

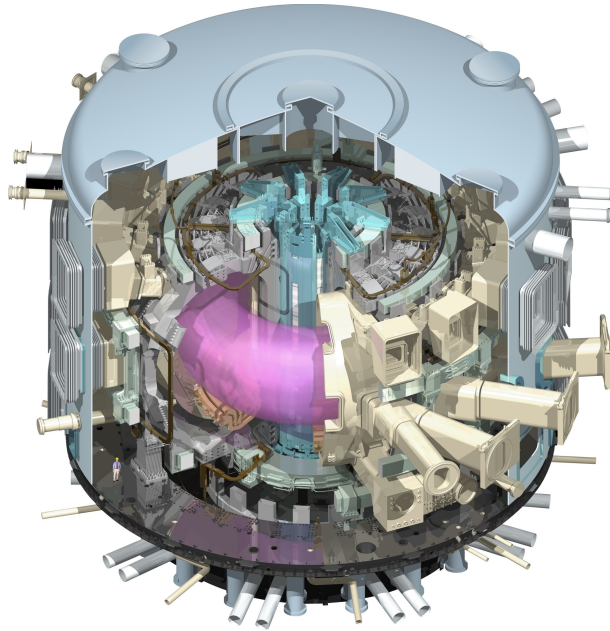
10401

## Fusion Energy and Plasma Physics

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**Abstract:** Plasma fusion for energy production is a growing scientific field with interests all over the globe as clean and sustainable energy as well as growing energy demands needs to be addressed. In the course “10401: Fusion energy and plasma physics” we deal with some of the fundamental engineering aspects of a commercial fusion energy power plant, as well as the research towards this goal. In this paper we first address a simple model for such a plant and how the model can be improved upon. Secondly we investigate how to diagnose plasma density with simple interferometry in the DTU tokamak reactor. Lastly we made measurements in the DTU fusor, and investigated some energy production aspects with such a device.

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## A. Intro

In recent years a large and quickly growing collaboration between plasma physicists and engineers have materialised into one of the most ambitious energy producing projects ever seen. The proof-of-concept plasma fusion tokamak reactor ITER<sup>1</sup> in Cadarache is currently taking shape in order to address the issue of growing energy demands and climate changes.

The mission is simple: Prove that plasma fusion is a viable source of electricity.

Whilst not being the most surmountable task, many researchers and institutions have gathered from across the world, including the Department of Physics at DTU.

In this paper, three assignments are solved as part of the course “10401 Fusion Energy and Fusion Plasma Physics”. Some key aspects of fusion plasma fuelled reactors are addressed and discussed in the assignments. The code repository for the report and matlab code can be found on Github: <https://github.com/rwiuff/10401> 

## I. A SIMPLE REACTOR MODEL

### A. Freidberg's simple reactor model

In the 5<sup>th</sup> chapter of the textbook by Freidberg<sup>2</sup>, he makes a simple model for designing a fusion reactor power plant. The model uses simple geometric and electromagnetic assumptions with little involvement of plasma physics. The variables put into the model are shown in Table I.

Symbol	Quantity
$n_{\text{flux fraction}}$	n flux in breeder end/n flux in breeder start [1]
$C_F$	Fixed cost proportionality constant [\$]
$C_I$	Nuclear island cost proportionality constant [ $\$ \cdot \text{W} \cdot \text{m}^{-3}$ ]
$P_E$	Desired output power [MW]
$P_W$	Maximum wall load [ $\text{MW} \cdot \text{m}^{-2}$ ]
$B_{\text{max}}$	Magnetic field at the edge of the coil [T]
$\sigma_{\text{max}}$	Tensile strength of the magnetic field coils [atm]
$\eta_t$	Energy conversion efficiency [1]

**Table I:** Variables in the Freidberg's model

Table II shows the output quantities from the model.

Symbol	Quantity	Obtained values
$b$	Blanket-and-shield thickness	1.20 m
$c$	Magnet coil thickness	0.799 m
$a$	Minor radius	2.01 m
$R_0$	Major radius	4.96 m
$A$	Aspect ratio	2.4670
$A_p$	Plasma surface	393 m <sup>2</sup>
$V_p$	Plasma volume	395 m <sup>3</sup>
$P_{\text{dens}}$	Power density	$4.97 \times 10^6 \text{ W} \cdot \text{m}^{-1}$
$p$	Plasma pressure	$7.37 \times 10^5 \text{ Pa}$
$n$	Particle density	$1.53 \times 10^{20} \text{ m}^{-3}$
$B_0$	Magnetic field at magnetic axis	4.57 T
$\beta$	Normalised plasma pressure	8.85%
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.14 s

**Table II:** Output quantities in the model in Freidberg's<sup>2</sup> along with the obtained values when inserting the parameters from Eq. (1)

This model has been implemented in a Matlab script provided for the course. The script

is shown included in Appendix A. As an example, the model is run with the following parameters:

$$\begin{aligned} n_{\text{flux fraction}} &= 0.01 & P_E &= 1000 \text{ MW} \\ P_W &= 4 \text{ MW} \cdot \text{m}^{-2} & B_{\text{max}} &= 13 \text{ T} \\ \sigma_{\text{max}} &= 3000 \text{ atm} & \eta_t &= 0.4 \end{aligned} \tag{1}$$

Note that  $C_F$  and  $C_I$  has been omitted as these serve no purpose for this assignment. It is not of interest how expensive the plant will be. Rather the geometries and physical quantities are of interest. The results from the model is given in Table II.

## B. Model sensitivity

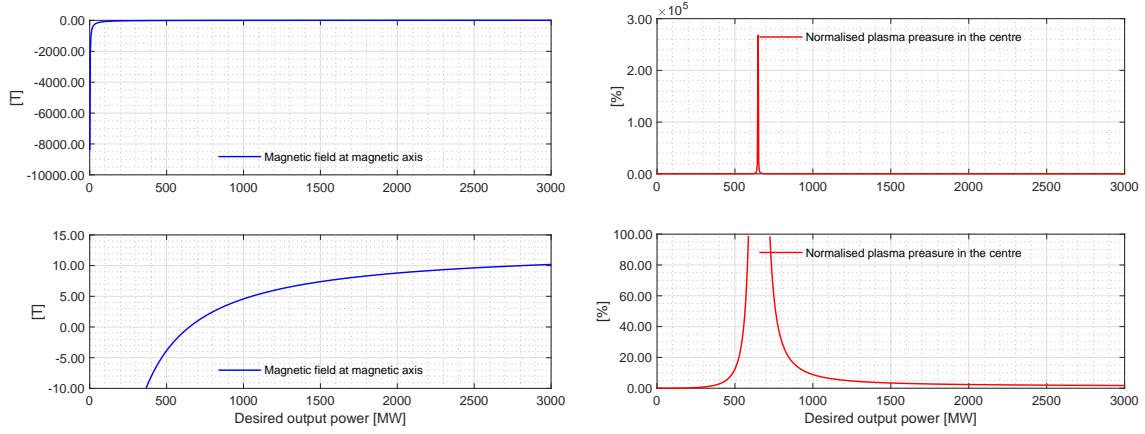
At this point Freidberg has provided a model that produce some reasonable results for a power plant. It could be interesting to see how this model behaves when some vital parameters are changed. In the last section the model used the parameters shown in Eq. (1). In Eq. (2) are listed some parameter ranges.

$$\begin{aligned} P_E &: 500 - 1000 \text{ MW} & P_W &: 0 - 10 \text{ MW} \cdot \text{m}^{-2} \\ B_{\text{max}} &: 0 - 20 \text{ T} & \sigma_{\text{max}} &: 2000 - 5000 \text{ atm} \end{aligned} \tag{2}$$

Appendix B includes Matlab code that does the following:

1. Take a variable range from Eq. (2).
2. Set all other input parameters as in Eq. (1).
3. Plot the model outputs as functions of the variable range.
4. Repeat with another range from Eq. (2).

The iteration showed some interesting breakdowns in the model. When varying the desired power output there was a breakdown in magnetic field strength at the magnetic axis as well as in the normalised plasma pressure. This is shown in Fig. 1. In this case, limits for what is physically possible broke and in these cases the model is unusable. The full gallery of model output is found in Appendix C.



(a) The magnetic field at magnetic axis (b) The normalised plasma pressure in the centre

**Figure 1:** Model breakdown when iterating over desired power output.

### C. Elliptic Cross section

Freidberg's model assume a circular cross section of the plasma. In reality this is not the case. We will now make a more realistic, yet still approximate reactor for an elliptic plasma cross section. In describing the geometry one refers to the elongation ratio:

$$\kappa = \frac{a_{\max}}{a_{\min}} \quad (3)$$

With  $a_{\max}$  the major radius and  $a_{\min}$  the minor radius of the ellipse. This parameter ensures a true elliptic cross section as defined by the equation,

$$\frac{x^2}{a_{\max}^2} + \frac{y^2}{a_{\min}^2} = 1 \quad (4)$$

which can be parameterised as

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} a_{\min} \cos \phi \\ \kappa a_{\min} \sin \phi \end{bmatrix} \quad (5)$$

Meanwhile the blanket must be implemented as an ellipse or swelled ellipse. The true ellipse results in a thickness difference throughout the blanket, while the second model results in a blanket of equal thickness throughout. To this, the choice of implementing the blanket as a true ellipse has been made, since it simplifies derivations a bit. Note however, that keeping a constant thickness is the preferable option as it will reduce the engineering

volume and hence the cost of the machine.

The outer layer is parameterised

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} (b + a_{\min}) \cos \phi \\ \kappa(b + a_{\min}) \sin \phi \end{bmatrix} \quad (6)$$

with  $b$  the blanket thickness at the minor ellipse axis. Choosing for now,  $a_{\min} = 2$ ,  $\kappa = 2$  and  $b = 1.2$ , Eq. (5) and Eq. (6) are plotted on Figure Fig. 2 along with the variation in thickness of the blanket.

Given Eq. (5) and Eq. (6), the engineering volume can easily be derived if  $c \cos(\phi)$  and  $\kappa c \sin(\phi)$  is added to the x and y-direction in Eq. (6) respectively, where  $c$  is the minimum thickness of the magnetic coils that provide the toroidal field.

The cross sectional area of an ellipse is  $A_e = \pi a_{\min} a_{\max}$  so the engineered volume becomes

$$\begin{aligned} V_I &= 2 \pi R_0 (A_{e, \text{outer}} - A_{e, \text{inner}}) \\ &= 2 \pi^2 R_0 ((a_{\min} + b + c)^2 - a_{\min}^2) \kappa \end{aligned} \quad (7)$$

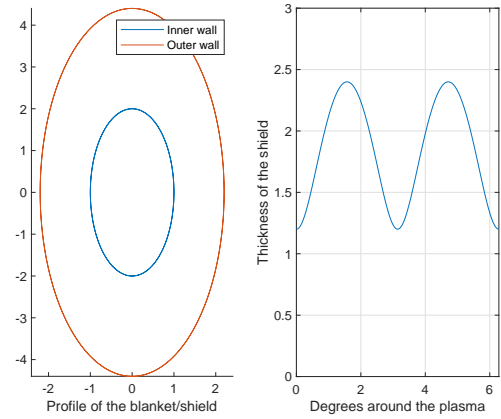
and the plasma volume is similarly calculated as  $V_P = 2 \pi^2 R_0 \kappa a_{\min}^2$ . The plasma surface area is a bit more tricky, but it can be approximated as

$$S_p = 2 \pi R_0 \pi (3(a_{\min} + \kappa a_{\min}) - \sqrt{(3a_{\min} + \kappa a_{\min})(a_{\min} + 3\kappa a_{\min})}) \quad (8)$$

Using the same arguments as in the book, the B-field in the centre is surprisingly unchanged when going to the elliptical model. Now  $c$  is also approximated, or rather overestimated using a slight change to Eq. (5.24) in the textbook. Since the force grows with  $a_{\min}$ , inserting  $\kappa a_{\min}$  instead yields an overestimation on the vertical force on the magnet. The tensile forces are the same, so the force balance leads to

$$c = \frac{2\xi}{1-\xi}(\kappa a + b) \quad (9)$$

with  $\xi = B_{sic}^2/4\mu_0\sigma_{\max}$ . These new equations are inserted in the code. The results are displayed in III. The plasma volume and surface area is of course increased as the plasma was made higher. This of course also results in a decreased power density. Overall, parameters such as  $B_0$ ,  $\beta$  and  $\tau_{E_{\min}}$  while changing a bit, they were not changed significantly.



**Figure 2:** The profile of the blanket-and-shield and the thickness as a function of the angle around origin,  $\phi$ .

Symbol	Quantity	Obtained values
$b$	Blanket-and-shield thickness	1.20 m
$c$	Magnet coil thickness	1.30 m
$a$	Minor radius	2.01 m
$R_0$	Major radius	4.96 m
$A$	Aspect ratio	2.4670
$A_p$	Plasma surface	609 m <sup>2</sup>
$V_p$	Plasma volume	793 m <sup>3</sup>
$P_{\text{dens}}$	Power density	$2.48 \times 10^6 \text{ W}\cdot\text{m}^{-1}$
$p$	Plasma pressure	$5.20 \times 10^5 \text{ Pa}$
$n$	Particle density	$1.08 \times 10^{20} \text{ m}^{-3}$
$B_0$	Magnetic field at magnetic axis	4.60 T
$\beta$	Normalised plasma pressure	6.17%
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.62 s

**Table III:**

*Output quantities from the elliptical model*

#### D. Main parameters for DEMO

Setting  $P_E = 2000$  in the elliptical model yields the output parameters seen in Table IV. Since  $R_0$  is directly proportional to the electric power, it increases linearly. The other geometric output parameters regarding areas and volumes also increase as a result.  $\beta$  has decreased a lot, so the plasma is not confined effectively in DEMO according to this model.

#### E. Designs for DEMO

With  $P_E$ ,  $\kappa = 2$  and  $A = R_0/a_{\text{min}} = 3 \Leftrightarrow R_0 = 3 a_{\text{min}}$ . This is implemented in the code and the results are displayed in Table V.  $\beta$  has increased a bit but only to 4.55%.  $R_0$  has been forced down, so the plasma volume etc. has decreased as well. Overall it seems like a smaller  $R_0$  while keeping  $a_{\text{min}}$  fixed is an improvement. Or in other words,  $A = 3$  is more desirable than  $A \approx 5$ .



Symbol	Quantity	Obtained values
$b$	Blanket-and-shield thickness	1.20 m
$c$	Magnet coil thickness	1.30 m
$a$	Minor radius	2.01 m
$R_0$	Major radius	9.95 m
$A$	Aspect ratio	4.9512
$A_p$	Plasma surface	$1.21 \times 10^3 \text{ m}^2$
$V_p$	Plasma volume	$1.59 \times 10^3 \text{ m}^3$
$P_{\text{dens}}$	Power density	$2.48 \times 10^6 \text{ W}\cdot\text{m}^{-1}$
$p$	Plasma pressure	$5.20 \times 10^5 \text{ Pa}$
$n$	Particle density	$1.08 \times 10^{20} \text{ m}^{-3}$
$B_0$	Magnetic field at magnetic axis	8.80 T
$\beta$	Normalised plasma pressure	1.69%
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.62 s

**Table IV:** Output quantities for DEMO using the elliptical model with  $P_E = 2\text{GW}$

### F. $A$ and $\kappa$ as free parameters

Designing the tokamak with  $\kappa$  and  $A$  as free parameters has led us to try and maximise profitability from  $V_P/A_P$  and  $\beta$ . Profitability in this context means that increasing the size of the tokamak leads to an increase in these parameters. However, there is a point when the change vs increase in size becomes constant. This means that we do not profit from increasing the size any longer.

$V_P$  and  $A_P$  were calculated earlier so

$$\begin{aligned} \frac{V_P}{A_P} &= \frac{2 \pi R_0 \kappa a_{\text{min}}^2}{2 \pi R_0 \pi (3(a + \kappa a_{\text{min}}) - \sqrt{(3a_{\text{min}} + \kappa a_{\text{min}})(a + 3\kappa a_{\text{min}})})} \\ &= \frac{\kappa a_{\text{min}}}{\pi (3 + 3\kappa - \sqrt{3 + \kappa} \sqrt{1 + 3\kappa})} \end{aligned} \quad (10)$$

The expression scales linearly with  $a_{\text{min}}$  so we set it equal to 1 m since it has no effect on the choice of  $\kappa$ . Differentiating with respect to  $\kappa$  and plotting is done, with the results shown in Fig. 3a. Considering the figure,  $\kappa = 2.5$  is chosen as the change is close enough to zero given this value.

Now work will be done towards choosing a value for  $A$ . First, for geometric reasons, a

Symbol	Quantity	Obtained values
$b$	Blanket-and-shield thickness	1.20 m
$c$	Magnet coil thickness	1.30 m
$a$	Minor radius	2.01 m
$R_0$	Major radius	6.03 m
$A$	Aspect ratio	3
$A_p$	Plasma surface	738 m <sup>2</sup>
$V_p$	Plasma volume	962 m <sup>3</sup>
$P_{\text{dens}}$	Power density	$4.09 \times 10^6 \text{ W} \cdot \text{m}^{-1}$
$p$	Plasma pressure	$6.68 \times 10^5 \text{ Pa}$
$n$	Particle density	$1.39 \times 10^{20} \text{ m}^{-3}$
$B_0$	Magnetic field at magnetic axis	6.07 T
$\beta$	Normalised plasma pressure	4.55%
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.26 s

**Table V:** Output quantities for DEMO using the elliptical model with  $P_E = 2$ ,  $\kappa = 2$  and  $A = 3$ .

minimum  $R_0$  is calculated. It is simply  $R_0 = a_{\text{min}} + b + c$  where  $a_{\text{min}}$  is to be determined,  $b = 1.2\text{m}$  and  $c = \frac{2\xi}{1-\xi}(\kappa a_{\text{min}} + b)$ . Meanwhile  $A$  is chosen to optimise  $\beta$ . Inserting  $R_0$  into  $B_0$  in the textbook yields

$$B_0 = \frac{2\xi(\kappa a_{\text{min}} + b) B_{\text{max}}}{((2\kappa - 1)a_{\text{min}} + a + b)} = \frac{178.75 a_{\text{min}} + 85.8}{55.5 + 36 a_{\text{min}}} \quad (11)$$

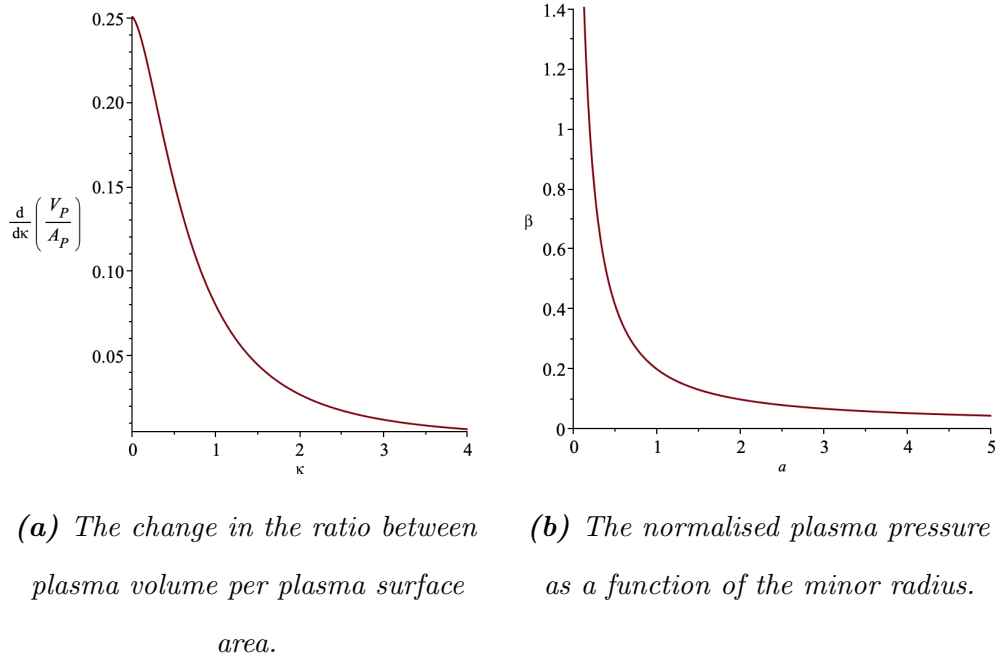
Where units has been disregarded and  $\xi = 0.11$ ,  $b = 1.2$ ,  $\kappa = 2.5$  and  $B_{\text{max}} = 13$  has been inserted. Meanwhile, inserting the textbook's expression for  $P_{\text{dens}}$  into the expression for  $p$  and inserting the input parameters yields

$$p = 1.04 \times 10^6 \sqrt{\frac{1}{\kappa a_{\text{min}}}} \quad (12)$$

Thus  $\beta$  becomes

$$\beta = \frac{2148 (a_{\text{min}} + 1.542)^2}{\sqrt{a_{\text{min}}} (178.8 a_{\text{min}} + 85.8)^2} \quad (13)$$

This function is plotted in Fig. 3b, and for  $\beta = 10\%$   $a = 1.94 \text{ m}$  is achieved. Therefore ( $R_0 = 1.94 \text{ m} + 1.20 \text{ m} + 2 \cdot 0.11/(1 - 0.11) \cdot 2.5 \cdot 1.94 \text{ m} + 1.20 \text{ m} = 4.63 \text{ m}$ ) which means



**Figure 3**

$A = 4.63 \text{ m}/1.94 \text{ m} = 2.39 \text{ m}$ . The output quantities after this optimisation is shown in Table VI

## II. DIAGNOSTICS VIA INTERFEROMETRY

When operating a fusion reactor a continuous process of diagnostics is necessary in order to optimise the plasma for the fusion process. One of the active diagnostic methods are interferometry. The goal here for this part of the paper is to measure the plasma electron density in the Danish Tokamak Undertaking reactor.

### A. Cut-off and Source frequency

Using a interferometer one can measure the electron density  $n_e$  of the plasma. The refractive index of electromagnetic waves depend on the electron density and plasma frequency  $\omega_p$  proportionally:

$$\omega_p^2 \propto n_e \quad (14)$$

For O-mode plasma waves, the refractive index is:

$$N_O = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \quad (15)$$

Symbol	Quantity	Obtained values
$b$	Blanket-and-shield thickness	1.20 m
$c$	Magnet coil thickness	1.51 m
$a$	Minor radius	1.94 m
$R_0$	Major radius	4.65 m
$A$	Aspect ratio	2.3966
$A_p$	Plasma surface	652 m <sup>2</sup>
$V_p$	Plasma volume	864 m <sup>3</sup>
$P_{\text{dens}}$	Power density	$4.55 \times 10^6 \text{ W}\cdot\text{m}^{-1}$
$p$	Plasma pressure	$7.05 \times 10^5 \text{ Pa}$
$n$	Particle density	$1.47 \times 10^{20} \text{ m}^{-3}$
$B_0$	Magnetic field at magnetic axis	4.21 T
$\beta$	Normalised plasma pressure	10%
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.19 s

**Table VI:** Output quantities for the elliptical model after  $\beta$  and  $A$  has been optimised

with  $\omega$  being the probing wave frequency. The plasma will reflect the beam if the plasma frequency is larger than the beam frequency so

$$\omega > \omega_p = \sqrt{n_e \frac{e^2}{\epsilon_0 m_{e0}}} \quad (16)$$

So for a given frequency, the plasma must not exceed a critical cut off electron density:

$$n_e < n_c = \omega^2 \frac{\epsilon_0 \cdot m_{e0}}{e^2} \quad (17)$$

which gives

$$N_O = \sqrt{1 - \frac{n_e}{n_c}} \quad (18)$$

With a probing frequency much higher than the plasma frequency and the critical density much higher than the electron density Eq. (18) can be approximated by:

$$N_O = \sqrt{1 - \frac{n_e}{n_c}} \approx 1 - \frac{1}{2} \frac{n_e}{n_c} - \frac{\omega_p^2}{2\omega^2} \quad (19)$$

With sufficient accuracy, the linear dependence of the O-mode refractive index on the electron density is obtained if the normalised quantities obey:

$$\frac{n_e}{n_c} \leq 0.4 \quad \frac{\omega_p}{\omega} \leq 0.6 \quad (20)$$

We must calculate the phase shift as one beam travels in vacuum by the length  $L_V$  and one wave travels in the plasma by the length  $L_P$ . The phase shift in terms of  $2\pi$  is equal to the optical difference divided by the wavelength. With the refractive index in vacuum,  $N_V = 1$ , this yields:

$$\begin{aligned}\frac{\Phi}{2\pi} &= \frac{\Delta L_{opt}}{\lambda} = \frac{\int_{x_1}^{x_2} (N_V - N_O(x')) dx'}{\lambda} \approx \frac{1}{2\lambda n_c} \int_0^x n_e(x') dx' \\ &= 4.48 \times 10^{-16} \left(\frac{\lambda}{\text{m}}\right) \int_0^x \left(\frac{n_e(x')}{\text{m}^{-3}}\right) \left(\frac{dx'}{\text{m}}\right)\end{aligned}\quad (21)$$

Assuming a Gaussian distribution, the electron density at  $\pm\infty$  is approximately equal to the densities just inside the reactor walls. Therefore

$$\int_{-\infty}^{\infty} n_e \exp\left(-\frac{(y-b)^2}{2c^2}\right) dy \approx n_e c \sqrt{2\pi} \quad (22)$$

With the density at the centre given as:

$$10^{16} \text{ m}^{-3} \leq n_e \leq 10^{18} \text{ m}^{-3}, \quad (23)$$

The  $c$  in Eq. (22) is the width of the Gaussian distribution and must fit inside the reactor.

The DTU tokamak has a minor diameter of 0.250 m. Thus

$$\begin{aligned}\frac{\Phi}{2\pi} &\approx 4.48 \times 10^{-16} \left(\frac{\lambda}{\text{m}}\right) n_e 0.250 \text{ m} \sqrt{2\pi} = 1.12 \times 10^{-16} n_e \left(\frac{\sqrt{2\pi} c}{\omega \text{m}}\right) \\ &= 8.416 \times 10^{-8} \left(\frac{n_e}{\omega \text{s}}\right)\end{aligned}\quad (24)$$

Remembering Eq. (17)

$$\omega^2 \frac{\epsilon_0 m_{e0}}{e^2} = \omega^2 \frac{8.85 \times 10^{-12} \text{ F}\cdot\text{m}^{-1} \cdot 9.11 \times 10^{-31} \text{ kg}}{1.60 \times 10^{-19} \text{ C}} \quad (25)$$

$\Downarrow$

$$n_e < 0.000314 \omega^2 \quad (26)$$

Where any units has been disregarded. We want the largest possible phase shift which means that the lower the frequency the better. However cutoff must first be taken into account. Since the cutoff is given by Eq. (26) and since we want to measure densities up to  $10^{18} \text{ m}^{-3}$  the minimum frequency of the wave is

$$\frac{\omega}{2\pi} > \frac{\sqrt{\frac{10^{18} \text{ m}^{-3}}{0.000314}}}{2\pi} \quad (27)$$

$\Downarrow$

$$f \approx 9 \text{ GHz} \quad (28)$$

Given the available emitters, the best emitter is therefore the one with  $f = 60 \text{ GHz}$ .

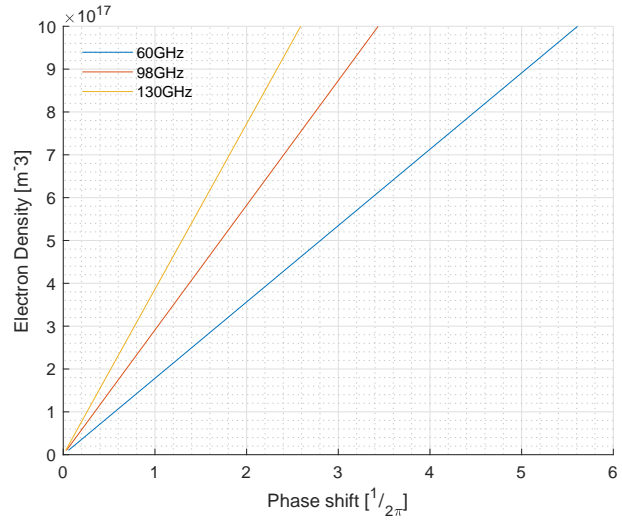
## B. Beam Phase shifts

The goal of the interferometer is to find the phase shift between the microwave beam in vacuum and in plasma. Our suggestion is a setup involving a interferometer emitting a beam through the centre of the reactor. The first measurement would be through vacuum to find the source phase. Afterwards phase measurements can be conducted with an active plasma in the reactor. So by first measuring the phase of the probing beam in vacuum, one can simply measure the phase shift.

We know from Eqs. (21), (22) and (24) that:

$$\frac{\Phi}{2\pi} = 8.416 \times 10^{-8} \left( \frac{n_e}{\omega_S} \right) \quad (29)$$

So for different average electron densities through the plasma, one can plot the resulting phase shifts. Using the code in Appendix E the plot in Fig. 4 is generated. The measured phase shift is then correlated to an electron density using Fig. 4.

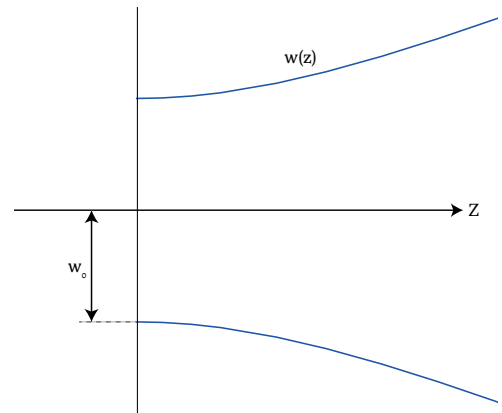


**Figure 4:** Electron densities for different wavelengths and phase shifts

## C. Evolving beam width

In our case we want to use a Gaussian microwave. Such beam propagates spatially as shown in Fig. 5.  $w_0$  is the initial beam waist determined by the source, and  $w(z)$  is the function describing the waist. This equation is:

$$w(z) = w_0 \sqrt{1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2} \quad (30)$$



From this equation one can track the spatial propagation of the wave. In this project three sources are given. The initial beam waist is set to  $w_0 = 0.0275$  m

**Figure 5:** Sketch of Gaussian beam propagation with indication of  $w_0$  and  $w(z)$ .

corresponding to half a reactor port. With

$$\frac{\lambda z}{\pi w_0^2} = \frac{9.543 \times 10^7 z}{w_0^2 f} \quad (31)$$

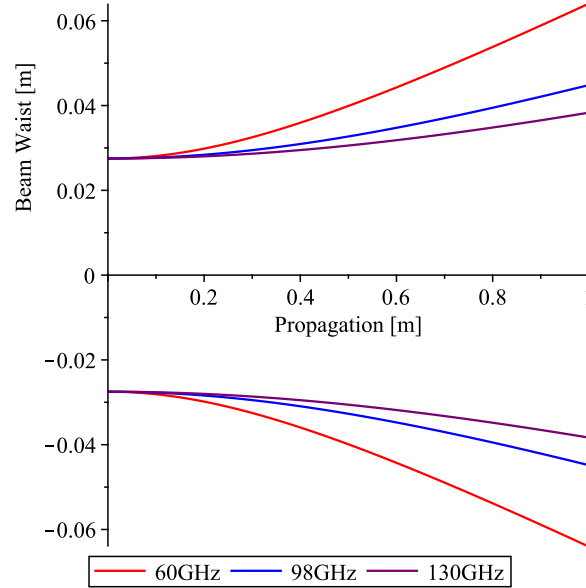
, and  $f$  the frequency of the emitter, the sources' beam waists are given as such:

$$60 \text{ GHz} : \quad w(z) = 0.0275 \sqrt{\left(1 + \frac{9.543 \times 10^7 z}{0.0275^2 \cdot 60 \times 10^9}\right)^2} \quad (32)$$

$$98 \text{ GHz} : \quad w(z) = 0.0275 \sqrt{\left(1 + \frac{9.543 \times 10^7 z}{0.0275^2 \cdot 98 \times 10^9}\right)^2} \quad (33)$$

$$130 \text{ GHz} : \quad w(z) = 0.0275 \sqrt{\left(1 + \frac{9.543 \times 10^7 z}{0.0275^2 \cdot 130 \times 10^9}\right)^2} \quad (34)$$

For the three given sources in this project, the beam waist has been plotted on Fig. 6



**Figure 6:** Beam Waist for the three microwave sources. The functions are Eqs. (32), (33) and (34).

It is undesirable for some of the beam to not reach the output port and instead propagate into the reactor wall, as this will result in a lesser signal strength. Therefore it can be necessary to deploy a Gaussian telescope.

#### D. Gaussian beam telescope interferometer

In order to control the beam waist, one can use a Gaussian beam telescope arrangement around the reactor. A lens is placed between the source and the reactor input port and

again between the output port and the receiver. From chapter 5 in “Fusion Plasma Diagnostics with mm-Waves”<sup>3</sup> the authors explain how such an arrangement can be obtained. Starting with the focal lengths of the lenses,

$$d = f_0 + f_1 \quad (35)$$

, where  $f_0$  and  $f_1$  are focal lengths, and  $d$  are the distance between the lenses. The resulting narrowest beam waist after the second lens are given as:

$$w_2 = \frac{f_1}{f_0} w_0 \quad (36)$$

The use of two lenses make the transformation wavelength independent (Eq 5.118)<sup>3</sup>, thus the only interaction with the wavelength of the probing beam will be that of the plasma itself. The distance to the minimum waist after the second lens is given as:

$$d_3 = \frac{f_1}{f_0} \left( f_0 + f_1 - \frac{f_1}{f_0} d_0 \right) \quad (37)$$

The waist in between the lenses is given as:

$$w_1 = \frac{\lambda f_0}{\pi w_0} \quad (38)$$

And the distance from the first lens to this waist is:

$$d_1 = \left( \frac{\frac{d_0}{f_0} - 1}{\frac{w_0^2 \pi}{f_0 \lambda} + \left( \frac{d_0}{f_0} - 1 \right)^2} + 1 \right) f_0 \quad (39)$$

Thus the distance from  $w_1$  to the second lens is:

$$d_2 = d - d_1 \quad (40)$$

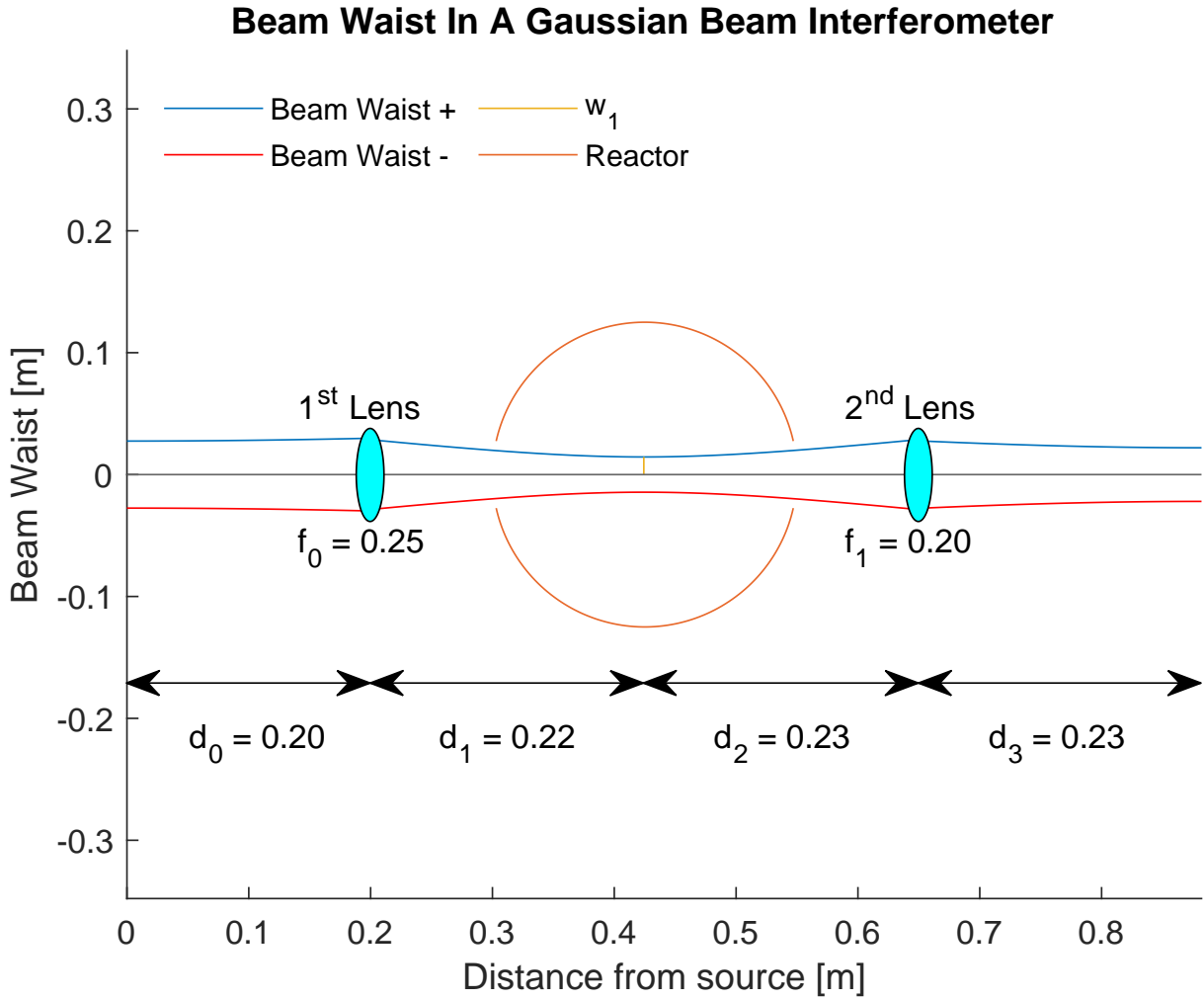
Knowing these variables and utilising Eq. (30) we can model a complete setup. Using the Matlab code in Appendix D, the following parameters are used:

$$\begin{aligned} r_p &= 0.0550 \text{ m} & r &= 0.125 \text{ m} & w_0 &= 0.0275 \text{ m} & freq &= 60 \text{ GHz} \\ d_0 &= 0.200 \text{ m} & d_r &= 0.100 \text{ m} & f_0 &= 0.250 \text{ m} & & 0.200 \text{ m} \end{aligned} \quad (41)$$

With  $r_p$  being the tokamak port radius,  $r$  the tokamak minor radius and  $d_r$  the distance between the first lens and the reactor wall. In the case of the Danish Tokamak Undertaking, two ports opposing each other is ideal for use with interferometry. In this example, the port radius,  $r_p$ , and the minor radius  $r$  are based on the ST-25 in the basement of building 309 at DTU. The script gave the results:

$$\begin{aligned} w_1 &= 0.0145 \text{ m} & w_2 &= 0.0220 \text{ m} \\ d_1 &= 0.224 \text{ m} & d_2 &= 0.226 \text{ m} & d_3 &= 0.232 \text{ m} \end{aligned} \quad (42)$$

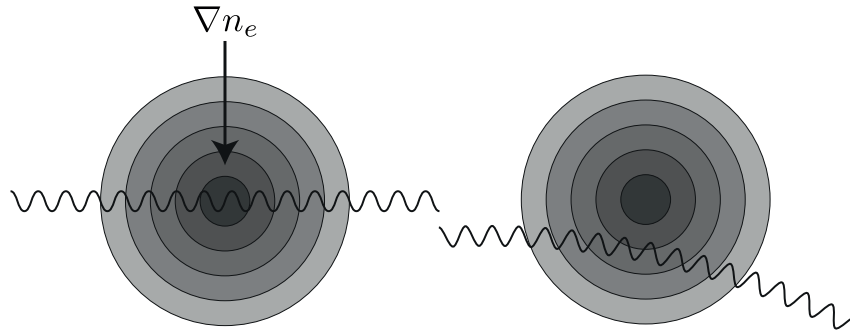




**Figure 7:** Gaussian beam telescope interferometer setup with input parameters shown in Eq. (41) and output parameters shown in Eq. (42)

Furthermore the code plots a sketch of the desired interferometer setup. The sketch can be seen in Fig. 7.

Lastly it should be mentioned that the beam waist confinement does not matter if the wave is refracted into the side of the reactor. As the plasma density varies from the centre and out, the plasma itself act as a variable lens with variable refractive indices. If the beam is emitted straight through the centre, the refractive indices will only shorten and lengthen the wavelength back to normal, however if the beam is emitted off centre, it will be refracted away from the centre. In such a setup, further calculations need to be conducted to determine variation in signal strength, phase and wavelength. The two described setups are depicted on Fig. 8.



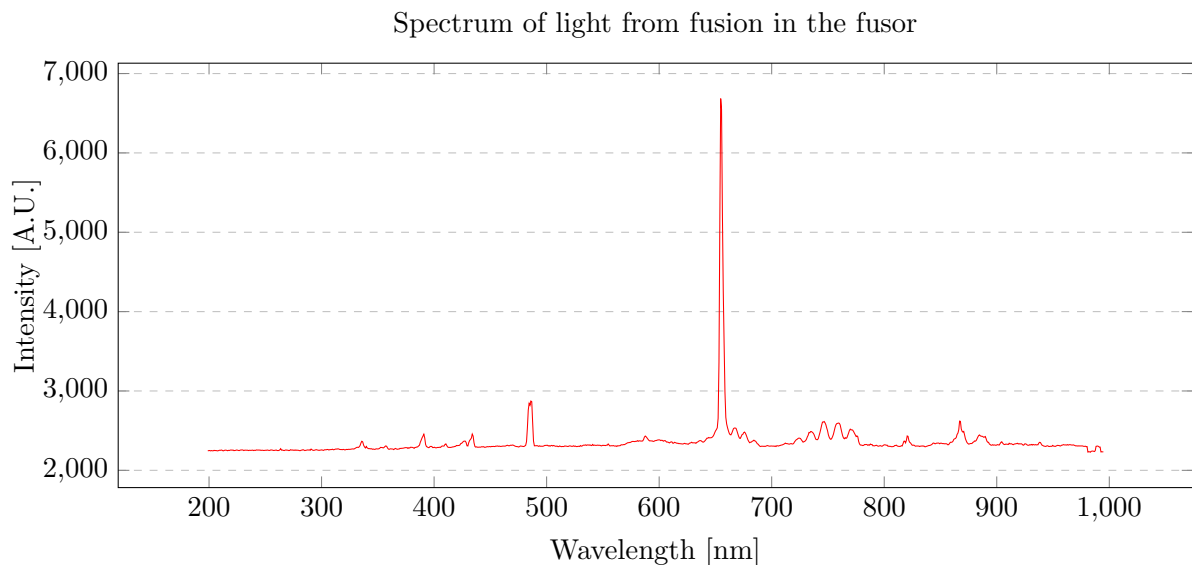
**Figure 8:** Refraction of the probing beam at centre (left) and off centre (right).

### III. FUSOR EXERCISE

The goal of this exercise is studying the inertial electrostatic confinement fusor at DTU. Measurements of neutron counts and the spectral line width are made as a function of the voltage and current. To this, the emission spectrum of the gas are also measured.

#### A. Plasma light emission and spectral line

When doing spectroscopy on a plasma of an unknown gas, optical dispersion splits up the spectrum of light into lines which are then recorded using a spectrometer. This detector measures the frequency/wavelength of the light and the intensity of the light. Thus a plot with the wavelength along the x-axis vs the intensity along the y-axis are produced. For our recordings a typical spectrum looked like that of Fig. 9.



**Figure 9:** Spectrum of light from fusion in the fusor

Each peak correspond with a constituent gas in the fusor. The peaks present on the spectrum are subject to broadening effects. Considering the Doppler effect, each photon's frequency will be shifted since the particles (which are the emitters) are moving at fast speeds (the Doppler shift increases with speed). Therefore if each particle were moving in the same direction, the entire intensity peak would be shifted. But since the particles are moving more or less isotropic around the centre of the device, the peaks are not shifted but rather broadened around the normal emission wavelength. To this, it is also noted that the Doppler broadening increases if particles are moving faster.

Considering Fig. 9, a spectrum is shown with central wavelengths around 656 nm, 486 nm and 433 nm which correspond to accepted values of hydrogen light emission wavelengths<sup>4</sup>. Meanwhile the spectral lines of Deuterium only differs by a factor of 1.000272<sup>5</sup>, which will not be detected using our methods. There are other peaks but those peaks are due to noise from impurities etc.

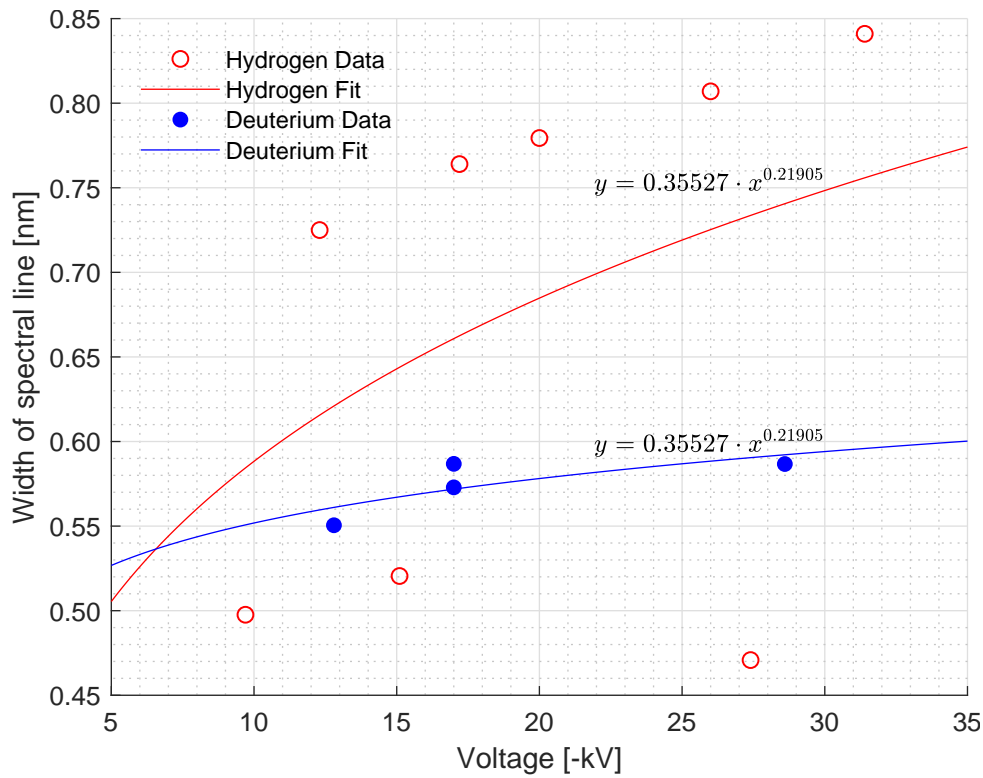
Since the speed of the particles increases with the voltage we expect the line broadening to do the same since  $\sigma = \lambda_0/c v_{1D}$ . While there seems to be enormous uncertainties on the hydrogen measurements, Fig. 10 shows a tendency to increase for both hydrogen and deuterium. Disregarding outliers, the hydrogen also seems to have a wider spectral line than deuterium which makes sense, due to the mass difference between the species. Meanwhile the uncertainties seen might be due to differences in pressure, which leads to differences in kinetic energy lost per time. Other uncertainties are most likely also present. For the width of a spectral line, it is known that:

$$\sigma = \Delta\lambda = \frac{\lambda_0}{c} v_{1D} \quad (43)$$

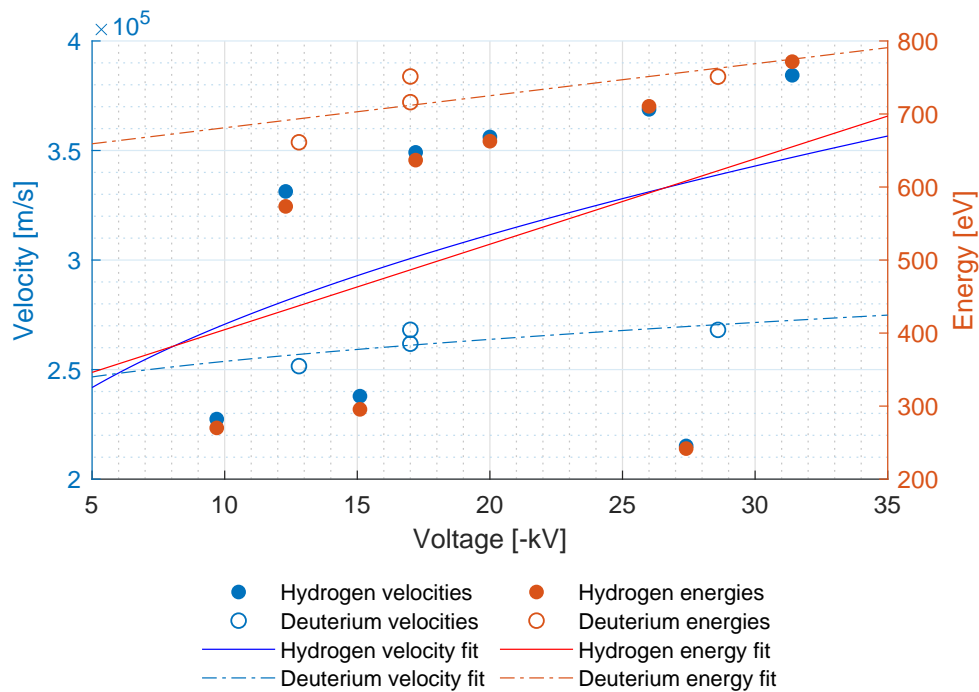
So for the obtained widths, the velocities can be estimated via Eq. (43). Once these points are obtained a curve is fitted to it. The fit is assumed to be linear, as velocity is proportional to voltage applied. The kinetic energies are also estimated via:

$$KE = \frac{1}{2}mv^2 \quad (44)$$

These points are also estimated and plotted along with a fit. The fit is assumed to be some constant, times the square root of the voltage as isolated from Eq. (44). This is done for both hydrogen and deuterium. The plots can be seen in Fig. 11.



**Figure 10:** The spectral line width as a function of applied voltage



**Figure 11:** The estimated velocities and kinetic energies for hydrogen and deuterium as functions of applied voltage

The four fits were computed to be:

$$\text{Velocity for hydrogen: } v = 3.12 \times 10^4 \sqrt{x} + 1.72 \times 10^5 \quad (45)$$

$$\text{Velocity for deuterium: } v = 0.7666 \times 10^4 \sqrt{x} + 2.295 \times 10^5 \quad (46)$$

$$\text{Kinetic energy for hydrogen: } KE = 11.7x + 287.7 \quad (47)$$

$$\text{Kinetic energy for deuterium: } KE = 4.387x + 637.3 \quad (48)$$

with  $x$  being the applied voltage. Of course the uncertainties from Fig. 10 carries over which is unfortunate. We expected the deuterium and hydrogen energies to be equal which was not the case, but the velocity of hydrogen seems to be larger than the deuterium atoms which is to be expected.

## B. Neutron production rate

The fusion reaction rate should increase with voltage and current where approximately every second reaction produce a neutron. Measurements of the neutron production rate are made and seen in Table VII. It is important to note that only 5 measurements are made, so the sample size is very small. There are 3 parameters that varies throughout the measurements which makes it hard to say anything about the data. There is an unknown dependency on the pressure (and on the voltage and current) since an increasing pressure will decelerate the particles overall, but it will also increase the amount of fuel available. The pressure varies from 1.40  $\mu\text{bar}$  to 2  $\mu\text{bar}$ . This should have an effect on the neutron rate, but let us disregard that fact for now. The data suggests that a current of 6 mA is needed to get decent reaction rates when the voltage is in the ranges measured. On top of that, the data also suggests that the voltage needed is between 24 V and 28 V. Note that the last suggestion is made at an approximately constant pressure.

Calculating the total neutron flux at the detector and the total neutron production rate is trivial given Cts./s,  $\eta_{\text{eff.}} = 15\%$ ,  $A_{\text{det}} = 6 \times 10^{-5} \text{ m}^2$  and  $R_{\text{det}} = 0.500 \text{ m}$ .

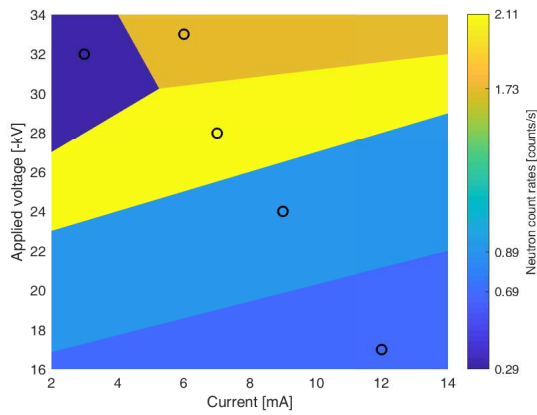
$$\Gamma_n = \frac{\text{Cts.}}{\text{s}} \frac{1}{\eta_{\text{det}}} \frac{1}{A_{\text{det}}} \quad (49)$$

And  $n/\text{s} = 4\pi R_{\text{det}} \Gamma_n$ . Now the power is equal to the product of the current and voltage, but there is a voltage drop across the resistor bank which must be taken into account. Meanwhile the fusion power  $P_{\text{fusion}} = n/\text{s} (E_n + E_\alpha)$  with  $E_n$ ,  $E_T$  being the energy release of the reaction that results in a neutron and a tritium particle respectively (disregarding the other resulting particles). For the remaining problems Table VII displays the data that are

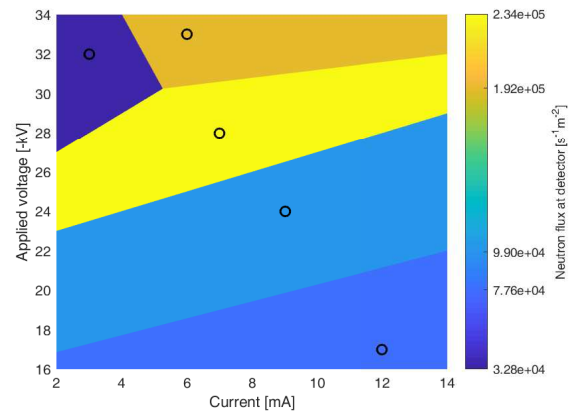
fitted in Fig. 12. On the Q-value; the tokamak reactor JET for example displays a Q-value of around 0.67<sup>6</sup>. This is approximately  $10^5$  times higher than the DTU fusor, so the inertial electrostatic confinement technology is way behind the magnetostatic confinement technology. Calculating the neutron production rates etc. is easy enough, however due to a very small amount of data points, it is hard to say anything meaningful about the fits. We do see an increase in rates when increasing the current and voltage. But the size of the contours should tell us that this conclusion is laughably optimistic. On top of that, the pressure is varying which is not taken into account. Thus our conclusion is that this data is close to, if not completely unusable.

p [μbar]	I [mA]	V[-kV]	P[W]	[Cts./s]	$\Gamma_n[s^{-1}\cdot m^{-2}]$	[n/s]	Q[10 <sup>-6</sup> ]
2	12	17	194	0.689	$0.766 \times 10^5$	$2.41 \times 10^5$	1.45
1.7	9	24	210	0.891	$0.990 \times 10^5$	$3.11 \times 10^5$	1.73
1.6	7	28	193	2.106	$2.34 \times 10^5$	$7.35 \times 10^5$	4.45
1.4	3	32	95.4	0.295	$0.328 \times 10^5$	$1.03 \times 10^5$	1.26
1.4	6	33	195	1.725	$1.92 \times 10^5$	$6.02 \times 10^5$	3.61

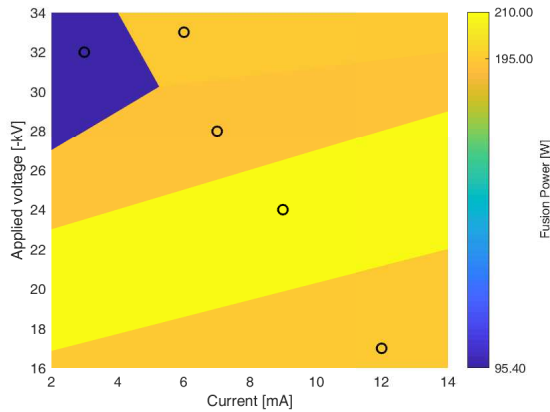
**Table VII:** Table containing the measured parameters for deuterium.



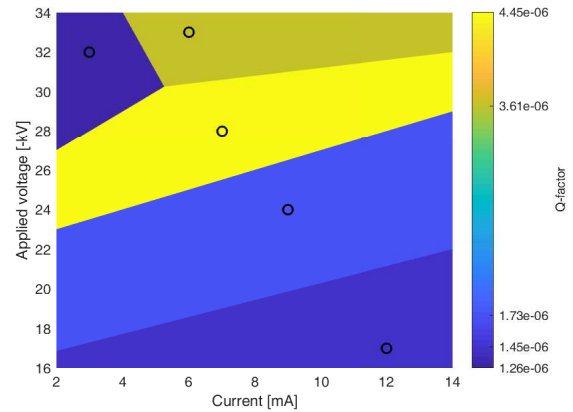
(a) Neutron count rates as functions of current and voltage



(b) The neutron flux at the detector as functions of current and voltage



(c) The fusion power as functions of current and voltage



(d) The Q-factor as functions of current and voltage

**Figure 12:** Fusor neutron count, flux, power and q-factor as functions of current and voltage

## IV. CONCLUSIONS

### A. Assignment 1: A simple reactor model

Freidberg's model produced some reasonable results for a simple tokamak design. The model would however break down when certain parameters would lie in certain regions. For example  $\beta$  would go to infinity when  $P_E$  would lie approximately between 500 MW and 1000 MW. Meanwhile after implementing the elliptical model and solving it for  $P_E = 2$  GW  $\beta$  would go to 1.69% which we deemed to low. Setting  $\kappa = 2$  and  $A = 3$  would increase  $\beta$  to 4.55%. Further optimising  $\beta$  and  $V_P/A_P$  would lead to  $\kappa = 2.5$  and  $A = 2.39$  using a desired  $\beta = 10\%$ .

### B. Assignment 2: Diagnostics via interferometry

Wanting to keep the emitter frequency above the plasma frequency resulted in a minimum frequency of 9 GHz given the calculations, this is the optimum frequency however the best emitter available is emitting at a frequency of 60 GHz so this emitter should be chosen. In order to make sure that the beam is controlled and that most of the beam is transmitted, a Gaussian telescope with a focal length of 0.250 m is implemented. It is also important that the beam hits the plasma surface perpendicular so refraction is avoided, which is why we strongly suggest such a setup.

### C. Assignment 3: Fusor exercise

While it is hard to conclude anything based on the very poor measurements, minor conclusions are somewhat possible. Barring the trivial measurement of the plasma constituents and focusing on the information the spectral line width should provide. While (again) measurements are very uncertain it does seem, as expected, that it measures that the velocity of the deuterium is smaller than for the hydrogen at a given voltage. This points towards the measurement being correct and the average ion energy as a function of voltage can then be calculated. While an increasing energy was derived with increasing voltage, the result did not correspond with expectations as the energies for the two species were not equal. On the neutron rates. A very optimistic conclusion of increasing neutron rates with voltage and current was achieved. The more important conclusion for now seems to be that an automatic pressure regulator must be implemented if an experiment is to be



precise. Furthermore, a much larger amount of data points must also be gathered.

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‡ Homepage of the Technical University of Denmark <http://www.dtu.dk/english/>;

🔗 Project Repository: <https://github.com/rwiuff/10401>

<sup>1</sup> <https://www.iter.org/>.

<sup>2</sup> Jeffrey P. Freidberg, *Plasma physics and fusion energy* (Cambridge University Press, 2007).

<sup>3</sup> Hans-Jürgen Hartfuß and Thomas Geist, *Fusion Plasma Diagnostics with mm-Waves* (Wiley-VCH, 2013).

<sup>4</sup> “Hydrogen spectral series” - Wikipedia (Visited January 22<sup>nd</sup>).

<sup>5</sup> “Deuterium#Spectroscopy” - Wikipedia (Visited January 22<sup>nd</sup>).

<sup>6</sup> “Fusion energy gain factor” - Wikipedia (Visited January 24<sup>th</sup>).

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## LISTINGS

# Appendices

## Appendix A: tokamakDTU\_asign\_1

```
1  %*****
2  % Name:          tokamakDTU
3  %
4  % Version:       1.0
5  %
6  % Purpose:       Contains the function 'tokamakDTU' which gives parameters
7  %                for a tokamak fusion power plant as output based on a
8  %                simplified model. The equations used are derived in
9  %                chapter 5 in Friedberg, Plasma physics and Fusion
10 %                Energy, 2007 (all references are referring thereto).
11 %
12 % To do (NOT for 10401 - Fusion Energi students):
13 %               1. Rewrite the code to a class (this is not done on purpose so
14 %               that the code is more readable for students not familiar
15 %               with classes). Can from that merge the files which takes
16 %               R_0/a and/or the ellipticity as an input into one file.
17 %
18 % Changelog:
19 %               1. December 2014:
20 %               Written by Michael Løiten based on a similar code written
21 %               as a bachelor project by Elias Pagh Sentius
22 %               mailto: mmag@fysik.dtu.dk
23 %*****
24
25 function [b, c, a, R_0, A, A_p, V_p, P_dens, p, n, B_0, beta, tau_E_min,...
26          C_per_watt] =...
27          tokamakDTU_asign_1(...
28          n_flux_fraction, C_F, C_I, P_E, P_W, B_max, sigma_max, eta_t)
```

```
29 %TOKAMAK_DTU Function which returns the parameters of a power plant
30 %
31 % Output parameters
32 %-----
33 % b          - Blanket/shield thickness [m]
34 % c          - Magnet coil thickness [m]
35 % a          - Minor radius [m]
36 % R_0        - Major radius [m]
37 % A          - Aspect ratio []
38 % A_p        - Plasma surface [m^2]
39 % V_p        - Plasma volume [m^3]
40 % P_dens     - Power density [W/m]
41 % p          - Plasma pressure [Pa]
42 % n          - Particle density [m^-3]
43 % B_0        - Magnetic field at magnetic axis [T]
44 % beta       - Plasma beta in the centre []
45 % tau_E_min  - Min confinement time for satisfaction of (p*tau_E)_min [s]
46 % C_per_watt - The cost of the powerplant [$]
47 %
48 % Input parameters
49 %-----
50 % n_flux_fraction - n flux in breeder end/n flux in breeder start []
51 % C_F          - Fixed cost propotionality constant [$]
52 % C_I          - Nuclear island cost propotionality constant [$W/m^3]
53 % P_E          - Desired output power [MW]
54 % P_W          - Maximum wall load [MW/m^2]
55 % B_max        - Magnetic field at the edge of the coil [T]
56 % sigma_max    - Tensile strenght of the magnetic field coils [atm]
57 % eta_t        - Energy conversion efficiency []
58
59
60 % The function starts by defining fixed constants
```

```

61 % Note that this is inefficient if we are looping over the function, but it
62 % makes the code easier to use, as these are not needed as input parameters
63
64 % Fixed constants
65 %#####
66 % Nuclear
67 %-----
68 % Energies
69 E_t      = 2.5e-8; % [MeV] Energy of slow (thermal) neutron (eq 5.6)
70 E_n      = 14.1;  % [MeV] Neutron energy after fusion (eq 2.17)
71 E_a      = 3.5;   % [MeV] alpha energy after fusion (eq 2.17)
72 E_Li     = 4.8;   % [MeV] Heat produced by breeding Li (under eq 4.31)
73 % Cross section and main free paths
74 sigma_v_avg = 3.0e-22; % [m^3/s] DT fusion cross section @ 15keV (table 5.2)
75 lambda_br   = 0.0031; % [m] Breeding mean free path (under eq 5.7)
76 lambda_sd   = 0.055;  % [m] Mean free path from sigma_sd (eq 5.3)
77
78 % Plasma physics
79 %-----
80 % Parameters for infinity gain at the minimum of p tau_E (eq 4.20)
81 T          = 15.0;  % [keV] Temperature for obtaining min tripple product
82 tripple_min = 8.3;  % [atm s] Min tripple prod to obtain Q=inf @ T=15 keV
83
84 % Natural constants
85 %-----
86 mu_0 = 4.0*pi*1e-7;      % Vacuum permeability [T*m/A]
87 e    = 1.602176565e-19;  % Elementary charge [C]
88 %#####
89
90
91 % Secondly we convert everything to SI units, so that the variables are
92 % easier to handle

```

```

93 % Again, this is computationally inefficient, but it suffices for our use
94 % Conversion to SI-units
95 %#####
96 % Conversion factors
97 W_per_MW      = 1.0e6;
98 Pa_per_atm    = 1.01325e5;
99 eV_per_keV    = 1.0e3;
100 eV_per_MeV    = 1.0e6;
101 J_per_eV      = e;
102 J_per_keV     = J_per_eV * eV_per_keV;
103 J_per_MeV     = J_per_eV * eV_per_MeV;
104 % Conversions
105 P_E           = P_E * W_per_MW;           % Desired output power
106 P_W           = P_W * W_per_MW;           % Wall Loading limit on first wall
107 E_t           = E_t * J_per_MeV;           % Energy of slow (thermal) neutron
108 E_n           = E_n * J_per_MeV;           % Neutron energy after fusion
109 E_a           = E_a * J_per_MeV;           % alpha energy after fusion
110 E_Li          = E_Li * J_per_MeV;           % Heat produced by breeding Li
111 sigma_max     = sigma_max * Pa_per_atm;    % Max allowable structural stress
112 T             = T * J_per_keV;             % Temperature for minimum p*tau_E
113 tripple_min   = tripple_min * Pa_per_atm;  % The Lawson parameter (p*tau_E)
114 %#####
115
116
117 % Calculate the geometrical factors
118 %-----
119 % Find the breeder thickness
120 b = get_b(lambda_sd, E_n, E_t, lambda_br, n_flux_fraction);
121 % Find the minor plasma radius and the coil thickness
122 [a, c] = get_a_and_c(B_max, mu_0, sigma_max, b);
123 % Find the major radius
124 R_0 = get_R_0(a, eta_t, E_n, E_a, E_Li, P_E, P_W);

```

```

125 % Find the resulting geometrical factors
126 A = R_0/a; % Aspect Ratio
127 A_p = (2.0*pi*a)*(2.0*pi*R_0); % Plasma surface area
128 V_p = (pi*a (2.0))*(2.0*pi*R_0); % Plasma volume
129
130
131 % Calculate the plasma physics parameters
132 %-----
133 % Find the power density in the plasma
134 P_dens = get_P_dens(E_a, E_n, E_Li, P_E, eta_t, V_p);
135 % Find the plasma pressure
136 p = get_p(E_a, E_n, P_dens, T, sigma_v_avg);
137 % Calculate the density from the definition of p under eq 5.36
138 n = p/(2.0*T);
139 % Find the magnetic field strength on the magnetic axis
140 B_0 = get_B_0(R_0,a,b,B_max);
141 % Find the plasma beta on the magnetic axis
142 beta = get_beta(p, B_0, mu_0);
143 % Find the minimum required confinement time from the definition of the
144 % minimum tripple product.
145 % NOTE: A higher confinement time is advantegous, and could in principle
146 % yield a smaller (and cheaper) reactor. However, the effect is not
147 % included in this model
148 tau_E_min = tripple_min/p;
149
150
151 % Calculate the cost
152 % (details about the cost can be found in the function get_a_and_c)
153 %-----
154 % Find the volume of the nuclear island
155 % (the material surrounding the plasma)
156 V_I = get_V_I(R_0,a,b,c);

```

```
157 % Find the reactor volume per power out
158 % In the current model, this is the only non-constant in the expression for
159 % cost per watt
160 V_I_per_P_E = V_I/P_E;
161 C_per_watt = get_C_per_watt(C_F, C_I, V_I_per_P_E);
162 end
163
164
165
166 function      = get_b(
167 %GET_B Calculates b from the need of slowing down and breeding neutrons
168
169 % Thickness of the moderator-breeding region so that 1 - n_flux_fraction
170 % have slowed down and undergone a breeding reaction
171 % [m]
172 % Equation 5.10
173 delta_x = 2.0*lambda_sd*...
174           log( 1.0-(1.0/2.0)*(E_n/E_t) (1.0/2.0)*...
175               (lambda_br/lambda_sd)*log( n_flux_fraction )...
176           );
177
178 % Set b from delta_x
179 % Friedberg argues above equation 5.11 that b should be between 1 and 1.5 m
180 % Therefore a self chose constant is set to 0.38
181 self_chosen_constant = 0.32;
182 b = delta_x + self_chosen_constant;
183 end
184
185
186
187
188 function      = get_a_and_c(
```



```

189 %GET_A_AND_C Calculates a and c
190
191 % c is obtained from requiring that the magnets are so thin that they are
192 % on the limit of the tensile strenght
193 % a is obtained from minimizing the costs
194
195 % xi defined when making the magnetic coil c as thin as possible
196 % Under equation 5.27
197 xi = B_max (2.0) / (4.0*mu_0*sigma_max);
198
199 % a is found from optimization of the cost, where
200 % total cost = fixed cost + nuclear island cost
201 %.....
202 % Fixed cost
203 %.....
204 % K_F = Fixed cost for building, turbines, generators etc (also applies to
205 % fusion, fission, fossil)
206 % Assumption: The fixed cost is proportional to power output:
207 % Equation 5.13
208 % K_F = C_F*P_E;
209 %.....
210 % Nuclear island cost (mainly cost of magnets, blanket and shield)
211 %.....
212 % Assumption: The proportional to reactor volume:
213 % Equation 5.14
214 % K_I = C_I*V_I;
215 % Equation 5.15
216 % V_I = 2.0*pi^(2.0) * R_0 * ( (a+b+c)^(2.0) - a^(2.0) ); % Reactor volume
217 %.....
218 % Cost per watt:
219 %.....
220 % Defined as C_p_watt = (K_F + K_I)/P_E, rewritten to

```

```

221 % C_p_watt = C_F + C_I*(V_I/P_E);
222 % Since the cost per watt contains two constants, we can minimize the
223 % V_I/P_E in order to optimize the cost
224 % Given by equation 5.20 inserted in 5.17
225 % Equation 5.21
226 % V_I_per_P_E = V_I/P_E; % Reactor volume per power out
227 % a is found by setting the derivative of V_I_per_P_E = 0
228 % Equation 5.29
229 a = ((1.0 + xi)/(2.0*xi (1.0/2.0))) * b;
230
231 % Knowing xi, a, and, b, we can calculate c
232 % c found by comparing tensile force and magnetic force working on the coil
233 % Equation 5.27
234 c = 2*xi/(1-xi)*(a+b);
235 end
236
237
238
239 function      = get_R_0(
240 %GET_R_0 Calculate the major radius
241
242 % Divide eq 5.18 (electric power out) by
243 % eq 5.19 (wall loading * area = total neutron production) and solve for R0
244 % Equation 5.20
245 R_0 = (1.0/(4.0*pi (2.0)*eta_t))*(E_n/(E_n + E_a + E_Li))*(P_E/(a*P_W));
246 end
247
248
249
250 function      = get_P_dens(
251 %GET_P_DENS Calculate the power density
252

```

```

253 % The power density is found by the sum of the power from the alphas plus
254 % the power from the neutrons, divided by the plasma volume
255 % Equation 5.35
256 P_dens = (E_a + E_n)/(E_a + E_n + E_Li)*P_E/(eta_t*V_p);
257 end
258
259
260
261 function      = get_B_0(
262 %GET_B_0 Calculte the magnetic field strength on the magnetic axis
263
264 % B_max is found in the edge of the magnet (at R = R_0-a-b)
265 % B0 is the magnetic field at R0
266 % As B propto 1/R. we have that B_0/B_max = (R_0-a-b)/R_0, which leads to
267 % Equation 5.42
268 B_0 = ((R_0-a-b)/R_0)*B_max;
269 end
270
271
272
273 function      = get_beta(
274 %GET_beta Calculte the magnetic field strength on the magnetic axis
275
276 % Plasma beta in the center (kinetical pressure over magnetical pressure):
277 % Equation 5.43
278 beta = p / (B_0^2/(2.0*mu_0));
279 end
280
281
282
283 function      = get_p(
284 %GET_P Calculate the plasma pressure

```

```
285
286 % Found from solving the sum of neutron and alpha power for n, and multiply
287 % the result with T
288 % Equation 5.37
289 p = ( (16.0/(E_a + E_n)) * P_dens ) (1.0/2.0)*...
290      (T (2.0)/sigma_v_avg) (1.0/2.0);
291 end
292
293
294
295 function      = get_V_I(
296 %GET_V_I Calculate the volume of the material surrounding the plasma
297
298 % Equation 5.15
299 V_I = 2.0*pi (2.0) * R_0 * ( (a+b+c) (2.0) - a (2.0) );
300 end
301
302
303
304 function      = get_C_per_watt(
305 %GET_C_PER_WATT Calculates the cost for one watt out from the power plant
306
307 % For details in how the cost is derived, see comments in the function
308 % get_a_and_c
309
310 C_per_watt = C_F + C_I*(V_I_per_P_E);
311 end
```

## Appendix B: IterateTokamakDTU

```
1 close all
2 clear all
3
4 titl = [ Desired output power [MW] , Maximum wall load [MW, m -2] , ...
5         Magnetic field at the edge of the coil [T] , ...
6         Tensile strenght of the magnetic field coils [atm] ];
7 foldertitl = [ PE , PW , Bmax , sigmamax ];
8
9 for d = 1:4
10     l = d; % Choose iteration: 1 = P_E, 2 = P_W, 3 = B_max, 4 = sigma_max
11     p0 = [];
12     x = [];
13     mkdir('..\\MatlabFigures', foldertitl(l))
14     printout = (sprintf('Created folder %s', foldertitl(l)));
15     disp(printout)
16     if l == 1
17         x = linspace(1, 3000, 1000);
18         for i = 1:length(x)
19             [b, c, a, R_0, A, A_p, V_p, P_dens, p, ...
20              n, B_0, beta, tau_E_min, C_per_watt] = ...
21             tokamakDTU_asign_1(0.01, 1, 2, x(i), 4, 13, 3000, 0.4);
22             q = [b, c, a, R_0, A, A_p, V_p, P_dens, p, ...
23                  n, B_0, beta, tau_E_min, C_per_watt];
24             p0 = cat(1, p0, q);
25         end
26     elseif l == 2
27         x = linspace(1, 10, 1000);
28         for i = 1:length(x)
29             [b, c, a, R_0, A, A_p, V_p, P_dens, p, ...
30              n, B_0, beta, tau_E_min, C_per_watt] = ...
```

```

31         tokamakDTU_asign_1(0.01, 1, 2, 1000, x(i), 13, 3000, 0.4);
32         q = [b, c, a, R_0, A, A_p, V_p, P_dens, p, ...
33             n, B_0, beta, tau_E_min, C_per_watt];
34         p0 = cat(1, p0, q);
35     end
36 elseif l == 3
37     x = linspace(10, 20, 1000);
38     for i = 1:length(x)
39         [b, c, a, R_0, A, A_p, V_p, P_dens, p, ...
40             n, B_0, beta, tau_E_min, C_per_watt] = ...
41             tokamakDTU_asign_1(0.01, 1, 2, 1000, 4, x(i), 3000, 0.4);
42         q = [b, c, a, R_0, A, A_p, V_p, P_dens, p, ...
43             n, B_0, beta, tau_E_min, C_per_watt];
44         p0 = cat(1, p0, q);
45     end
46 elseif l == 4
47     x = linspace(2000, 5000, 1000);
48     for i = 1:length(x)
49         [b, c, a, R_0, A, A_p, V_p, P_dens, p, ...
50             n, B_0, beta, tau_E_min, C_per_watt] = ...
51             tokamakDTU_asign_1(0.01, 1, 2, 1000, 4, 13, x(i), 0.4);
52         q = [b, c, a, R_0, A, A_p, V_p, P_dens, p, ...
53             n, B_0, beta, tau_E_min, C_per_watt];
54         p0 = cat(1, p0, q);
55     end
56 end
57 printout = (sprintf('Iterated tokamakDTU_asign_1.m %d of 4', d));
58 disp(printout)
59 b = p0(:, 1);
60 c = p0(:, 2);
61 a = p0(:, 3);
62 R_0 = p0(:, 4);

```

```
63     A = p0(:, 5);
64     A_p = p0(:, 6);
65     V_p = p0(:, 7);
66     P_dens = p0(:, 8);
67     p = p0(:, 9);
68     n = p0(:, 10);
69     B_0 = p0(:, 11);
70     beta = p0(:, 12);
71     beta = beta * 100;
72     tau_E_min = p0(:, 13);
73     C_per_watt = p0(:, 14);
74
75     f1 = figure;
76     ptau = plot(x, tau_E_min, 'color', 'r', 'LineWidth', 1);
77     xlabel(titl(1));
78     ylabel('[s]');
79     ytickformat('%0.2f');
80     grid on
81     grid minor
82     lgd = legend([ptau], ...
83         { Min confinement time for satisfaction of (p tau_E)_{min} });
84     legend('boxoff')
85     legend('Location', 'north')
86     figcount = 1;
87     printout = (sprintf('Created figure %d of 8', figcount));
88     disp(printout)
89
90     f2 = figure;
91     pb = plot(x, b, 'color', 'r', 'LineWidth', 1);
92     hold on
93     pc = plot(x, c, '--', 'color', 'b', 'LineWidth', 1);
94     pa = plot(x, a, '-.', 'color', 'm', 'LineWidth', 1);
```

```
95     pcpw = plot(x, C_per_watt, 'color', 'g', 'LineWidth', 1);
96     hold off
97     xlabel(titl(1));
98     ytickformat('%.2f');
99     lgd = legend([pb, pc, pa, pcpw], ...
100         { Blanket - shield, thickness, [m] , Magnet, coil, thickness, [m] , ...
101         Minor, radius, [m] , ...
102         Cost, per, watt, [ ] } ); % Figure legend
103     legend('boxoff')
104     if l == 3
105         legend('Location', 'north')
106     else
107         legend('Location', 'west')
108     end
109     grid on
110     grid minor
111     figcount = 2;
112     printout = (sprintf('Created figure %d of 8', figcount));
113     disp(printout)
114
115     f3 = figure;
116     hold on
117     yyaxis left
118     pA = plot(x, A, 'LineWidth', 1);
119     ylabel('Aspect ratio')
120     yyaxis right
121     pr_0 = plot(x, R_0, '--', 'LineWidth', 2);
122     ylabel('Major radius [m]')
123     hold off
124     xlabel(titl(1));
125     ytickformat('%.2f');
126     lgd = legend([pr_0, pA], ...
```



```
127     { Major, radius , Aspect, ratio, [] } );% Figure legend
128     legend('boxoff')
129     legend('Location', 'east')
130     grid on
131     grid minor
132     figcount = 3;
133     printout = (sprintf('Created figure %d of 8', figcount));
134     disp(printout)
135
136     f4 = figure;
137     pA_p = plot(x, A_p, 'LineWidth', 1);
138     hold on
139     pV_p = plot(x, V_p, '--', 'LineWidth', 2);
140     hold off
141     xlabel(titl(1));
142     ytickformat('%.2f');
143     lgd = legend([pA_p, pV_p], ...
144         { Plasma, surface, [m 2] , Plasma, volume, [m 3] } );% Figure legend
145     legend('boxoff')
146     legend('Location', 'east')
147     grid on
148     grid minor
149     figcount = 4;
150     printout = (sprintf('Created figure %d of 8', figcount));
151     disp(printout)
152
153     f5 = figure;
154     pP_dens = plot(x, P_dens, 'color', 'r', 'LineWidth', 1);
155     xlabel(titl(1));
156     ylabel(' [W m^-1] ');
157     ytickformat('%.2f');
158     grid on
```

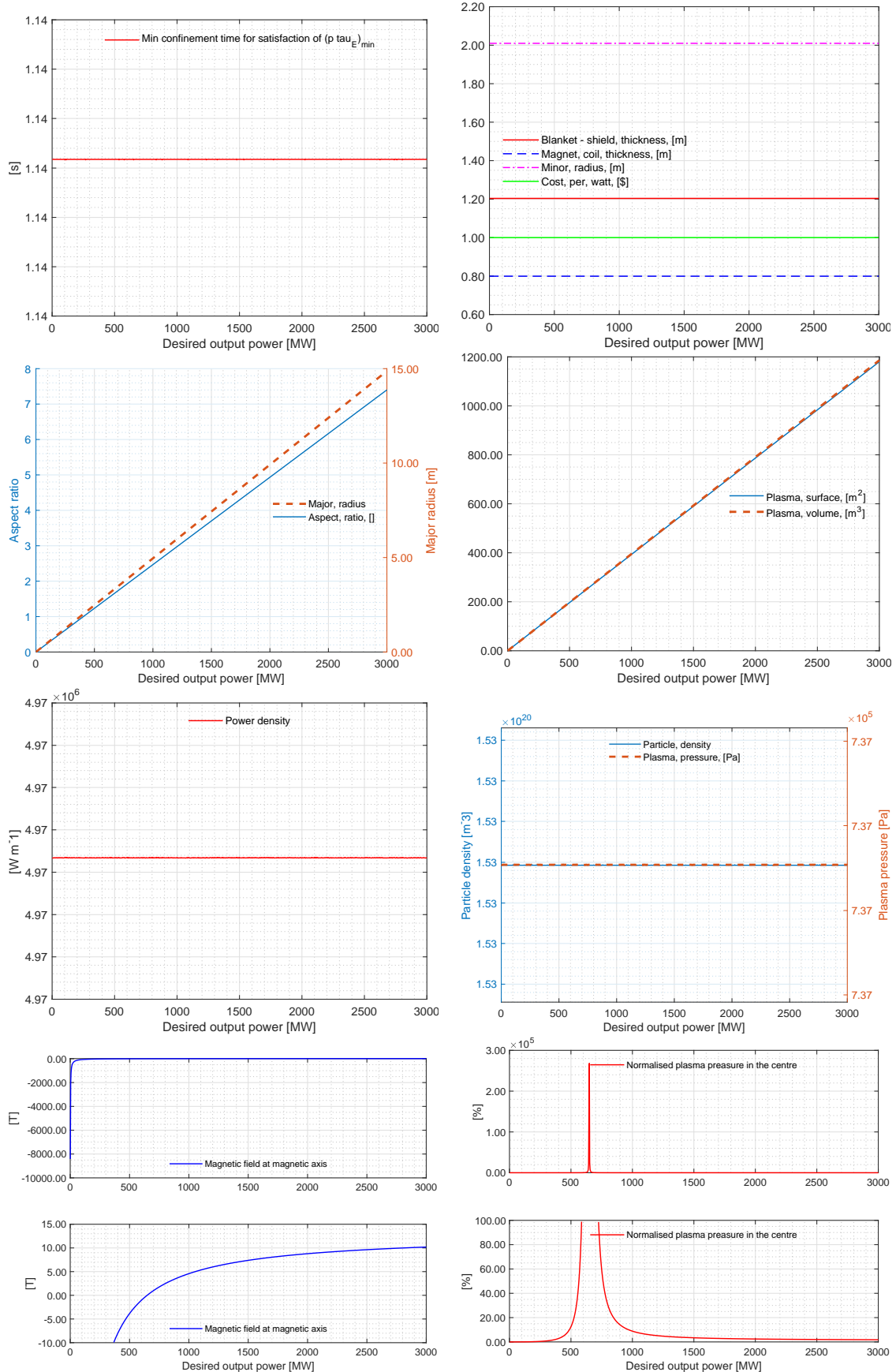
```
159     grid minor
160     lgd = legend([pP_dens], ...
161         { Power density });% Figure legend
162     legend('boxoff')
163     legend('Location', 'north')
164     figcount = 5;
165     printout = (sprintf('Created figure %d of 8', figcount));
166     disp(printout)
167
168     f6 = figure;
169     yyaxis left
170     pn = plot(x, n, 'LineWidth', 1);
171     ylabel('Particle density [m^-3]')
172     ytickformat('%.2f');
173     yyaxis right
174     p = plot(x, p, '--', 'LineWidth', 2);
175     ylabel('Plasma pressure [Pa]')
176     ytickformat('%.2f');
177     xlabel(titl(1));
178     lgd = legend([pn, p], ...
179         { Particle, density , Plasma, pressure, [Pa] });% Figure legend
180     legend('boxoff')
181     legend('Location', 'north')
182     grid on
183     grid minor
184     figcount = 6;
185     printout = (sprintf('Created figure %d of 8', figcount));
186     disp(printout)
187
188     if l == 1
189         f7 = figure;
190         subplot(2, 1, 1);
```

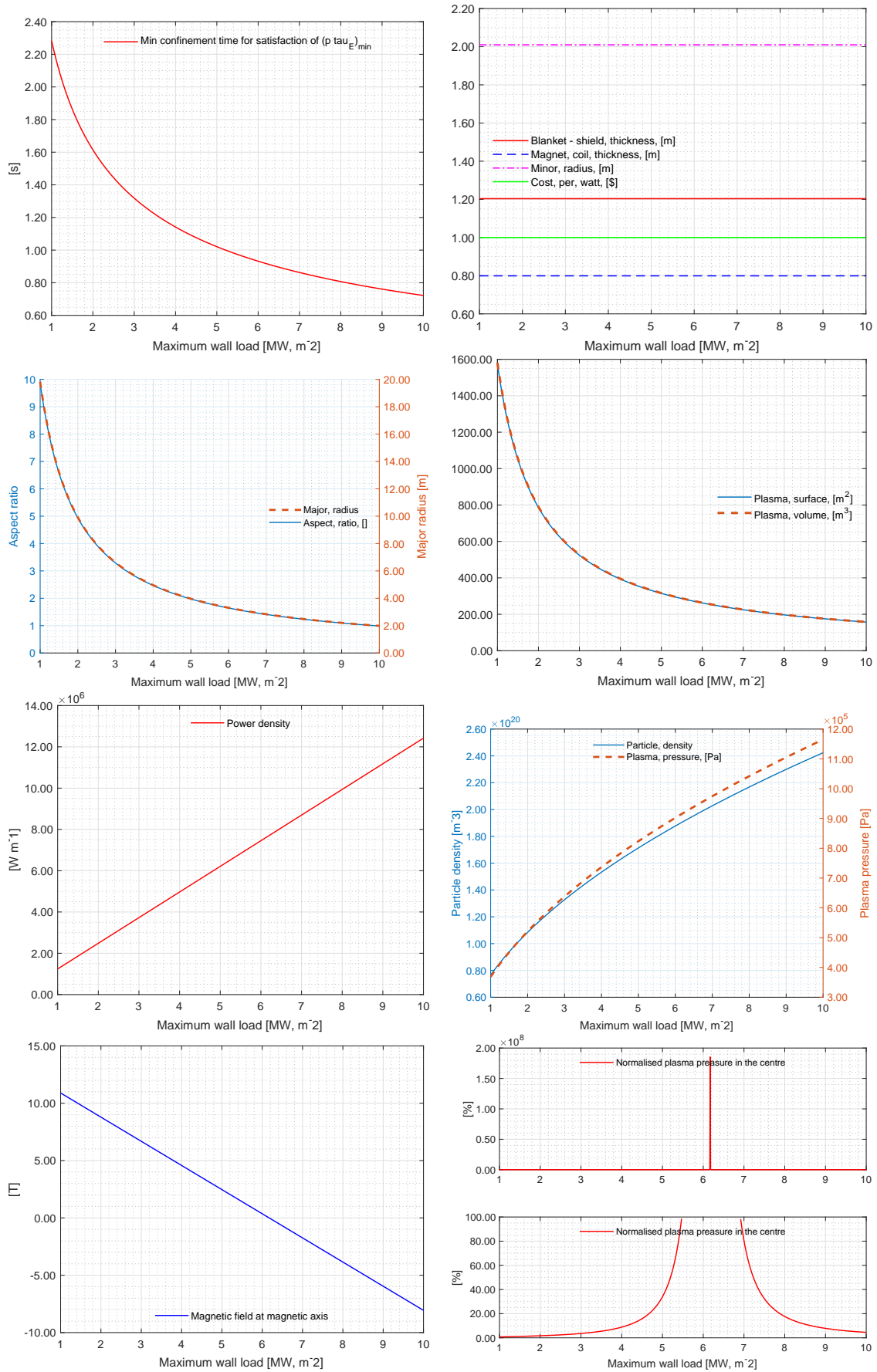
```
191     pB_0 = plot(x, B_0, 'color', 'b', 'LineWidth', 1);
192     ylabel('[T]');
193     ytickformat('%.2f');
194     grid on
195     grid minor
196     lgd = legend([pB_0], ...
197     { Magnetic field at magnetic axis });% Figure legend
198     legend('boxoff')
199     legend('Location', 'south')
200     subplot(2, 1, 2);
201     B_0(B_0 <= -10) = NaN;
202     pB_0 = plot(x, B_0, 'color', 'b', 'LineWidth', 1);
203     xlabel(titl(1));
204     ylabel('[T]');
205     ytickformat('%.2f');
206     grid on
207     grid minor
208     lgd = legend([pB_0], ...
209     { Magnetic field at magnetic axis });% Figure legend
210     legend('boxoff')
211     legend('Location', 'south')
212 else
213     f7 = figure;
214     pB_0 = plot(x, B_0, 'color', 'b', 'LineWidth', 1);
215     xlabel(titl(1));
216     ylabel('[T]');
217     ytickformat('%.2f');
218     grid on
219     grid minor
220     lgd = legend([pB_0], ...
221     { Magnetic field at magnetic axis });% Figure legend
222     legend('boxoff')
```

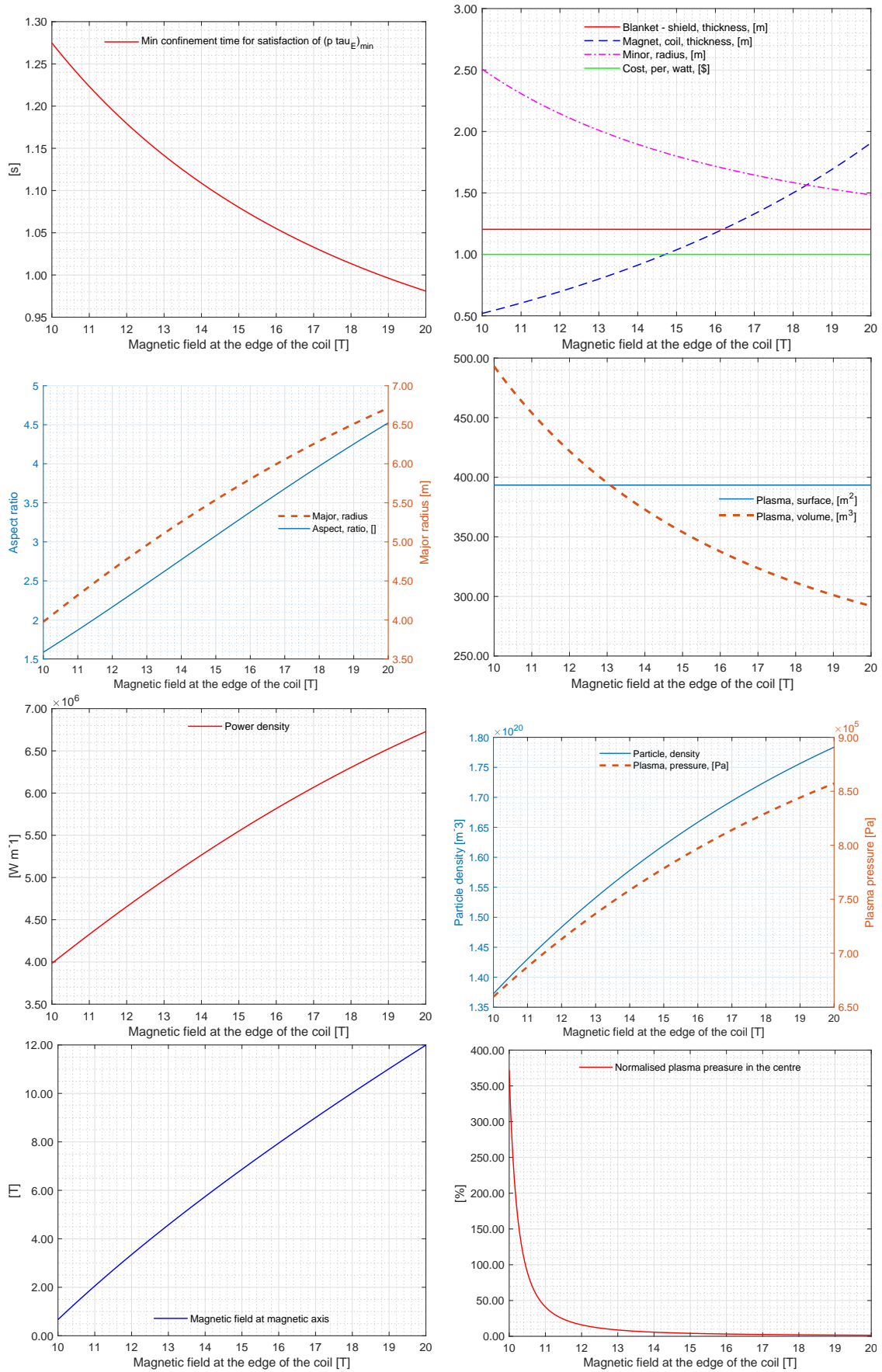
```
223     legend('Location', 'south')
224
225     end
226
227     figcount = 7;
228
229     printout = (sprintf('Created figure %d of 8', figcount));
230     disp(printout)
231
232     if l == 1 || l == 2
233
234         f8 = figure;
235
236         subplot(2, 1, 1);
237
238         pbeta = plot(x, beta, 'color', 'r', 'LineWidth', 1);
239
240         ylabel(' [%] ');
241
242         ytickformat('%.2f');
243
244         grid on
245
246         grid minor
247
248         lgd = legend([pbeta], ...
249
250         { Normalised plasma preasure in the centre });% Figure legend
251
252         legend('boxoff')
253
254         legend('Location', 'north')
255
256         subplot(2, 1, 2);
257
258         beta(beta >= 100) = NaN;
259
260         pbeta = plot(x, beta, 'color', 'r', 'LineWidth', 1);
261
262         xlabel(titl(1));
263
264         ylabel(' [%] ');
265
266         ytickformat('%.2f');
267
268         grid on
269
270         grid minor
271
272         lgd = legend([pbeta], ...
273
274         { Normalised plasma preasure in the centre });% Figure legend
275
276         legend('boxoff')
277
278         legend('Location', 'north')
279
280     else
281
282         f8 = figure;
283
284         pbeta = plot(x, beta, 'color', 'r', 'LineWidth', 1);
```

```
255     xlabel(titl(1));
256     ylabel(['[%]']);
257     ytickformat('%.2f');
258     grid on
259     grid minor
260     lgd = legend([pbeta], ...
261     { Normalised plasma preasure in the centre });% Figure legend
262     legend('boxoff')
263     legend('Location', 'north')
264 end
265 figcount = 8;
266 printout = (sprintf('Created figure %d of 8', figcount));
267 disp(printout)
268
269 fignames = [ f1 , f2 , f3 , f4 , f5 , f6 , f7 , f8 ];
270 figfigs = [f1, f2, f3, f4, f5, f6, f7, f8];
271
272
273 for i = 1:8
274     foldername = sprintf('../MatlabFigures/%s', foldertitl(1));
275     epsfilename = sprintf('%s.eps', fignames(i));
276     fullfilename = fullfile(foldername, epsfilename);
277     saveas(figfigs(i), fullfilename, 'epsc')
278     printout = (sprintf('Saved figure %d of 8', i));
279     disp(printout)
280 end
281
282 close all
283 printout = (sprintf('Completed iteration %d of 4', d));
284 disp(printout)
285 end
```

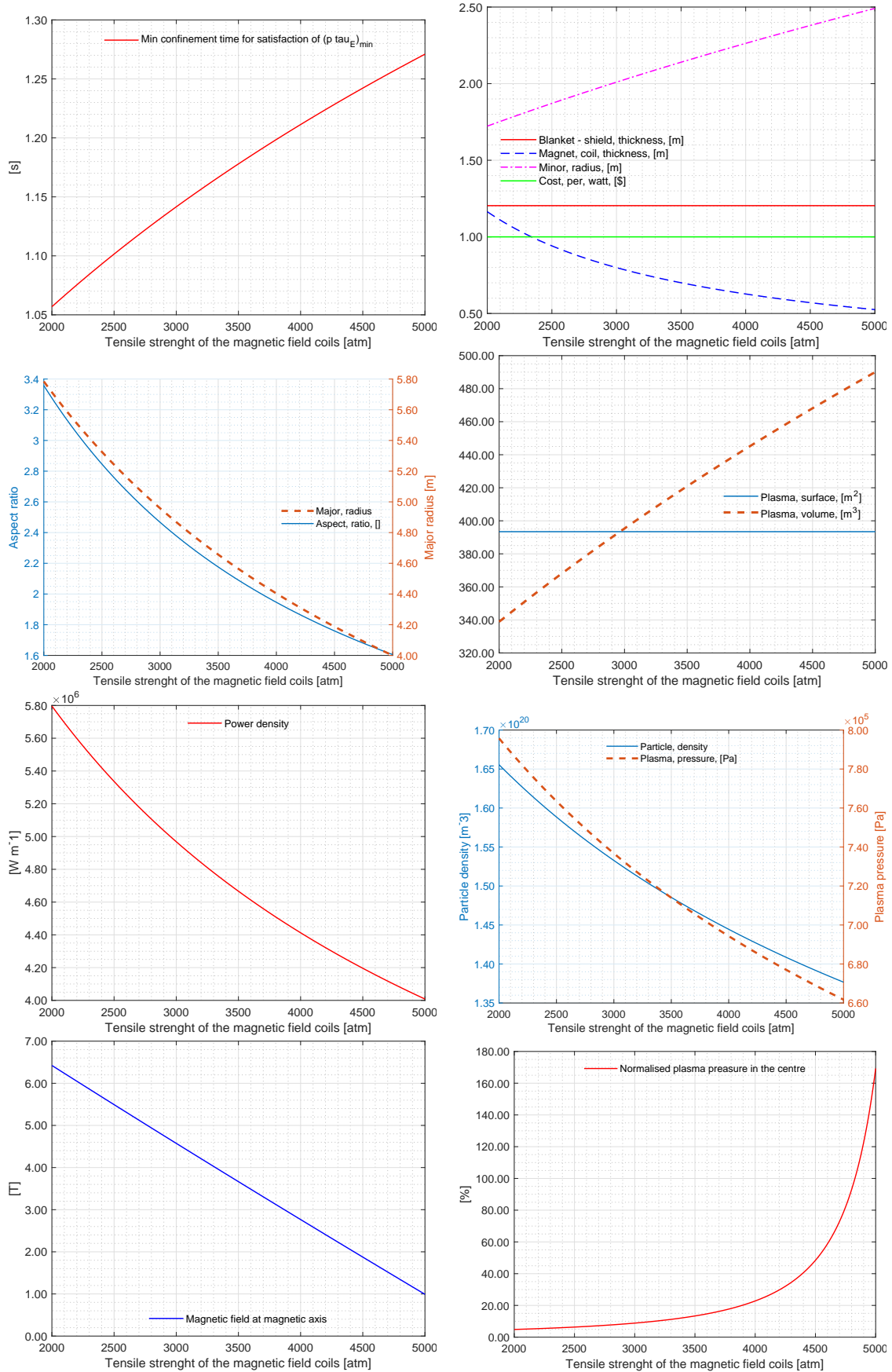
### Appendix C: Iterations over Freidberg's model











# Appendix D: interferometer.m

```

1 close all
2 clear all
3
4 mkdir('../MatlabFigures', 'Interferometer'); % Create save directory
5
6 %-----Input Parameters-----%
7 r_p = 0.055; % Reactor port opening [m]
8 w_0 = 0.0275; % Initial Beam Waist [m]
9 r = 0.125; % Minor tokamak radius [m]
10 freq = 60; % Frequency of probe beam [GHz]
11
12 d_0 = 0.20; % Distance between source and first lens
13 d_r = 0.10; % Distance between first lens and reactor wall
14 f_0 = 0.25; % Focal length of first lens
15 f_1 = 0.20; % Focal length of second lens
16 %-----%
17
18 %-----Distances and Lowest Beam Waists Calculations-----%
19 c = 299792458; % The spec of light [m/s]
20 lambda = c / (freq * 109); % Calculates the wavelength [m]
21 d = f_0 + f_1; % Calculates the distance between lenses
22 w_1 = (lambda * f_0) / ...
23     (pi * w_0); % Calculates the beam waist between lenses
24 d_1 = (((d_0 / f_0) - 1) / ((w_02) * pi) / ...
25     (f_0 * lambda) + ((d_0 / f_0) - 1)2) + 1) * ...
26     f_0; % Distance after 1st lens to lowest beam waist
27 w_2 = (f_1 / f_0) * w_0; % Calculates w_2 after 2nd second lens
28 d_2 = d - d_1; % Distance from w_2 to second lens
29 d_3 = f_1 / f_0 * ...
30     (f_0 + f_1 - ...

```

```

31     (f_1 / f_0) * d_0);% Distance to lowest beam waist after 2nd lens
32 %-----%
33
34 %-----Beam Waist Datapoints-----%
35 x_0 = linspace(0, d_0, 1000);% Plotpoints till first lens
36 y_0 = [];
37 for i = 1:1000
38     w = w_0 * ...
39         sqrt(1+((lambda * x_0(i)) / ...
40             (pi * w_0 ^ 2)) ^ 2);% Calculates the beam waist from source
41     y_0 = horzcat(y_0, w);
42 end
43
44 x_1 = linspace(0, d_1, 1000); % Plotpoints from first lens to w_1
45 y_1 = [];
46 for i = 1:1000
47     w = w_1 * ...
48         sqrt(1+((lambda * x_1(i)) / ...
49             (pi * w_1 ^ 2)) ^ 2);% Beam waist between w_1 and the 1st lens
50     y_1 = horzcat(y_1, w);
51 end
52 y_1 = fliplr(y_1); % Flips the y-plot points as the beam waist is declining
53 y = horzcat(y_0, y_1);
54
55 x_2 = linspace(0, d_2, 1000); % Plotpoints from w_1 to second lens
56 y_2 = [];
57 for i = 1:1000
58     w = w_1 * ...
59         sqrt(1+((lambda * x_2(i)) / ...
60             (pi * w_1 ^ 2)) ^ 2);% Beam waist between w_1 and the 2nd lens
61     y_2 = horzcat(y_2, w);
62 end

```

```

63
64 y = horzcat(y, y_2);
65
66 x_3 = linspace(0, d_3, 1000); % Plotpoints from 2nd lens to w_2
67 y_3 = [];
68 for i = 1:1000
69     w = w_2 * ...
70         sqrt(1+((lambda * x_3(i)) / ...
71             (pi * w_2^2))^2); % Evolving beam waist after the 2nd lens to w_2
72     y_3 = horzcat(y_3, w);
73 end
74 y_3 = fliplr(y_3); % Flips the y-plot points as the beam waist is declining
75 y = horzcat(y, y_3);
76
77 for i = 1:1000
78     x_1(i) = x_1(i) + d_0; % Creates x-axis data points
79 end
80 for i = 1:1000
81     x_2(i) = x_2(i) + d_0 + d_1; % Creates x-axis data points
82 end
83 for i = 1:1000
84     x_3(i) = x_3(i) + d_0 + d_1 + d_2; % Creates x-axis data points
85 end
86 x = horzcat(x_0, x_1, x_2, x_3);
87
88 Opy = -y; % Flip beam waist to plot 'Beam Waist -'
89 %-----%
90
91 %-----Beam Waist Propagation Plot-----%
92 l = figure; % Creates figure
93 hold on
94 axis equal

```

```

95 p1 = plot(x, y); % Plots Beam Waist+
96 p2 = plot(x, 0py, 'Color', 'red'); % Plots Beam Waist-
97 %p3 = xline(d_0, '--');
98 %xline(d_0+d_1+d_2, '--');
99 axPos = get(gca, 'Position'); % Get normalised axis coordinates
100 xMinMax = xlim; % Get normalised axis coordinates
101 yMinMax = ylim; % Get normalised axis coordinates
102 zAnn = axPos(1) + ((0 - xMinMax(1)) / (xMinMax(2) - xMinMax(1))) ...
103     * axPos(3); % Defines points relative to axis in normalised coordinates
104 d0Ann = axPos(1) + ((d_0 - xMinMax(1)) / (xMinMax(2) - xMinMax(1))) ...
105     * axPos(3); % Defines points relative to axis in normalised coordinates
106 d1Ann = axPos(1) + ((d_0 + d_1 - xMinMax(1)) / (xMinMax(2) - xMinMax(1))) ...
107     * axPos(3); % Defines points relative to axis in normalised coordinates
108 d2Ann = axPos(1) + ((d_0 + d_1 + d_2 - ...
109     xMinMax(1)) / (xMinMax(2) - xMinMax(1))) ...
110     * axPos(3); % Defines points relative to axis in normalised coordinates
111 d3Ann = axPos(1) + ((d_0 + d_1 + d_2 + d_3 - ...
112     xMinMax(1)) / (xMinMax(2) - xMinMax(1))) ...
113     * axPos(3); % Defines points relative to axis in normalised coordinates
114 yAnn = axPos(2) + ((0 - yMinMax(1)) / (yMinMax(2) - yMinMax(1))) ...
115     * axPos(4); % Defines points relative to axis in normalised coordinates
116 annotation('doublearrow', ...
117     [zAnn, d0Ann], [yAnn - 0.2, yAnn - 0.2]); % Annotates d_0
118 annotation('doublearrow', ...
119     [d0Ann, d1Ann], [yAnn - 0.2, yAnn - 0.2]); % Annotates d_1
120 annotation('doublearrow', ...
121     [d1Ann, d2Ann], [yAnn - 0.2, yAnn - 0.2]); % Annotates d_2
122 annotation('doublearrow', ...
123     [d2Ann, d3Ann], [yAnn - 0.2, yAnn - 0.2]); % Annotates d_3
124 annotation('ellipse', ...
125     [d0Ann - 0.01, yAnn - y_0(1000) * 1.5, 0.02, y_0(1000) * 3], ...
126     'FaceColor', 'cyan'); % Draws 1st lens

```

```

127 annotation('ellipse', ...
128     [d2Ann - 0.01, yAnn - y_0(1000) * 1.5, 0.02, y_0(1000) * 3], ...
129     'FaceColor', 'cyan');% Draws 2nd lens
130 text(d_0/4, -0.22, sprintf('d_0 = %0.2f', d_0)) % Distance annotation
131 text(d_0+(d_1 / 4), -0.22, ...
132     sprintf('d_1 = %0.2f', d_1))% Distance annotation
133 text(d_0+d_1+(d_2 / 4), -0.22, ...
134     sprintf('d_2 = %0.2f', d_2))% Distance annotation
135 text(d_0+d_1+d_2+(d_3 / 4), -0.22, ...
136     sprintf('d_3 = %0.2f', d_3))% Distance annotation
137 text(d_0-0.06, 0.06, '1^{st} Lens') % Lens annotation
138 text(d_0-0.06, -0.06, sprintf('f_0 = %0.2f', f_0)) % Lens annotation
139 text(d_0+d_1+d_2-0.06, 0.06, '2^{nd} Lens') % Lens annotation
140 text(d_0+d_1+d_2-0.06, -0.06, ...
141     sprintf('f_1 = %0.2f', f_1))% Lens annotation
142 w1x = (d_0 + d_1); % w_1 coordinate
143 w1x = repelem(w1x, 100); % w_1 x-coordinate array
144 w1y = linspace(0, w_1); % w_1 y-coordinate array
145 p4 = plot(w1x, w1y); % Plots w_1
146 yline(0, '-'); % Plots optical axis
147 xlabel('Distance from source [m]'); % x-axis label
148 ylabel('Beam Waist [m]'); % y-axis label
149 title('Beam Waist In A Gaussian Beam Interferometer'); % Figure title
150
151 topstart = 2 * asin(r_p/(2 * r)) / 2; % Calculates reactor plot coordinates
152 topend = (2 * pi - 2 * topstart) / 2; % Calculates reactor plot coordinates
153 bottomstart = topend + topstart * 2; % Calculates reactor plot coordinates
154 bottomend = 2 * pi - topstart; % Calculates reactor plot coordinates
155
156 top = linspace(topstart, ...
157     topend, 100);% Creates 100 datapoints for the top half reactor cutout
158 x_t = d_0 + d_r + r + r * cos(top); % x-parameter of circle

```

```

159 y_t = r * sin(top); % y-parameter of circle
160 bottom = linspace(bottomstart, ...
161     bottomend, 100); % Creates 100 datapoints for the bottom half reactor
162 x_b = d_0 + d_r + r + r * cos(bottom); % x-parameter of circle
163 y_b = r * sin(bottom); % y-parameter of circle
164 p5 = plot(x_t, y_t, 'color', ...
165     [0.911, 0.4100, 0.1700]); % Plot top half reactor
166 plot(x_b, y_b, 'color', ...
167     [0.911, 0.4100, 0.1700]); % Plot bottom half reactor
168 lgd = legend([p1, p2, p4, p5], ...
169     {'Beam Waist +', 'Beam Waist -', 'w_1', 'Reactor'}); % Figure legend
170 legend('boxoff')
171 legend('Location', 'northwest')
172 lgd.NumColumns = 2;
173 hold off
174 %-----%
175
176 %-----Saving figure as Encapsulated Postscript-----%
177 epsfilename = 'Interferometer.eps'; % Savename for the figure
178 foldername = sprintf('../MatlabFigures/Interferometer'); % Folder path
179 fullfilename = fullfile(foldername, epsfilename); % Filename path
180 saveas(1, fullfilename, 'eps') % Save the figure as eps
181 %-----%
182
183 %-----Print Outputs-----%
184 w_1
185 w_2
186 d_1
187 d_2
188 d_3
189 %-----%

```

## Appendix E: PhaseShiftDensity.m

```
1 close all;
2 clear all;
3
4 mkdir('../MatlabFigures', 'PhaseShift'); % Create save directory
5
6 %-----Input Parameters-----%
7 f_0 = 60; % 1st beam frequency [GHz]
8 f_1 = 98; % 2nd beam frequency [GHz]
9 f_2 = 130; % 3rd beam frequency [GHz]
10
11 n_0 = 10 (16); % Lower electron density
12 n_1 = 10 (18); % Higher electron density
13 %-----%
14
15 c = 299792458; % The speed of light [m/s]
16
17 y = linspace(n_0, n_1, 1000); % Datapoints between n_0 and n_1
18 Phi0 = []; % Phase shift array
19 Phi1 = []; % Phase shift array
20 Phi2 = []; % Phase shift array
21
22 for i = 1:1000 % Calculates Phase Shifts for frequency = f_0
23     Phi0 = horzcat(Phi0, (8.416*10-8)*((y(i) / (f_0 * 109))))/0.25);
24 end
25 for i = 1:1000 % Calculates Phase Shifts for frequency = f_1
26     Phi1 = horzcat(Phi1, (8.416*10-8)*((y(i) / (f_1 * 109))))/0.25);
27 end
28 for i = 1:1000 % Calculates Phase Shifts for frequency = f_2
29     Phi2 = horzcat(Phi2, (8.416*10-8)*((y(i) / (f_2 * 109))))/0.25);
30 end
```



```
31
32 l = figure; % Creates figure
33 hold on
34 p0 = plot(Phi0, y); % Plot electron density as function of phase shift
35 p1 = plot(Phi1, y); % Plot electron density as function of phase shift
36 p2 = plot(Phi2, y); % Plot electron density as function of phase shift
37 xlabel('Phase shift [ $\phi_1/\phi_2$ ]'); % x-axis label
38 ylabel('Electron Density [ $m^{-3}$ ]'); % y-axis label
39 %title('Electron Density as function of measured phase shift'); % Figure title
40 lgd = legend([p0, p1, p2], ...
41             {'60GHz', '98GHz', '130GHz'}); % Figure legend
42 legend('boxoff')
43 legend('Location', 'northwest')
44 grid on
45 grid minor
46 hold off
47
48 %-----Saving figure as Encapsulated Postscript-----%
49 epsfilename = 'PhaseShift.eps'; % Savename for the figure
50 foldername = sprintf('../MatlabFigures/PhaseShift'); % Folder path
51 fullfilename = fullfile(foldername, epsfilename); % Filename path
52 saveas(l, fullfilename, 'eps') % Save the figure as eps
53 %-----%
```