Statistical Mechanics

Based on lectures by Dr. Suvankar Dutta
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These notes are not endorsed by the lecturers, and I have modified them (often significantly) after lectures. They are nowhere near accurate representations of what was actually lectured, and in particular, all errors are almost surely mine.[1]

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[1] This is how Dexter Chua describes his lecture notes from Cambridge. I could not have described mine in any better way.

1 Ensemble

Definition 1 (Macroscopic variables)**.** The variables of a system that can be controlled externally or determined experimentally. They are also called thermodynamic variables.

For e.g., volume, pressure, temperature, etc.

Definition 2 (Microscopic variables)**.** These parameters are not under any control.

For e.g., the coordinates of a molecule of a gaseuos system.

Definition 3 (Ensemble)**.** Set of identical copies of the same system which are same at the macroscopic level but different at the microscopic level.

1.1 Phase Space

In the case of 1D motion of a single particle, the phase space is represented by the p, q–plane, where p, q represent the momentum and position of the paticle, respectively. The collection of points (q, p) that are possible classical states of the particle is called the phase space trajectory(or surface) of the particle. This trajectory can be found by

$$E = \mathcal{H}(q, p) = \frac{p^2}{2m} + V(q)$$

where E represents the classically allowed energy states of the system.

Example 1 (Classical Harmonic Oscillator)**.** In this case E is a constant of motion and $V(q) = -\frac{1}{2}m\omega^2 q^2$. So we have

$$\frac{p^2}{2m} - \frac{1}{2}m\omega^2 q^2 = E$$

This is an ellipse in the p, q–plane. The direction of the trajectory depends on the boundary conditions.

> Thus, the phase space trajectory tells us about the possible states of the particle(or, system) and how one state evolves into another.

Now, we extend this idea to N particles in 3 dimensions. This time instead of just two quantities p, q, we require the 3 position and 3 momentum coordinates of each of the N particles. Provided there are no additional constrains, in general, we require a $6N$ dimensional phase space to describe the states of this system. Thus, $E = \mathcal{H}$ represents a $6N - 1$ dimensional hypersurface in this phase space.

Note. $\mathcal{H}$ is a function of all the $6N$ positions and momenta.

Note. If E is piecewise continuous, we get $6N$ dimensional regions in the phase space.