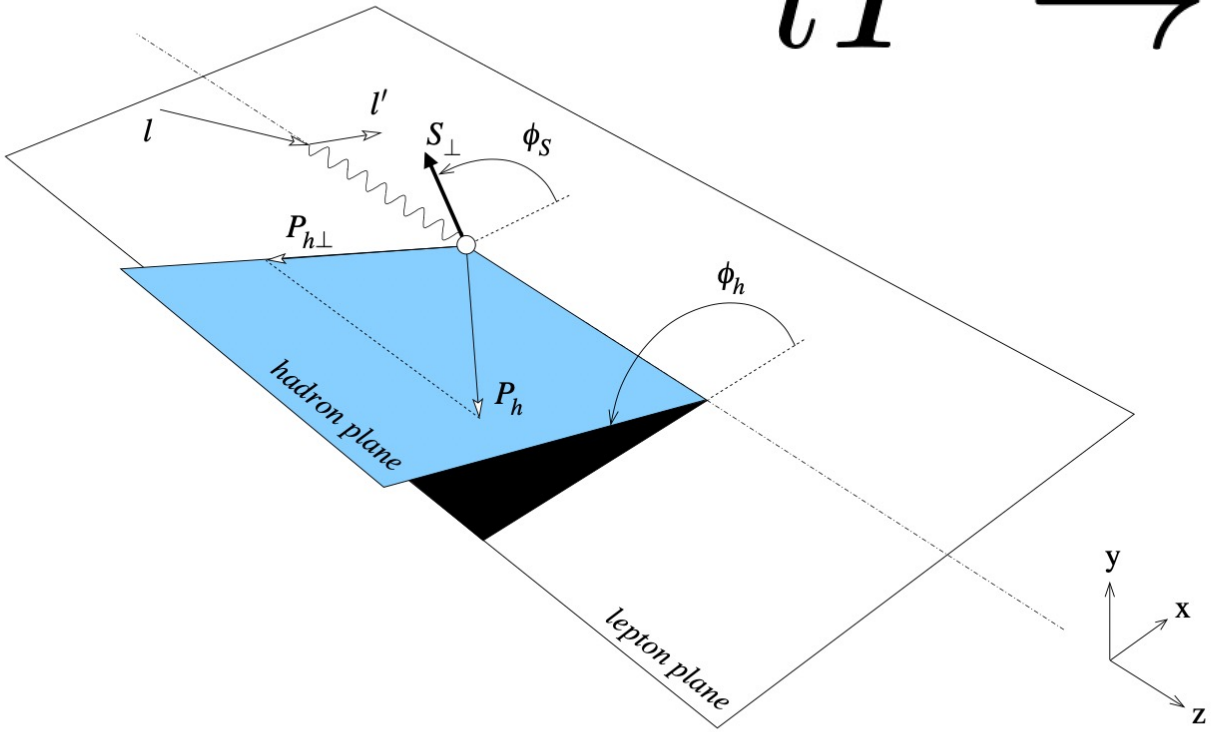
- PDFs inferred from experimental data
- Experiments are lengthy, expensive
- Helpful to have theory serve as solid base for these experiments
- Analyze various scattering processes to understand potential of experiment in determining PDFs

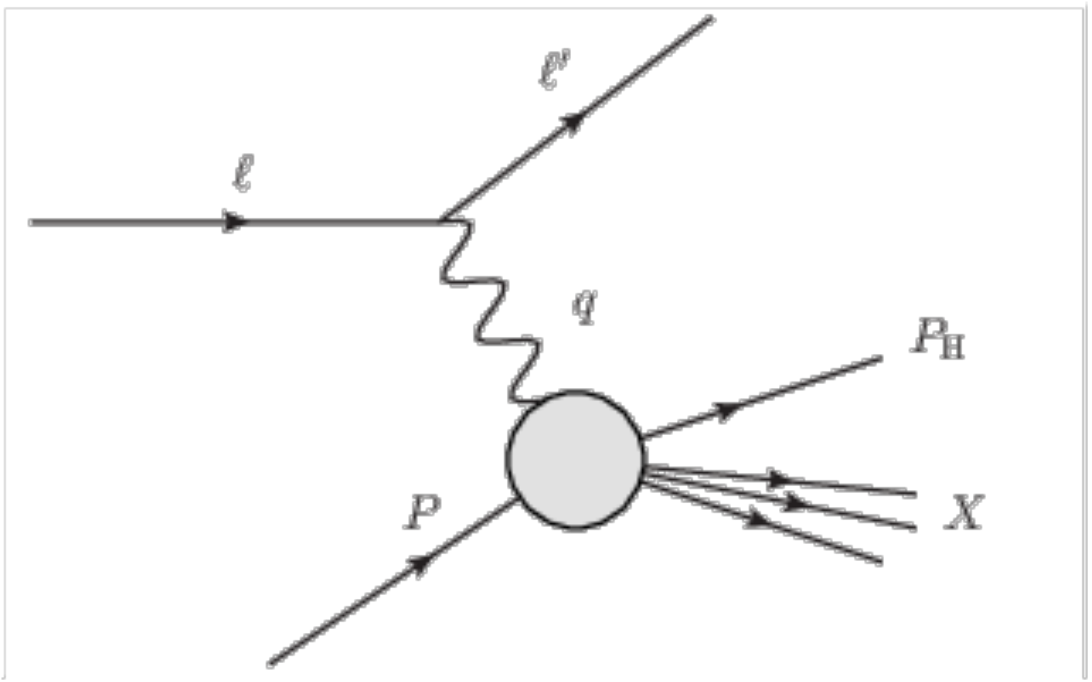
What have I done?

- Looked at semi-inclusive deep inelastic scattering (SIDIS) specifically for polarized gluon PDF (Δg) in cases of large transverse momentum of produced hadron

SIDIS

$$lP \to lhX$$

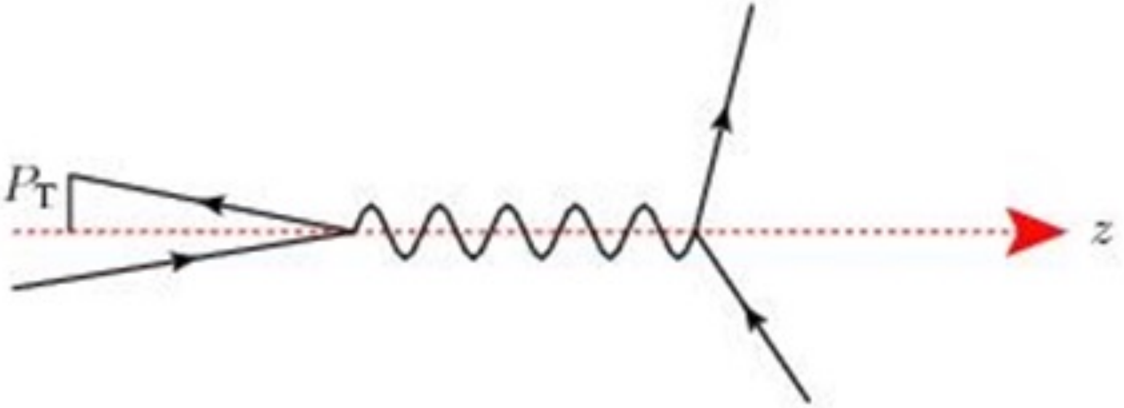




Kinematic Region of Validity

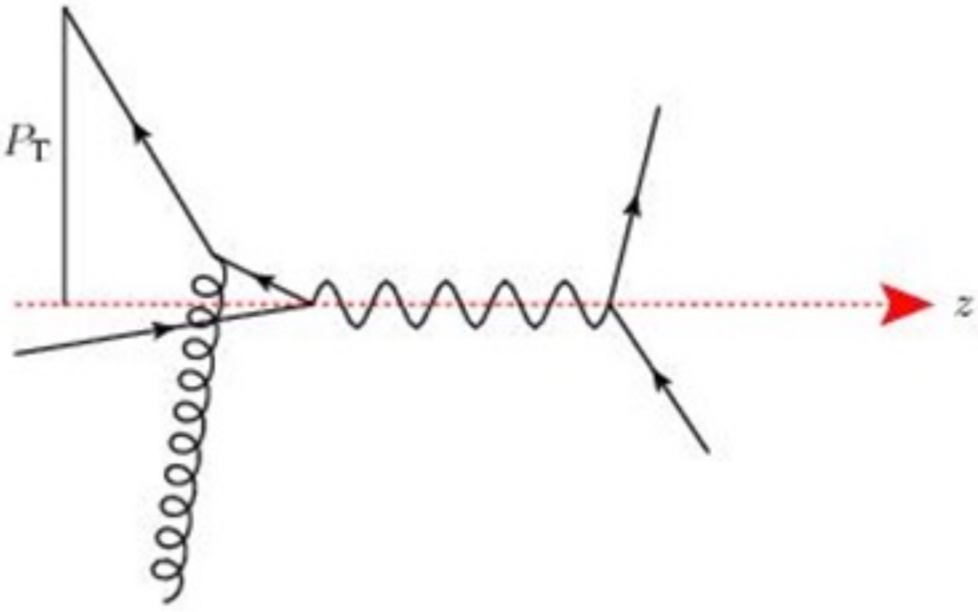
Small intrinsic transverse momentum in initial/final state

- Ignored in this analysis (TMDs)



Large transverse momentum from gluon interaction/production

- Allows access to gluon channel



Unpolarized Spin Products

$$\sum_{\lambda} u(p,\lambda)\bar{u}(p,\lambda) = \not{p} + m$$

$$\sum_{\lambda} \epsilon^{\mu}(p)\epsilon^{*\nu}(p) = -g^{\mu\nu}$$

Polarization

$$u(p,\lambda)\bar{u}(p,\lambda) = \frac{1}{2}(1+\lambda\gamma_5)\not{p}$$

$$\epsilon^{\mu}(p)\epsilon^{*\nu}(p) = \frac{1}{2}\left[-g^{\mu\nu} + \frac{i\lambda}{p\cdot Z}\epsilon^{\mu\nu\alpha\beta}p_\alpha Z_\beta\right]$$

Lepton Tensor

$$L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu} \ell \cdot \ell' + i\epsilon_{\mu\nu\alpha\beta} s^\alpha q^\beta)$$

Assuming that the lepton is longitudinally polarized: $s^\mu = \lambda_l \ell^\mu$

$$L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu} \ell \cdot \ell' - i\lambda_l \epsilon_{\mu\nu\alpha\beta} \ell^\alpha \ell'^\beta)$$

Hadron Tensor

Full decomposition includes 18 structure functions

- Long and ugly
- Not exactly sure what structures contribute

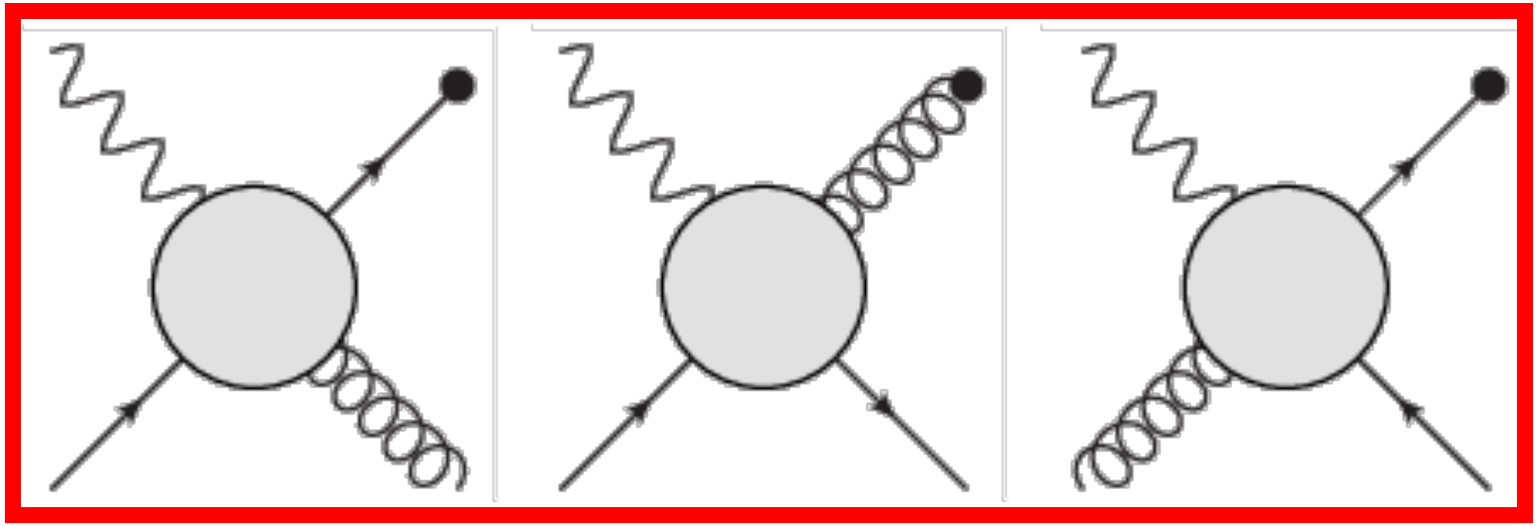
Simpler to calculate directly from squared matrix elements

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\quad \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\qquad + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\qquad + S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\qquad + S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\qquad + |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad\quad + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&\qquad + |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\},
\end{aligned}
$$

Collinear Factorization

$$W^{\mu\nu} = \int_z^1 \frac{\mathrm{d}\zeta}{\zeta^2} \int_x^1 \frac{\mathrm{d}\xi}{\xi} \boxed{\hat{W}^{\mu\nu}} \boxed{f_{i/P}(\xi) D_{H/j}(\zeta)} \longleftarrow \boxed{\mathcal{JAM}}$$

$$\boxed{\hat{W}^{\mu\nu} = \sum_{i,j} \mathcal{M}^{\mu} \mathcal{M}^{\dagger\nu}}$$



Kinematic Variables

Defined in terms of Lorentz invariant quantities

1. Hadronic Variables

$$x = \frac{Q^2}{2P \cdot q} \qquad y = \frac{P \cdot q}{P \cdot \ell}$$

$$z = \frac{P_{\mathrm{H}} \cdot P}{P \cdot q} \qquad q_{\mathrm{T}} = \frac{P_{\mathrm{H,T}}}{z}$$

2. Partonic Variables

$$p = \xi P \qquad k_1 = \frac{P_{\mathrm{H}}}{\zeta}$$

$$\hat{x} = \frac{Q^2}{2p \cdot q} = \frac{x}{\xi} \qquad \hat{z} = \frac{k_1 \cdot p}{p \cdot q}$$

Momentum Fractions

$$
\begin{aligned}
d\Pi &= \frac{dk_2}{(2\pi)2k_2^0}\delta^{(2)}(q+p-k_1-k_2) \\
&= \frac{2\pi\hat{x}}{Q^2}\delta\left((1-\hat{x})(1-\hat{z}) - \frac{\hat{x}\hat{z}q_{\mathrm{T}}^2}{Q^2}\right)
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{cases}
\zeta &= z\left(1 - \frac{\hat{x}}{\hat{x}-1}\frac{q_{\mathrm{T}}^2}{Q^2}\right) \\
\xi_{\min} &= x\left(1 + \frac{z}{(1-z)}\frac{q_{\mathrm{T}}^2}{Q^2}\right)
\end{cases}
$$

Measurables

$$\frac{d\sigma}{dx dy dz dq_{\mathrm{T}}} = \frac{1}{2}\left[\frac{d\sigma^{\overrightarrow{\Rightarrow}}}{dx dy dz dq_{\mathrm{T}}} + \frac{d\sigma^{\overleftarrow{\Leftarrow}}}{dx dy dz dq_{\mathrm{T}}}\right]$$

$$\frac{d\Delta\sigma}{dx dy dz dq_{\mathrm{T}}} = \frac{1}{2}\left[\frac{d\sigma^{\overrightarrow{\Rightarrow}}}{dx dy dz dq_{\mathrm{T}}} - \frac{d\sigma^{\overleftarrow{\Leftarrow}}}{dx dy dz dq_{\mathrm{T}}}\right]$$

Cross Sections

Double Spin Asymmetry

$$A_{\mathrm{LL}} = \frac{d\Delta\sigma}{d\sigma}$$

Squared Matrix Elements (Unpolarized)

Born A

$$\frac{\mathrm{d}\sigma_{\gamma q\to qg}}{\mathrm{d}q_T\ldots} = \frac{16\big[Q^4\big(\hat{x}^2\hat{z}^2\big(y^2-2y+2\big)+y^2-2y+2\big)+8Q^2\hat{x}^2\hat{z}^2q_T^2(1-y)+\hat{x}^2\hat{z}^2q_T^4\big(y^2-2y+2\big)\big]}{3\hat{x}y^2(\hat{x}-1)(Q^2\hat{z}-Q^2-\hat{z}q_T^2)}$$

Born B

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\gamma q\to gq}}{\mathrm{d}q_T\ldots} = -16\Big[&Q^4\big(\hat{x}^2\hat{z}^2\big(y^2-2y+2\big)-2\hat{x}^2\hat{z}y^2+2\hat{x}^2y^2-4\hat{x}\hat{z}(1-y)-2\hat{x}y^2+2y^2-4y+4\big)\\&+Q^2q_T^2\big(\hat{x}^2\hat{z}^2(1-y)+2\hat{x}^2\hat{z}y^2-2\hat{x}\hat{z}\big(y^2-2y+2\big)\big)\\&+2q_T^4\hat{x}^2\hat{z}^2\big(y^2-2y+2\big)\Big]\Big/3y^2(\hat{x}-1)\big(Q^2\hat{x}\hat{z}-Q^2\hat{x}+Q^2-\hat{x}\hat{z}q_T^2\big)\end{aligned}$$

Born C

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\gamma g\to q\bar{q}}}{\mathrm{d}q_T\ldots} = -2Q^2\Big[&Q^4\big(2\hat{x}^2\hat{z}^2\big(y^2-2y+2\big)-2\hat{x}^2\hat{z}y^2+2\hat{x}^2y^2-4\hat{x}\hat{z}(1-y)-2\hat{x}y^2+y^2-2y+2\big)\\&+Q^2q_T^2\big(16\hat{x}^2\hat{z}^2(1-y)+2\hat{x}^2\hat{z}y^2-2\hat{x}\hat{z}\big(y^2-2y+2\big)\big)\\&+2q_T^4\hat{x}^2\hat{z}^2\big(y^2-2y+2\big)\Big]\Big/\hat{x}y^2\big(Q^2\hat{z}-Q^2-\hat{z}q_T^2\big)\big(Q^2\hat{x}\hat{z}-Q^2\hat{x}+Q^2-\hat{x}\hat{z}q_T^2\big)\end{aligned}$$

Squared Matrix Elements (Polarized)

Born A

$$\frac{\mathrm{d}\Delta\hat{\sigma}_{\gamma q\to qg}}{\mathrm{d}q_{\mathrm{T}}\ldots} = \frac{16(2-y)\left(\hat{x}^2\hat{z}^2q_T^4 - \hat{x}^2\hat{z}^2Q^4 - Q^4\right)}{3\hat{x}y(\hat{x}-1)(Q^2\hat{z} - Q^2 - \hat{z}q_T^2)}$$

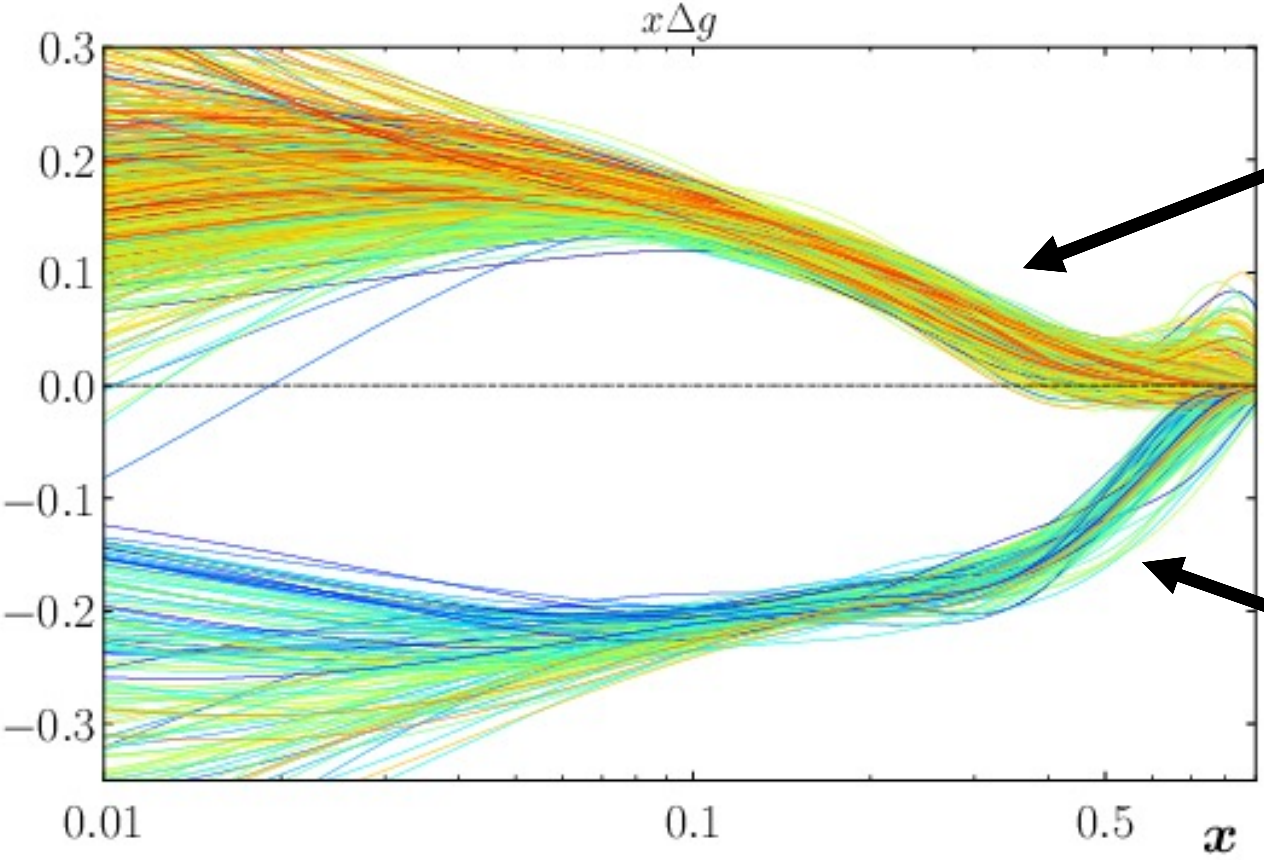
Born B

$$\frac{\mathrm{d}\Delta\hat{\sigma}_{\gamma q\to gq}}{\mathrm{d}q_T\ldots} = \frac{16\hat{x}(2-y)\left(Q^4\hat{x}\hat{z}^2 - 2Q^4\hat{x}\hat{z} + 2Q^4 - 2Q^2\hat{x}\hat{z}q_T^2 + 2Q^2\hat{z}q_T^2 - \hat{x}\hat{z}^2q_T^4\right)}{3y(\hat{x}-1)(Q^2\hat{x}\hat{z} - Q^2\hat{x} + Q^2 - \hat{x}\hat{z}q_T^2)}$$

Born C

$$\frac{\mathrm{d}\Delta\hat{\sigma}_{\gamma g\to q\bar{q}}}{\mathrm{d}q_T\ldots} = \frac{2Q^2(2-y)\left(2Q^4\hat{x}^2\hat{z}^2 - 2Q^4\hat{x}^2\hat{z} + 2Q^4\hat{x} - Q^4 - 2Q^2\hat{x}^2\hat{z}q_T^2 + 2Q^2\hat{x}\hat{z}q_T^2 - 2\hat{x}^2\hat{z}^2q_T^4\right)}{\hat{x}y(\hat{z}q_T^2 + Q^2 - Q^2\hat{z})(Q^2\hat{x}\hat{z} - Q^2\hat{x} + Q^2 - \hat{x}\hat{z}q_T^2)}$$

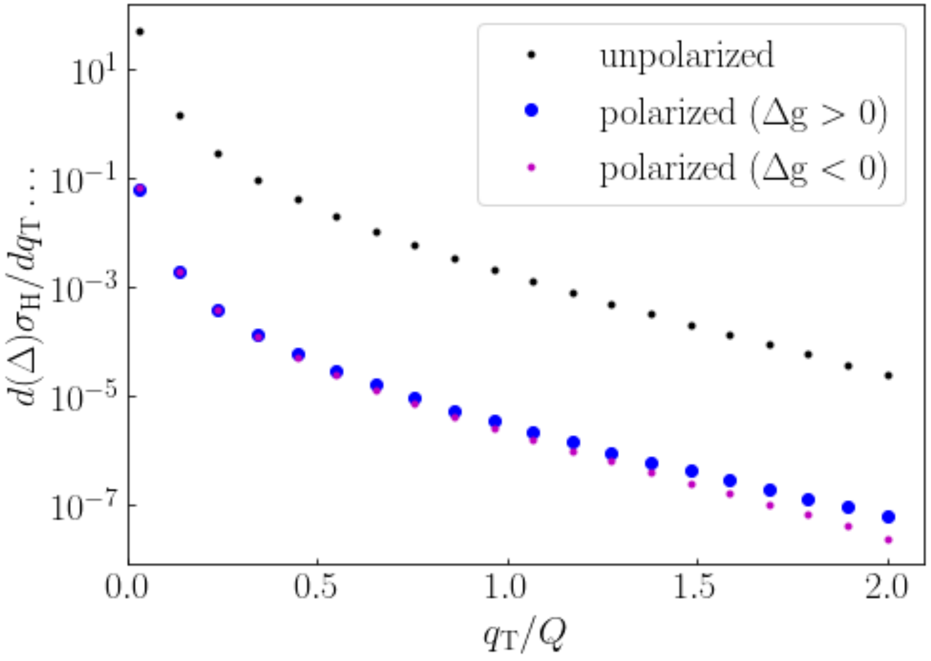
Gluon PPDF

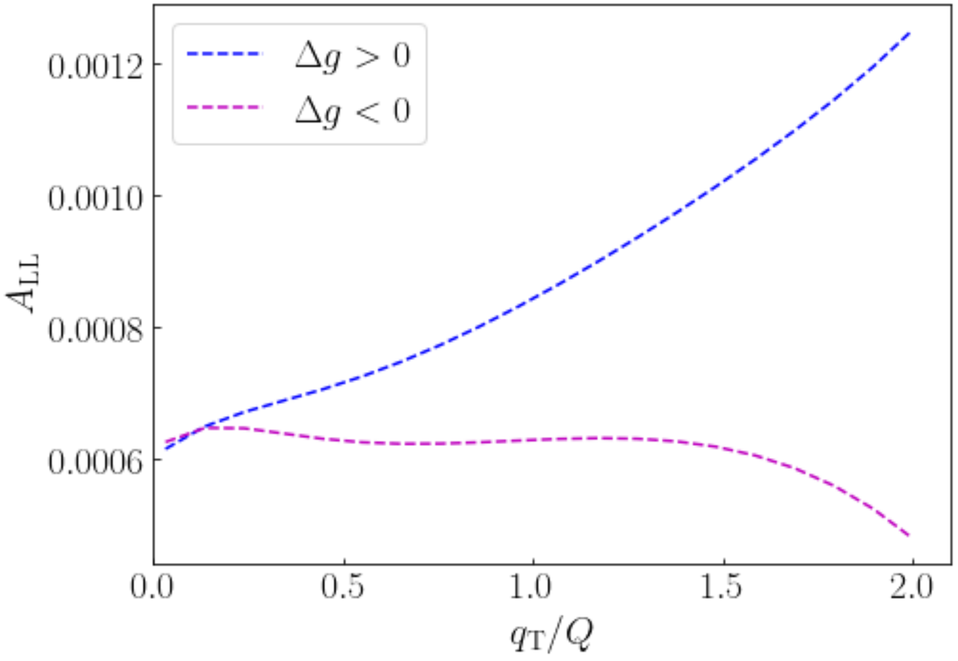


$$\Delta g > 0$$

$$\Delta g < 0$$

Test Plots





<u>Kinematics</u>

$\sqrt{s} = 140$ GeV

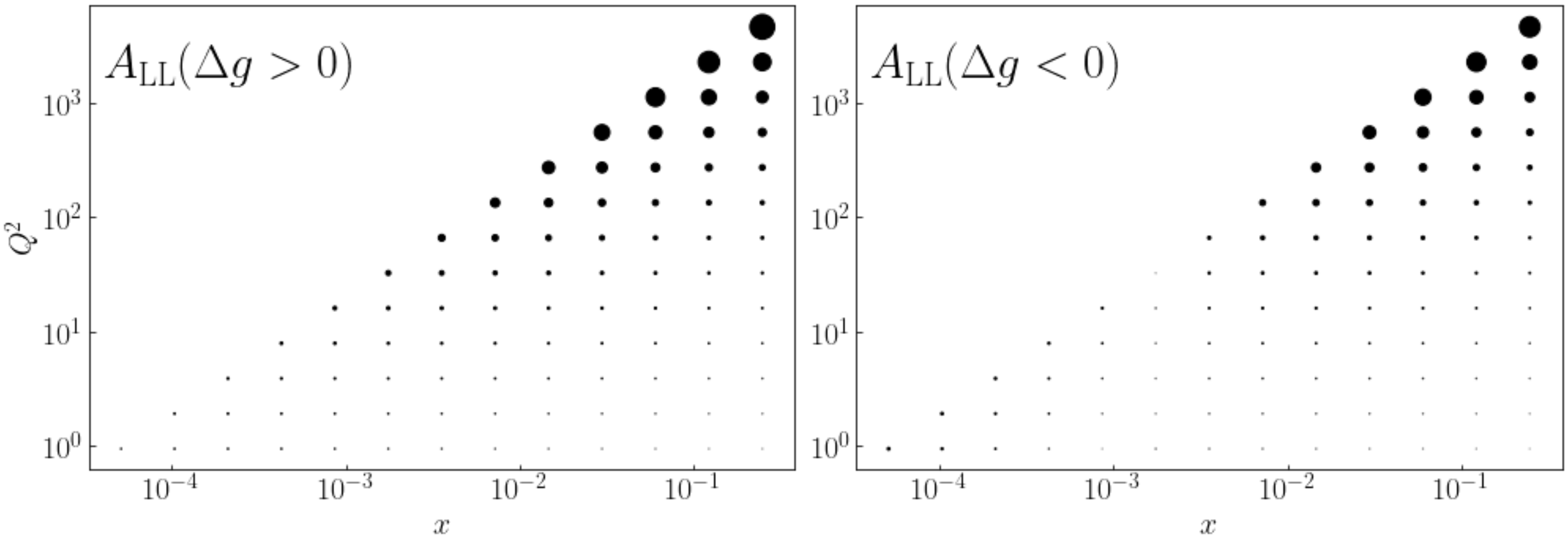
$Q^2 = 9.0$ GeV

$x = 0.1$

$z = 0.3$

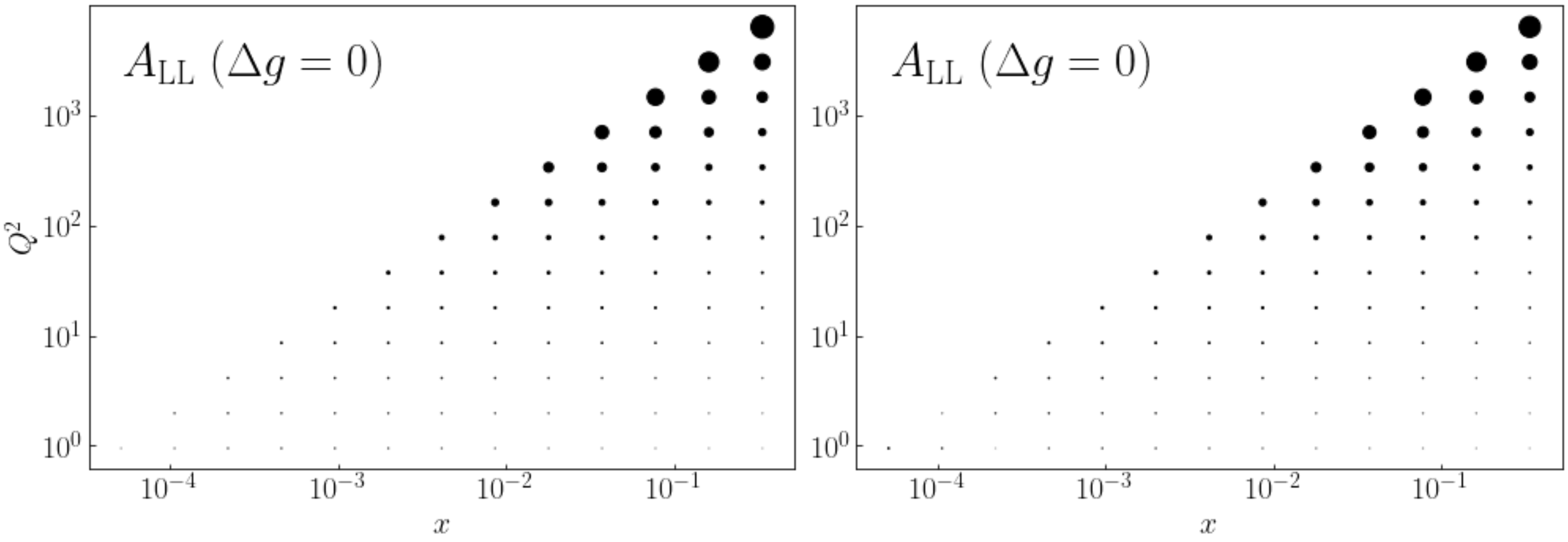
$q_{\mathrm{T}} \in \{0.1\ \mathrm{GeV}, 2Q\}$

Asymmetry across phase space

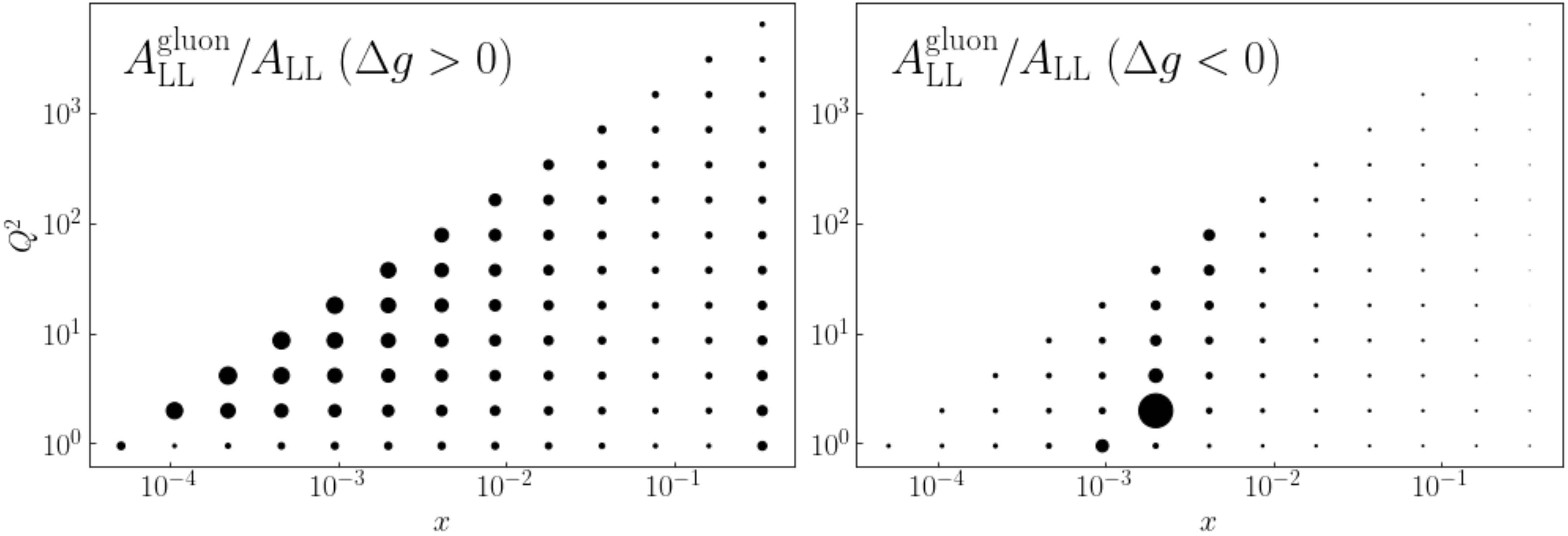


$z = 0.3$ and $q_{\rm T}/Q = 1$

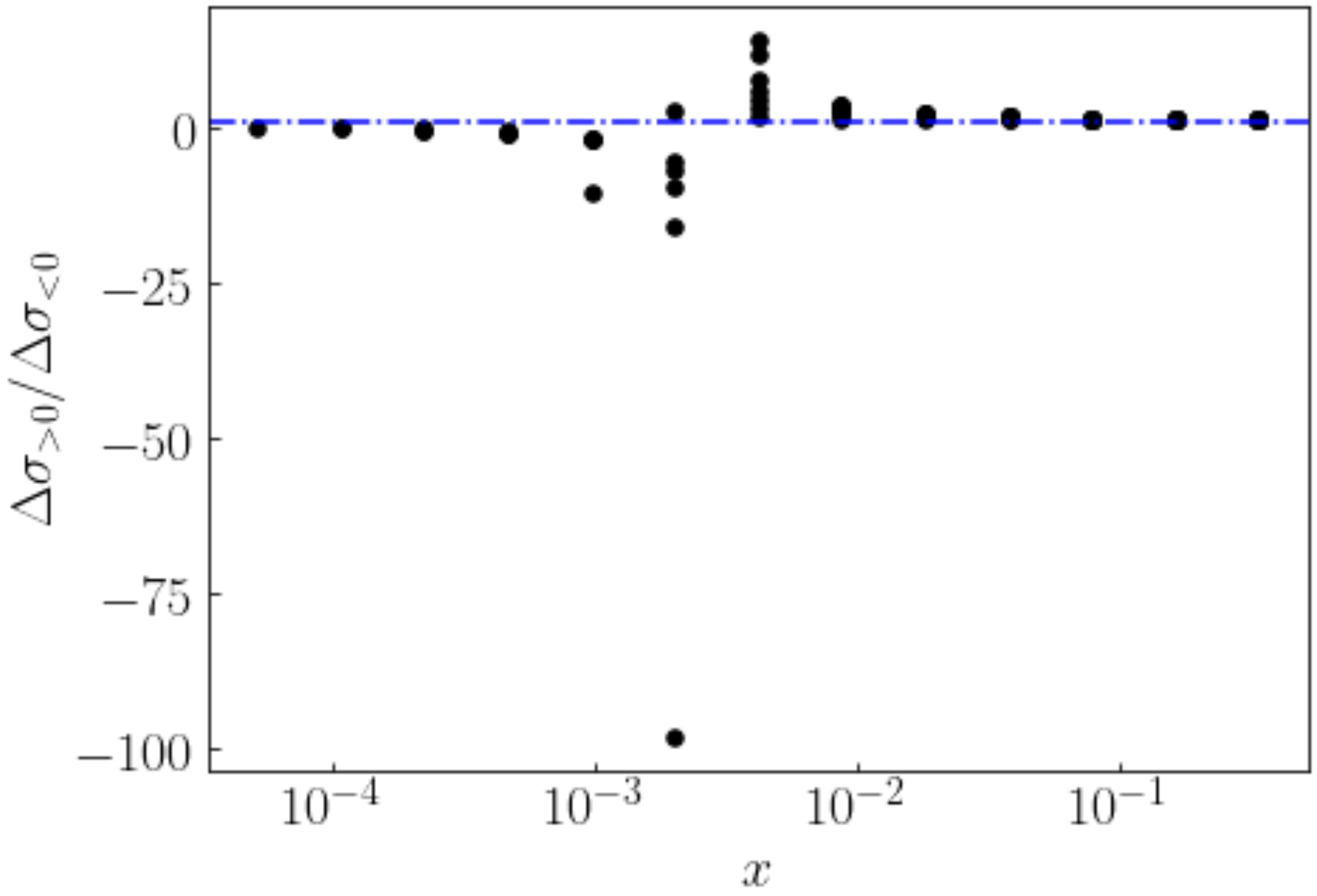
Quark contribution



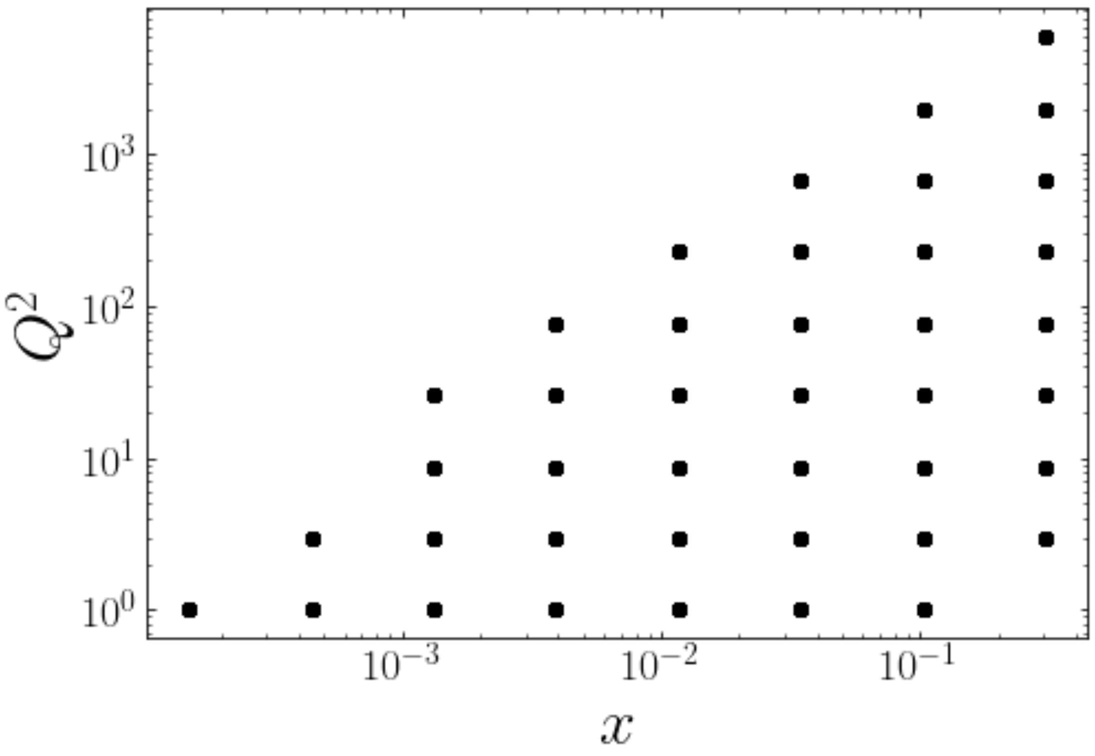
Gluon contribution

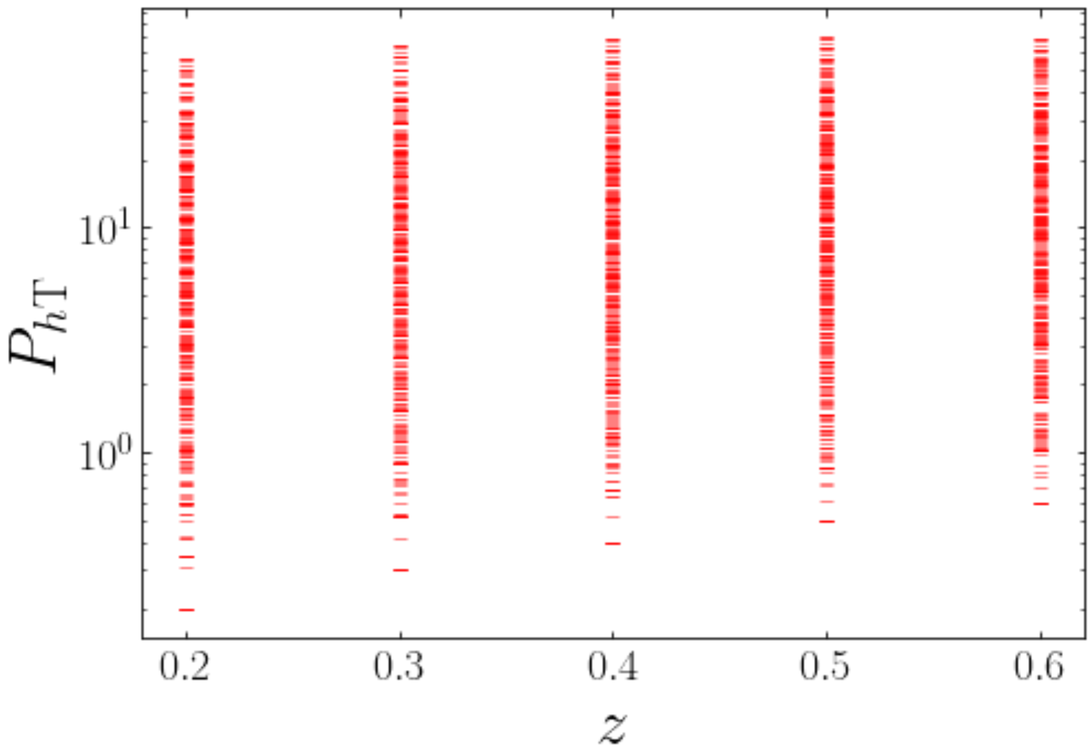


Comparison between gluon solutions



Project State





Recap

- REUs and other research internships are great opportunities for
  - Starting to do research
  - Exposure to active areas of physics
  - Learning new physics in general
  - Making contacts with other people
- Talked about historical development of proton as composite particle
- Discussed how theoretical nuclear physicists learn about proton through PDFs
- Covered how theoretical studies give insight into potential utility of experimental data
  - Especially related to SIDIS to access gluon polarization

Questions?