Statistical Mechanics

Based on lectures by Dr. Suvankar Dutta
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These notes are not endorsed by the lecturers, and I have modified them (often significantly) after lectures. They are nowhere near accurate representations of what was actually lectured, and in particular, all errors are almost surely mine.[1]

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[1] This is how Dexter Chua describes his lecture notes from Cambridge. I could not have described mine in any better way.

1 Mathematical Background

1.1 Dirac Delta Function

Definition 1 (Heaviside Step Function)**.**

$$\Theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

Definition 2 (Dirac Delta Function)**.** The set of "functions" $\delta_\epsilon : \mathbb{R} \to \mathbb{R}$ that satisfy

$$\lim_{\epsilon \to 0^+} \delta_\epsilon(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$$\lim_{\epsilon \to 0^+} \int_{-\infty}^{\infty} \delta_\epsilon(x)\ dx = 1$$

Example 1. The Gaussian

$$\delta_\epsilon(x) = \frac{1}{\epsilon\sqrt{\pi}} e^{-\frac{x^2}{\epsilon^2}}$$

is a Dirac delta function.

Notation 1.

$$\int_{-\infty}^{\infty} \delta(x)\ dx \equiv \lim_{\epsilon \to 0^+} \int_{-\infty}^{\infty} \delta_\epsilon(x)\ dx$$

Any integral involving the Dirac delta function must be interpreted as the corresponding integral of δ_ϵ as $\epsilon \to 0^+$.

Theorem 1 (Properties of the delta function)**.**

1. $\delta(-x) = \delta(x)$
2. $\delta(ax) = \frac{1}{|a|}\delta(x),\ a \neq 0$
3. $\int_{-\infty}^{\infty} \delta(x-c) f(x)\ dx = f(c)$, for any function $f : \mathbb{R} \to \mathbb{R}$.
4. $\delta(f(x)) = \sum_i \frac{1}{|f'(x_i)|}\delta(x - x_i)$ if $f(x_i) = 0$

1.2 Gaussian Integral

Theorem 2.

$$\int_{-\infty}^{\infty} e^{-x^2}\ dx = \sqrt{\pi}$$

Theorem 3 (Important Results of Gaussian integral)**.**

1. $\int_{-\infty}^{\infty} e^{-ax^2+bx}\ dx = e^{\frac{b^2}{4a}}\sqrt{\frac{\pi}{a}}$

Theorem 4. In general,

$$\int_0^{\infty} x^n e^{-ax^2}\ dx \propto a^{-\frac{n+1}{2}}$$

and in particular,

$$\int_0^{\infty} x^n e^{-ax^2}\ dx = \begin{cases} \frac{(n-1)(n-3)\cdots 3\cdot 1}{2^{\frac{n}{2}+1} a^{\frac{n}{2}}}\sqrt{\frac{\pi}{a}}, & n \text{ even} \\ \frac{[\frac{1}{2}(n-1)]!}{2a^{\frac{n+1}{2}}}, & n \text{ odd} \end{cases}$$

2 Ensemble

Definition 3 (Macroscopic variables)**.** The variables of a system that can be controlled externally or determined experimentally. They are also called thermodynamic variables.

For e.g., volume, pressure, temperature, etc.

Definition 4 (Microscopic variables)**.** These parameters are not under any control.

For e.g., the coordinates of a molecule of a gaseuos system.

Definition 5 (Ensemble)**.** Set of identical copies of the same system which are same at the macroscopic level but different at the microscopic level.