Exam presentation

Course 10401

Rasmus Kronborg Finnemann Wiuff (s163977)

January 17, 2019

DTU Tokamak Interferometer

Title	Diagnostics via interferometry
Goal # 1	To calculate the phase shift induced by O-mode plasma and investigate different microwave sources
Goal # 2	To design a lens system to accommodate the beam propagation in the small DTU tokamak

Fusor Spectral Lines

Title	Fusor exercise
Goal # 1	To measure spectral lines from fusion in the inertial electrostatic confinement fusor at DTU
Goal # 2	To estimate velocities and energies as functions of voltage

Cut off

$$\text{Proportionallity:} \quad \omega_p^2 \propto n_\text{e} \tag{1}$$

$$\text{Refreactive index:} \quad N_\text{O} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \tag{2}$$

The plasma is transparent if

$$\omega > \omega_p = \sqrt{n_\text{e} \frac{e^2}{\varepsilon_0 m_{\text{e}0}}} \tag{3}$$

For ω, n_e must not exceed cutoff density n_c:

$$n_\text{e} < n_\text{c} = \omega^2 \frac{\varepsilon_0 m_{\text{e}0}}{e^2} \tag{4}$$

$$n_\text{c} = \omega^2 \frac{8.85 \times 10^{-12}\,\text{F}\cdot\text{m}^{-1}\ 9.11 \times 10^{-31}\,\text{kg}}{1.60 \times 10^{-19}\,\text{C}} \tag{5}$$

Cut off

$$n_{\mathrm{e}} < 0.000314\omega^2 \tag{6}$$

Electron density range: 10^{16}-$10^{18}\,\mathrm{m}^{-3}$

$$\omega > \sqrt{\frac{10^{18}\,\mathrm{m}^{-3}}{0.000314}} \tag{7}$$

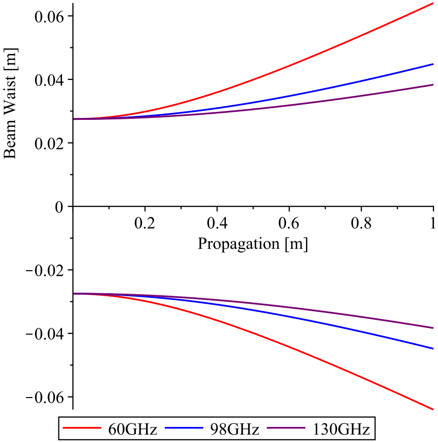
$$\Downarrow$$

$$f \approx 9\,\mathrm{GHz} \tag{8}$$

Four Sources

At a density of $10^{18}\,\mathrm{m}^{-3}$ the minimum frequency of the wave is: $f \approx 9\,\mathrm{GHz}$

- 2.45 GHz
- 60 GHz (Transparent)
- 98 GHz (Transparent)
- 130 GHz (Transparent)



Phase shift

$$N_{\mathrm{O}} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = \sqrt{1 - \frac{n_{\mathrm{e}}}{n_{\mathrm{c}}}} \tag{9}$$

Probing frequency much higher than plasma frequency, critical density much higher than electron density:

$$N_{\mathrm{O}} = \sqrt{1 - \frac{n_{\mathrm{e}}}{n_{\mathrm{c}}}} \tag{10}$$

$$\approx 1 - \frac{1}{2}\frac{n_{\mathrm{e}}}{\mathrm{c}} \tag{11}$$

$$= 1 - \frac{\omega_p^2}{2\omega^2} \tag{12}$$

The phase shift is given as:

$$\frac{\Phi}{2\pi} = \frac{\Delta L_{opt}}{\lambda} = \frac{\int_{x_1}^{x_2} (N_V - N_{\mathrm{O}}(x'))\, \mathrm{d}x'}{\lambda} \approx \frac{1}{2\lambda n_c} \int_0^x n_{\mathrm{e}}(x')\, \mathrm{d}x'$$
$$= 4.48 \times 10^{-16} \left(\frac{\lambda}{\mathrm{m}}\right) \int_0^x \left(\frac{n_{\mathrm{e}}(x')}{\mathrm{m}^{-3}}\right) \left(\frac{\mathrm{d}x'}{\mathrm{m}}\right) \quad (13)$$

Assuming a Gaussian density distribution:

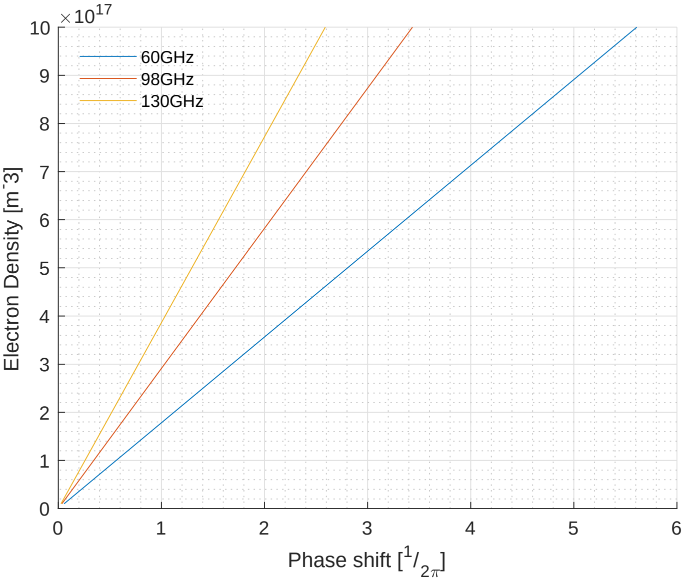
$$\int_{-\infty}^{\infty} n_{\mathrm{e}} \exp\left(-\frac{(y-b)^2}{2c^2}\right) \mathrm{d}y \approx n_{\mathrm{e}}\ c\ \sqrt{2\pi} \quad (14)$$

With tokamak diameter of 0.250 m:

$$\frac{\Phi}{2\pi} \approx 4.48 \times 10^{-16} \left(\frac{\lambda}{\mathrm{m}}\right) n_e 0.250\,\mathrm{m} \sqrt{2\pi} \quad (15)$$

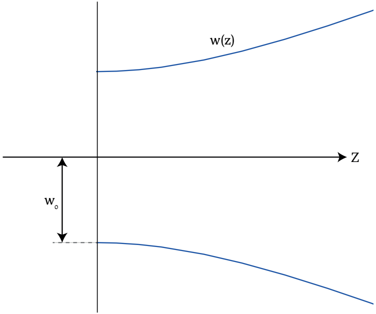
$$= 1.12 \times 10^{-16}\ n_e \left(\frac{\sqrt{2\pi} c}{\omega \mathrm{m}}\right) = 8.416 \times 10^{-8} \left(\frac{n_e}{\omega \mathrm{s}}\right) \quad (16)$$

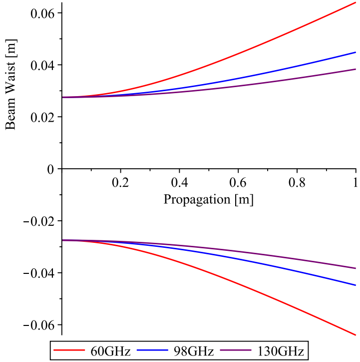
Phase shifts



Beam Waist:

$$w(z) = w_0(z)\sqrt{1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2} \tag{17}$$





Designing a Gaussian telescope Interferometer

$$d = f_0 + f_1 \tag{18}$$

$$w_2 = \frac{f_1}{f_0} w_0 \tag{19}$$

$$d_3 = \frac{f_1}{f_0}\left(f_0 + f_1 - \frac{f_1}{f_0} d_0\right) \tag{20}$$

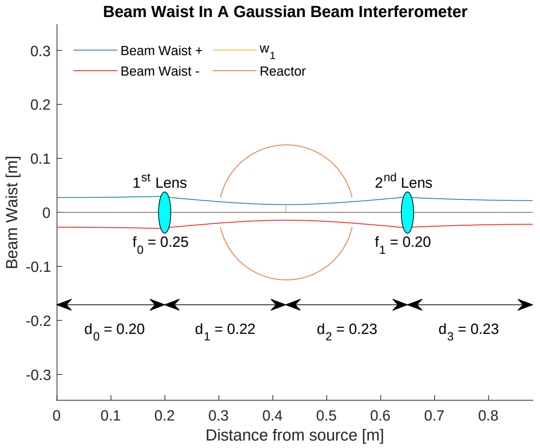
$$w_1 = \frac{\lambda f_0}{\pi w_0} \tag{21}$$

$$d_1 = \left(\frac{\frac{d_0}{f_0} - 1}{\frac{w_0^2 \pi}{f_0 \lambda} + \left(\frac{d_0}{f_0} - 1\right)^2} + 1\right) f_0 \tag{22}$$

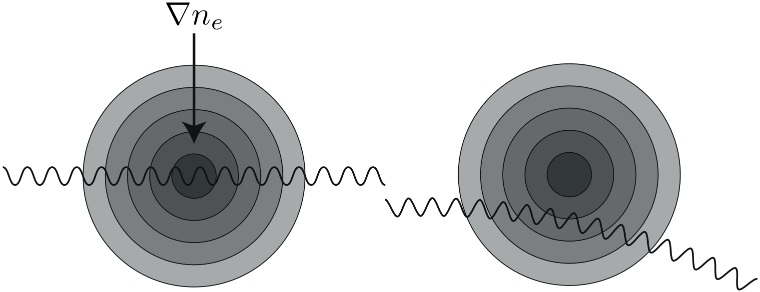
$$d_2 = d - d_1 \tag{23}$$

Gaussian Telescope

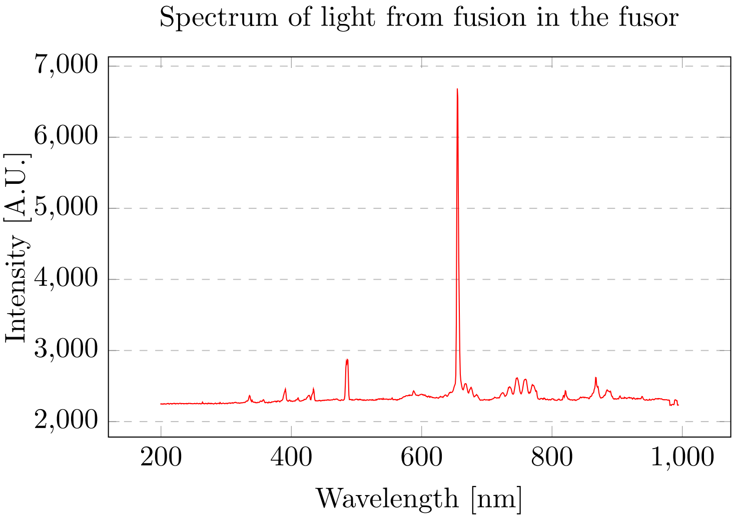
- Input parameters:
  $r_p = 0.0550\,\mathrm{m}$, $r = 0.125\,\mathrm{m}$,
  $w_0 = 0.0275\,\mathrm{m}$, $\mathrm{freq} = 60\,\mathrm{GHz}$,
  $d_0 = 0.200\,\mathrm{m}$, $d_r = 0.100\,\mathrm{m}$,
  $f_0 = 0.250\,\mathrm{m}$, $f_1 = 0.200\,\mathrm{m}$
- Output distances:
  $d_1 = 0.224\,\mathrm{m}$, $d_2 = 0.226\,\mathrm{m}$,
  $d_3 = 0.232\,\mathrm{m}$
- Output beamwaists:
  $w_1 = 0.0145\,\mathrm{m}$, $w_2 = 0.0220\,\mathrm{m}$
- Wavelenght: $\lambda = 0.005\,00\,\mathrm{m}$



Beware

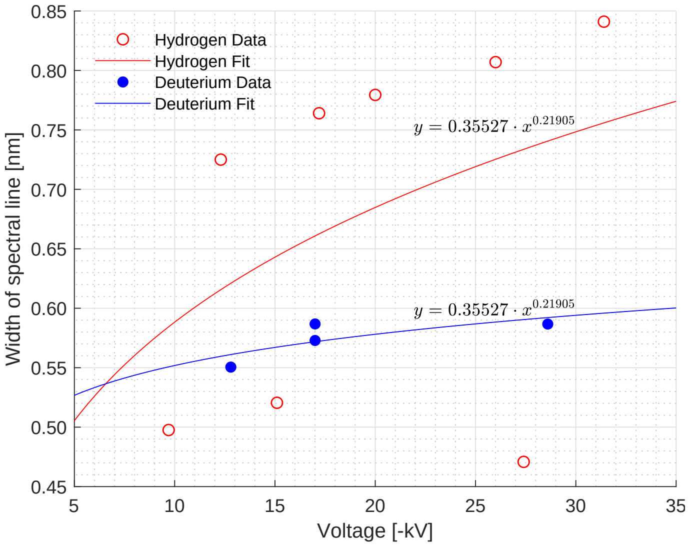


Spectrum



Spectrum of light from fusion in the fusor
7,000
6,000
5,000
4,000
3,000
2,000
Intensity [A.U.]
200
400
600
800
1,000
Wavelength [nm]

Spectral line width as function of applied voltage



Spectral line width:

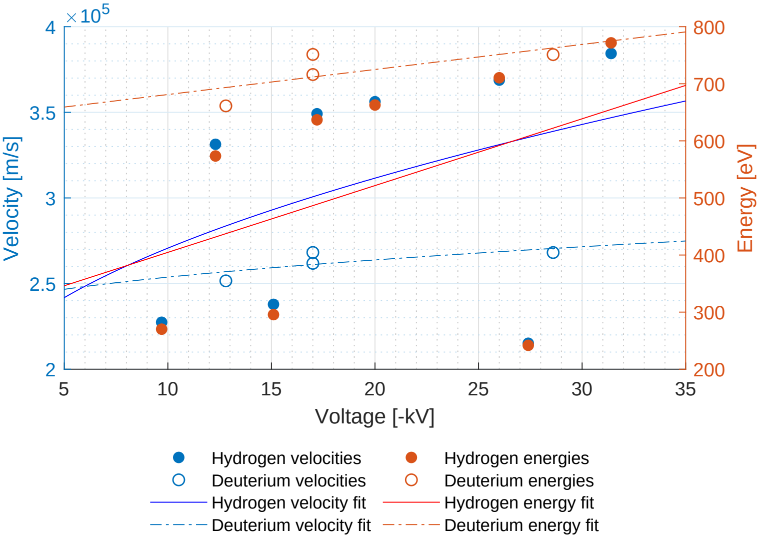
$$\sigma = \Delta\lambda = \frac{\lambda_0}{c} v_{1\mathrm{D}} \tag{24}$$

$$= \frac{\lambda_0}{c} \frac{kT}{m}^{1/2} \tag{25}$$

Kinetic energy:

$$\mathrm{KE} = \frac{1}{2} m v^2 \tag{26}$$

Energy and velocity



Questions

Rasmus Kronborg Finnemann Wiuff (s163977)

January 17, 2019

Energy and velocity fits

$$\begin{aligned}
\text{Velocity for hydrogen:} \quad & v = 3.12 \times 10^4 \sqrt{x} + 1.72 \times 10^5 \\
\text{Velocity for deuterium:} \quad & v = 0.7666 \times 10^4 \sqrt{x} + 2.295 \times 10^5 \\
\text{Kinetic energy for hydrogen:} \quad & KE = 11.7x + 287.7 \\
\text{Kinetic energy for deuterium:} \quad & KE = 4.387x + 637.3
\end{aligned}$$