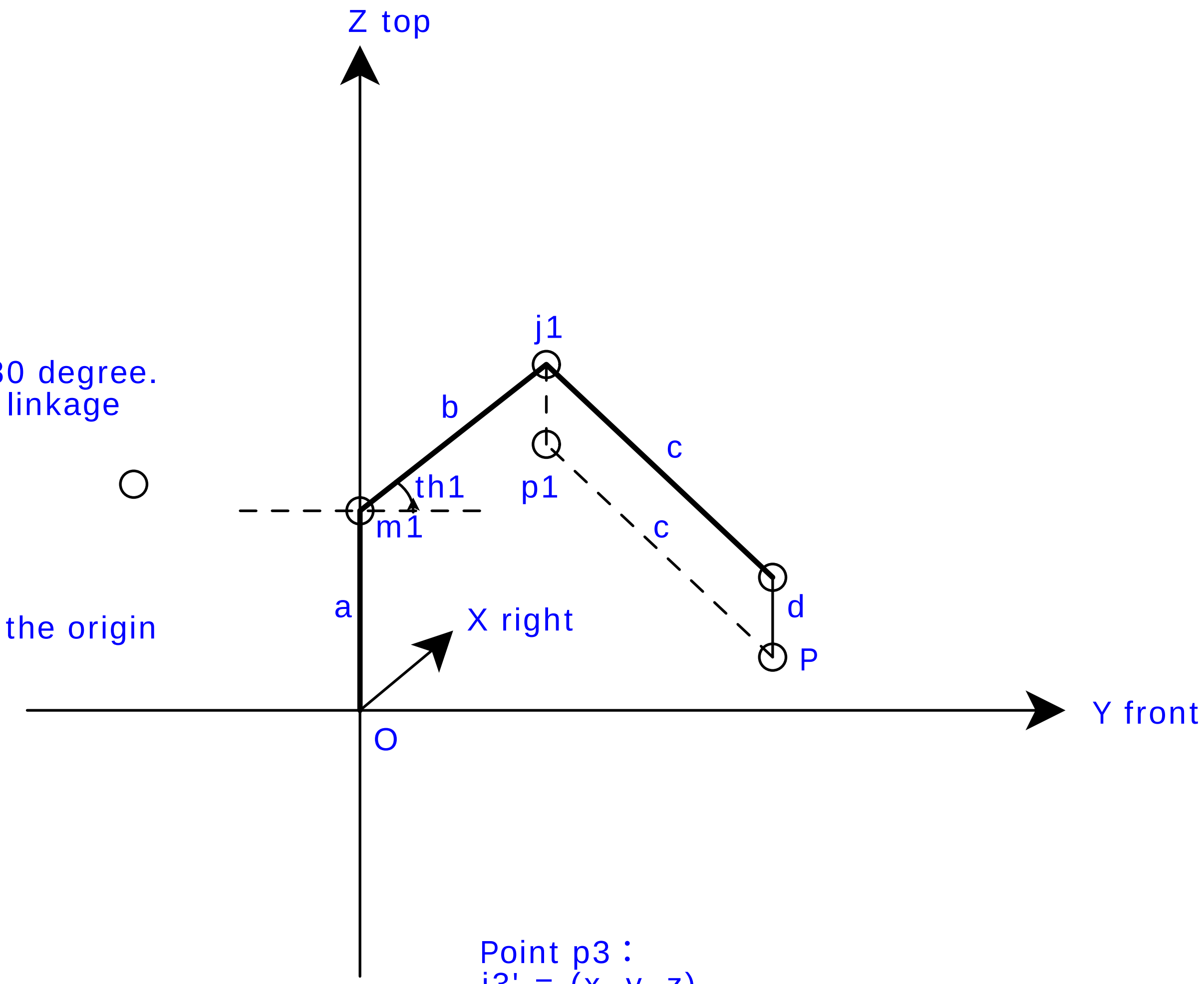


m1: Point of the top arm rotates around
th1: Angle of top arm rotates, from 0 to 180 degree.
j1: Joint center of top arm and the parallel linkage
P: Center of handle

m2, th2, j2 for the left arms
m3, th3, j3 for the right arms

and:
O denotes the center of m1, m2, m3 to be the origin

Point p1 :
j1 = (x, y, z)
where:
x = 0
y = b*cos(th1)
z = b*sin(th1) + a
and j1 translate to p1:
p1 = (x, y, z - d)

Point p2 :
j2' = (x, y, z)
where:
x = 0
y = b*cos(th2)
z = b*sin(th2) + a
j2' rotate 120 degree CCW around y axix
 to point j2 (x',y',z')
where:
x' = -z/sqrt(3)
y' = y
z' = -z/2
and j2 translate to p2:
p2 = (x'+d/sqrt(3), y', z'+d/2)

Point p3 :
j3' = (x, y, z)
where:
x = 0
y = b*cos(th3)
z = b*sin(th3) + a
j3' rotate 120 degree CW around y axix to j3(x',y',z')
where:
x' = z/sqrt(3)
y' = y
z' = -z/2
and j3 translate to p3:
p3 = (x'-d/sqrt(3), y', z'+d/2)

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|---|---|
| TITLE: Sheet_1 | REV: 1.0 |
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| Date: 2021-11-07 Drawn By: aallian | |

C(x, y, z) denotes the center of triangle
p1(x1, y1, z1),p2(x2, y2, z2) and p3(x3, y3, z3).

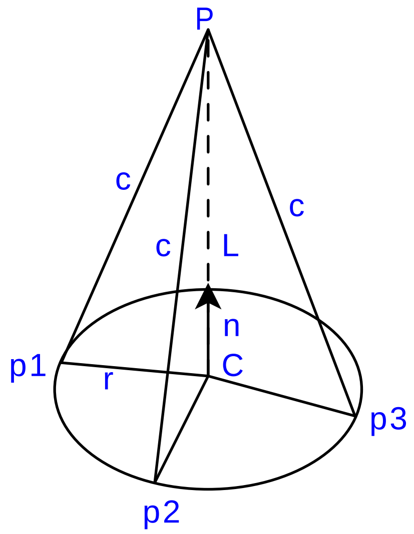
So CP and triangle p1,p2, p3 would be perpendicullar.

Let a1*x + b1*y + c1*z + d1 = 0    ------ (1)
denotes the plane where p1, p2, p3 lie on.
n(a1,b1,c1) dentoes the normal vector of the plane.

vector p1p2 = (x2-x1, y2-y1, z2-z1) = (ax, ay, az)
vector p1p3 = (x3-x1, y3-y1, z3-z1) = (bx, by, bz)

so n equals to the cross product of p1p2 and p1p3

n = p1p2 x p1p3 = (a1, b1, c1)
where:
a1 = ay*bz - az*by
b1 = az*bx - ax*bz
c1 = ax*by - ay*bx
from (1), we have:
d1 = -(a1*x + b1*y + c1*z) = -(a1*x1 + b1*y1 + c1*z1)  ---- (2)




The distance of point C to p1, p2, p3 are the same, so we have
r^2 = (x - x1)^2 + (y - y1)^2 + (z - z1)^2    ---- (3)
r^2 = (x - x2)^2 + (y - y2)^2 + (z - z2)^2    ---- (4)
r^2 = (x - x3)^2 + (y - y3)^2 + (z - z3)^2    ---- (5)

from (3) and (4), we have:
2(x2-x1)x + 2(y2-y1)y + 2(z2-z1) + x1^2 + y1^2 + z1^2 - x2^2 - y2^2 - z2^2 = 0 ----- (6)
let:
a2 = 2(x2-x1)
b2 = 2(y2-y1)
c2 = 2(z2-z1)
d2 = x1^2 + y1^2 + z1^2 - x2^2 - y2^2 - z2^2
(6) can be:
a2*x + b2*y + c2*z + d2 = 0    ------ (7)

similarlly from (3) and (5), we have:
a3*x + b3*y + c3*z + d3 = 0    ------ (8)
where:
a3 = 2(x3-x1)
b3 = 2(y3-y1)
c3 = 2(z3-z1)
d3 = x1^2 + y1^2 + z1^2 - x3^2 - y3^2 - z3^2

With (1), (7) and (8) we can solve point C(x,y,z)
where:
x = (b1*c2*d3 - b1*c3*d2 - b2*c1*d3 + b2*c3*d1 + b3*c1*d2 - b3*c2*d1)/d
y = (-a1*c2*d3 + a1*c3*d2 + a2*c1*d3 - a2*c3*d1 - a3*c1*d2 + a3*c2*d1)/d
z = (a1*b2*d3 - a1*b3*d2 - a2*b1*d3 + a2*b3*d1 + a3*b1*d2 - a3*b2*d1)/d
where:
d = (a1*b2*c3 - a1*b3*c2 - a2*b1*c3 + a2*b3*c1 + a3*b1*c2 - a3*b2*c1)

Python script to solve x0, y0, z0:

```
from sympy import *
a1, b1, c1, d1 = symbols('a1 b1 c1 d1')
a2, b2, c2, d2 = symbols('a2 b2 c2 d2')
a3, b3, c3, d3 = symbols('a3 b3 c3 d3')
x, y, z = symbols('x y z')
eq1 = Eq(d1,  a1 * x + b1 * y + c1 * z)
eq2 = Eq(d2,  a2 * x + b2 * y + c2 * z)
eq3 = Eq(d3,  a3 * x + b3 * y + c3 * z)
sol = solve([eq1, eq2, eq3], (x, y, z))
```

Solve point P:
Substitue x, y, z with C(x,y,z) in equation (3), we have:
r^2 = (x - x1)^2 + (y - y1)^2 + (z - z1)^2

CP and Cp1 are perpendicullar, so:
L = sqrt(c^2 - r^2)
Length of normal vector n: Ln = sqrt(a1^2 + b1^2 + c1^2)

Vector CP equals to normal vector with a scalling factor S = (+/-)L/Ln

And P(xp, yp, zp) can be solved, where:
xp = x + a1 * S
yp = y + b1 * S
zp = z + c1 * S
there are two solutions for P depending to the sign of S

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|---|---|---|
| TITLE: Sheet_2 | | REV: 1.0 |
| EasyEDA | Company: Your Company | Sheet: 1/1 |
| | Date: 2022-08-16    Drawn By: aallian | |