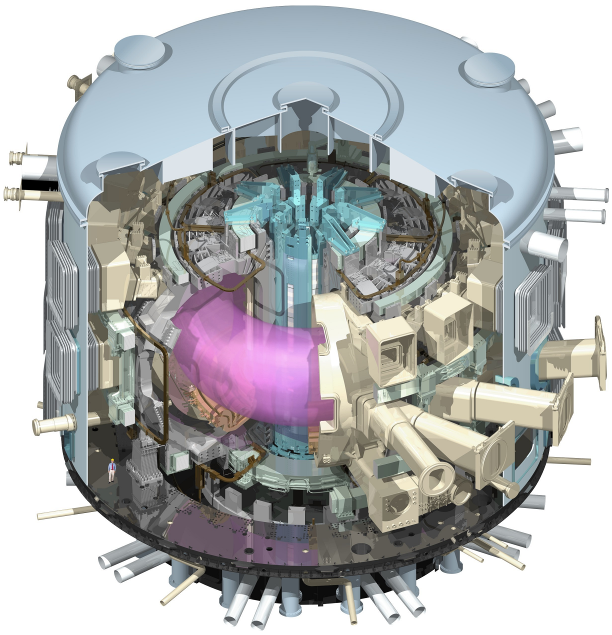
10401

Fusion Energy and Plasma Physics

Rasmus Kronborg Finnemann Wiuff (s163977)[*] and Nicklas Kihm (s143286)[†]

Technical University of Denmark[‡]

(Dated: January 24^{th} 2019)



Abstract: Abstract

CONTENTS

I. INTRO

In recent years a large and quickly growing collaboration between plasma physcisists and engineers have materialised in one of the most ambitious energy producing projects ever seen. The proof-of-concept plasma fusion tokamak reactor ITER[1] in Cadarache is

Symbol	Quantity
$n_{\text{flux fraction}}$	n flux in breeder end/n flux in breeder start []
C_F	Fixed cost propotionality constant [\$]
C_I	Nuclear island cost propotionality constant $\left[\$ \cdot \mathrm{W{\cdot}m^{-3}}\right]$
P_E	Desired output power [MW]
P_W	Maximum wall load $\left[\mathrm{MW{\cdot}m^{-2}}\right]$
$B_{\max}$	Magnetic field at the edge of the coil [T]
$\sigma_{\max}$	Tensile strenght of the magnetic field coils [atm]
η_t	Energy conversion efficiency []

Table I: Variables in the Freidberg's model

currently taking shape in order to adress the issue of growing energy demands and climate changes.
The mission is simple: Prove that plasma fusion is a viable source of electricity.
Whilst not being the most surmountable task, many researchers and institutions have gathered from across the world, including the Department of Physics at DTU.
In this paper, three assignments are solved as part of the course "10401 Fusion Energy and Fusion Plasma Physics". Some key aspects of fusion plasma fueled reactors are adressed and discussed in the assignments.

II. PART 1: A SIMPLE REACTOR MODEL

A. Freidberg's simple reactor model

In the 5^{th} chapter of the textbook by Freidberg[2], he makes a simple model for designing a fusion reactor power plant. The model uses simple geometric and electromagnetic assumptions with little involvement of plasma physics. The variables put into the model are shown in Table I. Table II shows the output quantities from the model. This model has been implemented in a matlab script provided for the course. The script is shown

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	0.799 m
a	Minor radius	2.01 m
R_0	Major radius	4.96 m
A	Aspect ratio	2.4670
A_p	Plasma surface	393 m^2
V_p	Plasma volume	395 m^3
P_{dens}	Power density	4.97×10^6 W·m^{-1}
p	Plasma pressure	7.37×10^5 Pa
n	Particle density	1.53×10^{20} m^{-3}
B_0	Magnetic field at magnetic axis	4.57 T
β	Normalised plasma pressure	8.85%
$\tau_{E_{\mathrm{min}}}$	Min confinement time for $(p \times \tau_E)_{\mathrm{min}}$	1.14 s

Table II: Output quantities in the model in Freidberg's[2] along with the obtained values when inserting the parameters from Eq. (1)

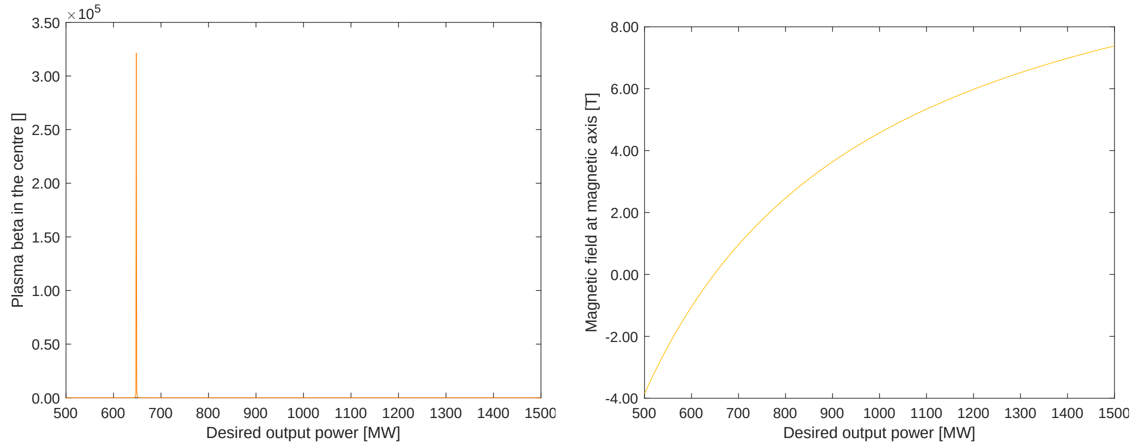
included in Appendix A. As an example, the model is run with the following parameters:

$$\begin{aligned} n_{\text{flux fraction}} &= 0.01 \qquad & P_E &= 1000 \text{ MW} \\ P_W &= 4 \text{ MW·m}^{-2} \qquad & B_{\max} &= 13 \text{ T} \\ \sigma_{\max} &= 3000 \text{ atm} \qquad & \eta_t &= 0.4 \end{aligned} \tag{1}$$

Note that C_F and C_I has been ommited as these serve no purpose for this assignment. It is not of interest how expensive the plant will be. Rather the geometries and physical quantities are of interest. The results from the model is given in Table II.

B. Model sensitivity

At this point Freidberg has provided a model that produce some reasonable results for a powerplant. It could be interesting to see how this model behaves when some vital parameters are changed. In the last section the model used the parameters shown in Eq. (1). Now the model will be iteratet over variations in the following parameters, while



(a) (b)

Figure 1: The evolution of the normalised plasma pressure (Fig. 1a) and the magnetic field strenght (Fig. 1b) when changing the desired power output.

retaining the rest. The variable parameters are the electric power P_E, the maximum wall loading P_W, the maximum magnetic field $B_{\max}$ and the maximum stress $\sigma_{\max}$. Appendix B includes the code for iterating the matlab model over various paremeters.

1. Change in desired power output.

For the rise in megawatts produced (500-1000 MW) there are a linear rise in aspect ratio, major radius, plasma surface and plasma volume. The rest of the parameters where constant except for the magnetic field strenght at the magnetic axis and the normalised plasma pressure. These are shown in Fig. 1.

C. Elliptic Cross section

Freidberg's model assumes a circular cross section of the plasma. In reality this is not the case, and as of such we will now make a more realistic, yet still approximate reactor for an elliptic plasma cross section. In describing the geometry one refers to the elongation ratio:

$$\kappa = \frac{a_{\max}}{a_{\min}} \tag{2}$$

With a_{max} the major radius and a_{min} the minor radius of the ellipse. This parameter ensures a true elliptic cross section as defined by the equation,

$$\frac{x^2}{a_{\text{max}}} + \frac{y^2}{a_{\text{min}}} = 1 \tag{3}$$

which can be parameterised as

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} a_{\text{min}} \cos\phi \\ \kappa a_{\text{min}} \sin\phi \end{bmatrix} \tag{4}$$

$$\tag{5}$$

Meanwhile the blanket must be implemented as an ellipse or swelled ellipse. The true ellipse results in a difference of thickness in the blanket while the second results in a blanket of equal thickness throughout the structure. To this the choice of implementing the blanket as a true ellipse has been made since it simplifies derivations a bit. Note however that keeping a constant thickness is the preferable option as it will reduce the engineering volume and hence the cost of the machine.

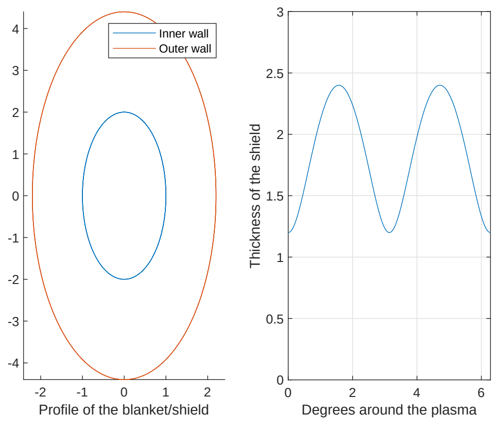


Figure 2: The profile of the blanket-and-shield and the thickness as a function of the angle around origo, ϕ.

The outher layer parameterised is

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} (b + a_{\text{min}}) \cos\phi \\ \kappa(b + a_{\text{min}}) \sin\phi \end{bmatrix} \tag{6}$$

with b the blanket thickness at the minor ellipse axis. Choosing for now, $a_{\text{min}} = 2$, $\kappa = 2$ and $b = 1.2$, 5 and 6 are plotted on Figure 2 along with the variation in thickness of the blanket.

Given 5 and 6 the engineering volume can easily be derived if $c\cos(\phi)$ and $\kappa\, c\sin(\phi)$ is added to the x and y-direction in 6 respectively, where c is the minimum thickness of the magnetic coils that provide the torroidal field.

The cross sectional area of an ellipse is $A_{\text{e}} = \pi\, a_{\text{min}}\, a_{\text{max}}$ so the engineered volume becomes

$$V_{\text{I}} = 2\pi R_0 \left(A_{\text{e, outer}} - A_{\text{e, inner}}\right) = 2\pi^2 R_0 \left((a_{\text{min}} + b + c)^2 - a_{\text{min}}^2\right) \kappa \tag{7}$$

and the plasma volume is similarly calculated as $V_{sip} = 2\pi^2 R_0\, a_{\text{min}}\, a_{\text{max}}$. The plasma surface area is a bit more tricky but the result is

$$S_{\text{p}} = 2\pi R_0 C_{sp} = 2\pi R_0\, 4\, a_{\text{min}} \int_0^{2\pi} \sqrt{1 - (1-\kappa^{-2})^{1/2} \sin^2\phi}\, d\phi \tag{8}$$

Symbol	Quantity	Obtained values
c	Magnet coil thickness	1.30 m
A_p	Plasma surface	1210 m^2
V_p	Plasma volume	793 m^3
P_{dens}	Power density	2.48×10^6 W$\cdot$m^{-1}
p	Plasma pressure	5.20×10^5 Pa
n	Particle density	1.08×10^{20} m^{-3}
β	Normalised plasma pressure	6.17%
$\tau_{E_{\mathrm{min}}}$	Min confinement time for $(p \times \tau_E)_{\mathrm{min}}$	1.62 s

Table III: Output quantities from the elliptical model

where the integral can be approximated after choosing κ. Thus our choice of κ yields $S_{\mathrm{p}} \approx 8\,\pi\,R_0\,a_{\mathrm{min}} \times 4.8$. The rest of the parameters in the code except c is approximated to be unchanged. Now c is also approximated, or rather overestimated using a slight change to eq. (5.24) in the textbook. The change is that since the force grows with a_{min} inserting $\kappa\,a_{\mathrm{min}}$ instead yields an overestimation on the vertical force on the magnet. The tensile forces are the same, so the force balance leads to

$$c = \frac{2\,\xi}{1-\xi}(\kappa\,a + b) \tag{9}$$

with $\xi = B_{sic}^2/4\,\mu_0\,\sigma_{\mathrm{max}}$. These new parameter equations are inserted in the code. The parameters that changed significantly are displayed in III. The most dramatic change is the plasma volume and surface area which increased by a factor of around 2 and 3 respectively, this makes sence as the plasma was made twice as high. The coil thickness was also changed significantly due to the overestimation while the rest of the parameters did not change much. This most likely means that the model needs more work to make the approximations more precise.

D. Main parameters for DEMO

Setting $P_{\mathrm{E}} = 2000$ in the elliptical model yields the output parameters seen in table IV. Since R_0 is directly proportional to the electric power this of course increases linearly. The other geometric output parameters regarding areas and volumes therefore also increases. β has decreased a lot, so the plasma is not confined effectively in DEMO.

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	1.30 m
a	Minor radius	2.01 m
R_0	Major radius	9.95 m
A	Aspect ratio	4.95
A_p	Plasma surface	2.41×10^3 m^2
V_p	Plasma volume	1.59×10^3 m^3
P_{dens}	Power density	2.48×10^6 W·m^{-1}
p	Plasma pressure	5.20×10^5 Pa
n	Particle density	1.08×10^{20} m^{-3}
B_0	Magnetic field at magnetic axis	8.80 T
β	Normalised plasma pressure	1.69%
$\tau_{E_{\min}}$	Min confinement time for $(p \times \tau_E)_{\min}$	1.62 s

Table IV: Output quantities for DEMO using the elliptical model with $P_{siE} = 2$

E. Designs for DEMO

With $\kappa = 2$ and $A = R_0/a_{\min} = 3 \Leftrightarrow R_0 = 3\, a_{\min}$ A is changed from 4.95 to 3 when going from Friedberg's model to this model. Meanwhile these changes yields the parameters seen in table V. Besides the obvious geometric changes which has decreased due to decreasing R_0. The pressure has increased while the magnetic field has decreased. Therefor a better confinement is achieved since this increases β. This is a positive, and decreasing the aspect ratio a bit is therefore recommended. Additional time could be spend on optimising this, but we are unfortunately unable to do so due to time and space restrictions.

F. *A* and *κ* as free parameters

For this assignment we have chosen to try and optimise β and V_{I} with A and κ as free parameters. β is optimised first. The magnetic field in the centre was approximated as

$$B_0 = \frac{R_0 - a - b}{R_0}\, B_{\max} = (1 - \frac{1}{A} - \frac{A\, b}{a_{\min}})\, B_{\max} \tag{10}$$

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	1.30 m
a	Minor radius	2.01 m
R_0	Major radius	6.03 m
A	Aspect ratio	3
A_p	Plasma surface	1.46×10^3 m^2
V_p	Plasma volume	962 m^3
P_{dens}	Power density	4.01×10^6 W·m^{-1}
p	Plasma pressure	6.68×10^5 Pa
n	Particle density	1.39×10^{20} m^{-3}
B_0	Magnetic field at magnetic axis	6.07 T
β	Normalised plasma pressure	4.55%
$\tau_{E_{\min}}$	Min confinement time for $(p \times \tau_E)_{\min}$	1.26 s

Table V: Output quantities for DEMO using the elliptical model with $P_{siE} = 2$, $\kappa = 2$ and setting $A = 3$

this yields

$$\beta = \frac{2\, p\mu_0}{(1 - \frac{1}{A} - \frac{A\, b}{a_{\min}})^2} \tag{11}$$

Differentiating with respect to A, setting the expression equal to zero and solving for A yields $A = \sqrt{a/b} = R_0/a_{\min} \Leftrightarrow R_0 = \sqrt{a^3/b}$. Meanwhile inserting this expression into eq. 7, substituting $a_{min} = a_{\max}/\kappa$, differentiating with respect to κ, setting the expression equal to zero and solving yields $a_{\max} = -(b+c)\,\kappa/6$ so $a_{\min} = -(b+c)/6$. Next Freidberg's expression for c is inserted and we solve for κ which yields

$$\kappa = \frac{6\, a_{\min}\, \xi - b\, \xi - 6\, a_{\min} - b}{2\, a_{\min}\, \xi} \tag{12}$$

κ must of course be positive(as must the expression for $a_{\min}$) so it really must be the size of this expression that is calculated. This is inserted into the code yielding the output parameters seen in table VI. While an extremely large β and a very small engineered volume is achieved, there is most likely other problems with this design.

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	1.30 m
a	Minor radius	2.01 m
R_0	Major radius	2.60 m
A	Aspect ratio	1.29
A_p	Plasma surface	630 m^2
V_p	Plasma volume	414 m^3
P_{dens}	Power density	9.49×10^6 $\mathrm{W{\cdot}m^{-1}}$
p	Plasma pressure	1.02×10^5 Pa
n	Particle density	2.12×10^{20} m^{-3}
B_0	Magnetic field at magnetic axis	-3.09 T
β	Normalised plasma pressure	26.9%
$\tau_{E_{\min}}$	Min confinement time for $(p \times \tau_E)_{\min}$	1.26 s

Table VI: Output quantities for the elliptical model after β and A has been optimised

III. PART 2: DIAGNOSTICS VIA INTERFEROMETRY

When operating a fusion reactor a continous process of diagnostics are necessary in order to optimise the plasma for the fusion process. One of the active diagnostic methods are interferometry. The goal here is to measure the plasma electron density in the Danish Tokamak Undertaking reactor. Using a interferometer one can measure the electron density n_e of the plasma. The refractive index of electromagnetic waves depend on the electron density and plasma frequency ω_p as such:

$$\omega_p^2 \propto n_e \tag{13}$$

For O-mode plasma waves the refractive index is:

$$N_{\mathrm{O}} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \tag{14}$$

with ω being the probing wave frequency. The plasma will reflect the beam if the plasma frequency is larger than the beam frequency so

$$\omega > \omega_p = \sqrt{n_e \frac{e^2}{\epsilon_0 m_{e0}}} \tag{15}$$

Therefore the critical electron density becomes

$$n_e < n_c = \omega^2 \frac{\epsilon_0 \cdot m_{e0}}{e^2} \tag{16}$$

which gives

$$N_{\mathrm{O}} = \sqrt{1 - \frac{n_e}{n_c}} \tag{17}$$

With a probing frequency much higher than the plasma frequency and the critical density much higher than the electron density Eq. (17) can be approximated by:

$$N_{\mathrm{O}} = \sqrt{1 - \frac{n_e}{n_c}} \approx 1 - \frac{1}{2}\frac{n_e}{n_c} 01 - \frac{\omega_p^2}{2\omega^2} \tag{18}$$

With sufficient accuracy, the linear dependence of the O-mode refractive index on the electron density is obtained if the normalised quantities obey:

$$\frac{n_e}{n_c} \leq 0.4 \quad \frac{\omega_p}{\omega} \leq 0.6 \tag{19}$$

We must calculate the phase shift as one beam travels in vacuum by the length L_V and one wave travels in the plasma by the length L_P. The phase shift in terms of 2π is equal to the optical difference divided by the wavelength. With the refractive index in vacuum, $N_V = 1$, this yields:

$$\frac{\Phi}{2\pi} = \frac{\Delta L_{opt}}{\lambda} = \frac{\int_{x_1}^{x_2} (N_V - N_{\mathrm{O}}(x'))\,\mathrm{d}x'}{\lambda} \approx \frac{1}{2\lambda n_c}\int_0^x n_e(x')\,\mathrm{d}x' \tag{20}$$

$$= 4.48 \times 10^{-16} \left(\frac{\lambda}{\mathrm{m}}\right) \int_0^x \left(\frac{n_e(x')}{\mathrm{m}^{-3}}\right)\left(\frac{\mathrm{d}x'}{\mathrm{m}}\right) \tag{21}$$

Assuming a Gaussian distribution, the electron density at $\pm\infty$ is approximately equal to the densities just inside the reactor walls. Therefore

$$\int_{-\infty}^{\infty} n_e \exp\left(-\frac{(y-b)^2}{2c^2}\right) \mathrm{d}y \approx n_e\ c\ \sqrt{2\pi} \tag{22}$$

With the density at the centre given as:

$$10^{16}\ \mathrm{m}^{-3} \leq n_e \leq 10^{18}\ \mathrm{m}^{-3}, \tag{23}$$

The c in Eq. (22) is the width of the Gaussian distribution and must fit inside the reactor. The DTU tokamak has a minor diameter of 0.250 m. Thus

$$\frac{\Phi(x)}{2\pi} \approx 4.48 \times 10^{-16}\left(\frac{\lambda}{\mathrm{m}}\right) 0.250\ \mathrm{m} n_e = 1.12 \times 10^{-16}\ n_e \left(\frac{2\pi\ 0.250\ \mathrm{m}}{\omega \mathrm{m}}\right) \tag{24}$$

$$= 1.12 \times 10^{-16}\ n_e \left(\frac{2\pi 3 \times 10^8\ \mathrm{s}^{-1}}{\omega}\right) \tag{25}$$

$$= 2.1112 \times 10^{-7} \left(\frac{n_e}{\omega \mathrm{s}}\right) \tag{26}$$

Remembering Eq. (16)

$$\omega^2 \frac{\epsilon_0 \cdot m_{e0}}{e^2} = \omega^2 \frac{8.85 \times 10^{-12}\ \mathrm{F{\cdot}m^{-1}} 9.11 \times 10^{-31}\ \mathrm{kg}}{1.60 \times 10^{-19}\ \mathrm{C}} \tag{27}$$

$$\Downarrow$$

$$n_e < 0.000314\omega^2 \tag{28}$$

Where any units has been disregarded. We want the largest possible phase shift which means that the lower the frequency the better. However cutoff must first be taken into account. Since the cutoff is given by Eq. (28) and since we want to measure densities up to $10^{18}\ \mathrm{m}^{-3}$ the minimum frequency of the wave is

$$\frac{\omega}{2\pi} > \frac{\sqrt{\frac{10^{18}\ \mathrm{m}^{-3}}{0.000314}}}{2\pi} \tag{29}$$

$$\Downarrow$$

$$f \approx 9\ \mathrm{GHz} \tag{30}$$

Given the available emitters, the best emitter is therefore the one with $f = 60\mathrm{GHz}$.

A. Evolving beam width

B. Gauss telescope arrangement

From chapter 5 in “Fusion Plasma Diagnostics with mm-Waves”[3] the authors argue that a Gauss telescope arrangement can alter the beam waist independently of wavelength. This motivates narrowing the interferometer beam waist with a lense system.

ACKNOWLEDGMENTS

The authors would like to thank...

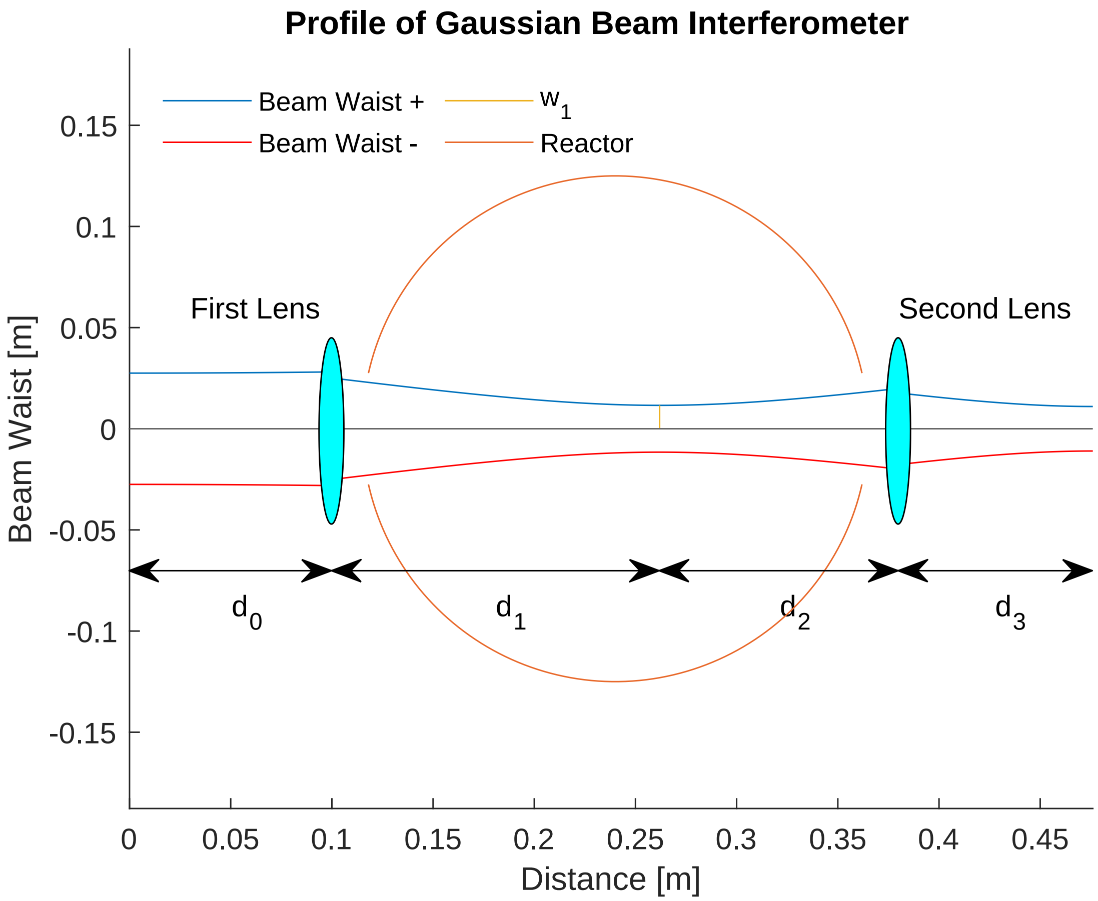


Figure 3: Awesome Gaussian Telescope

---

* E-mail at s163977@student.dtu.dk

† E-mail at spacrone@live.dk

‡ Homepage of the Technical University of Denmark http://www.dtu.dk/english/

1 `https://www.iter.org/`.

2 Jeffrey P. Freidberg, *Plasma physics and fusion energy* (Cambridge University Press, 2007).

3 Hans-Jürgen Hartfuß and Thomas Geist, *Fusion Plasma Diagnostics with mm-Waves* (Wiley-VCH, 2013).

LIST OF FIGURES

LIST OF TABLES

LISTINGS

Appendices

Appendix A: tokamakDTU_asign_1

```
%*************************************************************************
% Name:          tokamakDTU
%
% Version:       1.0
%
% Purpose:       Contains the function 'tokamakDTU' which gives parameters
%                for a tokamak fusion power plant as output based on a
%                simplified model. The equations used are derived in
%                chapter 5 in Friedberg, Plasma physics and Fusion
%                Energy, 2007 (all references are referring thereto).
%
% To do (NOT for 10401 - Fusion Energi students):
%             1. Rewrite the code to a class (this is not done on purpose so
%                that the code is more readable for students not familiar
%                with classes). Can from that merge the files which takes
%                R_0/a and/or the ellipticity as an input into one file.
%
% Changelog:
%             1. December 2014:
%                Written by Michael Løiten based on a similar code written
%                as a bachelor project by Elias Pagh Sentius
%                mailto: mmag@fysik.dtu.dk
%*************************************************************************

function [b, c, a, R_0, A, A_p, V_p, P_dens, p, n, B_0, beta, tau_E_min,...
          C_per_watt] =...
    tokamakDTU_asign_1(...
        n_flux_fraction, C_F, C_I, P_E, P_W, B_max, sigma_max, eta_t)
```

```
%TOKAMAK_DTU Function which returns the parameters of a power plant
%
% Output parameters
%-----------------
% b           - Blanket/shield thickness [m]
% c           - Magnet coil thickness [m]
% a           - Minor radius [m]
% R_0         - Major radius [m]
% A           - Aspect ratio []
% A_p         - Plasma surface [m^2]
% V_p         - Plasma volume [m^3]
% P_dens      - Power density [W/m]
% p           - Plasma pressure [Pa]
% n           - Particle density [m^-3]
% B_0         - Magnetic field at magnetic axis [T]
% beta        - Plasma beta in the centre []
% tau_E_min   - Min confinement time for satisfaction of (p*tau_E)_min [s]
% C_per_watt - The cost of the powerplant [$]
%
% Input parameters
%------------------
% n_flux_fraction - n flux in breeder end/n flux in breeder start []
% C_F             - Fixed cost propotionality constant [$]
% C_I             - Nuclear island cost propotionality constant [$W/m^3]
% P_E             - Desired output power [MW]
% P_W             - Maximum wall load [MW/m^2]
% B_max           - Magnetic field at the edge of the coil [T]
% sigma_max       - Tensile strenght of the magnetic field coils [atm]
% eta_t           - Energy conversion efficiency []


% The function starts by defining fixed constants
```

```
% Note that this is inefficient if we are looping over the function, but it
% makes the code easier to use, as these are not needed as input parameters

% Fixed constants
%##########################################################################
% Nuclear
%--------
% Energies
E_t         = 2.5e-8; % [MeV] Energy of slow (thermal) neutron (eq 5.6)
E_n         = 14.1;   % [MeV] Neutron energy after fusion (eq 2.17)
E_a         = 3.5;    % [MeV] alpha energy after fusion (eq 2.17)
E_Li        = 4.8;    % [MeV] Heat produced by breeding Li (under eq 4.31)
% Cross section and main free paths
sigma_v_avg = 3.0e-22;% [m^3/s] DT fusion cross section @ 15keV (table 5.2)
lambda_br   = 0.0031; % [m] Breeding mean free path (under eq 5.7)
lambda_sd   = 0.055;  % [m] Mean free path from sigma_sd (eq 5.3)

% Plamsa physics
%---------------
% Parameters for infinity gain at the minimum of p tau_E (eq 4.20)
T            = 15.0;  % [keV] Temparature for obtaining min tripple product
tripple_min  = 8.3;   % [atm s] Min tripple prod to obtain Q=inf @ T=15 keV

% Natural constants
%------------------
mu_0 = 4.0*pi*1e-7;        % Vacuum permeability [T*m/A]
e    = 1.602176565e-19;   % Elementary charge [C]
%##########################################################################


% Secondly we convert everything to SI units, so that the variables are
% easier to handle
```

```
% Again, this is computationally inefficient, but it suffices for our use
% Conversion to SI-units
%##########################################################################
% Conversion factors
W_per_MW      = 1.0e6;
Pa_per_atm    = 1.01325e5;
eV_per_keV    = 1.0e3;
eV_per_MeV    = 1.0e6;
J_per_eV      = e;
J_per_keV     = J_per_eV * eV_per_keV;
J_per_MeV     = J_per_eV * eV_per_MeV;
% Conversions
P_E           = P_E * W_per_MW;           % Desired output power
P_W           = P_W * W_per_MW;           % Wall Loading limit on first wall
E_t           = E_t * J_per_MeV;          % Energy of slow (thermal) neutron
E_n           = E_n * J_per_MeV;          % Neutron energy after fusion
E_a           = E_a * J_per_MeV;          % alpha energy after fusion
E_Li          = E_Li * J_per_MeV;         % Heat produced by breeding Li
sigma_max     = sigma_max * Pa_per_atm;   % Max allowable structural stress
T             = T * J_per_keV;            % Temparature for minimum p*tau_E
tripple_min   = tripple_min * Pa_per_atm;  % The Lawson parameter (p*tau_E)
%##########################################################################


% Calculate the geometrical factors
%-----------------------------------
% Find the breeder thickness
b = get_b(lambda_sd, E_n, E_t, lambda_br, n_flux_fraction);
% Find the minor plasma radius and the coil thickness
[a, c] = get_a_and_c(B_max, mu_0, sigma_max, b);
% Find the major radius
R_0 = get_R_0(a, eta_t, E_n, E_a, E_Li, P_E, P_W);
```

```
% Find the resulting geometrical factors
A   = R_0/a;                        % Aspect Ratio
A_p = (2.0*pi*a)*(2.0*pi*R_0);     % Plasma surface area
V_p = (pi*a^(2.0))*(2.0*pi*R_0);  % Plasma volume


% Calculate the plasma physics parameters
%---------------------------------------
% Find the power density in the plasma
P_dens = get_P_dens(E_a, E_n, E_Li, P_E, eta_t, V_p);
% Find the plasma pressure
p = get_p(E_a, E_n, P_dens, T, sigma_v_avg);
% Calculate the density from the definition of p under eq 5.36
n = p/(2.0*T);
% Find the magnetic field strength on the magnetic axis
B_0 = get_B_0(R_0,a,b,B_max);
% Find the plasma beta on the magnetic axis
beta = get_beta(p, B_0, mu_0);
% Find the minimum reqiured confinement time from the definition of the
% minimum tripple product.
% NOTE: A higher confinement time is advantegous, and could in principle
% yield a smaller (and cheaper) reactor. However, the effect is not
% included in this model
tau_E_min = tripple_min/p;


% Calculate the cost
% (details about the cost can be found in the function get_a_and_c)
%-------------------
% Find the volume of the nuclear island
% (the material surrounding the plasma)
V_I = get_V_I(R_0,a,b,c);
```

```
% Find the reactor volume per power out
% In the current model, this is the only non-constant in the expression for
% cost per watt
V_I_per_P_E = V_I/P_E;
C_per_watt = get_C_per_watt(C_F, C_I, V_I_per_P_E);
end



function [b] = get_b(lambda_sd, E_n, E_t, lambda_br, n_flux_fraction)
%GET_B Calculates b from the need of slowing down and breeding neutrons

% Thickness of the moderator-breeding region so that 1 - n_flux_fraction
% have slowed down and undergone a breeding reaction
% [m]
% Equation 5.10
delta_x = 2.0*lambda_sd*...
          log( 1.0-(1.0/2.0)*(E_n/E_t)^(1.0/2.0)*...
               (lambda_br/lambda_sd)*log( n_flux_fraction )...
              );

% Set b from delta_x
% Friedberg argues above equation 5.11 that b should be between 1 and 1.5 m
% Therefore a self chose constant is set to 0.38
self_chosen_constant = 0.32;
b = delta_x + self_chosen_constant;
end




function [a,c] = get_a_and_c(B_max, mu_0, sigma_max, b)
```

```
%GET_A_AND_C Calculates a and c

% c is obtained from requiring that the magnets are so thin that they are
% on the limit of the tensile strenght
% a is obtained from minimizing the costs

% xi defined when making the magnetic coil c as thin as possible
% Under equation 5.27
xi = B_max^(2.0) / (4.0*mu_0*sigma_max);

% a is found from optimization of the cost, where
% total cost = fixed cost + nuclear island cost
%...........
% Fixed cost
%...........
% K_F = Fixed cost for building, turbines, generators etc (also applies to
% fusion, fission, fossil)
% Assumption: The fixed cost is proportional to power output:
% Equation 5.13
% K_F = C_F*P_E;
%...........
% Nuclear island cost (mainly cost of magnets, blanket and shield)
%............
% Assumption: The proportional to reactor volume:
% Equation 5.14
% K_I = C_I*V_I;
% Equation 5.15
% V_I = 2.0*pi^(2.0) * R_0 * ( (a+b+c)^(2.0) - a^(2.0) ); % Reactor volume
%.............
% Cost per watt:
%.............
% Defined as C_p_watt = (K_F + K_I)/P_E, rewritten to
```

```
% C_p_watt = C_F + C_I*(V_I/P_E);
% Since the cost per watt contains two constants, we can minimize the
% V_I/P_E in order to optimize the cost
% Given by equation 5.20 inserted in 5.17
% Equation 5.21
% V_I_per_P_E = V_I/P_E;  % Reactor volume per power out
% a is found by setting the derivative of V_I_per_P_E = 0
% Equation 5.29
a = ((1.0 + xi)/(2.0*xi^(1.0/2.0))) * b;

% Knowing xi, a, and, b, we can calculate c
% c found by comparing tensile force and magnetic force working on the coil
% Equation 5.27
c = 2*xi/(1-xi)*(a+b);
end



function [R_0] = get_R_0(a, eta_t, E_n, E_a, E_Li, P_E, P_W)
%GET_R_0 Calculate the major radius

% Divide eq 5.18 (electric power out) by
% eq 5.19 (wall loading * area = total neutron production) and solve for R0
% Equation 5.20
R_0 = (1.0/(4.0*pi^(2.0)*eta_t))*(E_n/(E_n + E_a + E_Li))*(P_E/(a*P_W));
end



function [P_dens] = get_P_dens(E_a, E_n, E_Li, P_E, eta_t, V_p)
%GET_P_DENS Calculate the power density

```

```
% The power density is found by the sum of the power from the alphas plus
% the power from the neutrons, divided by the plasma volume
% Equation 5.35
P_dens = (E_a + E_n)/(E_a + E_n + E_Li)*P_E/(eta_t*V_p);
end



function [B_0] = get_B_0(R_0, a, b, B_max)
%GET_B_0 Calculte the magnetic field strength on the magnetic axis

% B_max is found in the edge of the magnet (at R = R_0-a-b)
% B0 is the magnetic field at R0
% As B propto 1/R. we have that B_0/B_max = (R_0-a-b)/R_0, which leads to
% Equation 5.42
B_0 = ((R_0-a-b)/R_0)*B_max;
end



function [beta] = get_beta(p, B_0, mu_0)
%GET_beta Calculte the magnetic field strength on the magnetic axis

% Plasma beta in the center (kinetical pressure over magnetical pressure):
% Equation 5.43
beta = p / (B_0^2/(2.0*mu_0));
end



function [p] = get_p(E_a, E_n, P_dens, T, sigma_v_avg)
%GET_P Calculate the plasma pressure
```

```
% Found from solving the sum of neutron and alpha power for n, and multiply
% the result with T
% Equation 5.37
p = ( (16.0/(E_a + E_n)) * P_dens )^(1.0/2.0)*...
     (T^(2.0)/sigma_v_avg)^(1.0/2.0);
end



function [V_I] = get_V_I(R_0, a,b,c)
%GET_V_I Calculate the volume of the material surrounding the plasma

% Equation 5.15
V_I = 2.0*pi^(2.0) * R_0 * ( (a+b+c)^(2.0) - a^(2.0) );
end



function [C_per_watt] = get_C_per_watt(C_F, C_I, V_I_per_P_E)
%GET_C_PER_WATT Calculates the cost for one watt out from the power plant

% For details in how the cost is derived, see comments in the function
% get_a_and_c

C_per_watt = C_F + C_I*(V_I_per_P_E);
end
```

Appendix B: IterateTokamakDTU

```
close;
clear;

titl = ["Desired output power [MW]", "Maximum wall load [MW m^-2]",...
    "Magnetic field at the edge of the coil [T]",...
    "Tensile strenght of the magnetic field coils [atm]"];
foldertitl = ["DesiredOutputPower", "MaximumWallLoad",...
    "MagneticFieldAtTheEdgeOfTheCoil",...
    "TensileStrenghtOfTheMagneticFieldCoils"];
l = 5;
p1 = [];
x = [];
mkdir('../MatlabFigures', foldertitl(l))

for i = 2000:5000
    [b, c, a, R_0, A, A_p, V_p, P_dens, p,...
        n, B_0, beta, tau_E_min, C_per_watt] =...
        tokamakDTU_asign_1(0.01, 1, 2, 1000, 4, 13, 3000, 0.4);
    q=[b, c, a, R_0, A, A_p, V_p, P_dens, p,...
        n, B_0, beta, tau_E_min, C_per_watt];
    p1=cat(1,p1,q);
    x=cat(1,x,i);
end

T = ["Blanket-shield thickness [m]", "Magnet coil thickness [m]"...
    "Minor radius [m]", "Major radius [m]", "Aspect ratio []"...
    "Plasma surface [m^2]", "Plasma volume [m^3]", "Power density [W m^-1]"...
    "Plasma pressure [Pa]", "Particle density [m^-3]",...
    "Magnetic field at magnetic axis [T]", "Plasma beta in the centre []"...
    "Min confinement time for satisfaction of (p tau_E)_min [s]",...
```

```
    "The cost of the powerplant [$]"];
fileT = ["Blanket-shieldThickness", "MagnetCoilThickness"...
    "MinorRadius", "MajorRadius", "AspectRatio"...
    "PlasmaSurface", "PlasmaVolume", "PowerDensity"...
    "PlasmaPressure", "ParticleDensity",...
    "MagneticFieldAtMagneticAxis", "PlasmaBetaInTheCentre"...
    "MinConfinementTime",...
    "TheCostOfThePowerplant"];
for k = 1:14
    q = figure;
    y = p1(:,k);
    CM = jet(14);
    plot(x, y,'color', CM(k,:));
    ylabel(T(k));
    xlabel(titl(l));
    ytickformat('%.2f');
    epsfilename = sprintf('%s.eps',fileT(k));
    foldername = sprintf('../MatlabFigures/%s', foldertitl(l));
    fullfilename = fullfile(foldername,epsfilename);
    saveas(q, fullfilename, 'epsc')
end
```