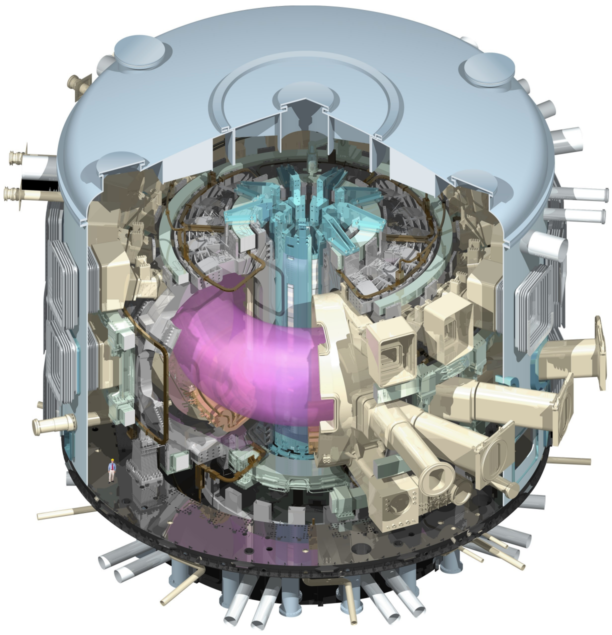
10401

Fusion Energy and Plasma Physics

Rasmus Kronborg Finnemann Wiuff (s163977)[*] and Nicklas Kihm (s143286)[†]

Technical University of Denmark[‡]

(Dated: January 24^{th} 2019)



Abstract: Abstract

CONTENTS

I. INTRO

In recent years a large and quickly growing collaboration between plasma physcisists and engineers have materialised in one of the most ambitious energy producing projects ever seen. The proof-of-concept plasma fusion tokamak reactor ITER[1] in Cadarache is currently taking shape in order to adress the issue of growing energy demands and climate changes.
The mission is simple: Prove that plasma fusion is a viable source of electricity.
Whilst not being the most surmountable task, many researchers and institutions have gathered from across the world, including the Department of Physics at DTU.
In this paper, three assignments are solved as part of the course "10401 Fusion Energy and Fusion Plasma Physics". Some key aspects of fusion plasma fueled reactors are adressed and discussed in the assignments.

II. PART 1: A SIMPLE REACTOR MODEL

A. Freidberg's simple reactor model

In the 5th chapter of the textbook by Freidberg[2], he makes a simple model for designing a fusion reactor power plant. The model uses simple geometric and electromagnetic assumptions with little involvement of plasma physics. The variables put into the model are shown in Table I. Table II shows the output quantities from the model. This model has been implemented in a matlab script provided for the course. The script is shown included in Appendix A. As an example, the model is run with the following parameters:

$$\begin{aligned} n_{\text{flux fraction}} &= 0.01 & P_E &= 1000 \text{ MW} \\ P_W &= 4 \text{ MW}{\cdot}\text{m}^{-2} & B_{\max} &= 13 \text{ T} \\ \sigma_{\max} &= 3000 \text{ atm} & \eta_t &= 0.4 \end{aligned} \tag{1}$$

Note that C_F and C_I has been ommited as these serve no purpose for this assignment. It is not of interest how expensive the plant will be. Rather the geometries and physical quantities are of interest. The results from the model is given in Table II.

Symbol	Quantity
$n_{\text{flux fraction}}$	n flux in breeder end/n flux in breeder start []
C_F	Fixed cost propotionality constant [$]
C_I	Nuclear island cost propotionality constant $\left[\$ \cdot \text{W}\cdot\text{m}^{-3}\right]$
P_E	Desired output power [MW]
P_W	Maximum wall load $\left[\text{MW}\cdot\text{m}^{-2}\right]$
B_{max}	Magnetic field at the edge of the coil [T]
σ_{max}	Tensile strenght of the magnetic field coils [atm]
η_t	Energy conversion efficiency []

Table I: Variables in the Freidberg's model

B. Model sensitivity

At this point Freidberg has provided a model that produce some reasonable results for a powerplant. It could be interesting to see how this model behaves when some vital parameters are changed. In the last section the model used the parameters shown in Eq. (1). Now the model will be iteratet over variations in the following parameters, while retaining the rest. The variable parameters are the electric power P_E, the maximum wall loading P_W, the maximum magnetic field B_{max} and the maximum stress σ_{max}. Appendix B includes the code for iterating the matlab model over various paremeters.

1. Change in desired power output.

For the rise in megawatts produced (500-1000 MW) there are a linear rise in aspect ratio, major radius, plasma surface and plasma volume. The rest of the parameters where constant except for the magnetic field strenght at the magnetic axis and the normalised plasma pressure. These are shown in **??**.

C. Elliptic Cross section

Freidberg's model assumes a circular cross section of the plasma. In reality this is not the case, and as of such we will now make a more realistic, yet still approximate reactor for an elliptic plasma cross section. In describing the geometry one refers to the elongation

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	0.799 m
a	Minor radius	2.01 m
R_0	Major radius	4.96 m
A	Aspect ratio	2.4670
A_p	Plasma surface	393 m^2
V_p	Plasma volume	395 m^3
P_{dens}	Power density	4.97×10^6 $\mathrm{W{\cdot}m^{-1}}$
p	Plasma pressure	7.37×10^5 Pa
n	Particle density	1.53×10^{20} $\mathrm{m^{-3}}$
B_0	Magnetic field at magnetic axis	4.57 T
β	Normalised plasma pressure	8.85%
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.14 s

Table II: Output quantities in the model in Freidberg's[2] along with the obtained values when inserting the parameters from Eq. (1)

ratio:

$$\kappa = \frac{a_{\text{max}}}{a_{\text{min}}} \tag{2}$$

With a_{max} the major radius and a_{min} the minor radius of the ellipse. This parameter ensures a true elliptic cross section as defined by the equation,

$$\frac{x^2}{a_{\text{max}}} + \frac{y^2}{a_{\text{min}}} = 1 \tag{3}$$

which can be parameterised as

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} a_{\text{min}} \cos\phi \\ \kappa a_{\text{min}} \sin\phi \end{bmatrix} \tag{4}$$

Meanwhile the blanket must be implemented as an ellipse or swelled ellipse. The true ellipse results in a difference of thickness in the blanket while the second results in a blanket of equal thickness throughout the structure. To this, the choice of implementing the blanket as a true ellipse has been made, since it simplifies derivations a bit. Note however, that keeping a constant thickness is the preferable option as it will reduce the engineering volume and hence the cost of the machine.

The outher layer is parameterised

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} (b + a_{\text{min}}) \cos\phi \\ \kappa(b + a_{\text{min}}) \sin\phi \end{bmatrix} \tag{5}$$

with b the blanket thickness at the minor ellipse axis. Choosing for now, $a_{\text{min}} = 2$, $\kappa = 2$ and $b = 1.2$, 4 and 5 are plotted on Figure 1 along with the variation in thickness of the blanket.

Given 4 and 5, the engineering volume can easily be derived if $c\,\cos(\phi)$ and $\kappa\, c\,\sin(\phi)$ is added to the x and y-direction in 5 respectively, where c is the minimum thickness of the magnetic coils that provide the torroidal field.

The cross sectional area of an ellipse is $A_{\text{e}} = \pi\, a_{\text{min}}\, a_{\text{max}}$ so the engineered volume becomes

$$\begin{aligned} V_{\text{I}} &= 2\,\pi\, R_0\, (A_{\text{e, outer}} - A_{\text{e, inner}}) \\ &= 2\,\pi^2\, R_0\, \left((a_{\text{min}} + b + c)^2 - a_{\text{min}}^2\right)\, \kappa \end{aligned} \tag{6}$$

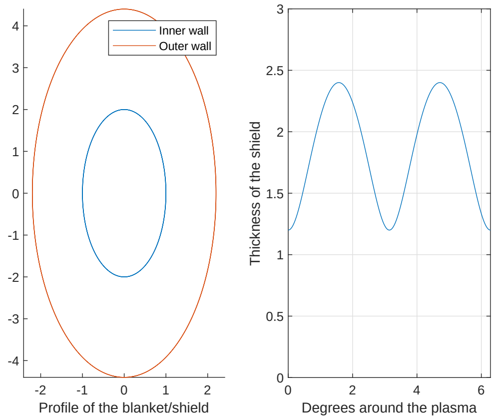


Figure 1: The profile of the blanket-and-shield and the thickness as a function of the angle around origo, ϕ.

and the plasma volume is similarly calculated as $V_{\text{P}} = 2\,\pi^2\, R_0\, \kappa\, a_{\text{min}}^2$. The plasma surface area is a bit more tricky, but it can be approximated as

$$S_{\text{p}} = 2\,\pi\, R_0\, \pi\, (3\,(a_{\text{min}} + \kappa\, a_{\text{min}}) - \sqrt{(3\, a_{\text{min}} + \kappa\, a_{\text{min}})\,(a_{\text{min}} + 3\,\kappa\, a_{\text{min}})}) \tag{7}$$

Using the same arguments as in the book, the B-field in the centre is surprisingly unchanged when going to the elliptical model. Now c is also approximated, or rather overestimated using a slight change to eq. (5.24) in the textbook. Since the force grows with a_{min} inserting $\kappa\, a_{\text{min}}$ instead yields an overestimation on the vertical force on the magnet. The tensile forces are the same, so the force balance leads to

$$c = \frac{2\,\xi}{1 - \xi}(\kappa\, a + b) \tag{8}$$

with $\xi = B_{sic}^2 / 4\,\mu_0\,\sigma_{\text{max}}$. These new equations are inserted in the code. The results are displayed in III. The plasma volume and surface area is of course increased as the plasma was made higher. This of course also results in a decreased power density. Overall, parameters such as B_0, β and $\tau_{\text{E}_{\text{min}}}$ while changing a bit, they were not changed significantly.

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	1.30 m
a	Minor radius	2.01 m
R_0	Major radius	4.96 m
A	Aspect ratio	2.4670
A_p	Plasma surface	609 m^2
V_p	Plasma volume	793 m^3
P_{dens}	Power density	2.48×10^6 $\mathrm{W{\cdot}m^{-1}}$
p	Plasma pressure	5.20×10^5 Pa
n	Particle density	1.08×10^{20} m^{-3}
B_0	Magnetic field at magnetic axis	4.60 T
β	Normalised plasma pressure	6.17%
$\tau_{E_{\mathrm{min}}}$	Min confinement time for $(p \times \tau_E)_{\mathrm{min}}$	1.62 s

Table III:
Output quantities from the elliptical model

D. Main parameters for DEMO

Setting $P_{\mathrm{E}} = 2000$ in the elliptical model yields the output parameters seen in table IV. Since R_0 is directly proportional to the electric power this of course increases linearly. The other geometric output parameters regarding areas and volumes also increase as a result. β has decreased a lot, so the plasma is not confined effectively in DEMO.

E. Designs for DEMO

With P_{E}, $\kappa = 2$ and $A = R_0/a_{\mathrm{min}} = 3 \Leftrightarrow R_0 = 3\,a_{\mathrm{min}}$. This is implemented in the code and the results are displayed in V. β has increased a bit but only to 4.55%. R_0 has been forced down, so the plasma volume etc. has decreased as well. Overall it seems like a smaller R_0 while keeping a_{min} fixed is an improvement. Or in other words, $A = 3$ is more desirable than $A \approx 5$.

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	1.30 m
a	Minor radius	2.01 m
R_0	Major radius	9.95 m
A	Aspect ratio	4.9512
A_p	Plasma surface	1.21×10^3 m^2
V_p	Plasma volume	1.59×10^3 m^3
P_{dens}	Power density	2.48×10^6 W·m^{-1}
p	Plasma pressure	5.20×10^5 Pa
n	Particle density	1.08×10^{20} m^{-3}
B_0	Magnetic field at magnetic axis	8.80 T
β	Normalised plasma pressure	1.69%
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.62 s

Table IV: Output quantities for DEMO using the elliptical model with $P_{\text{E}} = 2$GW

F. A and κ as free parameters

Designing the tokamak with κ and A as free parameters has led us to try and maximise profitability from V_P/A_P and β. Profitability in this context means that increasing the size of the tokamak leads to an increase in these parameters. However, there is a point when the change vs increase in size becomes constant. This means that we do not profit from increasing the size any longer.

V_{P} and A_{P} were calculated earlier so

$$\begin{aligned} \frac{V_{\text{P}}}{A_{\text{P}}} &= \frac{2\,\pi\,R_0\,\kappa\,a_{\text{min}}^2}{2\,\pi\,R_0\,\pi\,(3\,(a+\kappa\,a_{\text{min}}) - \sqrt{(3\,a_{\text{min}}+\kappa\,a_{\text{min}})\,(a+3\,\kappa\,a_{\text{min}})})} \\ &= \frac{\kappa\,a_{\text{min}}}{\pi\,(3+3\,\kappa-\sqrt{3+\kappa}\,\sqrt{1+3\,\kappa})} \end{aligned} \tag{9}$$

The expression scales linearly with a_{min} so we set it equal to 1m since it has no effect on the choice of κ. Differentiating with respect to κ and plotting is done, with the results shown in Figure 2a. Considering the figure, $\kappa = 2.5$ is chosen as the change is close enough to zero given this value.

Now work will be done towards choosing a value for A. First for geometric reasons a

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	1.30 m
a	Minor radius	2.01 m
R_0	Major radius	6.03 m
A	Aspect ratio	3
A_p	Plasma surface	738 m^2
V_p	Plasma volume	962 m^3
P_{dens}	Power density	4.09×10^6 W·m^{-1}
p	Plasma pressure	6.68×10^5 Pa
n	Particle density	1.39×10^{20} m^{-3}
B_0	Magnetic field at magnetic axis	6.07 T
β	Normalised plasma pressure	4.55%
$\tau_{E_{\min}}$	Min confinement time for $(p \times \tau_E)_{\min}$	1.26 s

Table V: Output quantities for DEMO using the elliptical model with $P_{siE} = 2$, $\kappa = 2$ and setting $A = 3$

minimum R_0 is calculated. It is simply $R_0 = a_{\min} + b + c$ where a_{min} is to be determined, $b = 1.2$m and $c = \frac{2\xi}{1-\xi}(\kappa\, a_{\min} + b)$. Meanwhile A is chosen to optimize β. Inserting R_0 into B_0 in the textbook yields

$$B_0 = \frac{2\,\xi\,(\kappa\, a_{\min} + b)\, B_{max}}{((2\,\kappa - 1)\, a_{\min} + a + b)} = \frac{178.75\, a_{\min} + 85.8}{55.5 + 36\, a_{\min}} \tag{10}$$

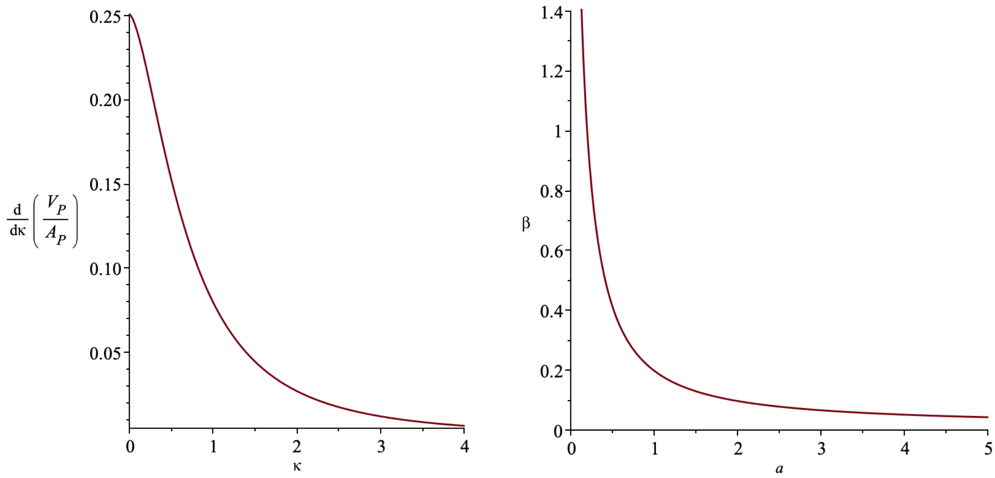
Where units has been disregarded and $\xi = 0.11$, $b = 1.2$, $\kappa = 2.5$ and $B_{\max} = 13$ has been inserted. Meanwhile, inserting the textbook's expression for P_{dens} into the expression for p and inserting the input parameters yields

$$p = 1.042\text{E}6 \sqrt{\frac{1}{\kappa\, a_{\min}}} \tag{11}$$

Thus β becomes

$$\beta = \frac{2148\,(a_{\min} + 1.542)^2}{\sqrt{a_{\min}}\,(178.8\, a_{\min} + 85.8)^2} \tag{12}$$

This function is plotted in Fig. 2b and for ($\beta = 10\%$ $a = 1.938$m) is achieved. Therefore $R_0 = 1.9378\text{m} + 1.2\text{m} + 2 \cdot 0.11/(1 - 0.11) \cdot 2.5 \cdot 1.938\text{m} + 1.2\text{m} = 4.632\text{m}$ which means $A = 4.632\text{m}/1.938\text{m} = 2.390$



(a)

(b)

Figure 2

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	1.30 m
a	Minor radius	2.01 m
R_0	Major radius	2.60 m
A	Aspect ratio	1.29
A_p	Plasma surface	630 m^2
V_p	Plasma volume	414 m^3
P_{dens}	Power density	9.49×10^6 W·m^{-1}
p	Plasma pressure	1.02×10^5 Pa
n	Particle density	2.12×10^{20} m^{-3}
B_0	Magnetic field at magnetic axis	−3.09 T
β	Normalised plasma pressure	26.9%
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.26 s

Table VI: Output quantities for the elliptical model after β and A has been optimised

III. PART 2: DIAGNOSTICS VIA INTERFEROMETRY

When operating a fusion reactor a continous process of diagnostics is necessary in order to optimise the plasma for the fusion process. One of the active diagnostic methods are interferometry. The goal here for this part of the paper is to measure the plasma electron density in the Danish Tokamak Undertaking reactor.

A. Cutt-off and Source frequency

Using a interferometer one can measure the electron density n_e of the plasma. The refractive index of electromagnetic waves depend on the electron density and plasma frequency ω_p proportionally:

$$\omega_p^2 \propto n_e \tag{13}$$

For O-mode plasma waves the refractive index is:

$$N_{\mathrm{O}} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \tag{14}$$

with ω being the probing wave frequency. The plasma will reflect the beam if the plasma frequency is larger than the beam frequency so

$$\omega > \omega_p = \sqrt{n_e \frac{e^2}{\epsilon_0 m_{e0}}} \tag{15}$$

So for a given frequency, the plasma must not exceed a critical cut off electron density:

$$n_e < n_c = \omega^2 \frac{\epsilon_0 \cdot m_{e0}}{e^2} \tag{16}$$

which gives

$$N_{\mathrm{O}} = \sqrt{1 - \frac{n_e}{n_c}} \tag{17}$$

With a probing frequency much higher than the plasma frequency and the critical density much higher than the electron density Eq. (17) can be approximated by:

$$N_{\mathrm{O}} = \sqrt{1 - \frac{n_e}{n_c}} \approx 1 - \frac{1}{2}\frac{n_e}{n_c}01 - \frac{\omega_p^2}{2\omega^2} \tag{18}$$

With sufficient accuracy, the linear dependence of the O-mode refractive index on the electron density is obtained if the normalised quantities obey:

$$\frac{n_e}{n_c} \leq 0.4 \quad \frac{\omega_p}{\omega} \leq 0.6 \tag{19}$$

We must calculate the phase shift as one beam travels in vacuum by the length L_V and one wave travels in the plasma by the length L_P. The phase shift in terms of 2π is equal to the optical difference divided by the wavelength. With the refractive index in vacuum, $N_V = 1$, this yields:

$$\begin{aligned}\frac{\Phi}{2\pi} &= \frac{\Delta L_{opt}}{\lambda} = \frac{\int_{x_1}^{x_2}\left(N_V - N_{\mathrm{O}}(x')\right)\mathrm{d}x'}{\lambda} \approx \frac{1}{2\lambda n_c}\int_0^x n_e(x')\,\mathrm{d}x' \\ &= 4.48\times10^{-16}\left(\frac{\lambda}{\mathrm{m}}\right)\int_0^x\left(\frac{n_e(x')}{\mathrm{m}^{-3}}\right)\left(\frac{\mathrm{d}x'}{\mathrm{m}}\right)\end{aligned} \tag{20}$$

Assuming a Gaussian distribution, the electron density at $\pm\infty$ is approximately equal to the densities just inside the reactor walls. Therefore

$$\int_{-\infty}^{\infty} n_e \exp\left(-\frac{(y-b)^2}{2c^2}\right)\mathrm{d}y \approx n_e\ c\ \sqrt{2\pi} \tag{21}$$

With the density at the centre given as:

$$10^{16}\ \mathrm{m}^{-3} \leq n_e \leq 10^{18}\ \mathrm{m}^{-3}, \tag{22}$$

The c in Eq. (21) is the width of the Gaussian distribution and must fit inside the reactor. The DTU tokamak has a minor diameter of 0.250 m. Thus

$$\begin{aligned}\frac{\Phi}{2\pi} &\approx 4.48\times10^{-16}\left(\frac{\lambda}{\mathrm{m}}\right)n_e 0.250\ \mathrm{m}\sqrt{2\pi} = 1.12\times10^{-16}\ n_e\left(\frac{\sqrt{2\pi}c}{\omega\mathrm{m}}\right) \\ &= 8.416\times10^{-8}\left(\frac{n_e}{\omega\mathrm{s}}\right)\end{aligned} \tag{23}$$

Remembering Eq. (16)

$$\omega^2\frac{\epsilon_0\ m_{e0}}{e^2} = \omega^2\frac{8.85\times10^{-12}\ \mathrm{F{\cdot}m^{-1}}\ 9.11\times10^{-31}\ \mathrm{kg}}{1.60\times10^{-19}\ \mathrm{C}} \tag{24}$$

$$\Downarrow$$

$$n_e < 0.000314\omega^2 \tag{25}$$

Where any units has been disregarded. We want the largest possible phase shift which means that the lower the frequency the better. However cutoff must first be taken into account. Since the cutoff is given by Eq. (25) and since we want to measure densities up to 10^{18} m^{-3} the minimum frequency of the wave is

$$\frac{\omega}{2\pi} > \frac{\sqrt{\frac{10^{18}\ \mathrm{m}^{-3}}{0.000314}}}{2\pi} \tag{26}$$

$$\Downarrow$$

$$f \approx 9\ \mathrm{GHz} \tag{27}$$

Given the available emitters, the best emitter is therefore the one with $f = 60$GHz.

B. Beam Phaseshifts

The goal of the interferometer is to find the phaseshift between the microwave beam in vacuum and in plasma. Our suggestion is a setup involving a interferometer emitting a beam through the center of the reactor. The first measurement would be through vacuum to find the source phase. Afterwards phase measurements can be conducted with an active plasma in the reactor. So by first measuring the phase of the probing beam in vacuum, one can simply measure the phase shift.

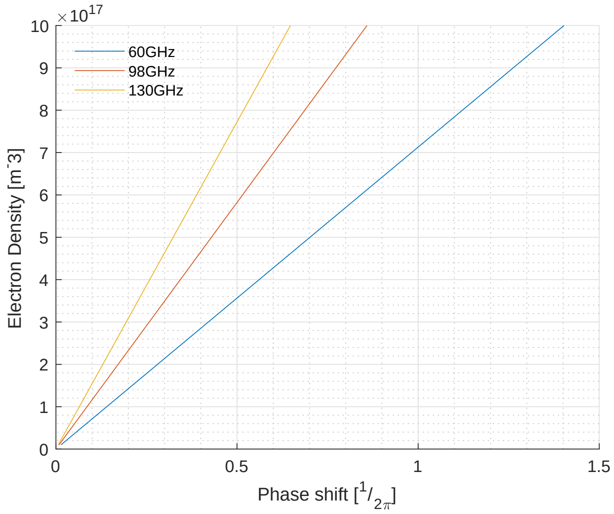


Figure 3: Electron densities for different wavelengths and phaseshifts

We know from Eqs. (20), (21) and (23) that:

$$\frac{\Phi}{2\pi} = 8.416 \times 10^{-8}\left(\frac{n_e}{\omega \mathrm{s}}\right) \tag{28}$$

So for different average electron densities through the plasma, one can plot the resulting phaseshifts. Using the code in Appendix D the plot in Fig. 3 is generated. The measured phase shift is then correlated to an electron density using Fig. 3.

C. Evolving beam width

In our case we want to use a Gaussian microwave. Such beam propagates spatially as shown in Fig. 4. w_0 is the beam initial beam waist determined by the source and $w(z)$ is the functino describing the waist. This equation is:

$$w(z) = w_0(z)\sqrt{1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2} \tag{29}$$

From this equation one can track the spatial propagation of the wave. In this project three sources are given. The initial beam waist is set to $w_0 = 0{\cdot}0275\mathrm{m}$ corresponding with half a reactor port. With

$$\frac{\lambda z}{\pi w_0^2} = \frac{9.542690316 \times 10^7 z}{w_0^2 f} \tag{30}$$

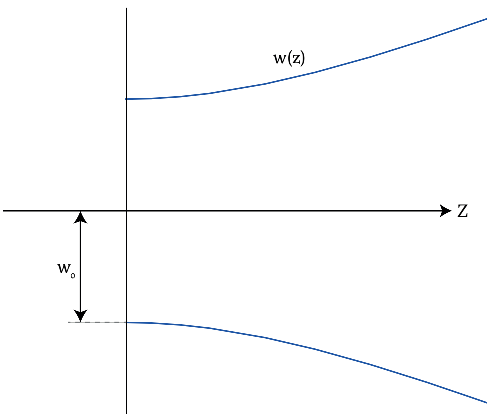


Figure 4: Sketch of Gaussian beam propagation with indication of w_0 and $w(z)$.

, and f being the frequency, the sources' beam waists are given as such:

$$60\text{ GHz}: \quad w(z) = 0.0275\sqrt{\left(1 + \frac{9.542690316 \times 10^7 z}{0.0275^2\ 60 \times 10^9}\right)^2} \tag{31}$$

$$98\text{ GHz}: \quad w(z) = 0.0275\sqrt{\left(1 + \frac{9.542690316 \times 10^7 z}{0.0275^2\ 98 \times 10^9}\right)^2} \tag{32}$$

$$130\text{ GHz}: \quad w(z) = 0.0275\sqrt{\left(1 + \frac{9.542690316 \times 10^7 z}{0.0275^2\ 130 \times 10^9}\right)^2} \tag{33}$$

For the three given sources in this project, the beam waist has been plotted on Fig. 5

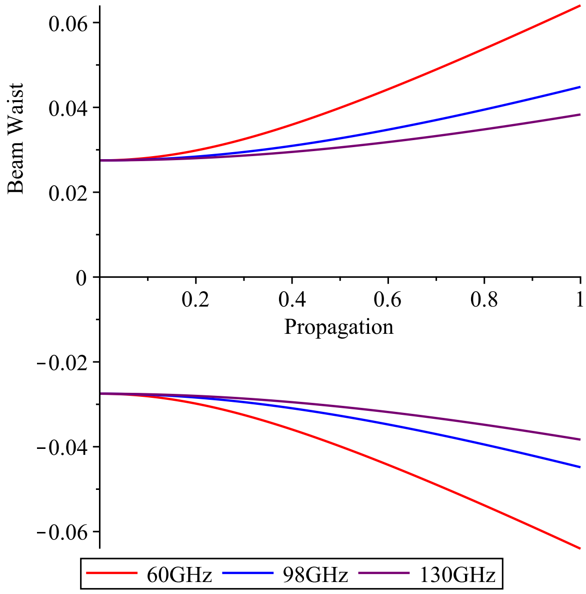


Figure 5: Beam Waist for the three microwave sources. The functions are Eqs. (31), (32) and (33).

It is undesireable for some of the beam to not reach the output port and instead propagate into the reactor wall, as this will result in a lesser signal strength. Therefore it can be nesessary to deploy a gaussian telescope.

D. Gaussian beam telescope interferometer

In order to control the beam waist, one can use a Gaussian beam telescope arrangement around the reactor. A lens is placed between the source and the reactor input port and again between the output port and the reciever. From chapter 5 in "Fusion Plasma Diagnostics with mm-Waves"[3] the authors explains how such an arrangement can be obtained. Starting with the focal lengths of the lenses,

$$d = f_0 + f_1 \tag{34}$$

, where f_0 and f_1 are focal lengths, and d are the distance between the lenses. The resulting narrowest beam waist after the second lens are given as:

$$w_2 = \frac{f_1}{f_0} w_0 \tag{35}$$

This making the transformation wavelength independent[3] (Eq 5.118). The distance to this waist is given as:

$$d_3 = \frac{f_1}{f_0}\left(f_0 + f_1 - \frac{f_1}{f_0} d_0\right) \tag{36}$$

The waist in between the lenses is given as:

$$w_1 = \frac{\lambda f_0}{\pi w_0} \tag{37}$$

And the distance from the first lens to this waist is:

$$d_1 = \left(\frac{\frac{d_0}{f_0} - 1}{\frac{w_0^2 \pi}{f_0 \lambda} + \left(\frac{d_0}{f_0} - 1\right)^2} + 1\right) f_0 \tag{38}$$

Thus the distance from w_1 to the second lens is:

$$d_2 = d - d_1 \tag{39}$$

Knowing these variables and utilising Eq. (29) we can model a complete setup. Using the matlab code in Appendix C, the following parameters were input:

$$\begin{aligned} r_p &= 0.0550 \text{ m} \quad r = 0.125 \text{ m} \quad w_0 = 0.0275 \text{ m} \quad freq = 60 \text{ GHz} \\ d_0 &= 0.200 \text{ m} \quad d_r = 0.100 \text{ m} \quad f_0 = 0.250 \text{ m} \qquad 0.200 \text{ m} \end{aligned} \tag{40}$$

With r_p being the tokamak port radius, r the tokamak minor radius and d_r the distance between the first lens and the reactor wall. The script gave the results:

$$\begin{aligned} w_1 &= 0.0145 \text{ m} \quad w_2 = 0.0220 \text{ m} \\ d_1 &= 0.224 \text{ m} \quad d_2 = 0.226 \text{ m} \quad d_3 = 0.232 \text{ m} \end{aligned} \tag{41}$$

Furtermore the code plots a sketch of the desired interferometer setup. The sketch can be seen in Fig. 6.

Lastly it should be mentioned that the beam waist confinement does not matter if the

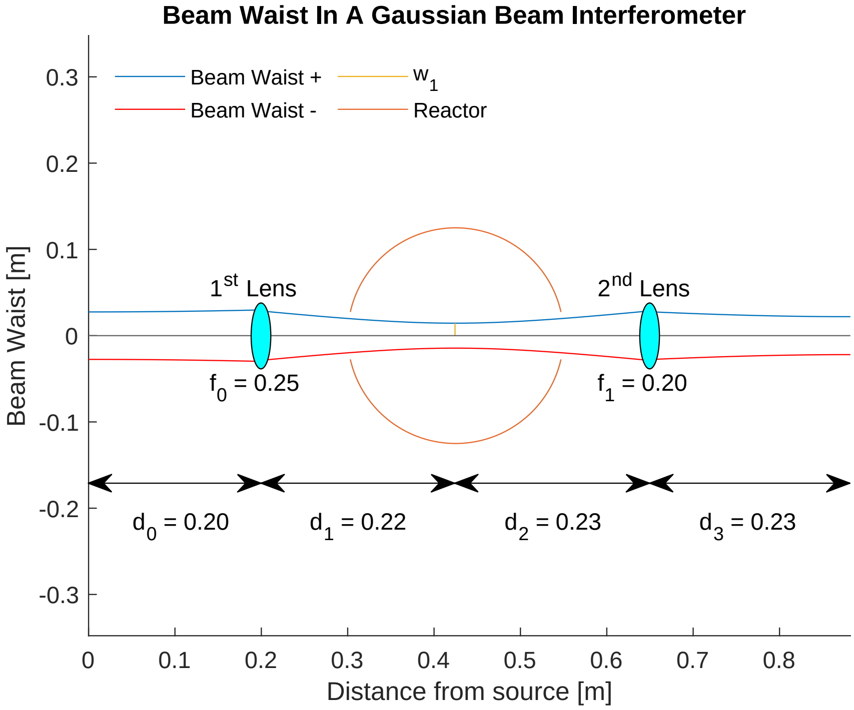


Figure 6: Gaussian beam telecsope interferometer setup with input parameters shown in Eq. (40) and output parameters shown in Eq. (41)

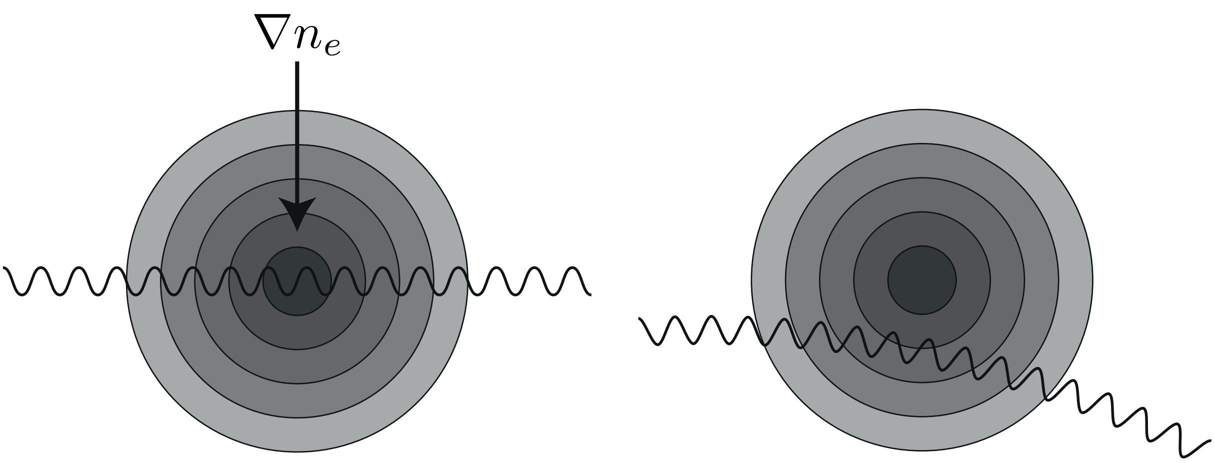


Figure 7: Refraction of the probing beam at center and off center.

wave is refracted into the side of the reactor. As the plasma density varies from the center and out, the plasma itself act as a variable lens with variable refractive indices. If the beam is emitted straight through the center, the refractive indices will only shorten and lengthen the wavelength back to normal, however if the beam is emmited off center, it will be refracted away from the center. In such a setup, further calculations need to be conducted to determine variation in signal strength, phase and wavelength. The two described setups are depicted on Fig. 7.

IV. PART 3: FUSOR EXERCISE

Studying the Inertial electrostatic confinement fusor at DTU. Measurements of neutron counts and the spectral line width are made as a function of the voltage and current. To this, the emission spectrum of the gas are also measured.

A. Plasma light emission and spectral line

When doing spectroscopy on a plasma of an unknown gas. Optical dispersion splits up the spectrum of light into lines which are then recorded using a detector. This detector measures the frequency/wavelength of the light and the intensity of the light. Thus a plot with the wavelength along the x-axis vs the intensity along the y-axis are produced. Meanwhile peaks are present on the recording where each peak are subject to broadening effects. Considering the Doppler effect, each photon's frequency will be shifted since the particles(which are the emitters) are moving at fast speeds(the Doppler shift increases with speed). Therefore if each particle where moving in the same direction the entire intensity peak would be shifted. But since the particles are moving more or less isotropically, the peaks are not shifted but rather broadened around the normal emission wavelength. To this it is also noted that the Doppler broadening increases if particles are moving faster. Considering Fig. 8, a spectral line is shown with a central wavelength of around 656 nm which correspond to the accepted value of hydrogen light emission[4]. Meanwhile the spectral lines of Deuterium only differs by a factor of 1.000272[5].

TODO LIST

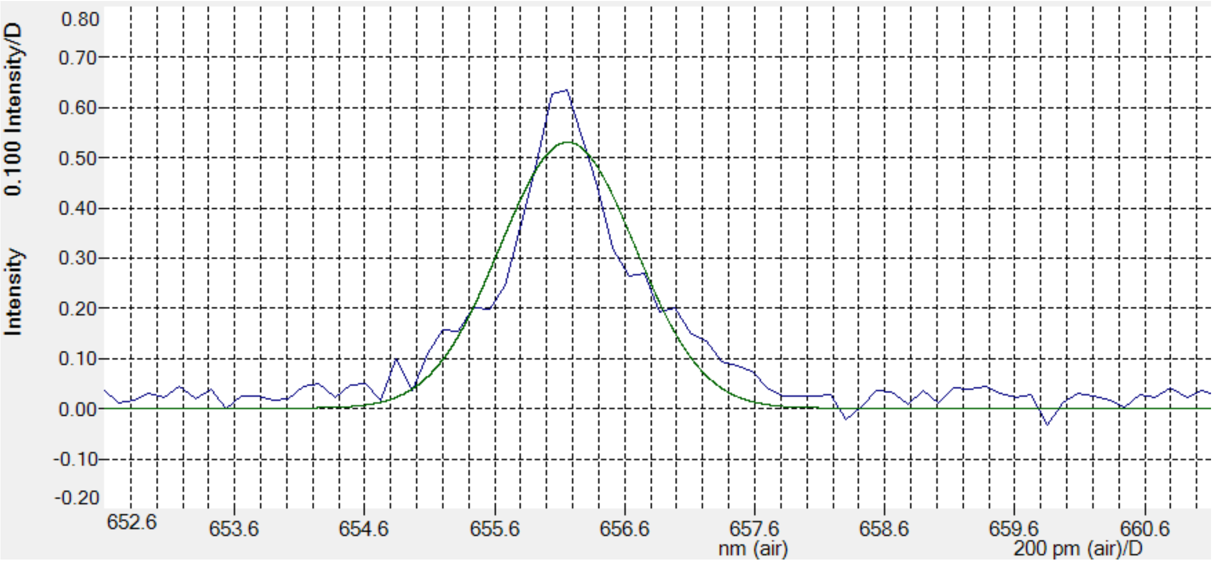


Figure 8: Spectral line for deuterium measured in the fusor.

---

* E-mail at s163977@student.dtu.dk

† E-mail at spacrone@live.dk

‡ Homepage of the Technical University of Denmark http://www.dtu.dk/english/;

Project Repository: https://github.com/rwiuff/10401

[1] https://www.iter.org/.

[2] Jeffrey P. Freidberg, *Plasma physics and fusion energy* (Cambridge University Press, 2007).

[3] Hans-Jürgen Hartfuß and Thomas Geist, *Fusion Plasma Diagnostics with mm-Waves* (Wiley-VCH, 2013).

[4] "Hydrogen spectral series" - Wikipedia (Visited January 22nd).

[5] "Deuterium#Spectroscopy" - Wikipedia (Visited January 22nd).

LIST OF FIGURES

LIST OF TABLES

LISTINGS

Appendices

Appendix A: tokamakDTU_asign_1

```
%***********************************************************************
% Name:          tokamakDTU
%
% Version:       1.0
%
% Purpose:       Contains the function 'tokamakDTU' which gives parameters
%                for a tokamak fusion power plant as output based on a
%                simplified model. The equations used are derived in
%                chapter 5 in Friedberg, Plasma physics and Fusion
%                Energy, 2007 (all references are referring thereto).
%
% To do (NOT for 10401 - Fusion Energi students):
%             1. Rewrite the code to a class (this is not done on purpose so
%                that the code is more readable for students not familiar
%                with classes). Can from that merge the files which takes
%                R_0/a and/or the ellipticity as an input into one file.
%
% Changelog:
%             1. December 2014:
%                Written by Michael Løiten based on a similar code written
%                as a bachelor project by Elias Pagh Sentius
%                mailto: mmag@fysik.dtu.dk
%***********************************************************************

function [b, c, a, R_0, A, A_p, V_p, P_dens, p, n, B_0, beta, tau_E_min,...
          C_per_watt] =...
    tokamakDTU_asign_1(...
        n_flux_fraction, C_F, C_I, P_E, P_W, B_max, sigma_max, eta_t)
```

```
%TOKAMAK_DTU Function which returns the parameters of a power plant
%
% Output parameters
%------------------
% b          - Blanket/shield thickness [m]
% c          - Magnet coil thickness [m]
% a          - Minor radius [m]
% R_0        - Major radius [m]
% A          - Aspect ratio []
% A_p        - Plasma surface [m^2]
% V_p        - Plasma volume [m^3]
% P_dens     - Power density [W/m]
% p          - Plasma pressure [Pa]
% n          - Particle density [m^-3]
% B_0        - Magnetic field at magnetic axis [T]
% beta       - Plasma beta in the centre []
% tau_E_min  - Min confinement time for satisfaction of (p*tau_E)_min [s]
% C_per_watt - The cost of the powerplant [$]
%
% Input parameters
%-------------------
% n_flux_fraction - n flux in breeder end/n flux in breeder start []
% C_F             - Fixed cost propotionality constant [$]
% C_I             - Nuclear island cost propotionality constant [$W/m^3]
% P_E             - Desired output power [MW]
% P_W             - Maximum wall load [MW/m^2]
% B_max           - Magnetic field at the edge of the coil [T]
% sigma_max       - Tensile strenght of the magnetic field coils [atm]
% eta_t           - Energy conversion efficiency []


% The function starts by defining fixed constants
```

```
% Note that this is inefficient if we are looping over the function, but it
% makes the code easier to use, as these are not needed as input parameters

% Fixed constants
%##########################################################################
% Nuclear
%--------
% Energies
E_t          = 2.5e-8; % [MeV] Energy of slow (thermal) neutron (eq 5.6)
E_n          = 14.1;   % [MeV] Neutron energy after fusion (eq 2.17)
E_a          = 3.5;    % [MeV] alpha energy after fusion (eq 2.17)
E_Li         = 4.8;    % [MeV] Heat produced by breeding Li (under eq 4.31)
% Cross section and main free paths
sigma_v_avg = 3.0e-22;% [m^3/s] DT fusion cross section @ 15keV (table 5.2)
lambda_br   = 0.0031; % [m] Breeding mean free path (under eq 5.7)
lambda_sd   = 0.055;  % [m] Mean free path from sigma_sd (eq 5.3)

% Plamsa physics
%---------------
% Parameters for infinity gain at the minimum of p tau_E (eq 4.20)
T             = 15.0;  % [keV] Temparature for obtaining min tripple product
tripple_min  = 8.3;    % [atm s] Min tripple prod to obtain Q=inf @ T=15 keV

% Natural constants
%------------------
mu_0 = 4.0*pi*1e-7;        % Vacuum permeability [T*m/A]
e    = 1.602176565e-19;   % Elementary charge [C]
%##########################################################################


% Secondly we convert everything to SI units, so that the variables are
% easier to handle
```

```
% Again, this is computationally inefficient, but it suffices for our use
% Conversion to SI-units
%##########################################################################
% Conversion factors
W_per_MW      = 1.0e6;
Pa_per_atm    = 1.01325e5;
eV_per_keV    = 1.0e3;
eV_per_MeV    = 1.0e6;
J_per_eV      = e;
J_per_keV     = J_per_eV * eV_per_keV;
J_per_MeV     = J_per_eV * eV_per_MeV;
% Conversions
P_E           = P_E * W_per_MW;          % Desired output power
P_W           = P_W * W_per_MW;          % Wall Loading limit on first wall
E_t           = E_t * J_per_MeV;         % Energy of slow (thermal) neutron
E_n           = E_n * J_per_MeV;         % Neutron energy after fusion
E_a           = E_a * J_per_MeV;         % alpha energy after fusion
E_Li          = E_Li * J_per_MeV;        % Heat produced by breeding Li
sigma_max     = sigma_max * Pa_per_atm;  % Max allowable structural stress
T             = T * J_per_keV;           % Temparature for minimum p*tau_E
tripple_min   = tripple_min * Pa_per_atm;  % The Lawson parameter (p*tau_E)
%##########################################################################


% Calculate the geometrical factors
%-----------------------------------
% Find the breeder thickness
b = get_b(lambda_sd, E_n, E_t, lambda_br, n_flux_fraction);
% Find the minor plasma radius and the coil thickness
[a, c] = get_a_and_c(B_max, mu_0, sigma_max, b);
% Find the major radius
R_0 = get_R_0(a, eta_t, E_n, E_a, E_Li, P_E, P_W);
```

```
% Find the resulting geometrical factors
A   = R_0/a;                          % Aspect Ratio
A_p = (2.0*pi*a)*(2.0*pi*R_0);     % Plasma surface area
V_p = (pi*a (2.0))*(2.0*pi*R_0);  % Plasma volume


% Calculate the plasma physics parameters
%----------------------------------------
% Find the power density in the plasma
P_dens = get_P_dens(E_a, E_n, E_Li, P_E, eta_t, V_p);
% Find the plasma pressure
p = get_p(E_a, E_n, P_dens, T, sigma_v_avg);
% Calculate the density from the definition of p under eq 5.36
n = p/(2.0*T);
% Find the magnetic field strength on the magnetic axis
B_0 = get_B_0(R_0,a,b,B_max);
% Find the plasma beta on the magnetic axis
beta = get_beta(p, B_0, mu_0);
% Find the minimum reqiured confinement time from the definition of the
% minimum tripple product.
% NOTE: A higher confinement time is advantegous, and could in principle
% yield a smaller (and cheaper) reactor. However, the effect is not
% included in this model
tau_E_min = tripple_min/p;


% Calculate the cost
% (details about the cost can be found in the function get_a_and_c)
%-------------------
% Find the volume of the nuclear island
% (the material surrounding the plasma)
V_I = get_V_I(R_0,a,b,c);
```

```
% Find the reactor volume per power out
% In the current model, this is the only non-constant in the expression for
% cost per watt
V_I_per_P_E = V_I/P_E;
C_per_watt = get_C_per_watt(C_F, C_I, V_I_per_P_E);
end



function      = get_b(                                              )
%GET_B Calculates b from the need of slowing down and breeding neutrons

% Thickness of the moderator-breeding region so that 1 - n_flux_fraction
% have slowed down and undergone a breeding reaction
% [m]
% Equation 5.10
delta_x = 2.0*lambda_sd*...
          log( 1.0-(1.0/2.0)*(E_n/E_t) (1.0/2.0)*...
               (lambda_br/lambda_sd)*log( n_flux_fraction )...
              );

% Set b from delta_x
% Friedberg argues above equation 5.11 that b should be between 1 and 1.5 m
% Therefore a self chose constant is set to 0.38
self_chosen_constant = 0.32;
b = delta_x + self_chosen_constant;
end




function       = get_a_and_c(                         )
```

```
%GET_A_AND_C Calculates a and c

% c is obtained from requiring that the magnets are so thin that they are
% on the limit of the tensile strenght
% a is obtained from minimizing the costs

% xi defined when making the magnetic coil c as thin as possible
% Under equation 5.27
xi = B_max (2.0) / (4.0*mu_0*sigma_max);

% a is found from optimization of the cost, where
% total cost = fixed cost + nuclear island cost
%..........
% Fixed cost
%..........
% K_F = Fixed cost for building, turbines, generators etc (also applies to
% fusion, fission, fossil)
% Assumption: The fixed cost is proportional to power output:
% Equation 5.13
% K_F = C_F*P_E;
%..........
% Nuclear island cost (mainly cost of magnets, blanket and shield)
%...........
% Assumption: The proportional to reactor volume:
% Equation 5.14
% K_I = C_I*V_I;
% Equation 5.15
% V_I = 2.0*pi^(2.0) * R_0 * ( (a+b+c)^(2.0) - a^(2.0) ); % Reactor volume
%.............
% Cost per watt:
%.............
% Defined as C_p_watt = (K_F + K_I)/P_E, rewritten to
```

```
% C_p_watt = C_F + C_I*(V_I/P_E);
% Since the cost per watt contains two constants, we can minimize the
% V_I/P_E in order to optimize the cost
% Given by equation 5.20 inserted in 5.17
% Equation 5.21
% V_I_per_P_E = V_I/P_E;  % Reactor volume per power out
% a is found by setting the derivative of V_I_per_P_E = 0
% Equation 5.29
a = ((1.0 + xi)/(2.0*xi (1.0/2.0))) * b;

% Knowing xi, a, and, b, we can calculate c
% c found by comparing tensile force and magnetic force working on the coil
% Equation 5.27
c = 2*xi/(1-xi)*(a+b);
end



function       = get_R_0(                          )
%GET_R_0 Calculate the major radius

% Divide eq 5.18 (electric power out) by
% eq 5.19 (wall loading * area = total neutron production) and solve for R0
% Equation 5.20
R_0 = (1.0/(4.0*pi (2.0)*eta_t))*(E_n/(E_n + E_a + E_Li))*(P_E/(a*P_W));
end



function          = get_P_dens(                          )
%GET_P_DENS Calculate the power density

```

```
% The power density is found by the sum of the power from the alphas plus
% the power from the neutrons, divided by the plasma volume
% Equation 5.35
P_dens = (E_a + E_n)/(E_a + E_n + E_Li)*P_E/(eta_t*V_p);
end



function       = get_B_0(                )
%GET_B_0 Calculte the magnetic field strength on the magnetic axis

% B_max is found in the edge of the magnet (at R = R_0-a-b)
% B0 is the magnetic field at R0
% As B propto 1/R. we have that B_0/B_max = (R_0-a-b)/R_0, which leads to
% Equation 5.42
B_0 = ((R_0-a-b)/R_0)*B_max;
end



function        = get_beta(            )
%GET_beta Calculte the magnetic field strength on the magnetic axis

% Plasma beta in the center (kinetical pressure over magnetical pressure):
% Equation 5.43
beta = p / (B_0 2/(2.0*mu_0));
end



function      = get_p(                          )
%GET_P Calculate the plasma pressure
```

```
% Found from solving the sum of neutron and alpha power for n, and multiply
% the result with T
% Equation 5.37
p = ( (16.0/(E_a + E_n)) * P_dens ) (1.0/2.0)*...
     (T (2.0)/sigma_v_avg) (1.0/2.0);
end



function       = get_V_I(          )
%GET_V_I Calculate the volume of the material surrounding the plasma

% Equation 5.15
V_I = 2.0*pi (2.0) * R_0 * ( (a+b+c) (2.0) - a (2.0) );
end



function              = get_C_per_watt(                    )
%GET_C_PER_WATT Calculates the cost for one watt out from the power plant

% For details in how the cost is derived, see comments in the function
% get_a_and_c

C_per_watt = C_F + C_I*(V_I_per_P_E);
end
```

Appendix B: IterateTokamakDTU

```
close;
clear;

titl = [ Desired output power [MW] ,  Maximum wall load [MW m -2] ,...
     Magnetic field at the edge of the coil [T] ,...
     Tensile strenght of the magnetic field coils [atm] ];
foldertitl = [ DesiredOutputPower ,  MaximumWallLoad ,...
     MagneticFieldAtTheEdgeOfTheCoil ,...
     TensileStrenghtOfTheMagneticFieldCoils ];
l = 5;
p1 = [];
x = [];
mkdir('../MatlabFigures', foldertitl(l))

for i = 2000:5000
    [b, c, a, R_0, A, A_p, V_p, P_dens, p,...
        n, B_0, beta, tau_E_min, C_per_watt] =...
        tokamakDTU_asign_1(0.01, 1, 2, 1000, 4, 13, 3000, 0.4);
    q=[b, c, a, R_0, A, A_p, V_p, P_dens, p,...
        n, B_0, beta, tau_E_min, C_per_watt];
    p1=cat(1,p1,q);
    x=cat(1,x,i);
end

T = [ Blanket-shield thickness [m] ,  Magnet coil thickness [m] ...
     Minor radius [m] ,  Major radius [m] ,  Aspect ratio [] ...
     Plasma surface [m 2] ,  Plasma volume [m 3] ,  Power density [W m -1] ...
     Plasma pressure [Pa] ,  Particle density [m -3] ,...
     Magnetic field at magnetic axis [T] ,  Plasma beta in the centre [] ...
     Min confinement time for satisfaction of (p tau_E)_min [s] ,...
```

```
     The cost of the powerplant [ ] ];
fileT = [ Blanket-shieldThickness ,  MagnetCoilThickness ...
     MinorRadius ,  MajorRadius ,  AspectRatio ...
     PlasmaSurface ,  PlasmaVolume ,  PowerDensity ...
     PlasmaPressure ,  ParticleDensity ,...
     MagneticFieldAtMagneticAxis ,  PlasmaBetaInTheCentre ...
     MinConfinementTime ,...
     TheCostOfThePowerplant ];
for k = 1:14
    q = figure;
    y = p1(:,k);
    CM = jet(14);
    plot(x, y,'color', CM(k,:));
    ylabel(T(k));
    xlabel(titl(l));
    ytickformat('%.2f');
    epsfilename = sprintf('%s.eps',fileT(k));
    foldername = sprintf('../MatlabFigures/%s', foldertitl(l));
    fullfilename = fullfile(foldername,epsfilename);
    saveas(q, fullfilename, 'epsc')
end
```

Appendix C: interferometer.m

```
close all
clear all

mkdir('../MatlabFigures', 'Interferometer'); % Create save directory

%--------------------------Input Parameters-------------------------------%
r_p = 0.055; % Reactor port opening [m]
w_0 = 0.0275; % Initial Beam Waist [m]
r = 0.125; % Minor tokamak radius [m]
freq = 60; % Frequency of probe beam [GHz]

d_0 = 0.20; % Distance between source and first lens
d_r = 0.10; % Distance between first lens and reactor wall
f_0 = 0.25; % Focal length of first lens
f_1 = 0.20; % Focal length of second lens
%-------------------------------------------------------------------------%

%----------Distances and Lowest Beam Waists Calculations------------------%
c = 299792458; % The speec of light [m/s]
lambda = c / (freq * 10 9); % Calculates the wavelength [m]
d = f_0 + f_1; % Calculates the distance between lenses
w_1 = (lambda * f_0) / ...
    (pi * w_0);% Calculates the beam waist between lenses
d_1 = (((d_0 / f_0) - 1) / ((w_0 (2) * pi) / ...
    (f_0 * lambda) + ((d_0 / f_0) - 1) (2)) + 1) * ...
    f_0;% Distance after 1st lens to lowest beam waist
w_2 = (f_1 / f_0) * w_0; % Calculates w_2 after 2nd second lens
d_2 = d - d_1; % Distance from w_2 to second lens
d_3 = f_1 / f_0 *...
    (f_0 + f_1 -...
```

```
    (f_1 / f_0) * d_0);% Distance to lowest beam waist after 2nd lens
%------------------------------------------------------------------------%

%-----------------------Beam Waist Datapoints----------------------------%
x_0 = linspace(0, d_0, 1000);% Plotpoints till first lens
y_0 = [];
for i = 1:1000
    w = w_0 * ...
        sqrt(1+((lambda * x_0(i)) / ...
        (pi * w_0 2)) 2);% Calculates the beam waist from source
    y_0 = horzcat(y_0, w);
end

x_1 = linspace(0, d_1, 1000); % Plotpoints from first lens to w_1
y_1 = [];
for i = 1:1000
    w = w_1 * ...
        sqrt(1+((lambda * x_1(i)) / ...
        (pi * w_1 2)) 2);% Beam waist between w_1 and the 1st lens
    y_1 = horzcat(y_1, w);
end
y_1 = fliplr(y_1); % Flips the y-plot points as the beam waist is declining
y = horzcat(y_0, y_1);

x_2 = linspace(0, d_2, 1000); % Plotpoints from w_1 to second lens
y_2 = [];
for i = 1:1000
    w = w_1 * ...
        sqrt(1+((lambda * x_2(i)) / ...
        (pi * w_1 2)) 2);% Beam waist between w_1 and the 2nd lens
    y_2 = horzcat(y_2, w);
end
```

```

y = horzcat(y, y_2);

x_3 = linspace(0, d_3, 1000); % Plotpoints from 2nd lens to w_2
y_3 = [];
for i = 1:1000
    w = w_2 * ...
        sqrt(1+((lambda * x_3(i)) / ...
        (pi * w_2 2)) 2);% Evolving beam waist after the 2nd lens to w_2
    y_3 = horzcat(y_3, w);
end
y_3 = fliplr(y_3); % Flips the y-plot points as the beam waist is declining
y = horzcat(y, y_3);

for i = 1:1000
    x_1(i) = x_1(i) + d_0; % Creates x-axis data points
end
for i = 1:1000
    x_2(i) = x_2(i) + d_0 + d_1; % Creates x-axis data points
end
for i = 1:1000
    x_3(i) = x_3(i) + d_0 + d_1 + d_2; % Creates x-axis data points
end
x = horzcat(x_0, x_1, x_2, x_3);

Opy = -y; % Flip beam waist to plot 'Beam Waist -'
%-----------------------------------------------------------------------%

%---------------------Beam Waist Propagation Plot-------------------------%
l = figure; % Creates figure
hold on
axis equal
```

```
p1 = plot(x, y); % Plots Beam Waist+
p2 = plot(x, Opy, 'Color', 'red'); % Plots Beam Waist-
%p3 = xline(d_0, '--');
%xline(d_0+d_1+d_2, '--');
axPos = get(gca, 'Position'); % Get normalised axis coordinates
xMinMax = xlim; % Get normalised axis coordinates
yMinMax = ylim; % Get normalised axis coordinates
zAnn = axPos(1) + ((0 - xMinMax(1)) / (xMinMax(2) - xMinMax(1))) ...
    * axPos(3);% Defines points relative to axis in normalised coordinates
d0Ann = axPos(1) + ((d_0 - xMinMax(1)) / (xMinMax(2) - xMinMax(1))) ...
    * axPos(3);% Defines points relative to axis in normalised coordinates
d1Ann = axPos(1) + ((d_0 + d_1 - xMinMax(1)) / (xMinMax(2) - xMinMax(1))) ...
    * axPos(3);% Defines points relative to axis in normalised coordinates
d2Ann = axPos(1) + ((d_0 + d_1 + d_2 - ...
    xMinMax(1)) / (xMinMax(2) - xMinMax(1))) ...
    * axPos(3);% Defines points relative to axis in normalised coordinates
d3Ann = axPos(1) + ((d_0 + d_1 + d_2 + d_3 - ...
    xMinMax(1)) / (xMinMax(2) - xMinMax(1))) ...
    * axPos(3);% Defines points relative to axis in normalised coordinates
yAnn = axPos(2) + ((0 - yMinMax(1)) / (yMinMax(2) - yMinMax(1))) ...
    * axPos(4);% Defines points relative to axis in normalised coordinates
annotation('doublearrow', ...
    [zAnn, d0Ann], [yAnn - 0.2, yAnn - 0.2]);% Annotates d_0
annotation('doublearrow', ...
    [d0Ann, d1Ann], [yAnn - 0.2, yAnn - 0.2]);% Annotates d_1
annotation('doublearrow', ...
    [d1Ann, d2Ann], [yAnn - 0.2, yAnn - 0.2]);% Annotates d_2
annotation('doublearrow', ...
    [d2Ann, d3Ann], [yAnn - 0.2, yAnn - 0.2]);% Annotates d_3
annotation('ellipse', ...
    [d0Ann - 0.01, yAnn - y_0(1000) * 1.5, 0.02, y_0(1000) * 3], ...
    'FaceColor', 'cyan');% Draws 1st lens
```

```
annotation('ellipse', ...
    [d2Ann - 0.01, yAnn - y_0(1000) * 1.5, 0.02, y_0(1000) * 3], ...
    'FaceColor', 'cyan');% Draws 2nd lens
text(d_0/4, -0.22, sprintf('d_0 = %0.2f', d_0)) % Distance annotation
text(d_0+(d_1 / 4), -0.22, ...
    sprintf('d_1 = %0.2f', d_1))% Distance annotation
text(d_0+d_1+(d_2 / 4), -0.22, ...
    sprintf('d_2 = %0.2f', d_2))% Distance annotation
text(d_0+d_1+d_2+(d_3 / 4), -0.22, ...
    sprintf('d_3 = %0.2f', d_3))% Distance annotation
text(d_0-0.06, 0.06, '1^{st} Lens') % Lens annotation
text(d_0-0.06, -0.06, sprintf('f_0 = %0.2f', f_0)) % Lens annotation
text(d_0+d_1+d_2-0.06, 0.06, '2^{nd} Lens') % Lens annotation
text(d_0+d_1+d_2-0.06, -0.06, ...
    sprintf('f_1 = %0.2f', f_1))% Lens annotation
w1x = (d_0 + d_1); % w_1 coordinate
w1x = repelem(w1x, 100); % w_1 x-coordinate array
w1y = linspace(0, w_1); % w_1 y-coordinate array
p4 = plot(w1x, w1y); % Plots w_1
yline(0, '-'); % Plots optical axis
xlabel('Distance from source [m]'); % x-axis label
ylabel('Beam Waist [m]'); % y-axis label
title('Beam Waist In A Gaussian Beam Interferometer'); % Figure title

topstart = 2 * asin(r_p/(2 * r)) / 2; % Calculates reactor plot coordinates
topend = (2 * pi - 2 * topstart) / 2; % Calculates reactor plot coordinates
bottomstart = topend + topstart * 2; % Calculates reactor plot coordinates
bottomend = 2 * pi - topstart; % Calculates reactor plot coordinates

top = linspace(topstart, ...
    topend, 100);% Creates 100 datapoints for the top half reactor cutout
x_t = d_0 + d_r + r + r * cos(top); % x-parameter of circle
```

```
y_t = r * sin(top); % y-parameter of circle
bottom = linspace(bottomstart, ...
    bottomend, 100);% Creates 100 datapoints for the bottom half reactor
x_b = d_0 + d_r + r + r * cos(bottom); % x-parameter of circle
y_b = r * sin(bottom); % y-parameter of circle
p5 = plot(x_t, y_t, 'color', ...
    [0.911, 0.4100, 0.1700]);% Plot top half reactor
plot(x_b, y_b, 'color', ...
    [0.911, 0.4100, 0.1700]);% Plot bottom half reactor
lgd = legend([p1, p2, p4, p5], ...
    {'Beam Waist +', 'Beam Waist -', 'w_1', 'Reactor'});% Figure legend
legend('boxoff')
legend('Location', 'northwest')
lgd.NumColumns = 2;
hold off
%-------------------------------------------------------------------------%

%------------------Saving figure as Encapsulated Postscript---------------%
epsfilename = 'Interferometer.eps'; % Savename for the figure
foldername = sprintf('../MatlabFigures/Interferometer'); % Folder path
fullfilename = fullfile(foldername, epsfilename); % Filename path
saveas(1, fullfilename, 'epsc') % Save the figure as eps
%-------------------------------------------------------------------------%

%--------------------------Print Outputs----------------------------------%
w_1
w_2
d_1
d_2
d_3
%-------------------------------------------------------------------------%
```

Appendix D: PhaseShiftDensity.m

```
close all;
clear all;

mkdir('../MatlabFigures', 'PhaseShift'); % Create save directory

%--------------------------Input Parameters-------------------------------%
f_0 = 60; % 1st beam frequency [GHz]
f_1 = 98; % 2nd beam frequency [GHz]
f_2 = 130; % 3rd beam frequency [GHz]

n_0 = 10 16; % Lower electron density
n_1 = 10 18; % Hihger electron density
%-------------------------------------------------------------------------%

c = 299792458; % The speec of light [m/s]

y = linspace(n_0, n_1, 1000); % Datapoints between n_0 and n_1
Phi0 = []; % Phase shift array
Phi1 = []; % Phase shift array
Phi2 = []; % Phase shift array

for i = 1:1000 % Calculates Phase Shifts for frequency = f_0
    Phi0 = horzcat(Phi0, 8.416e-8*(y(i) / (f_0 * 10 9)));
end
for i = 1:1000 % Calculates Phase Shifts for frequency = f_1
    Phi1 = horzcat(Phi1, 8.416e-8*(y(i) / (f_1 * 10 9)));
end
for i = 1:1000 % Calculates Phase Shifts for frequency = f_2
    Phi2 = horzcat(Phi2, 8.416e-8*(y(i) / (f_2 * 10 9)));
end
```

```

l = figure; % Creates figure
hold on
p0 = plot(Phi0, y); % Plot electron denisty as function of phase shift
p1 = plot(Phi1, y); % Plot electron denisty as function of phase shift
p2 = plot(Phi2, y); % Plot electron denisty as function of phase shift
xlabel('Phase shift [^{1}/_{2\pi}]'); % x-axis label
ylabel('Electron Density [m^-3]'); % y-axis label
%title('Electron Density as function of measured phase shift'); % Figure title
lgd = legend([p0, p1, p2], ...
    {'60GHz', '98GHz', '130GHz'});% Figure legend
legend('boxoff')
legend('Location', 'northwest')
grid on
grid minor
hold off

%-------------------Saving figure as Encapsulated Postscript---------------%
epsfilename = 'PhaseShift.eps'; % Savename for the figure
foldername = sprintf('../MatlabFigures/PhaseShift'); % Folder path
fullfilename = fullfile(foldername, epsfilename); % Filename path
saveas(l, fullfilename, 'epsc') % Save the figure as eps
%--------------------------------------------------------------------------%
```