

ASSIGNMENT 1 & 2

COURSE 10401

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Assignment 1

TITLE Fusion power plant

GOAL Design a tokamak using a modified version of the model presented in Freidberg's book.

Assignment 2

TITLE Design of interferometer for DTU tokamak

GOAL # 1 To calculate the phase shift induced by O-mode radiation and investigate different microwave sources

GOAL # 2 To design a lens system to accommodate the beam propagation in the small DTU tokamak

FUSION POWER PLANT- FREIDBERG'S MODEL

Freidberg assumes a circular plasma cross section. Output parameters presented in the textbook are calculated given a few input parameters.

INPUT PARAMETERS

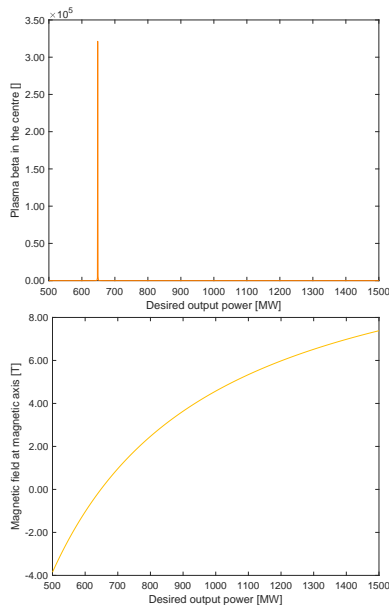
Symbol	Quantity	
n_{flux} fraction	n flux in breeder end/n flux in breeder start	[]
C_F	Fixed cost propotionality constant	[\$]
C_I	Nuclear island cost propotionality constant	[\$ · W/m ³]
P_E	Desired output power	[MW]
P_W	Maximum wall load	[MW/m ²]
B_{max}	Magnetic field at the edge of the coil	[T]
σ_{max}	Tensile strenght of the magnetic field coils	[atm]
η_t	Energy conversion efficiency	[]

OUTPUT PARAMETERS

Symbol	Quantity	Obtained values	
b	Blanket-and-shield thickness	1.2037	m
c	Magnet coil thickness	0.7993	m
a	Minor radius	2.0098	m
R_0	Major radius	4.9583	m
A	Aspect ratio	2.4670	
A_p	Plasma surface	393.4152	m ²
V_p	Plasma volume	395.3509	m ³
P_{dens}	Power density	4.9685e06	W/m
p	Plasma pressure	7.3672e05	Pa
n	Particle density	1.5327e20	1/m ³
B_0	Magnetic field at magnetic axis	4.5744	T
β	Normalised plasma pressure	8.85	%
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.1415	s

MODEL SENSITIVITY

P_E , P_W , B_{\max} and σ_{\max} are changed. Many output parameters were unaffected or changed linearly. But some interesting results were found.

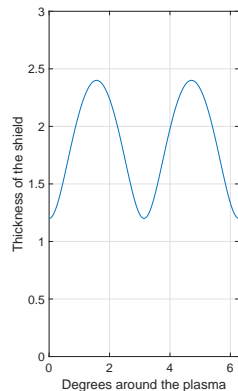
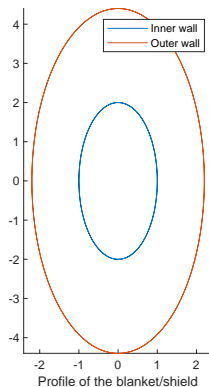


ELLIPTIC PLASMA-CHANGES IN MODEL

Now the plasma is modeled as an ellipse.
The thickness of the blanket is not constant.
Rather the outer and inner ellipses are kept
as true ellipses.

$$V_I = 2 \pi^2 R_0 ((a_{\min} + b + c)^2 - a_{\min}^2) \kappa \quad (1)$$

$$c = \frac{2\xi}{1-\xi} (\kappa a_{\min} + b) \quad (2)$$



ELLIPTICAL MODEL OUTPUT PARAMETERS

Symbol	Quantity	Obtained values
c	Magnet coil thickness	1.30 m \uparrow
A_p	Plasma surface	1210 m $^2\uparrow$
V_p	Plasma volume	793 m $^3\uparrow$
P_{dens}	Power density	2.48×10^6 W/m \downarrow
p	Plasma pressure	5.20×10^5 Pa \downarrow
n	Particle density	1.08×10^{20} /m $^3\downarrow$
β	Normalised plasma pressure	6.17% \downarrow
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.62 s \uparrow

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	1.30 m
a	Minor radius	2.01 m
R_0	Major radius	9.95 m \uparrow
A	Aspect ratio	4.95 \uparrow
A_p	Plasma surface	$2.41 \times 10^3 \text{ m}^2 \uparrow$
V_p	Plasma volume	$1.59 \times 10^3 \text{ m}^3 \uparrow$
P_{dens}	Power density	$2.48 \times 10^6 \text{ W/m}$
p	Plasma pressure	$5.20 \times 10^5 \text{ Pa}$
n	Particle density	$1.08 \times 10^{20} / \text{m}^3$
B_0	Magnetic field at magnetic axis	8.80 T \uparrow
β	Normalised plasma pressure	1.69% \downarrow
τE_{min}	Min confinement time for $(p \times \tau E)_{\text{min}}$	1.62 s

DEMO OUTPUT PARAMETERS WITH $\kappa = 2$ AND $A = 3$

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	1.30 m
a	Minor radius	2.01 m
R_0	Major radius	6.03 m ↓
A	Aspect ratio	3 ↓
A_p	Plasma surface	$1.46 \times 10^3 \text{ m}^2$ ↓
V_p	Plasma volume	962 m^3 ↓
P_{dens}	Power density	$4.01 \times 10^6 \text{ W/m}$ ↑
p	Plasma pressure	$6.68 \times 10^5 \text{ Pa}$ ↑
n	Particle density	$1.39 \times 10^{20} / \text{m}^3$ ↑
B_0	Magnetic field at magnetic axis	6.07 T ↓
β	Normalised plasma pressure	4.55% ↑
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.26 s ↓

We have chosen to optimise the engineering volume and β using κ and A as free parameters. This yields optimum R_0 and κ

$$R_0 = \sqrt{a^3/b} \quad (3)$$

$$\kappa = \frac{6 a_{\min} \xi - b \xi - 6 a_{\min} - b}{2 a_{\min} \xi} \quad (4)$$

DEMO OUTPUT PARAMETERS WITH κ AND A AS FREE PARAMETERS

Symbol	Quantity	Obtained values
b	Blanket-and-shield thickness	1.20 m
c	Magnet coil thickness	1.30 m
a	Minor radius	2.01 m
R_0	Major radius	2.60 m ↓
A	Aspect ratio	1.29 ↓
A_p	Plasma surface	630 m ²
V_p	Plasma volume	414 m ³ ↓
P_{dens}	Power density	9.49×10^6 W/m ↓
$p \downarrow$	Plasma pressure	1.02×10^5 Pa
n	Particle density	2.12×10^{20} /m ³ ↓
B_0	Magnetic field at magnetic axis	-3.09 T ↓
β	Normalised plasma pressure	26.9% ↑
$\tau_{E_{\text{min}}}$	Min confinement time for $(p \times \tau_E)_{\text{min}}$	1.26 s

$$n_e < n_c = \omega^2 \frac{\epsilon_0 \cdot m_{e0}}{e^2} \quad (5)$$

$$\omega^2 \frac{\epsilon_0 \cdot m_{e0}}{e^2} = \omega^2 \frac{8.85 \times 10^{-12} \text{ F/m} \cdot 9.11 \times 10^{-31} \text{ kg}}{1.60 \times 10^{-19} \text{ C}} \quad (6)$$

⇓

$$n_e < 0.000314 \omega^2 \quad (7)$$

At densities of $10^{18}/\text{m}^3$ the minimum frequency is:

$$\frac{\omega}{2\pi} > \frac{\sqrt{\frac{10^{18}/\text{m}^3}{0.000314}}}{2\pi} \quad (8)$$

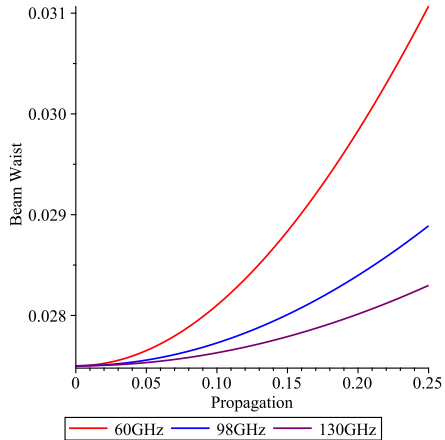
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$$f \approx 9 \text{ GHz} \quad (9)$$

FOUR SOURCES

At a density of $10^{18}/\text{m}^3$ the minimum frequency of the wave is:
 $f \approx 9 \text{ GHz}$

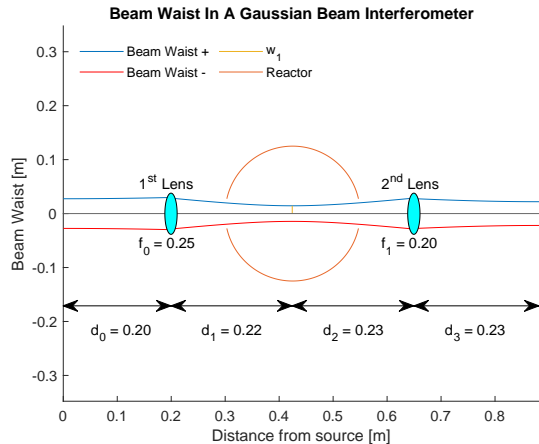
- ▶ 2.45 GHz
- ▶ 60 GHz (Transparent)
- ▶ 98 GHz (Transparent)
- ▶ 130 GHz (Transparent)



Frequency [GHz]	Phase at $[2\pi]$	
	$n_e = 10 \times 10^{16}/\text{m}^3$	Phase at $n_e = 10 \times 10^{18}/\text{m}^3$
2.45	0.8620	86.17
60	0.0352	3.519
98	0.0215	2.154
130	0.0162	1.624

GAUSSIAN TELESCOPE

- ▶ Input parameters:
 $w_0 = 0.0275 \text{ m}$, $f_{req} = 60 \text{ GHz}$,
 $d_0 = 0.200 \text{ m}$, $f_0 = 0.250 \text{ m}$,
 $f_1 = 0.200 \text{ m}$
- ▶ Output distances:
 $d_1 = 0.220 \text{ m}$, $d_2 = 0.230 \text{ m}$,
 $d_3 = 0.230 \text{ m}$
- ▶ Output beamwaists:
 $w_1 = 0.0145 \text{ m}$, $w_2 = 0.0220 \text{ m}$
- ▶ Wavelength: $\lambda = 0.00500 \text{ m}$



QUESTIONS

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