Inference of Seismogenic Stresses using Earthquake Slip Data

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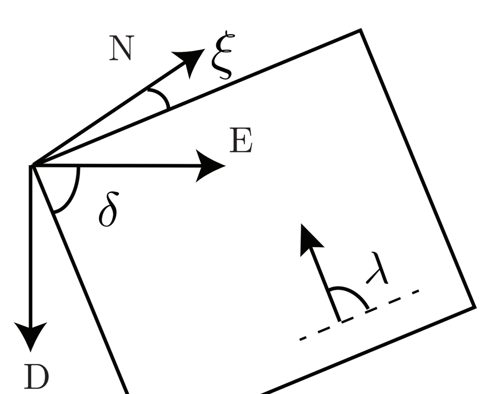
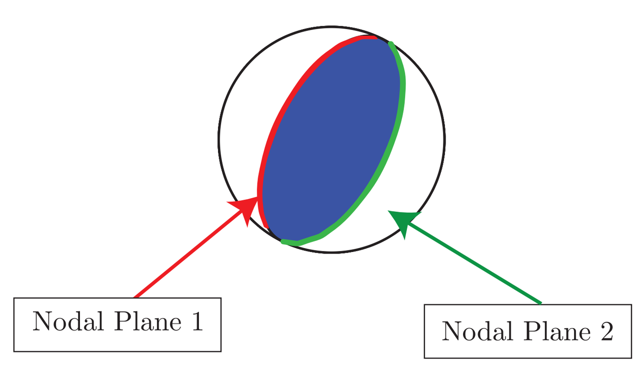
Abstract

The quantification of seismogenic stresses is crucial in the understanding earthquake dynamics and has a direct implication for earthquake hazard forecasting; however, direct measurement of seismogenic stresses is exceedingly rare. Focal mechanisms have been used to infer the state of stress in the upper crust and are most commonly used to constrain the orientations of the principal stress axes, with the relative magnitudes of the principal stresses less well constrained by focal mechanisms (e.g., Michael, 1984; Arnold & Townend, 2007). Traditional estimation methods have relied on a least-squares-based approach (e.g., Michael, 1984), seeking to find the stress solution with the highest likelihood. Recently, Bayesian-based estimation methods have been proposed (e.g., Arnold & Townend, 2007; Styron & Hetland, 2015) that have allowed for the inclusion of a-priori information, which can account for observational errors in the focal mechanisms along with providing robust estimates on the uncertainties of the posterior distributions. We develop a Markov-Chain Monte Carlo (MCMC) method to determine posterior distributions of stress parameters on groups of clustered focal mechanisms and then apply a Bayes' Information Criterion in order to constrain the state of stress and stress heterogeneity in a given region.

Focal Mechanisms



Fault Plane Representation



Focal Mechanism

Tensor representation of the mechanism of an earthquake graphically depicted as a lower hemispherical projection of the quadrants of motion. Focal Mechansims are determined by inverting seismic waveform data.

Physically there is only one fault plane, however, there are two possible nodal plane solutions for a given event as there is ambiguity in identifying the fault plane for which slip occurred from an orthogonal, auxiliary plane.

Focal mechanisms are parameterized by three angular values:

$$\left.\begin{array}{l}\text{1) Nodal plane strike: } \xi \\ \text{2) Nodal plane dip: } \delta \\ \text{3) Slip (rake) on nodal plane: } \lambda\end{array}\right\} \boxed{\text{Data: } \vec{d} = (\xi, \delta, \lambda)}$$

References

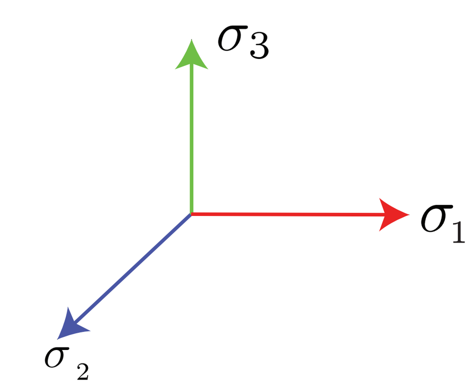
Arnold, R., & Townend, J. (2007). A Bayesian approach to estimating tectonic stress from seismological data. Geophysical Journal International, 170(3), 1336–1356. https://doi.org/10.1111/j.1365-246X.2007.03485.x

Michael, A. J., (1984). Determination of stress from slip data: faults and folds. J.Geophys. Res., 89, B13, 11,517-11,526, doi: 10.1029/JB089iB13p11517.

Styron, R. H., & Hetland, E. A. (2015). The weight of the mountains: Constraints on tectonic stress, friction, and Fluid pressure in the 2008 Wenchuan earthquake from estimates of topographic loading. Journal of Geophysical Research: Solid Earth https://doi.org/10.1002/2014JB011338.

Inversion

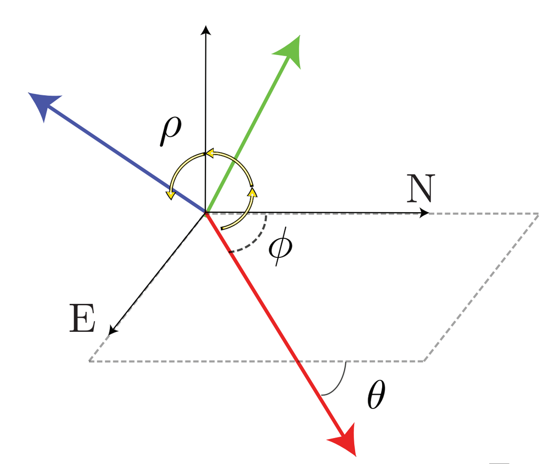
Principal Coordinate System



$$\sigma = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_1(R+\Delta-\Delta R) & 0 \\ 0 & 0 & \sigma_1 R \end{pmatrix}$$

(a)

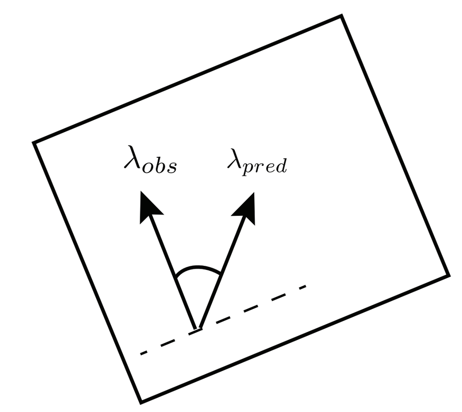
Geographic Coordinate System



$$\sigma_{geo} = \Lambda(\phi,\theta,\rho)\,\sigma\,\Lambda(\phi,\theta,\rho)^{\mathbf{T}}$$

(b)

Fault Plane Coordinate System



$$\sigma_{fault} = \Lambda(\xi,\delta)\sigma\Lambda(\xi,\delta,)^{\mathbf{T}}$$

(c)

Figure 1: Coordinate System Transformations

Figure 1(a): Stress tensor in the principal coordinate system. Figure 1(b): Stress tensor rotated into the geographic coordinate system using the Euler angles (ϕ,θ,ρ) Figure 1(c): Stress tensor rotated onto the fault plane using the Euler angles corresponding to the strike, ξ , and dip, δ, of the fault plane, which results in a predicted slip direction

Model Parameters

Stress Orientation Parameters: Set of Euler Angles $(\phi,\theta,\rho) \in$ S3 where ϕ is the trend of the most compressive stress (σ_1) measured clockwise relative to North, θ is the plunge of σ_1 and is measured downward from the horizontal, and ρ is measured counter-clockwise about σ_1.

Stress Magnitude Parameters: The set of stress ratios $(R , \Delta) \in$ [0, 1] where $R = \frac{\sigma_3}{\sigma_1}$ and $\Delta = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}$

We seek to determine the set of model parameters which minimize the residual between the observed rake and the slip rake predicted for a given stress tensor and fault geometry. We assume that a fault will slip in the direction of the maximum shear stress resolved on that fault. A Markov-Chain Monte Carlo is used to estimate the model parameters by performing two coupled random walks, one on a hypersphere of the Euler angles describing the orientations of the principal stresses, and one in a bounded plane of the magnitudes of the relative stress magnitudes. A trial stress tensor is either accepted or rejected as a sample of the posterior in accordance to the Metropolis algorithm, using a Von-Mises Fisher distribution as the likelihood function of the slip rake.

Likelihood Function

$$\mathcal{L} = A\exp(\sum_{n=1}^{N}\kappa_n\cos(\lambda_n^{(obs)} - \lambda_n^{(pred)}))$$

where κ is a concentration parameter, and A is a normalization constant`

κ can be related to the uncertainty of an angular parameter through the relation

$$Var(\lambda^{(obs)}) = -2\ln(\frac{I_1(\kappa)}{I_0(\kappa)})$$

where I is the modified Bessel function

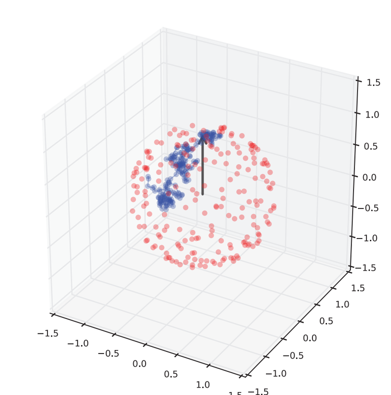
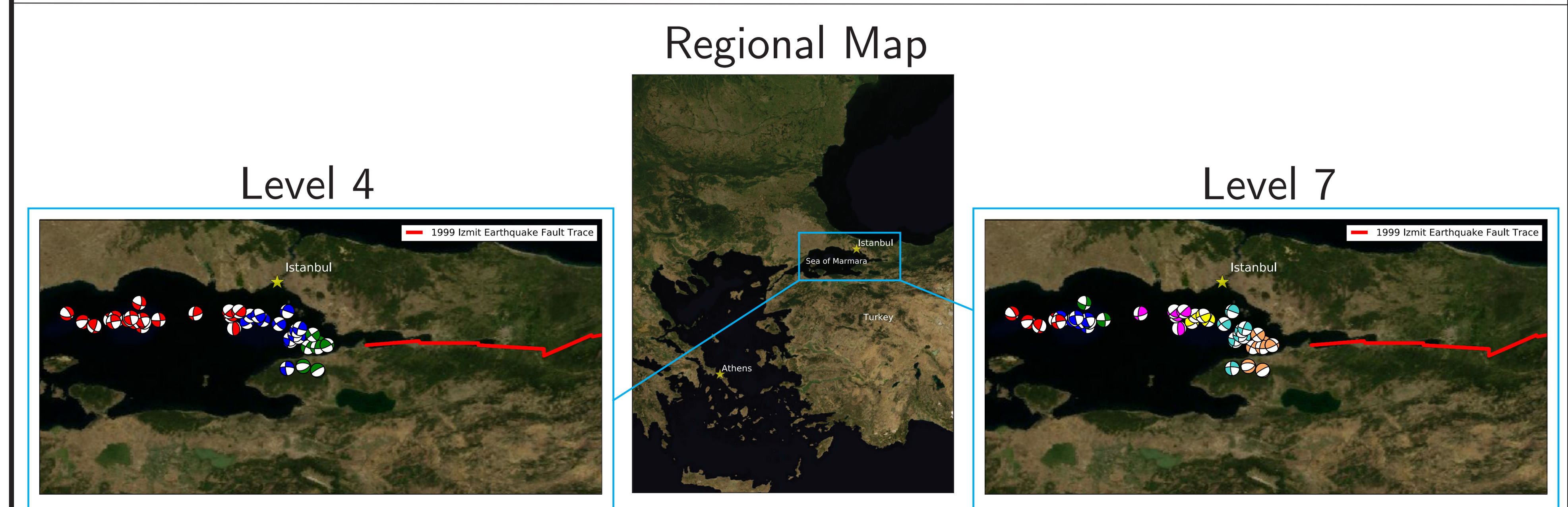


Figure 2: Von-Mises Fisher Distribution

Comparison of two random walks on S2 using a Von-Mises Fisher Distribution. Red points correspond to points sampled with a concentration parameter of 1. Blue points correspond to points sampled with a concentration parameter of 100. The black vector indicates the starting location (0, 0, 1) of the walk. As κ decreases points become distributed uniformly over the sphere.

Stress Heterogeneity

Diversity in focal mechanism catalogs and observed complex fault structures have been interpreted to be indicative of a spatially heterogeneous stress field in the Earth's crust and also could reflect heterogeneities in structure. We seek to quantify and constrain the degree of heterogeneity of the underlying stress field using focal mechanisms. Earthquakes can be clustered at various levels based on relative location and fault type. We assume that all earthquakes within each cluster are a result of a single stress. The validity of each partitioning of the earthquakes and their associated stress is then compared using Bayes Information Criterion where, roughly speaking, the optimal model would have the lowest BIC.



(a) (b)

Figure 3: Clustered Events from the North Anatolian Fault

Comparison of two levels of clustered events. Figure 3(a) has 4 clusters and Figure 3(b) has 7 clusters. Each color represents a cluster of focal mechanisms. Events shown are from the North Anatolian Fault near the time of the 1999 Izmit Earthquake. Each event has a magnitude less than 5.4.

Clustering

Events are clustered using an agglomerative hierarchical clustering algorithm with similarity matrix defined as

$$S_{ij} = D_{ij} + w|\tau_i - \tau_j|$$

where Dij is the Euclidean distance between epicenters, w is a weighting term, and $\tau \in$ [-1, 1] is the fault type parameter

Model Selection

Bayes Information Criterion (BIC) is used to determine which model of the state of stress can best describe the observed events. The BIC for a candidate model is defined as

$$BIC = 2\ln(\mathcal{L}_{max}) - \nu\ln(\alpha)$$

where ν is the number of model parameters, α is the number of observations, and $\mathcal{L}_{max}$ is the maximum likelihood estimate. We seek to quantify the underlying stress field and to determine whether the stress field is uniform (i.e a single state of stress described by 5 parameters) or whether the stress field is piece-wise smooth (i.e k >1 stress states, described by 5k parameters). The BIC can then be rewritten as

$$BIC = 2\ln(\exp(\sum_{j=1}^{k}\sum_{n=1}^{N}\kappa_{jn}\cos(\lambda_{jn}^{(obs)} - \lambda_{jn}^{(pred)}))) - 5k\ln(F)$$

where j is the group number, n is a given focal mechanism is the jth group, N is the number of focal mechanisms in the jth group, and F is the total number of focal mechanisms.

Nomenclature

Cluster
Collection of focal mechanisms determined through a clustering algorithm

Group
Collection of focal mechanisms clusters that are assumed to have been caused by the same stress

Model
Collection of complete, non-overlapping, focal mechanisms groups.