Very Very Informal Notes for Cameron and Nate from Dan

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Abstract

These notes are written quite informally and they are meant to summarize the types of probability distributions that will construct a decently "randomly" formalized Θ function for my Cornell paper. This manuscript was written in only a couple of hours time, and I therefore apologize for many likely examples of carelessness in my QUITE INFORMAL extension of [3]'s results.

1 Crudely Composed Notes

I will use the phrase "probability distribution" quite informally in our discussion. The goal is thus to "invent" *any type* of Lebesgue measure that will assure that there is a probability bounded below by some tiny constant c where [3]'s Θ function will produce a consistent self-justifying formalism. *Even if that probability is tiny,* any discovered lower bound $c > 0$, will assure that the IQFS(β) formalism is consistent when β holds true under the Standard Model. This is because IQFS(β)'s Group 0, 1 and 2 axioms trivially hold true under the standard model and its final Group-3 axiom sentence cannot be proven false when our probability framework can generate a model where it holds with an explicit probability lower bound lower bound of $c > 0$.

(In other words since ZF Set Theory can formalize the validity of Gödel's Completeness Theorem, the existence of some model satisfying a probability lower bound lower bound $c > 0$ will be sufficient for establishing that the Group-3 axiom's self-justification statement will not be contradicted.)

Please allow me to be informal here because I am trying to quickly compose a rough approximation of working notes, without delving into a bevy of tedious details.

Let us recall that page 11 of [3] defines the $\theta(x)$ function-mapping to be an operation that maps powers of 2 onto powers of 2 subject to the following rules:

$$\forall \; x \quad \text{Power}(x) \quad \Rightarrow \quad \text{Power}(\; \theta(x) \;) \tag{1}$$

$$\forall \; x \quad \theta(x) \; \neq \; 1 \tag{2}$$

$$\forall \; x \quad \forall \; y \quad [\; x \; \neq \; y \; \wedge \; \text{Power}(x) \;] \Rightarrow \quad \theta(x) \; \neq \; \theta(y) \tag{3}$$

$$\forall \; x \quad \neg \; \text{Power}(x) \quad \Rightarrow \quad \theta(x) \; = \; 0 \tag{4}$$

We want our probability distribution to have the property that any recursively defined function has a zero probability of occurring. Thus the countable set of all recursively defined functions will also have a probability zero of occurring. BUT YET the $\theta(x)$ primitive will grow at a slow enough rate that it is incapable of producing a fatal diagonalizing contradiction, while satisfying the crucial constraints in Lines (1)- (3).

I can immediately think of three likely ways of doing this. *It is (?) possible all three methods will work, successfully.* Among the three plausible methods, Method A is the simplest procedure, and Method C is the most complicated. The virtue of Method C is that it is the one that I am most confident about, although its procedure is a complex hybrid of methods A and B.

All three of these randomized methods will first generate the value of $\theta(1)$, and then calculate in chronological oder the values of $\theta(2)$, $\theta(4)$, $\theta(8)$ etc., This chronological order is important because once $\theta(2^i)$ is assigned a values of 2^K then all $j > i$ are forbidden by rule 3 from mapping $\theta(2^j)$ onto 2^K . This Rule 3 will be called the **Exclusion Principle** during our discussion of Methods A-C below:

1.1 Method A: The "Pairing Method"

The Pairing method will begin by finding the two smallest powers of 2, at least as large as 2, where no $j < i$ has $\theta(2^j)$ correspond to one of these two powers of 2, which we write as 2^{K_1} and 2^{K_2}. It will then set $\theta(2^i)$ equal to one of these two values, with each quantity receiving a 50 % probability of occurring.

It would not surprise me if this Pairing rule (by itself) would be sufficient to produce our needed final result. It might, however, be problematic because it would cause $\theta(2^i)$ to always satisfy the following possibly excessively tight constraint of:

$$\forall \;\; x \qquad \theta(x) \;\leq\; 4\, x^2 \tag{5}$$

1.2 Method B: The Almost-Stochastic Independence Method

This method will assume that we have an infinite set of real numbers greater than zero, p_1 , p_2 , p_3 , ... such that p_j is the probability that $\theta(2^0) = 2^j$. (The easiest example [1] of this is when $p_j = 2^{-j}$.) Then for each subsequent power of 2, denoted as 2^L, the same probability distribution will be used, except that the "Exclusion Principle" will be followed in precluding repetitions from occurring. Thus, let the prior powers of 2 be mapped onto the quantities of 2^{K_0} 2^{K_1}, ... $2^{K_{L-1}}$ and let S be defined as below:

$$S = \sum_{i=0}^{L-1} p_{k_i} \tag{6}$$

Then assuming 2^j is not one of the previously taken positions of 2^{K_0} 2^{K_1}, ... $2^{K_{L-1}}$, the method B will have hold $\theta(2^L) = 2^j$ with an obviously adjusted probability of

$$\frac{p_j}{1-S} \tag{7}$$

1.3 Hybridizations of Methods A and B

Again, we would not be surprised if either Methods A or B (or both) would establish the conjectured self-justification property. However, there are also other plausible hybrid methods that would establish

[1] If $p_j = 2^{-j}$ then $\sum p_j = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 1$.

this property. Here are two examples

$Method\!-\!C\!-\!1$. We follow Method A's approach with probability 0.5 and method B's approach with probability 0.5. Thus, either of Method A's two choices occur with a precise probability of $\frac{1}{4}$, and Method B's setting of $\theta(2^L) = 2^j$ occurs with an exact probability of:

$$\frac{p_j}{2\,(1-S)} \tag{8}$$

$Method\!-\!C\!-\!2$. We follow Method A's approach with probability of S and method B's approach with probability $1-S$. Thus, either of Method A's two choices will hold with probability of $\frac{S}{2}$ and Method B's setting of $\theta(2^L) = 2^J$ occurs with a probability of exactly p_j .

2 Basic Strategy

The basic strategy is to show that if axiom basis β holds valid in the standard model then it is impossible for a minimal proof of $0 = 1$ to exist, Providing, just an overview, this should be able to be proved, *roughly,* by contradiction.

In particular, if β holds valid in the standard model then all the Group 0, 1 and 2 axioms will hold valid under the Standard Model. Hence if the consistency preservation property fails, then the Group-3 *"I am consistent" axiom* must be false under the standard model. Thus, the proof of the falsification of the Group-3 axiom must be able to self-reference itself and thereby establish its own incorrectness.

At this juncture, one must use a natural encoding for the Gödel numbers of a proof (such as what was formally given in [1, 2]'s examples of Gödel encodings). The proof of the existence of such a proof will exceed the proof's length by a factor $\lambda > 4$ (or more) under all natural encodings of proofs (including the examples given in [1, 2]). But the point is that our formalism *has only the* Θ operation as a primitive, for representing growth. Thus, the natural probability distributions from the prior section should establish something to the effect that there is a probability lower bound of $c > 0$, such that an excessively fast growth-rate will be impossible.

Thus leaving aside many messy details, this lower bound will assure that there is stochastic model where growth is precluded at a fast enough rate for some demonstrated model to form the type of counterexample that Gödel's Completeness Theorem needs to show that the *"I am consistent" axiom* needs for corroborating its claim for self-justification.

This result differs from my earlier work in that it needs ZF Set Theory (rather than Peano Arithmetic) to corroborate what I call the Consistency Preservation Property. That is fine and legal because the system IQFS(β) affirms its own consistency via a 1-sentence axiom. Thus, ZF Set Theory's knowledge about Lebesgue measures should, likely, indicate IQFS(β) is a competent enough formalism to make no false claims.

My apologies that the preceeding short summary is not a formal proof. It merely outlines, *very roughly*, what I have in mind.

References

[1] Willard, D. E.: "Self-verifying systems, the incompleteness theorem and the tangibiltiy reflection principle", in *Journal of Symbolic Logic* 66 (2001) pp. 536-596. (Unlike our later articles, [1, 2] spends some time explaining how we generate our analogs of Gödel numbers for itemized sentences and proofs.)

[2] Willard, D. E.: "A generalization of the second incompleteness theorem and some exceptions to it". *Annals of Pure and Applied Logic* 141 (2006) pp. 472-496. (Unlike our later articles, [1, 2] spends some time explining how we generate our analogs of Gödel numbers for itemized sentences and proofs.)

[3] Willard, D. E.: On how the introducing of a new θ function symbol into arithmetic's formalism is germane to devising axiom systems that can appreciate fragments of their own Hilbert consistency. *Cornell Archives arXiv Report* 1612.08071v5 (2017).

[4] Willard, D. E.: "On the Tender Line Separating Generalizations and Boundary-Case Exceptions for the Second Incompleteness Theorem under Semantic Tableaux Deduction", a talk given on January 7 at the LFCS 2020 conference. Early version in Volume 11972 of Springer's LNCS series, and longer version sent to Cameron and Nate.