Scientific Notes of Dan Willard Notarized on August 22, 2017

Comment: A 30-minute invited talk summarizing the contents of this article was presented on June 18, 2017 at the NEEPS-2017 Conference at Binghamton University. The Version-1 June 27 (2017) draft of this paper was similar to the 30-minute talk I gave at Binghamton University, and it was disseminated nine days after this talk. This second draft is more detailed and polished because it contains Section 4's additional evidence.

ABSTRACCT: At a theoretical level, the Trivers-Willard Sex Ratio Hypothesis applies to both avian species and mammals. This article, however, conjectures that at the statistical level, sex ratio effects are likely to produce sharper numerical variations among birds than among mammals. We explain this greater statistical variation should likely have beneficial implications for increasing the efficiency of world-wide poultry egg (and perhaps also meat) production.

1 Introduction

The Trivers-Willard Hypothesis is an extension of Fisher's Sex Equilibrium paradigm [15, 61]. Fisher had noted that essentially all vertebrate species will invest a roughly equal amount of energy raising male and female offspring because such a paradigm corresponds to the favored stabilizing equilibrium state that Darwinian Evolution gravitates towards over an extended period of time. The Trivers-Willard Hypothesis (TW) was based on the observation that some mating couples provide a setting (or have genes) more useful to male reproductive success, while others will be more supportive to a female's reproductive success. TW predicted that it would be useful, accordingly, for Darwinian Evolution to attempt to guess which settings are more useful to which sex's reproductive success, and then to adjust the sex ratio to reflect the strategy that maximizes long-term reproductive success. This prediction will be called the *Generalized Version* of the Trivers-Willard Hypothesis.

I conceived of this version of the TW during a 25-minute bicycle ride, after taking in the Spring of 1968, a philosophy course at SUNY Stony Brook. Its reading material included the Desmond Morris book entitled *The Naked Ape* [35]. During that fateful bicycle ride, I also reconstructed Fisher's general sex ratio paradigm (without knowing that Fisher conceived of this idea almost forty years earlier). I also noticed that the preceding "Generalized" theory would imply that a mating couple should be more likely to produce male offspring when they are in better health condition (or have an above average nutritional diet). This latter observation was a consequence of the well known anthropological observation that competitive success benefits the reproduction rate of a male more than a female because a male is capable of inseminating simultaneously several females.

At the conclusion of this 1968 bicycle ride, I was left with a quandary as to what to do next? This is because I was uncertain what part of the "Generalized" theory was new, if any part of it was actually new? There were several occasions in the past when I developed theories that I learned, later, others had discovered substantially earlier [1] . In the end, I decided to make no effort to publish the Generalized Theory in 1968 because I thought it likely that either someone had thought of a similar idea earlier, or it would be difficult to persuade the academic community that these principles were sound. (My reluctance to publish this "Generalized" theory was further amplified by two family tragedies, where my father experienced a heart attack and my mother was diagnosed with cancer during that same year of 1968. My mother was diagnosed with cancer, actually, only a few weeks after my bicycle ride. I doubt I would have had the temperament to discover the General Theory, had the chronology of these two events been physically reversed.)

The remainder of the history behind the TW discovery has been told by Robert Trivers. In 1970, I attended Harvard University as a graduate student and audited a course taught by Irven Devore, where Robert Trivers was a teaching assistant. In one lecture during that course, Trivers reviewed Fisher's Sex Ratio Principle, and he mentioned in a subsequent second lecture that it was known that upper class income families were statistically more likely to have male offspring. I was delighted when I heard the first of Trivers's two lectures because I had inadvertently reconstructed Fisher's 50/50 sex ratio principle during my earlier 1968 bicycle ride. I explained to Trivers, subsequently after his second lecture, my explanation for the statistical paradigms he mentioned. Trivers warmly encouraged me to publish this result.

Distracted by the illness of both my parents, as well as the burden of preparing for Harvard's notoriously hard Ph D Qualifying exam in Mathematics, I did not pursue the TW project further. As a consequence, Bob Trivers helpfully wrote up the manuscript of what would ultimately become our announced result. This joint paper [48] did later become a classic article, according to the MacArthur award-winning philosopher Rebecca Goldstein [21], within sociobiology's broader and ever-expanding literature.

A period of 49 years has now transpired between the current date and the occasion when I took that fateful 25-minute bicycle ride in 1968. It is accurate to state that no single 25-minute investment of my time (conjoined with Robert Trivers excellent and meticulously diligent write-up of our joint paper [48]) did produce a greater impact on the academic community from my on-going research. Thus on 23 June 2017, Google Scholar recorded that there were 3,338 citations to [48]'s research, *except for one hilarious error*, that deserves to be in a Woody Allen movie. It will make the academic community laugh at the silly incompetence of the careless computer engineers, who embedded an amusing, almost schizophrenic bug into what was supposed to be their sagacious Google-Scholar software [2] .

In any case, this little *"Google Bug"* is of small importance because the significance of the TW article is well known. For instance, there have been four recent mentions of this article in the popular news media [10, 37, 38, 40]. The latter has included one year-2017 article in the *Sunday Week in Review* section of the *New York Times* [38] .

Our purpose in the current short note will be to achieve three goals. The first will be to suggest that avian species are likely to follow the predictions of the TW hypothesis with greater statistical accuracy than do mammals. A second objective will be to suggest that this prediction is likely to have beneficial implications in enhancing the efficiency of poultry farms. A third goal, confined to §5, will be to provide the reader with a brief summary of my research into symbolic logic and into Gödel's Incompleteness theorem. This research has, traditionally, been treated as a subfield of mathematics and philosophy. But as we shall explain, it also has nontrivial implications for anthropology and psychology, as well.

2 A New Amendment to the Trivers-Willard Hypothesis for Avian Species

Our interest in applying the TW hypothesis to avian species was initially stimulated by an article by Nancy Burley [5]. It studied the behavior of Australian Zebra Finches, and found that their propensity to produce male offspring would be enhanced if colored bands were attached to their legs that made the males look more attractive and the females less attractive. The reverse sex ratio would be produced if the bands had polar opposite types of sexual attraction features.

I had not predicted such an effect. However with retrospection, I do have an interesting explanation as to why avian species seem to follow the predictions of the Generalized TW Hypothesis with more statistical accuracy than mammals.

It is because sperm type determines the sex of mammal offspring, while it is unfertized egg type that is the control agent among birds. In particular, it is well known that it is *the type*, X or Y, of sperm which determines the sex among mammals (e.g. an XX offspring is a daughter and an XY offspring is a son). In contrast among birds, a "ZZ" genetic mix produces a male offspring, and a "ZW" mixture corresponds to a female. The latter implies that it is the *unfertilized egg* (rather than donated sperm) that functionally determines the sex type for a bird.

Among both mammals and birds, the TW Hypothesis predicts that Darwinian Evolution has an incentive to guess which sex of offspring is likely to produce more grandchildren for a mating couple. The engine, however, to determine which type of sperm will first reach the egg is complicated, when a male mammal donates several million competing sperms, at once. The comparable engine for sex ratio determination among birds is, presumably, much simpler because only a small number of "Z" or "W" unfertized eggs are deposited by the mother for the purposes of being fertilized by a male.

This distinction suggests it will likely be *substantially easier* for avian species to gain full dexterous control over the sex of their offspring than the analogous paradigm, occurring among mammal species.

Our suggested amendment to the TW Hypothesis will probably be very difficult to empirically check for its correctness. It would require a meticulous study that compares various species of mammals to sundry species of birds. It would, however, be theoretically interesting if Avian species were found to obey the predictions of the

---

[1] For example during a bicycle ride in 1964, I reconstructed many of the famous integration formulas, taught in a Freshman Calculus course, without knowing about the famous mathematical research done 200 years earlier. The resulting series of brain-storming thoughts actually resulted in my discovering of Euler's irrational number of "e", without knowing what Euler had done. Humorously, I naively called this number "w" (for "Willard's constant") before learning, to my naive teen-age chagrin, that Euler discovered approximately two centuries earlier a comparable irrational number he called "e".

[2] The amusing error is that neither Dan Willard nor Robert Trivers were listed by Google-Scholar as the authors of the article [48], as recently as 23 June 2017. Instead, "James A. McKanna" is listed as the author of an article with the same title and page numbers as [48]. This error occurred because *Science Magazine* listed, in 1973, the authors of its articles on the specified article's last page (rather than on its first page). Moreover, McKanna's 1973 paper ended on the first column of the same page 90, whose second and third columns were occupied by the TW article. Thus, the supposedly sagacious Google-Scholar software had gone amusingly schizophrenic, when it tried to guess who was the actual author of this particular article with an unusual quantity of 3,338 citations ? (This error persisted during all the Winter and Spring months of the year 2017. In fairness to Google, their error was corrected on June 26, 2017, shortly after I gave my June 18 talk at NEEPS-2017. We are not sure exactly why, but Google Scholar's software has made this persistent error and then corrected it repeatedly, on several occasions, during the last few years.)

3   Poultry Farms

It is well known that the efficiency of Poultry Farms shall increase if more female chickens are born. In that case, egg production will quickly increase, and also poultry meat production should also likely increase, somewhat.

It is apparent that if chickens do function similarly to Australian Zebra Finches, then a mating couple will produce more female offspring, if they are artificially endowed with female color features.

Moreover, a large variety of other techniques are likely to be also available for influencing the sex of offspring. For instance, if an excess of male rooster-like sounds were pumped into a chicken farm then it is probable that more females will be born (because the illusion of an excess supply of males would have been temporarily created).

One drawback of such strategies is that the inbred supply of farm animals will naturally evolve in a direction *that is exactly the opposite* to a farmer's intentions. This is because the classic local farm animal population will degrade, spontaneously, in a direction towards a 50/50 sex ratio, according to Fisher's Sex Equilibrium argument [15].

A useful remedy is for a concerned farmer to keep a log of which chickens come from a genetic lineage producing more female offspring — and to encourage those particular chickens to breed.

Unfortunately, such a log would require a labor-intensive effort to maintain, thus undermining its cost effectiveness. Fortunately, there is a solution to this challenge in the modern computer age. A unique computerized bar-code name identifier could be attached to each chicken, and a robot could ascertain that the correct genetic line of chickens are breeding.

In other words, we are suggesting that a computerized algorithm could maintain some type of desired protocol to enhance the ratio of female offspring, and that this protocol will probably be cost effective in the new age of computerized robotics that is now emerging. In any case, it is evident that the efficiency of poultry production should be increased if farmers could gain better control of the sex of raised chickens.

4   Supporting Data

A diversity of articles, by several authors, will probably be necessary to confirm our twin conjectures, suggesting that:

A. Avian species do follow the predictions of the Trivers-Willard hypothesis with greater accuracy than do mammals.

B. This paradigm can increase the efficiency of poultry production.

There is, however, adequate evidence in the already-published literature to make these two conjectures quite credible.

It firstly should be noted that there are some species of laboratory animals, drawn from the Vertebrate kingdom, where human experimenters have gained essentially 100% control of the manipulated sex of the studied offspring. For instance, it has been observed that reptiles and amphibians neither follow the mammal XX/XY or the bird ZZ/ZW chromosomal method to control the sex of offspring. Instead, both sexes have identical chromosome structures, and it has been observed that laboratory scientists can gain 100% control of the sex of offspring in 14 different genres of turtles by changing the incubation temperatures for turtle eggs. (Thus, [3, 4, 14, 49] observed that a 25 Celsius incubation temperature produces an all-male rate of offspring, while a 31 Celsius incubation temperature leads to an all-female population among 14 different tested genres of turtles.) A similar temperature-control effect has also been observed to occur in several species of reptiles [2].

Exact analogs of the preceding paradigm will not apply to mating chickens, since poultry uses a ZZ/ZW model, where a ZZ animal is genetically a male and ZW is female. It has, however, been observed that lowering the incubation temperature does increase the frequency of female births, partly because of sex-differential mortality rates and also because some genetically male (e.g. ZZ) chickens possess a female anatomy and own an observed female-like ability to lay eggs [7, 22, 23, 32], if the incubating temperature is lowered soon after the ZZ-egg is fertilized.

There is a serious interest in the poultry industry to increase the ratio of female births, as noted by [34, 39, 43, 44, 62] among other sources. Several published articles have studied the implications of TW hypothesis for avian species [1, 5, 6, 9, 11, 12, 18, 29, 30, 31, 41, 45, 46, 47, 50, 51], and they have noted it has had a documented measured effect. We suspect there in not yet enough available unambiguous evidence to determine whether our conjecture (A) is precisely correct (e.g. that the TW hypothesis has significantly greater implications among birds, than among mammals). This particular topic should be investigated in much greater detail in the future.

If our conjecture (B) is correct (that poultry farms can have their productivity increased by even a few percentage points through a better understanding of the implications of the TW hypothesis) then the avian version of the TW hypothesis deserves as much study as the 3,335 articles that have already examined its implications for mostly mammals.

There is one particular experiment that I would recommend be undertaken. It is known that the sex ratios within human societies change in the aftermath of wars [24, 27]. *Would the same be true among poultry species?* That is, what would happen if mating chickens repeatedly heard sounds from an electronic speaker of tape-recordings from a cock-fight?

The preceding experiment is a little awkward and embarrassing to undertake, but the scientific information that it supplied could actually be quite valuable and sobering.

5   Historic Context of this Research

The first chapter of this article had mentioned that Willard's main contribution to the TW paper [48] consisted of a flash insight that I developed during a short bicycle ride when I was 20 years old. The curiosity of many readers may have been stirred by this fact. Some readers may, perhaps, begin to wonder what other intellectual projects I have worked on, subsequently, in the aftermath of [48]'s publication.

Essentially, my research has had two focal points. Prior to 1992, my focus was on mainly classical topics concerning computer algorithm design. My best known work in this area consisted of a joint study with Fredman to determine the optimal cost for computerized sorting and related searching methodologies. Our joint work showed that the then-commonly-held presumption that computerized sorting could run no faster than in $O(N \, Log(N))$ time was incorrect (i.e. a theoretical speed-up for sorting and searching was demonstrated in [16, 17]). These two projects produced four papers (if one counts separately their journal and conference publications). It is reported in Google Scholar that 920 academic citations to these four variations of our work had subsequently appeared. Moreover, the *1991 Annual Report of the National Science Foundation* [35] cited this particular "Fusion Tree" investigation as the *chronologically first* among only six projects that were mentioned in its 1991 *Mathematics and Computer Science* section.

Starting in 1993, I started publishing papers [52]-[60] about Gödel's historic Incompleteness Theorem. Gödel's work has traditionally been of interest to researchers in fields as broadly diverse as mathematics, philosophy and computing (as a reader can quickly surmise by looking at any one of Gödel's biographies [8, 20, 63]). His "First Incompleteness Theorem" indicated there existed no systematic manner to categorize all the technically true statements *in even the simplest branch* of mathematics. Gödel's "Second Incompleteness Theorem" indicated conventional logical systems are also unable to confirm their own consistency, in a fully formal sense [13, 33].

There is no question that both these incompleteness results are rigorously correct, but they raise the question about whether Darwinian Evolution might favor the evolving of a more advanced specie of primate, that finds it adaptive to employ an *unconventional mode of thought*, in order to maintain some type (7) of specially *modified knowledge* of its own consistency. The latter topic was the stimulus for our on-going investigations in [52]-[60]. These articles proposed a variety of unconventional revisions of arithmetic's formal axiomatic structure. They found these delicate revisions could preserve most of the pragmatic content of traditional arithmetic, *while simultaneously providing* at least some type of philosophically meaningful, albeit partially diluted, formalized appreciation of their own internal self consistency.

The best and indeed preferred paper to examine first, in the preceding 24-year long series of papers, is our final article [60]. The Remark 7.5 of [60] mentions that we suspect some variation of our proposed "IQFS" formalism has applications to anthropology, psychology and philosophy, as well as to linguistics. We do not suggest this paper is of easy reading. A reader can, however, at least partially appreciate [60]'s gist, when its examination is conjoined with also a reading of at least some select parts of the books [8, 13, 20, 33, 63].

We do not want to overstate this point, but the Remark 7.5 of [60] does indicate that our newly proposed "indeterminate function symbol" θ should have implications for each of the fields of anthropology, psychology and philosophy, as well as linguistics. A wide spectrum of readers is, thus, encouraged to, at least, glance briefly at [60]'s discussion.

6   Concluding Remarks

The main purpose of this article was to introduce our proposed avian amendment to the TW Sex Ratio Theory. This article also included §5's brief summary of our other research, during the last 44 years, because we suspected some readers would find its short review to be informative, as well.

The main reason the observations in this short note will be of interest is because it is possible that the world-wide egg (and plausibly also poultry meat) production could undergo an approximate minimal 2-3 percentage or greater increase, if the number of born female chicklets is significantly enhanced. Such a difference will, certainly, not resolve world-wide famine challenges. It would, however, be a useful development, beneficial to mankind.

ACKNOWLEDGMENT: I thank Glenn Geher for his useful suggestion, conveyed on June 18 to me at the NEEPS-2017 conference [19], that my article should also discuss the relevance of fascinating aspects of sex ratio effects, that have been documented for a variety of different species of turtles.

References

[1] C. Alonso-Alvarez and A. Velando, "Female body conditon and brood sex ratio in Yellow-legged Bulls Larus cachinnans", *British Ornithologists' Union* 145 (2003), pp.220-226.

[2] J.J. Bull, " Sex determination in reptiles", *Quarterly Review in Biology*, 55 (1980), pp. 3-21.

[3] J.J. Bull and R.V. Vogt, "Temperature-dependent sex determination in turtles", *Science* (New Series Volume 206), December 7, 1979, pp. 1186-1188.

[4] J.J. Bull, R.V. Vogt and M. G. Bullmer, "Heritability of sex ratio in turtles with environmental sex determination", *Evolution* 36:2 (1982), pp. 333-341.

[5] N. Burley, "Sex ratio manipulation and selection for attractiveness", *Science* (New Series, Volume 211), February 13, 1981, pp.721-722.

[6] N. Burley, "Sex ratio manipulation in color-banded populations of zebra-finches", *Evolution* 40 (1986), pp. 1191-1206.

[7] J. Chue and C. A. Smith, "Sex determination and sexual differetiation in the avian model", the *FEBS Journal* 278 (2011), pp. 1027-1034. Available on-line via Google.

[8] J. W. Dawson, *Logical Dilemmas: The life and work of Kurt Gödel*, AKPeters, 1997

[9] C. Dijkstra, S. Daan and J. B. Burker, "Seasonal variation in the sex ratios of kestrel broods", *Functional Ecology* 4:2 (1990), pp.143-147.

[10] *The Economist*, "Sons and mothers", July 11, 2009 (London), page 83 (no author given).

[11] H. Ellegren, L. Gustafsson and B. C. Sheldon, "Sex ratio adjustment in relation to paternal attractiveness in a wild bird population", *Proceedings of National Academy of Sciences* 93 (1996), pp. 11723-11728.

[12] S. T. Emlen. J. M. Emlen and S. A. Levin, "Sex ratio selection in species with helpers-at-the-nest", *American Naturalist* 127:1 (1986) pp. 1-8.

[13] H. B Enderton, *Mathematical Introduction Logic*, Academic Press, 2011

[14] M. A. Ewert, D. R. Jackson and C. R. Nelson, "Patterns of temperature-dependent sex determination in turtles", *Journal of Experimental Zoology* 270 (1994), pp. 3-15.

[15] R. A. Fisher, *The Genetical Theory of Natural Selection*, Clarendon, Oxford, 1930.

[16] M. L. Fredman and D. E. Willard, "Surpassing the information theoretic barrier with fusion trees", *The Journal of Computer and Systems Sciences* 47 (1993), pp. 424-433.

[17] M. L. Fredman and D. E. Willard, "Transdichotomous algorithms for minimum spanning trees and shortest paths", *The Journal of Computer and Systems Sciences* 48 (1994), pp. 533-551.

[18] L. Gilbert, A. N. Rutstein, N. Hazon and J. A. Graves, "Sex biased investments in yolk androgens depends on female quality and laying order in zebra finches", *Naturwissenschaften* 92:4 (2005), pp. 174-181.

[19] Glenn Geher, Private Communications on June 18 at the NEEPS-2017 conference, summarizing the literature about sex determination in species of turtles.

[20] R. Goldstein, *Incompleteness: The Proof and Paradox of Kurt Gödel*, Norton Press, 2005.

[21] R. Goldstein, private communications in 2012 during a visit by Goldstein to the SUNY-Albany campus. During those communications, Goldstein told me that it was both her impression (and those of her husband Steven Pinker) that the TW article [48] played a major role in the historic development of sociobiology because it was one of the first examples of a sociobiology article making purely theoretical predictions, that were later statistically verified with much corroborating evidence.

[22] A. Goth and D. T. Booth, "Temperature dependent sex ratio in a bird", *Biology Letters* (Royal Society), 1:1 ((March 22 2005), pp. 31-33. Available on-line via Google.

[23] *The Independent*, "A drop in temperature can change the sex of chickens", an author is unspecified for this on-line news article, but it does quote remarks by Prof Mark Ferguson from the University of Manchester (10 September 1997). Available on-line via Google.

[24] W. H. James, "The variations of human sex ratio at birth during and after wars and their potential implications", *Journal of Theoretical Biology* 257:1 (2009), pp. 116-123,.

[25] S. Kananzawa, "Big and tall parents have more sons: Further generalizatons of the Trivers-Willard hypothesis", *Journal of Theoretical Biology* 235 (2005), pp. 583-590.

[26] S. Kananzawa, "Beautiful parents have more daughters: A further implication of the Trivers-Willard hypothesis (gTWH)", *Journal of Theoretical Biology* 244 (2007), pp. 133-140.

[27] S. Kananzawa, "Big and tall soldiers are more likely to survive a battle: A possible explanation for the 'returning soldier effect' on the secondary sex ratio", *Human Reproduction* 22:11 (2007), pp. 3002-3008.

[28] S. Kananzawa and N. L. Segal, "Same-sex twins are taller than opposite-sex twins (but only if breast-fed): Possible evidence for sex bias in human breast milk", *Journal of Experimental Child Psychology*, available on-line 9 Janaury 2017 and to be published in the near future, (Also accessible on Satoshi Kananzawa's home page.)

[29] R. Kilner, "Primary and scondary sex ratio manipulation by zebra finches", *Animal Behavior* 56 (1998), pp.155-164.

[30] J. Komdeur, "Facutative sex ratio bias in the offspring of seychelles warblers", *Proceedings Biological Sciences* 263:1370 (1996), pp. 661-666 (Royal Society)

[31] J. Komdeur, J. Daan, S. Tinbergen and C. Mateman, "Extreme adaptive modification in sex ratio of seychelles warbler's eggs", *Nature* 385 (1997), pp. 522-524.

[32] M. W. McDonald, "Effects of temperature storage and age of fowl eggs on hatchability and sex ratio, growth and viability of the chickens", *Australian Journal of Agricultural Science* 11:4 (1960), pp. 664-672. Available on-line via Google.

[33] E. Mendelson, *Introduction to Mathematical Logic*, CRC Press, 2010.

[34] *My Pet Chicken* website (no author specified), "How can I incubate eggs that will hatch female chicks only?" Available on-line via Google.

[35] D. Morris, *The Naked Ape*, Penguin Publishing House, 1967.

[36] *National Science Foundation 1991 Annual Report*, "Mathematics and computer science section". (The Fredman-Willard "fusion tree" methodology of [16, 17] was the chronolgically first among six research projects mentioned in this section.)

[37] *The New York Times*, "Survival of species linked to lopsided sex ratios" (by Walter Sullivan), front page of science section, February 16, 1981.

[38] *The New York Times*, "Does breast milk have a sex bias" (by Nancy L Segal and Satoshi Kanazawa) in Sunday Review section, January 21, 2017. (See [28] for a fascinating scientific study that documents in more detail the evdence germane to this newspaper article.)

[39] *Permies.com* website (no author specified), "Can we hatch more hens and fewer roosters?" Available on-line via Google.

[40] *Psychology Today*, "10 politically incorrect truths about human nature" (by Alan S. Miller and Satoshi Kanazawa), August 2007, pp. 88-95. (The Trivers-Willard Hypothesis was "Politically Incorrect Truth # 6"; see the articles [25, 26] for some scientific data germane to this paper's summarized discussion.)

[41] C. B. Patterson and J. M. Emlen, "Variation in nestling sex ratios in the yellow-headed blackbird", *The American Naturalist* 115:5 (1980) pp. 743-747,

[42] J. R. Roche, J. M. Lee and D. B. Berry, "Pre-conception energy balance and secondary sex ratio – Partial support of the Trivers-Willard hypothesis in dairy cows", *Journal of Dairy Science* 89:6 (2006) pp. 2119-2125.

[43] Y. Scott-Reid, *Munchies* website, "A scientist has figured out how to determine chickens' sex before they hatch", April 8, 2015. Available on-line via Google.

[44] W. Seltzer, "The method of controlling the sex of avian embryo, improving embryo hachability and improving viability of the hatched chick", *United States Patent:* 2,733,482, June 19, 1953.

[45] B. C. Sheldon, "Recent studies of avian sex ratios", *Heredity* 80(1998), pp. 397-402.

[46] E. Svenson and J. Nilsson, "Mate quality affects sex ratio in blue tits", *Proceedings on Biological Sciences* 263:1368 (1996), pp. 357-361 (Royal Society)

[47] E. Szasz, D. Kiss and B. Rosivall, "Sex ratio adjustments in birds", *Ornis Hungarica*, 20:1 (2012), pp. 26-36.

[48] R. L. Trivers and D. E. Willard, "Natural selection of parental ability to vary sex ratio of offspring", *Science* (New Series, Volume 179), January 5, 1973, pp. 90-92,

[49] R. C. Vogt and J. J. Bull, "Temperature controlled sex determination in turtles: ecological and behavorial aspects", *Herpetologica* 38:1 (1982), pp. 156-164.

[50] S. A. West and B. C. Sheldon, "Constraints in the evolution of sex ratio adjustment", *Science* 295 (March 1 2002), pp. 1685-1688.

[51] K. L. Wiebe and G. R. Bortolotti, "Facutative sex ratio manipulation in American kestrels", *Behavioral Ecology and Sociobiology* 30:6 (1992), pp.379-386.

[52] D. E. Willard, "Self-verifying axiom systems", *Computational Logic and Proof Theory: The Third Kurt Gödel Colloquium* (1993), Springer-Verlag LNCS#713, 325-336.

[53] D. E. Willard, "Self-verifying systems, the incompleteness theorem and the tangibility reflection principle", in *Journal of Symbolic Logic* 66 (2001), pp. 536-596.

[54] D. E. Willard, "How to extend the semantic tableaux and cut-free versions of the second incompleteness theorem almost to Robinson's arithmetic Q", *Journal of Symbolic Logic* 67 (2002), pp. 465-496.

[55] D. E. Willard, "An exploration of the partial respects in which an axiom system recognizing solely addition as a total function can verify its own consistency", *Journal of Symbolic Logic* 70 (2005), pp. 1171-1209.

[56] D. E. Willard, "A generalization of the second incompleteness theorem and some exceptions to it". *Annals of Pure and Applied Logic* 141 (2006), pp. 472-496.

[57] D. E. Willard, "On the available partial respects in which an axiomatization for real valued arithmetic can recognize its consistency", *Journal of Symbolic Logic* 71 (2006) pp. 1189-1199.

[58] D. E. Willard, "Passive induction and a solution to a Paris-Wilkie question", *Annals of Pure and Applied Logic* 146(2007), pp. 124-149.

[59] D. E. Willard, "On the broader epistemological significance of self-justifying axiom systems", *Proceedings of 21st Wollic Conference*, Springer Verlag LNCS 8652 (2014), pp. 221-236.

[60] D. E. Willard, "On how the introducing of a new θ function symbol into arithmetic's formalism is germane to devising axiom systems that can appreciate fragments of their own Hilbert consistency", http://arxiv.org/abs/1612.08071 a Cornell Archives report.

[61] E. O. Wilson, *Sociobiology: The New Synthesis*, Harvard University Press, 1975.

[62] B. Yilmaz-Dirkmen and S. Dirkmen, "A morphometric method of sexing white layer eggs", *Brazilian Journal of Poultry Science*, 15:3 (2013), pp. 203-210. Available on-line via Google.

[63] P. Yourgrau, *A World Without Time: The Forgotten Legacy of Gödel and Einstein*, Basic Books, 2005.

On How the Introducing of a New θ Function Symbol Into Arithmetic's Formalism Is Germane to Devising Axiom Systems that Can Appreciate Fragments of Their Own Hilbert Consistency

Dan E. Willard*

State University of New York at Albany

Abstract

It is well known that the combined work of Pudlák and Solovay [?, ?], enhanced by some added techniques of Nelson and Wilkie-Paris [?, ?], implies no reasonable axiom system can verify its own Hilbert consistency, when it recognizes Successor as a total function and treats addition and multiplication as 3-way relations (as Example ?? will explain). These considerations will lead us to examine unconventional axiomatizations for arithmetic that continue to view addition and multiplication as 3-way relations, but which replace the successor function symbol with an entirely new operator, called the " θ " primitive.

This θ operator will allow us to encode any integer n by a term T_n whose length will exceed the $O[\,\text{Log}(n)\,]$ length of a binary encoding only by the relatively small magnitudes formalized by Propositions ?? and ??. This paradigm will be significant because the combination of our Appendix A and prior published results in [56] will provide good reasons for conjecturing the θ primitive can be used to construct logics that can verify their own consistency under the desired setting of a Hilbert-styled deductive methodology.

It is likely that this paradigm can be combined with our prior results from [56] to construct axiom systems that are seriously diluted but able to verify their Hilbert-style consistency in some interesting fragmentary respects.

Keywords: Gödel's Second Incompleteness Theorem, Consistency, Hilbert's Second Open Question, Hilbert-styled Deduction (and its Frege-like analogs).