# Assignment 1 & 2

## Course 10401


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**Assignment 1**

TITLE Fusion power plant

GOAL Design a tokamak using a modified version of the model presented in Freidberg's book.

**Assignment 2**

TITLE Design of interferometer for DTU tokamak

GOAL # 1 To calculate the phase shift induced by O-mode radiation and investigate different microwave sources

GOAL # 2 To design a lens system to accommodate the beam propagation in the small DTU tokamak

# Fusion power plant- Freidberg's model

Freidberg assumes a circular plasma cross section. Output parameters presented in the textbook are calculated given a few input parameters.

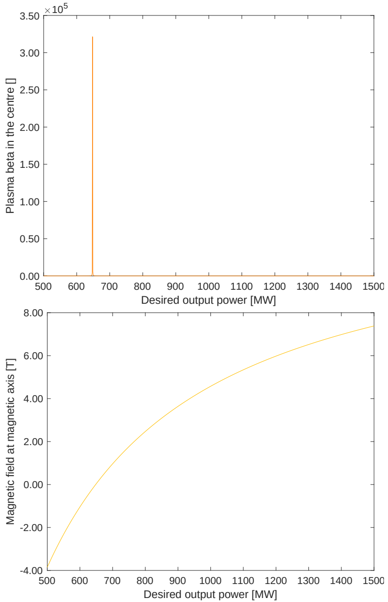
# Input parameters

| Symbol | Quantity | |
|---|---|---|
| $n_{\text{flux fraction}}$ | n flux in breeder end/n flux in breeder start | [] |
| $C_F$ | Fixed cost propotionality constant | [$] |
| $C_I$ | Nuclear island cost propotionality constant | $[\$ \cdot \mathrm{W/m^3}]$ |
| $P_E$ | Desired output power | [MW] |
| $P_W$ | Maximum wall load | $[\mathrm{MW/m^2}]$ |
| $B_{\max}$ | Magnetic field at the edge of the coil | [T] |
| $\sigma_{\max}$ | Tensile strenght of the magnetic field coils | [atm] |
| $\eta_t$ | Energy conversion efficiency | [] |

| Symbol | Quantity | Obtained values | |
|---|---|---|---|
| $b$ | Blanket-and-shield thickness | 1.2037 | m |
| $c$ | Magnet coil thickness | 0.7993 | m |
| $a$ | Minor radius | 2.0098 | m |
| $R_0$ | Major radius | 4.9583 | m |
| $A$ | Aspect ratio | 2.4670 | [] |
| $A_p$ | Plasma surface | 393.4152 | m$^2$ |
| $V_p$ | Plasma volume | 395.3509 | m$^3$ |
| $P_{\mathrm{dens}}$ | Power density | 4.9685e06 | W/m |
| $p$ | Plasma pressure | 7.3672e05 | Pa |
| $n$ | Particle density | 1.5327e20 | 1/m$^3$ |
| $B_0$ | Magnetic field at magnetic axis | 4.5744 | T |
| $\beta$ | Normalised plasma pressure | 8.85 | % |
| $\tau_{E_{\mathrm{min}}}$ | Min confinement time for $(p \times \tau_E)_{\mathrm{min}}$ | 1.1415 | s |

# Model sensitivity

$P_\mathsf{E}$, $P_\mathsf{W}$, $B_{\max}$ and $\sigma_{\max}$ are changed. Many output parameters were unaffected or changed linearly. But some interesting results were found.

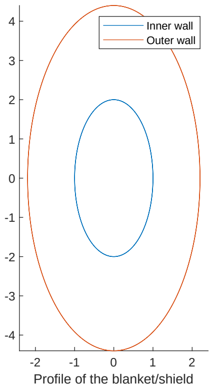


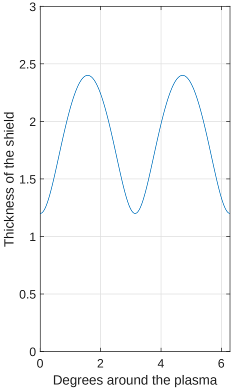
# Elliptic plasma-changes in model

Now the plasma is modeled as an ellipse. The thickness of the blanket is not constant. Rather the outer and inner ellipses are kept as true ellipses.

$$V_{\mathsf{l}} = 2\,\pi^2\,R_0\,((a_{\min} + b + c)^2 - a_{\min}^2)\,\kappa \quad (1)$$

$$c = \frac{2\,\xi}{1-\xi}\,(\kappa\,a_{\min} + b) \quad (2)$$



# Elliptical model output parameters

| Symbol | Quantity | Obtained values |
|---|---|---|
| $c$ | Magnet coil thickness | $1.30\,\mathrm{m}\uparrow$ |
| $A_p$ | Plasma surface | $1210\,\mathrm{m}^2\uparrow$ |
| $V_p$ | Plasma volume | $793\,\mathrm{m}^3\uparrow$ |
| $P_{\text{dens}}$ | Power density | $2.48 \times 10^6\,\mathrm{W/m}\downarrow$ |
| $p$ | Plasma pressure | $5.20 \times 10^5\,\mathrm{Pa}\downarrow$ |
| $n$ | Particle density | $1.08 \times 10^{20}/\mathrm{m}^3\downarrow$ |
| $\beta$ | Normalised plasma pressure | $6.17\%\downarrow$ |
| $\tau_{E_{\min}}$ | Min confinement time for $(p \times \tau_E)_{\min}$ | $1.62\,\mathrm{s}\uparrow$ |

| Symbol | Quantity | Obtained values |
|---|---|---|
| $b$ | Blanket-and-shield thickness | 1.20 m |
| $c$ | Magnet coil thickness | 1.30 m |
| $a$ | Minor radius | 2.01 m |
| $R_0$ | Major radius | 9.95 m↑ |
| $A$ | Aspect ratio | 4.95↑ |
| $A_p$ | Plasma surface | $2.41 \times 10^3$ m$^2$↑ |
| $V_p$ | Plasma volume | $1.59 \times 10^3$ m$^3$↑ |
| $P_{\text{dens}}$ | Power density | $2.48 \times 10^6$ W/m |
| $p$ | Plasma pressure | $5.20 \times 10^5$ Pa |
| $n$ | Particle density | $1.08 \times 10^{20}$/m$^3$ |
| $B_0$ | Magnetic field at magnetic axis | 8.80 T↑ |
| $\beta$ | Normalised plasma pressure | 1.69%↓ |
| $\tau_{E_{\text{min}}}$ | Min confinement time for $(p \times \tau_E)_{\text{min}}$ | 1.62 s |

# DEMO output parameters with $\kappa = 2$ and $A = 3$

| Symbol | Quantity | Obtained values |
|---|---|---|
| $b$ | Blanket-and-shield thickness | 1.20 m |
| $c$ | Magnet coil thickness | 1.30 m |
| $a$ | Minor radius | 2.01 m |
| $R_0$ | Major radius | 6.03 m $\downarrow$ |
| $A$ | Aspect ratio | 3 $\downarrow$ |
| $A_p$ | Plasma surface | $1.46 \times 10^3$ m$^2$ $\downarrow$ |
| $V_p$ | Plasma volume | 962 m$^3$ $\downarrow$ |
| $P_{\text{dens}}$ | Power density | $4.01 \times 10^6$ W/m $\uparrow$ |
| $p$ | Plasma pressure | $6.68 \times 10^5$ Pa $\uparrow$ |
| $n$ | Particle density | $1.39 \times 10^{20}$/m$^3$ $\uparrow$ |
| $B_0$ | Magnetic field at magnetic axis | 6.07 T $\downarrow$ |
| $\beta$ | Normalised plasma pressure | 4.55% $\uparrow$ |
| $\tau_{E_{\min}}$ | Min confinement time for $(p \times \tau_E)_{\min}$ | 1.26 s $\downarrow$ |

# DEMO WITH $\kappa$ AND $A$ AS FREE PARAMETERS

We have chosen to optimise the engineering volume and $\beta$ using $\kappa$ and $A$ as free parameters. This yields optimum $R_0$ and $\kappa$

$$R_0 = \sqrt{a^3/b} \tag{3}$$

$$\kappa = \frac{6\,a_{\min}\,\xi - b\,\xi - 6\,a_{\min} - b}{2\,a_{\min}\,\xi} \tag{4}$$

# DEMO output parameters with $\kappa$ and $A$ as free parameters

| Symbol | Quantity | Obtained values |
|---|---|---|
| $b$ | Blanket-and-shield thickness | 1.20 m |
| $c$ | Magnet coil thickness | 1.30 m |
| $a$ | Minor radius | 2.01 m |
| $R_0$ | Major radius | 2.60 m $\downarrow$ |
| $A$ | Aspect ratio | 1.29 $\downarrow$ |
| $A_p$ | Plasma surface | 630 $m^2$ |
| $V_p$ | Plasma volume | 414 $m^3$ $\downarrow$ |
| $P_{\text{dens}}$ | Power density | $9.49 \times 10^6$ W/m $\downarrow$ |
| $p \downarrow$ | Plasma pressure | $1.02 \times 10^5$ Pa |
| $n$ | Particle density | $2.12 \times 10^{20}/m^3$ $\downarrow$ |
| $B_0$ | Magnetic field at magnetic axis | $-3.09$ T $\downarrow$ |
| $\beta$ | Normalised plasma pressure | 26.9% $\uparrow$ |
| $\tau_{E_{\text{min}}}$ | Min confinement time for $(p \times \tau_E)_{\text{min}}$ | 1.26 s |

# Cut off

$$n_e < n_c = \omega^2 \frac{\epsilon_0 \cdot m_{e0}}{e^2} \tag{5}$$

$$\omega^2 \frac{\epsilon_0 \cdot m_{e0}}{e^2} = \omega^2 \frac{8.85 \times 10^{-12}\,\mathrm{F/m} 9.11 \times 10^{-31}\,\mathrm{kg}}{1.60 \times 10^{-19}\,\mathrm{C}} \tag{6}$$

$$\Downarrow$$

$$n_e < 0.000314\omega^2 \tag{7}$$

At densities of $10^{18}/\mathrm{m}^3$ the minimum frequency is:

$$\frac{\omega}{2\pi} > \frac{\sqrt{\frac{10^{18}/\mathrm{m}^3}{0.000314}}}{2\pi} \tag{8}$$

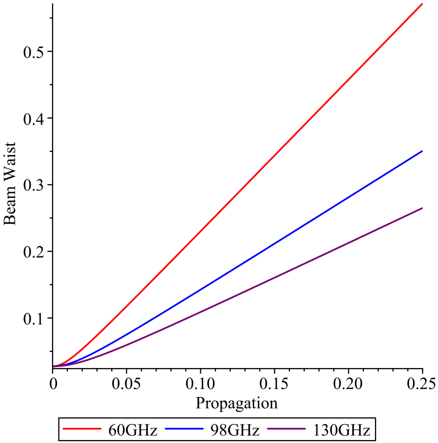
$$\Downarrow$$

$$f \approx 9\,\mathrm{GHz} \tag{9}$$

# Four Sources

At a density of $10^{18}/\mathrm{m}^3$ the minimum frequency of the wave is: $f \approx 9\,\mathrm{GHz}$

- 2.45 GHz (Transparent)
- 60 GHz
- 98 GHz
- 130 GHz

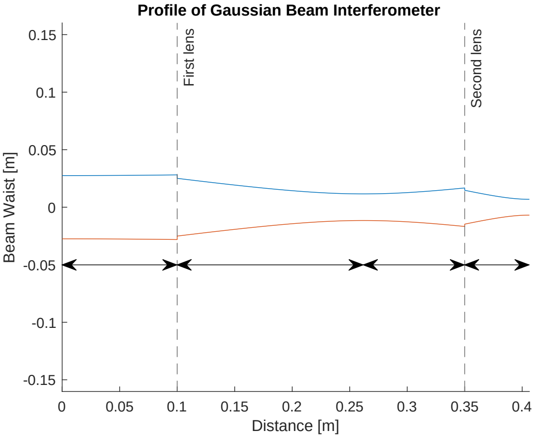


# Phaseshifts

| Frequency [GHz] | Phase at$[2\pi]$ $n_e = 10 \times 10^{16}/\mathrm{m}^3$ | Phase at $n_e = 10 \times 10^{18}/\mathrm{m}^3$ |
|---|---|---|
| 2.45 | 0.8620 | 86.17 |
| 60 | 0.0352 | 3.519 |
| 98 | 0.0215 | 2.154 |
| 130 | 0.0162 | 1.624 |

# Gaussian Telescope

Input parameters: $w_0$, $freq$, $d_0$, $f_0$, $f_1$ Output distances: $d_1 = 0.1619$, $d_2 = 0.0881$, $d_3 = 0.0563$ Output beamwaists: $w_1 = 0.0116$, $w_2 = 0.0069$ Wavelenght: $\lambda = 0.005$



# Questions

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