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Focal Mechanism Inversion: A Markov-Chain Monte Carlo Approach

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Introduction

Focal mechanisms have been used to infer the state of stress in the upper crust (e.g., Michael, 1984; Arnold & Towned, 2007). Focal mechanisms are most commonly used to constrain the just the orientations of the principal stress axes, with the relative magnitudes of the principal stresses less well constrained by focal mechanisms. Traditional estimation methods have relied a least-squares-based approach (e.g., Michael, 1984), seeking to find the stress solution with the highest likelihood. Recently, Bayesian-based estimation methods have been proposed (e.g., Arnold & Townend, 2007; Styron & Hetland, 2015). We employ a Markov-Chain Monte Carlo (MCMC) method to determine posterior distributions of stress parameters, including orientations and relative magnitudes of principal stresses, in order to provide additional insight and constraints on the state of stress.

Methods

We seek to determine the set of model parameters which minimize the residual between the observed rake and the slip rake predicted for a given stress tensor and fault geometry. We assume that a fault will slip in the direction of the maximum shear stress resolved on that fault. A Markov-Chain Monte Carlo is used to estimate the model parameters by performing two coupled random walks, one on a hypersphere of the Euler angles describing the orientations of the principal stresses, and one in a bounded plane of the magnitudes of the relative stress magnitudes. A trial stress tensor is either accepted or rejected as a sample of the posterior in accordance to the Metropolis-Hastings algorithm, using a Von-Mises Fisher distribution as the likelihood function of the slip rake.

Definitions

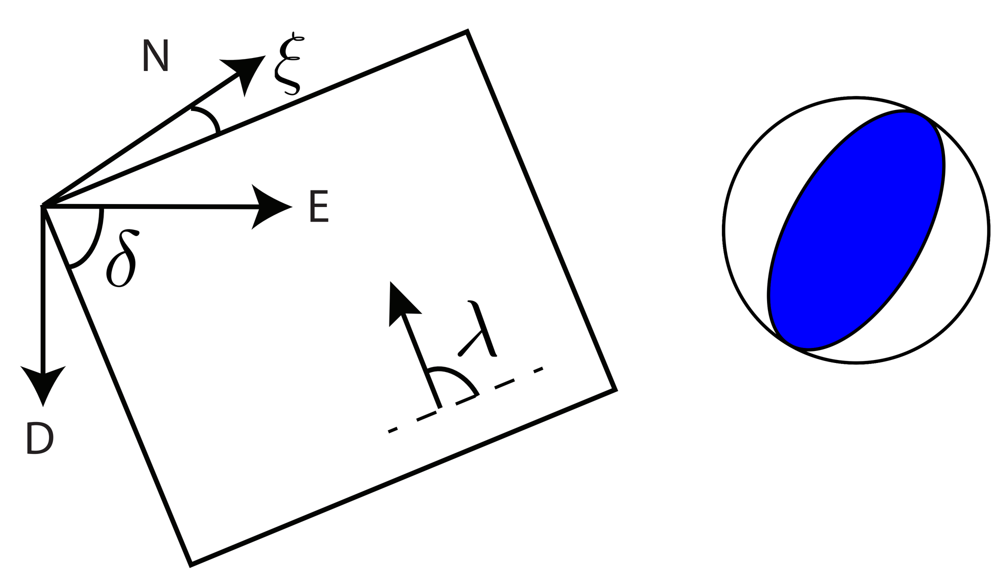
Data: $\vec{d} = (\xi, \delta, \lambda)$

Model Parameters: $\vec{m} = (R, \Delta, \phi_{MCS}, \theta_{MCS}, \rho_{MCS})$

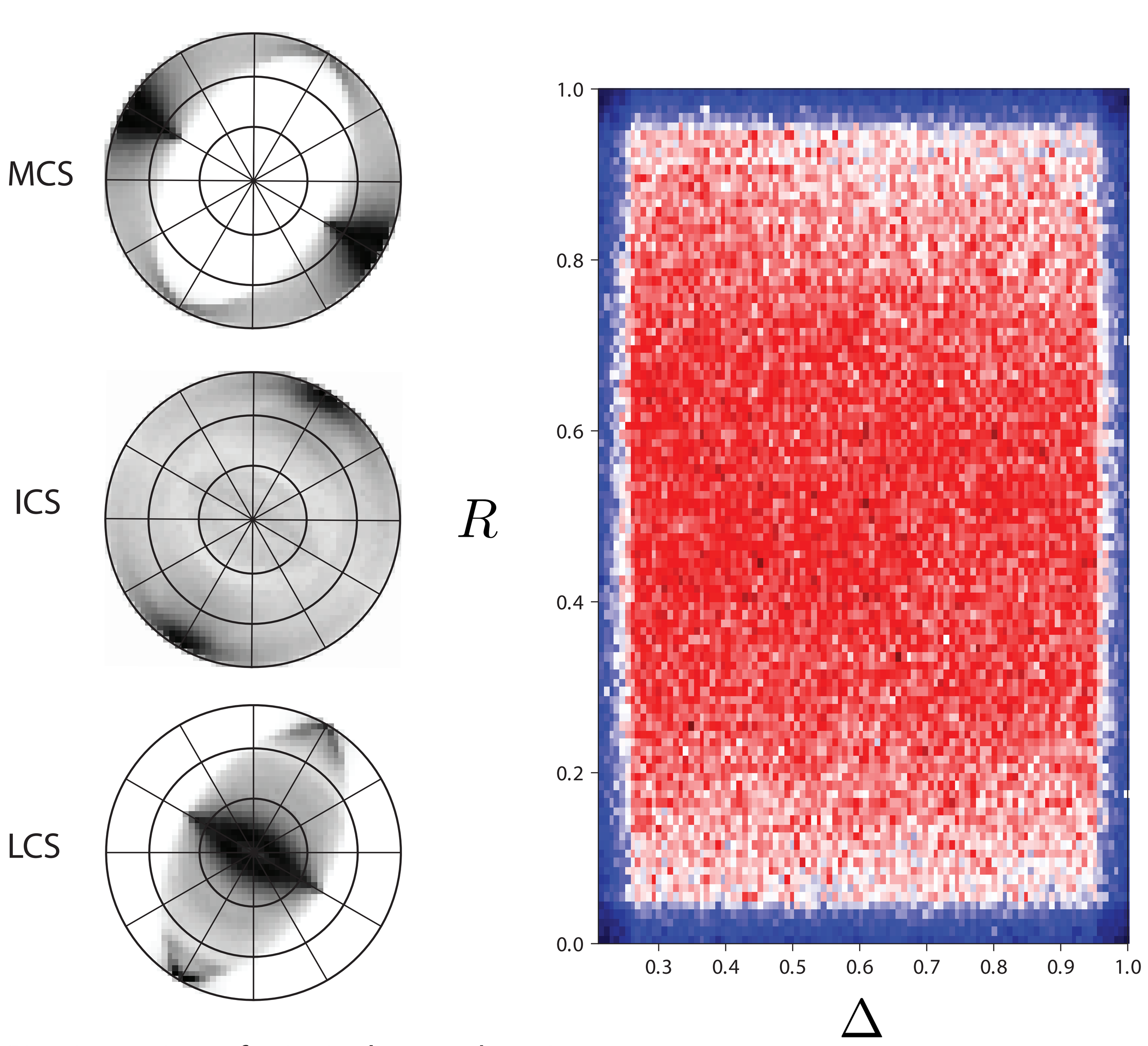
where $R = \dfrac{\sigma_3}{\sigma_1}$, $\Delta = \dfrac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}$

Likelihood Function: $\mathcal{L} = \exp(\kappa \sum_{i=1}^{N_{focal}} \cos(\Delta\lambda_i))$

Input: 1) Strike, Dip, and Rake of N number of Focal Mechanisms
2) Uncertaintiy in fault geometry and rake
3) Nodal Plane Information (Choose nodal plane or use either)



Output: 1) Estimated orientations of MCS, LCS, and ICS
2) Estimated values of R and Delta

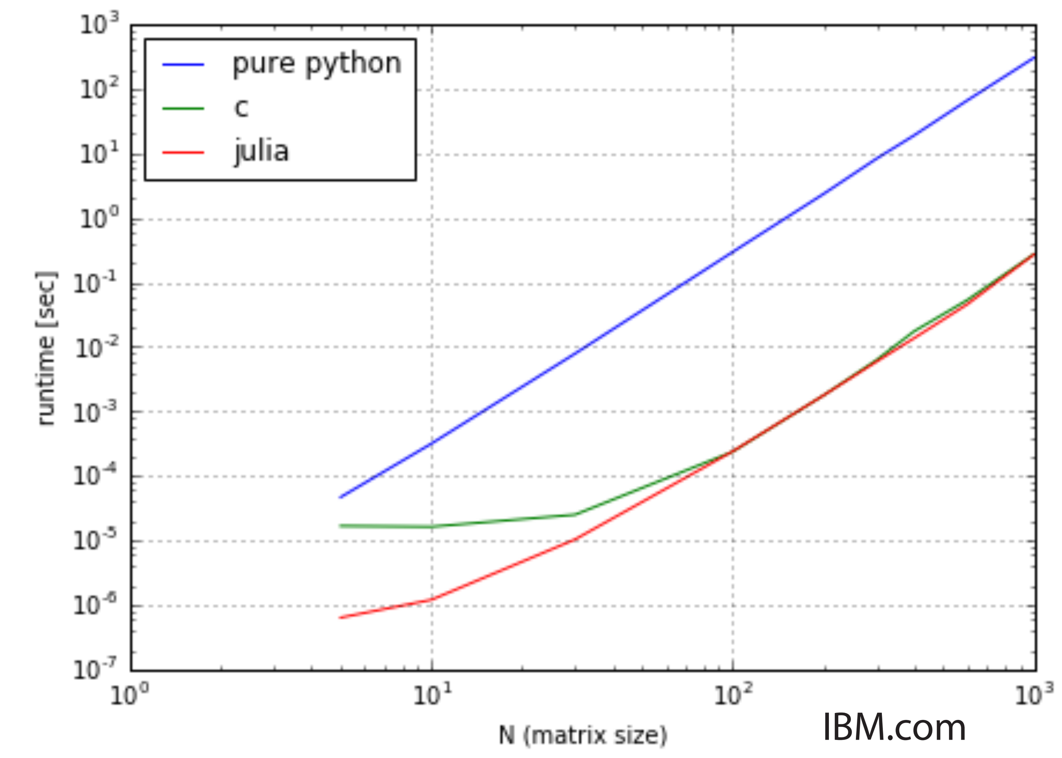


Piercing points of principal stress directions shown as a Lambert projection on the lower hemisphere of a unit sphere are shown above.

Why Julia?

Julia is a high level programming language that was first launched in 2012. Julia is geared to applied mathematics and simulation on high performance computing resources. It is a general-purpose language, with integration to both python and C without the need for external wrappers.

Speed Comparison of LU Factorization*



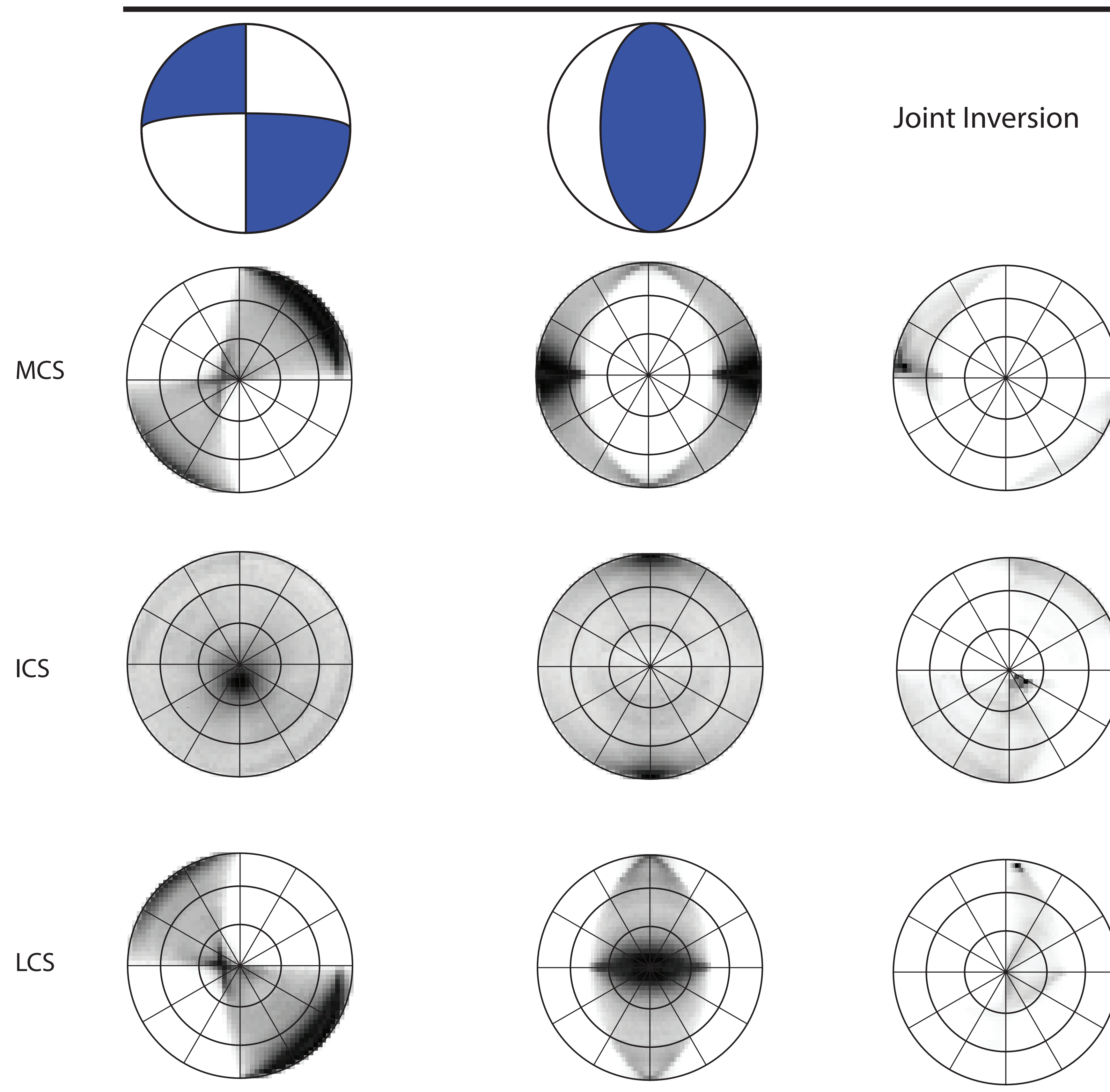
* Speed tests of LU Factorization were performed by IBM

- Fast
- Similar syntax to MATLAB makes it easy to learn and easy to port code
- Designed for scientific computing
- No external API's needed
- Nested for loops were found to be ~284 times faster using Julia compared to Python

For more information on Julia see:

Bezanson, J., Edelman, A., Karpinski, S., & Shah, V. B. Julia: A fresh approach to numerical computing. SIAM Review, 59, 65–98 (2017).

Inversion of Synthetic Focal Mechanisms



Moving Forward

- Use BIC and/or AIC to quantify the degree to which focal mechanisms support stress heterogeneity in the crust.
- Provide estimates of stress orientation and magnitudes, along with well-defined uncertainties, which can supplement geodetic measurements and provide additional constraints on intersesimic deformation models and earthquake physics simulations

Acknowledgments

Arnold, R., & Townend, J. (2007). A Bayesian approach to estimating tectonic stress from seismological data. Geophysical Journal International, 170(3), 1336–1356. https://doi.org/10.1111/j.1365-246X.2007.03485.x

Michael, A. J., (1984). Determination of stress from slip data: faults and folds. J.Geophys. Res., 89, B13, 11,517-11,526, doi: 10.1029/JB089iB13p11517.

Styron, R. H., & Hetland, E. A. (2015). The weight of the mountains: Constraints on tectonic stress, friction, and fluid pressure in the 2008 Wenchuan earthquake from estimates of topographic loading. Journal of Geophysical Research: Solid Earth https://doi.org/10.1002/2014JB011338.