Mathematical Foundations

MFE Spring 2021

Assignment 2

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Solved in team: No

1. Stochastic Matrices

Given the below transition matrix

In [1]:
```
import numpy as np
import pandas as pd
np.set_printoptions(linewidth=100)
np.set_printoptions(suppress=True, threshold=np.inf)
pd.set_option('display.float_format', lambda x: '%.8f' % x)
l = [[0.90788, 0.08291, 0.00716, 0.00102, 0.00102, 0, 0, 0],
     [0.00103, 0.91219, 0.07851, 0.00620, 0.00103, 0.00103, 0, 0],
     [0.00924, 0.02361, 0.90041, 0.05441, 0.00719, 0.00308, 0.00103, 0.00103],
     [0, 0.00318, 0.05938, 0.86947, 0.05302, 0.011166, 0.00117, 0.00212],
     [0, 0.00110, 0.00659, 0.07692, 0.80549, 0.08791, 0.00989, 0.01209],
     [0, 0.00114, 0.00227, 0.00454, 0.06470, 0.82747, 0.04086, 0.05902],
     [0, 0, 0.00456, 0.01251, 0.02275, 0.12856, 0.60637, 0.22526],
     [0, 0, 0, 0, 0, 0, 0, 1.0]]
phi = np.array(l)
ix = ['AAA', 'AA', 'A', 'BBB', 'BB', 'B', 'CCC', 'Default']
df = pd.DataFrame(data = phi, columns=ix, index=ix)
df
```

Out[1]:

	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	0.90788000	0.08291000	0.00716000	0.00102000	0.00102000	0.00000000	0.00000000	0.00000000
AA	0.00103000	0.91219000	0.07851000	0.00620000	0.00103000	0.00103000	0.00000000	0.00000000
A	0.00924000	0.02361000	0.90041000	0.05441000	0.00719000	0.00308000	0.00103000	0.00103000
BBB	0.00000000	0.00318000	0.05938000	0.86947000	0.05302000	0.01116600	0.00117000	0.00212000
BB	0.00000000	0.00110000	0.00659000	0.07692000	0.80549000	0.08791000	0.00989000	0.01209000
B	0.00000000	0.00114000	0.00227000	0.00454000	0.06470000	0.82747000	0.04086000	0.05902000
CCC	0.00000000	0.00000000	0.00456000	0.01251000	0.02275000	0.12856000	0.60637000	0.22526000
Default	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000

(a) What is the probability that a bond over two years moves from an AA rating to Default?

In [2]:
```
phi_sq = phi.dot(phi)
print(phi_sq[0,7])
```

```
2.1869e-05
```

$$\Phi^2 = \begin{bmatrix} 0.82439765 & 0.15107542 & 0.01952391 & 0.00279497 & 0.0019386 & 0.00020851 & 0.00001866 & 0.00002187 \\ 0.0026001 & 0.83405164 & 0.14269188 & 0.01540298 & 0.00273011 & 0.00219344 & 0.00014039 & 0.00016725 \\ 0.01673292 & 0.04374602 & 0.81594788 & 0.0970349 & 0.01540669 & 0.00671822 & 0.0018126 & 0.0025735 \\ 0.00055195 & 0.00713869 & 0.10572522 & 0.76337229 & 0.08998565 & 0.0239456 & 0.0027685 & 0.00558802 \\ 0.00006202 & 0.00238986 & 0.01614041 & 0.12972614 & 0.65885373 & 0.14570529 & 0.01765208 & 0.0294145 \\ 0.00002215 & 0.00212241 & 0.00489407 & 0.01332257 & 0.10684028 & 0.6957062 & 0.05923424 & 0.11785559 \\ 0.00004213 & 0.00031903 & 0.00805551 & 0.02104446 & 0.04113371 & 0.18648815 & 0.37318187 & 0.36974478 \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. \end{bmatrix}$$

$$\Phi^2[0, 7] = 0.002187\%$$

(b) What is the long-term (stationary) distribution, $\mathbf{p}$, of the ratings for a corporate bond with the above transition matrix (assuming that it has a very long maturity state)?

In [3]:

```python
new_phi = phi
converged = False
for i in range(100000):
    old_phi = new_phi
    new_phi = old_phi.dot(phi)
    if np.allclose(new_phi, old_phi, atol=1e-10):
        print(f'converged after {i} iterations!')
        print(pd.DataFrame(new_phi, columns=ix, index=ix))
        converged = True
        break

if not converged:
    print('failed to converge')
```

```
converged after 1460 iterations!
               AAA         AA          A        BBB         BB          B  \
AAA     0.00000000 0.00000000 0.00000001 0.00000001 0.00000000 0.00000000
AA      0.00000000 0.00000000 0.00000001 0.00000001 0.00000000 0.00000000
A       0.00000000 0.00000000 0.00000001 0.00000000 0.00000000 0.00000000
BBB     0.00000000 0.00000000 0.00000001 0.00000000 0.00000000 0.00000000
BB      0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
B       0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
CCC     0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
Default 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000

               CCC    Default
AAA     0.00000000 0.98875433
AA      0.00000000 0.98882510
A       0.00000000 0.98889601
BBB     0.00000000 0.98763573
BB      0.00000000 0.99281930
B       0.00000000 0.99625655
CCC     0.00000000 0.99786619
Default 0.00000000 1.00000000
```

Multiplying Φ by itself 10000 times is converging to:

$$\Phi^{1000} \approx \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Implying that over the very long-term, according to this markov process, the probability of default is $\approx$ 100% no matter the starting state.

$$\mathbf{p} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Portfolio Optimization

$$u_0 = \mathbf{h}^T \mu \tag{1}$$

$$\mathbf{h}^T \mathbf{1} = 1.00 \tag{2}$$

$$U(\mathbf{h}) = \mathbf{h}^T \mu - \frac{\gamma}{2} \mathbf{h}^T \Sigma \mathbf{h} \tag{3}$$

$$A = \mathbf{1}^T \Sigma^{-1} \mathbf{1} \qquad B = \mathbf{1}^T \Sigma^{-1} \mu \qquad C = \mu^T \Sigma^{-1} \mu \qquad \Delta = AC - B^2 > 0 \qquad \mathbf{w_A} = \frac{1}{A} \Sigma^{-1} \mathbf{1} \qquad \mathbf{w_B} = \frac{1}{B} \Sigma^{-1} \mu$$

(a) Derive a formula for the optimal portfolio, $\mathbf{h}$, satisfying (1) and (2) and maximizing (3). Express the formula as a function of A, B, C, Δ, μ_0, w_A and w_B

$$L(\mathbf{h}, \lambda_1, \lambda_2) = \mathbf{h}^T \mu - \frac{\gamma}{2} \mathbf{h}^T \Sigma \mathbf{h} + \lambda_1(\mathbf{h}^T \mathbf{1} - 1) + \lambda_2(\mathbf{h}^T \mu - \mu_0)$$

$$\frac{\partial L}{\partial \mathbf{h}} = \mu - \gamma \Sigma \mathbf{h} + \lambda_1 \mathbf{1} + \lambda_2 \mu$$

$$\mu - \gamma \Sigma \mathbf{h} + \lambda_1 \mathbf{1} + \lambda_2 \mu = \mathbf{0}$$

$$-\gamma \Sigma \mathbf{h} = -\mu - \lambda_1 \mathbf{1} - \lambda_2 \mu$$

$$\Sigma \mathbf{h} = \frac{1}{\gamma} \mu + \frac{\lambda_1}{\gamma} \mathbf{1} + \frac{\lambda_2}{\gamma} \mu$$

$$\Sigma^{-1}\Sigma\mathbf{h} = \frac{1}{\gamma}\Sigma^{-1}\mu + \frac{\lambda_1}{\gamma}\Sigma^{-1}\mathbf{1} + \frac{\lambda_2}{\gamma}\Sigma^{-1}\mu$$

$$\mathbf{h} = \frac{1}{\gamma}\Sigma^{-1}\mu + \frac{\lambda_1}{\gamma}\Sigma^{-1}\mathbf{1} + \frac{\lambda_2}{\gamma}\Sigma^{-1}\mu \tag{4}$$

Will solve the two below equations for λ_1 and λ_2

$$\mathbf{1}^T\mathbf{h} = \frac{1}{\gamma}\mathbf{1}^T\Sigma^{-1}\mu + \frac{\lambda_1}{\gamma}\mathbf{1}^T\Sigma^{-1}\mathbf{1} + \frac{\lambda_2}{\gamma}\mathbf{1}^T\Sigma^{-1}\mu = 1 \qquad \mu^T\mathbf{h} = \frac{1}{\gamma}\mu^T\Sigma^{-1}\mu + \frac{\lambda_1}{\gamma}\mu^T\Sigma^{-1}\mathbf{1} + \frac{\lambda_2}{\gamma}\mu^T\Sigma^{-1}\mu = \mu_0$$

$$\frac{1}{\gamma}\mathbf{1}^T\Sigma^{-1}\mu + \frac{\lambda_1}{\gamma}\mathbf{1}^T\Sigma^{-1}\mathbf{1} + \frac{\lambda_2}{\gamma}\mathbf{1}^T\Sigma^{-1}\mu = 1$$

$$B + \lambda_1 A + \lambda_2 B = \gamma$$

$$\lambda_1 A = \gamma - B - \lambda_2 B$$

$$\lambda_1 = \frac{\gamma - B - B\lambda_2}{A} \tag{5}$$

Plugging this result into the second equation above

$$\frac{1}{\gamma}\mu^T\Sigma^{-1}\mu + \frac{\lambda_1}{\gamma}\mu^T\Sigma^{-1}\mathbf{1} + \frac{\lambda_2}{\gamma}\mu^T\Sigma^{-1}\mu = \mu_0$$

$$\frac{1}{\gamma}C + \left(\frac{\gamma - B - B\lambda_2}{A}\right)\left(\frac{\mu^T\Sigma^{-1}\mathbf{1}}{\gamma}\right) + \frac{\lambda_2}{\gamma}C = \mu_0$$

Note: since Σ is symmetric, Σ^{-1} is also symmetric $\implies \mu^T\Sigma^{-1}\mathbf{1} = \mathbf{1}^T\Sigma^{-1}\mu = B$

$$\frac{1}{\gamma}C + \left(\frac{\gamma - B - B\lambda_2}{A}\right)\left(\frac{B}{\gamma}\right) + \frac{\lambda_2}{\gamma}C = \mu_0$$

$$AC + B\gamma - B^2 - \lambda_2 B^2 + \lambda_2 AC = \mu_0 A\gamma$$

$$\lambda_2(AC - B^2) = \mu_0 A\gamma - AC - B\gamma + B^2$$

$$\lambda_2 = \frac{\mu_0 A\gamma - AC - B\gamma + B^2}{\Delta}$$

Plugging λ_2 into (5) above.

$$\lambda_1 = \frac{1}{A}(\gamma - B - B\lambda_2) \tag{5}$$

$$\lambda_1 = \frac{1}{A}\left[\gamma - B - B\left(\frac{\mu_0 A\gamma - AC - B\gamma + B^2}{\Delta}\right)\right]$$

$$A\Delta\lambda_1 = \Delta\gamma - \Delta B - B(\mu_0 A\gamma - AC - B\gamma + B^2)$$

$$A\Delta\lambda_1 = \Delta\gamma - \Delta B - \mu_0\gamma AB + ABC + \gamma B^2 - B^3$$

$$\lambda_1 = \frac{\Delta\gamma - \Delta B - \mu_0\gamma AB + ABC + \gamma B^2 - B^3}{A\Delta}$$

$$\lambda_1 = \frac{\Delta\gamma - \gamma B^2 + \mu_0\gamma AB}{A\Delta}$$

Now plugging λ_1 and λ_2 back into (4)

$$\mathbf{h} = \frac{1}{\gamma}\Sigma^{-1}\mu + \frac{\lambda_1}{\gamma}\Sigma^{-1}\mathbf{1} + \frac{\lambda_2}{\gamma}\Sigma^{-1}\mu \tag{4}$$

$$\gamma\mathbf{h} = \Sigma^{-1}\mu + \lambda_1\Sigma^{-1}\mathbf{1} + \lambda_2\Sigma^{-1}\mu$$

$$\gamma\mathbf{h} = \Sigma^{-1}\mu + \frac{\Delta\gamma - \Delta B - \mu_0\gamma AB + ABC + \gamma B^2 - B^3}{\Delta}\mathbf{w}_A + \frac{\mu_0 A\gamma - AC - B\gamma + B^2}{\Delta}\Sigma^{-1}\mu$$

$$\Delta\gamma\mathbf{h} = \Delta\Sigma^{-1}\mu + (\Delta\gamma - \Delta B - \mu_0\gamma AB + ABC + \gamma B^2 - B^3)\mathbf{w}_A + (\mu_0 A\gamma - AC - B\gamma + B^2)\Sigma^{-1}\mu$$

$$\Delta\gamma\mathbf{h} = \Delta\Sigma^{-1}\mu + ((AC - B^2)\gamma - B(AC - B^2) - \mu_0\gamma AB + ABC + \gamma B^2 - B^3)\mathbf{w}_A + (\mu_0 A\gamma - AC - B\gamma + B^2)\Sigma^{-1}\mu$$

$$\Delta\gamma\mathbf{h} = \Delta\Sigma^{-1}\mu + (\gamma AC - \mu_0\gamma AB)\mathbf{w}_A + (\mu_0 A\gamma - \Delta - B\gamma)\Sigma^{-1}\mu$$

$$\frac{\Delta\gamma}{B}\mathbf{h} = \Delta\mathbf{w}_B + \frac{\gamma\mathbf{w}_A}{B}(AC - \mu_0 AB) + \mathbf{w}_B(-\Delta - \gamma B + \mu_0 A\gamma)$$

$$\frac{\Delta\gamma}{B}\mathbf{h} = \frac{\gamma\mathbf{w}_A}{B}(AC - \mu_0 AB) + \gamma\mathbf{w}_B(\mu_0 A - B)$$

$$\mathbf{h} = \mathbf{w}_A\frac{(AC - \mu_0 AB)}{\Delta} + \mathbf{w}_B\frac{(\mu_0 AB - B^2)}{\Delta}$$

(b) Use your results from (a) to write σ_h^2 as a quadratic function of μ_0 using only the parameters A, B, C and Δ. This function is known as the minimum variance frontier.</center>

$$\sigma_h^2 = \mathbf{h}^T\Sigma\mathbf{h}$$

$$\sigma_h^2 = \left[\mathbf{w}_A\frac{(AC - \mu_0 AB)}{\Delta} + \mathbf{w}_B\frac{(\mu_0 AB - B^2)}{\Delta}\right]^T\Sigma\left[\mathbf{w}_A\frac{(AC - \mu_0 AB)}{\Delta} + \mathbf{w}_B\frac{(\mu_0 AB - B^2)}{\Delta}\right]$$

$$\sigma_h^2 = \left[\mathbf{w}_A \frac{(AC - \mu_0 AB)}{\Delta} + \mathbf{w}_B \frac{(\mu_0 AB - B^2)}{\Delta}\right]^T \Sigma \left[\Sigma^{-1}\mathbf{1}\frac{(AC - \mu_0 AB)}{A\Delta} + \Sigma^{-1}\mathbf{1}\frac{(\mu_0 AB - B^2)}{B\Delta}\right]$$

$$\sigma_h^2 = \left[\mathbf{w}_A \frac{(AC - \mu_0 AB)}{\Delta} + \mathbf{w}_B \frac{(\mu_0 AB - B^2)}{\Delta}\right]^T \left[\mathbf{1}\frac{(AC - \mu_0 AB)}{A\Delta} + \vec{\mu}\frac{(\mu_0 AB - B^2)}{B\Delta}\right]$$

$$\sigma_h^2 = \left[\left[\Sigma^{-1}\mathbf{1}\frac{(AC - \mu_0 AB)}{A\Delta}\right]^T + \left[\Sigma^{-1}\vec{\mu}\frac{(\mu_0 AB - B^2)}{B\Delta}\right]^T\right] \left[\mathbf{1}\frac{(AC - \mu_0 AB)}{A\Delta} + \vec{\mu}\frac{(\mu_0 AB - B^2)}{B\Delta}\right]$$

$$\sigma_h^2 = \left[\left[\mathbf{1}^T \Sigma^{-1^T}\frac{(AC - \mu_0 AB)}{A\Delta}\right] + \left[\vec{\mu}^T \Sigma^{-1^T}\frac{(\mu_0 AB - B^2)}{B\Delta}\right]\right] \left[\mathbf{1}\frac{(AC - \mu_0 AB)}{A\Delta} + \vec{\mu}\frac{(\mu_0 AB - B^2)}{B\Delta}\right]$$

$$\sigma_h^2 = \mathbf{1}^T \Sigma^{-1^T}\mathbf{1}\frac{(AC - \mu_0 AB)^2}{(A\Delta)^2} + \mathbf{1}^T \Sigma^{-1^T}\vec{\mu}\frac{(AC - \mu_0 AB)(\mu_0 AB - B^2)}{AB\Delta^2} + \vec{\mu}^T \Sigma^{-1^T}\mathbf{1}\frac{(\mu_0 AB - B^2)(AC - \mu_0 AB)}{AB\Delta^2} + \vec{\mu}^T \Sigma^{-1^T}\vec{\mu}\frac{(\mu_0 AB - B^2)^2}{(B\Delta)^2}$$

$$\sigma_h^2 = A\frac{(AC - \mu_0 AB)^2}{(A\Delta)^2} + B\frac{(AC - \mu_0 AB)(\mu_0 AB - B^2)}{AB\Delta^2} + B\frac{(\mu_0 AB - B^2)(AC - \mu_0 AB)}{AB\Delta^2} + C\frac{(\mu_0 AB - B^2)^2}{(B\Delta)^2}$$

$$\sigma_h^2 = \frac{(AC - \mu_0 AB)^2}{A\Delta^2} + 2\frac{(AC - \mu_0 AB)(\mu_0 AB - B^2)}{A\Delta^2} + C\frac{(\mu_0 AB - B^2)^2}{(B\Delta)^2}$$

$$\sigma_h^2 = \frac{A^2C^2 - 2A^2BC\mu_0 + \mu_0^2 A^2 B^2}{A\Delta^2} + \frac{2A^2BC\mu_0 - 2\mu_0^2 A^2 B^2 - 2ACB^2 + 2AB^3\mu_0}{A\Delta^2} + \frac{A^2B^2C\mu_0^2 - 2AB^3C\mu_0 + B^4C}{(B\Delta)^2}$$

$$\sigma_h^2 = \frac{AC^2 - 2ABC\mu_0 + \mu_0^2 AB^2}{\Delta^2} + \frac{2ABC\mu_0 - 2\mu_0^2 AB^2 - 2CB^2 + 2B^3\mu_0}{\Delta^2} + \frac{A^2C\mu_0^2 - 2ABC\mu_0 + B^2C}{\Delta^2}$$

$$\sigma_h^2 = \frac{(A^2C - AB^2)}{\Delta^2}\mu_0^2 + \frac{2B^3 - 2ABC}{\Delta^2}\mu_0 + \frac{AC^2 - B^2C}{\Delta^2}$$

$$\sigma_h^2 = A\frac{(AC - B^2)}{\Delta^2}\mu_0^2 + 2B\frac{B^2 - AC}{\Delta^2}\mu_0 + C\frac{AC - B^2}{\Delta^2}$$

$$\sigma_h^2 = \frac{A}{\Delta}\mu_0^2 - \frac{2B}{\Delta}\mu_0 + \frac{C}{\Delta}$$

(c) Now assume the investor actually chooses μ_0 to maximize (6), given risk aversion coefficient γ. Use your previous results to write the optimal μ_0 as a function of the previous parameters and γ.

$$U(\mathbf{h}) = \mu_0 - \frac{\gamma}{2}\sigma_{\mathbf{h}}^2$$

$$U(\mathbf{h}) = \mu_0 - \frac{\gamma}{2}\left(\frac{A}{\Delta}\mu_0^2 - \frac{2B}{\Delta}\mu_0 + \frac{C}{\Delta}\right)$$

$$\frac{\partial U}{\partial \mu_0} = 1 - \frac{\gamma}{2}\left(\frac{2A}{\Delta}\mu_0 - \frac{2B}{\Delta}\right) = 0$$

$$\frac{\gamma}{2}\left(\frac{2A}{\Delta}\mu_0 - \frac{2B}{\Delta}\right) = 1$$

$$\frac{\gamma A}{\Delta}\mu_0 - \frac{\gamma B}{\Delta} = 1$$

$$\frac{\gamma A}{\Delta}\mu_0 = 1 + \frac{\gamma B}{\Delta}$$

$$\mu_0 = \frac{\Delta}{\gamma A} + \frac{\gamma B}{\gamma A} = \frac{\Delta + \gamma B}{\gamma A}$$

3. Portfolio Optimization with a Risk-Free Asset

> **(a) Show that there is a so-called one-fund separation in the market in this case, in that all investors, regardless of their γ, will hold the same stock market portfolio:**

$$\mathbf{w} = \frac{1}{B - AR}\Sigma^{-1}(\mu - R\mathbf{1})$$

$$U(\mathbf{h}) = R + \mathbf{h}^T(\mu - R\mathbf{1}) - \frac{\gamma}{2}\mathbf{h}^T\Sigma\mathbf{h}$$

$$\frac{\partial U}{\partial h} = \mu - R\mathbf{1} - \gamma\Sigma\mathbf{h} = 0$$

$$\gamma\Sigma\mathbf{h} = \mu - R\mathbf{1}$$

$$\mathbf{h} = \frac{1}{\gamma}\Sigma^{-1}(\mu - R\mathbf{1})$$

Let $w =$ the stock market portfolio.

$$w = \frac{\mathbf{h}}{\mathbf{1}^T\mathbf{h}} = \frac{\Sigma^{-1}(\mu - R\mathbf{1})}{\mathbf{1}^T\Sigma^{-1}(\mu - R\mathbf{1})} = \frac{\Sigma^{-1}(\mu - R\mathbf{1})}{\mathbf{1}^T\Sigma^{-1}\mu - R\mathbf{1}^T\Sigma^{-1}\mathbf{1}} = \frac{\Sigma^{-1}(\mu - R\mathbf{1})}{B - AR}$$

> **(b) What is the relationship between an investor's γ and the expected return of the chosen portfolio (including the bond) in this case?**

$$\lim_{\gamma\to\infty}\mathbf{h} = \lim_{\gamma\to\infty}\frac{1}{\gamma}\Sigma^{-1}(\mu - R\mathbf{1}) = 0$$

$$\lim_{\mathbf{h}\to 0} R + \mathbf{h}^T(\mu - R\mathbf{1}) = R$$

As gamma gets larger the weights on the stock portfolio get smaller driving the expected return to R

Conversley if γ goes to zero the weights on the stock portfolio will get larger and the expected return will be driven by the expected returns of the stocks in the portfolio thus giving the portfolio greater and greater risk.

4. The Capital Asset Pricing Model (CAPM):

> **(a) Using the below, show that CAPM relationship holds.**

$$\mu_n - R = \beta_n(\mu_{market} - R) \qquad \beta_n = \frac{Cov(\tilde{\mu}_n, \tilde{\mu}_{market})}{\sigma^2_{market}} \qquad \beta = \frac{1}{\mathbf{w}^T\Sigma\mathbf{w}}\Sigma\mathbf{w}$$

$$w = \frac{1}{B - AR}\Sigma^{-1}(\mu - R\mathbf{1}) \tag{11}$$

$$(B - AR)\Sigma w = \Sigma\Sigma^{-1}(\mu - R\mathbf{1})$$

$$(B - AR)\Sigma w = \mu - R\mathbf{1} \tag{12}$$

$$(B - AR)\beta\mathbf{w}^T\Sigma w = \beta\mathbf{w}^T(\mu - R\mathbf{1})$$

$$(B - AR)\frac{\Sigma\mathbf{w}}{\mathbf{w}^T\Sigma\mathbf{w}}\mathbf{w}^T\Sigma w = \beta\mathbf{w}^T(\mu - R\mathbf{1})$$

$$(B - AR)\Sigma\mathbf{w} = \beta\mathbf{w}^T(\mu - R\mathbf{1})$$

$$\mu - R\mathbf{1} = \beta\mathbf{w}^T(\mu - R\mathbf{1}) \because (12)$$

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In [ ]:
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