[Institution]

**A Defense of One-World Quantum Physics**

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## Contents

## 0.1 A description of Kent's Interpretation of Quantum Physics

In this section I will provide an account of Kent's interpretation of quantum physics focusing on the ideas Kent presents in his 2014 paper.[1] This section is primarily descriptive. We'll wait until the next section to consider how Kent's interpretation addresses the issues Butterfield raises.

Kent's interpretation of quantum physics has some similarities in common with the pilot wave interpretation. Firstly, there is no quantum state collapse in Kent's interpretation. Secondly, some additional values beyond standard quantum theory (i.e. in addition to the quantum state) are included in Kent's interpretation. And thirdly, Kent's interpretation is a one-world interpretation of quantum physics. I'll consider these three features of Kent's interpretation in some detail as I describe his theory. I'll then present an account of his toy model that provides a simple example of how the ideas of his theory fit together.

### 0.1.1 The No-collapse Feature of Kent's Interpretation

We first consider the no-collapse feature of Kent's interpretation. This is a feature that belongs both to the many worlds interpretation and to the pilot wave interpretation. In all three interpretations, the quantum state deterministically evolves according to the Schrödinger equation. The Schrödinger equation itself describes how a quantum state evolves over time when there are no outside influences. The precise formula for the Schrödinger equation need not concern us here, but all we need to know is that the Schrödinger equation determines a so-called **unitary operator** $U(t', t)$.

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[1] Adrian Kent, "Lorentzian Quantum Reality: Postulates and Toy Models," 2014, https://doi.org/10.1098/rsta.2014.0241, eprint: arXiv:1411.2957.

What this means is that if a system is in a state $|\psi\rangle$ at time $t$, then it will be in the state $|\psi'\rangle = U(t',t)\,|\psi\rangle$ at time $t'$. A unitary operator $U$ has the property that if $|\psi'\rangle = U\,|\psi\rangle$ and $|\chi'\rangle = U\,|\chi\rangle$, then

{unitarycond}
$$\langle\chi'|\psi'\rangle = \langle\chi|\psi\rangle\,.^2 \tag{1}$$

Under the Copenhagen interpretation, a system will evolve unitarily for the most part, but there will typically be a non-unitary change in the state describing the system whenever there is a measurement.[3] However, in non-collapse models such as the pilot wave interpretation, the many-worlds interpretation, and Kent's interpretation, the quantum state always evolves unitarily.

[2]A unitary operator $U$ must also be linear so that for any two states $|\psi\rangle$ and $|\phi\rangle$ and complex numbers $\alpha$ and $\beta$, we have
$$U(\alpha\,|\psi\rangle + \beta\,|\phi\rangle) = \alpha U\,|\psi\rangle + \beta U\,|\phi\rangle\,,$$
and furthermore, a unitary operator must have the property that it is invertible: there is a linear operator $U^{-1}$ such that $UU^{-1}$ and $U^{-1}U$ are the identity operator $I$, i.e. $U^{-1}U\,|\psi\rangle = UU^{-1}\,|\psi\rangle = |\psi\rangle$ for any state $|\psi\rangle$.

[3]Note that to say that the change in a state is non-unitary when a measurement is made is not to say that there is a non-unitary collapse operator that maps the quantum state to an eigenstate of some observable. Such a mapping would not make sense, since the collapse is not deterministic given the initial state. However, one could have a well-defined mapping from a time value $t$ to the quantum state of the system $|\psi(t)\rangle$ at time $t$. We then say that a system changes unitarily if and only if there is a unitary operator $U(t_1,t_0)$ for any two times $t_0$ and $t_1$ such that whenever the state of the system at time $t_0$ is given by $|\psi(t_0)\rangle$, then the state of the system at time $t_1$ must be given by $|\psi(t_1)\rangle = U(t_1,t_0)\,|\psi(t_0)\rangle$, and that for an intermediate time $t$, $U(t_1,t_0) = U(t_1,t)U(t,t_0)$. So to say that the change in a state is non-unitary when a measurement is made is to say that the state $|\psi(t)\rangle$ describing the system does not change unitarily in the process of making a measurement. Now to see why this is the case under the Copenhagen interpretation, we suppose that at time $t_0$ a system is in the state $|\psi(t_0)\rangle$ and that as long as no measurements are made up until a time $t \geq t_0$, the state evolves to a state $|\psi^{(U)}(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle$ where $U(t,t_0)$ is a unitary operator determined by Schrödinger's equation. Furthermore, we suppose that there is a measurable quantity with which we associate an observable $\hat{O}$ so that whenever the state of the system is an eigenstate of $\hat{O}$, the value of the measurable quantity for the system will be a determinate value and equal to the corresponding eigenvalue of $\hat{O}$. At time $t_0$, we can express $|\psi(t_0)\rangle$ as a linear combination
$$|\psi(t_0)\rangle = \sum_i c_i\,|s_i(t_0)\rangle$$
where the $|s_i(t_0)\rangle$ are eigenstates of $\hat{O}$ with distinct eigenvalues. As long as no measurement is made, this will evolve as
$$|\psi^{(U)}(t)\rangle = \sum_i c_i U(t,t_0)\,|s_i(t_0)\rangle\,.$$
We assume that as the state $|s_i(t_0)\rangle$ evolves to the state $|s_i(t_1)\rangle$ from time $t_0$ to $t_1$, it remains an eigenstates of $\hat{O}$ with approximately the same eigenvalue. This assumption is based on the principle

### 0.1.2 The Additional Values of Kent's Interpretation

additional

Secondly, like the pilot wave interpretation, some additional values beyond standard quantum theory (i.e. in addition to the quantum state[4]) are included in Kent's interpretation. In the pilot wave interpretation, these additional values are the positions and momenta of all the particles, whereas in Kent's interpretation, the additional values specify the mass-energy density on a three-dimensional distant future hypersurface in spacetime. We refer to this hypersurface as $S$.

To describe the nature of this three-dimensional hyperspace $S$, we will need some terminology and notation used in special relativity. A **spacetime location** is a point $(x^1, x^2, x^3)$ in three-dimensional space at a particular instant of time $t$, and hence described by four numbers $(x^0, x^1, x^2, x^3)$ where $x^0 = ct$ and where $c$ is the speed of light.[5] We will use the convention of boldface type to depict spatial locations,

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that in practice, performing a measurement is not instantaneous, but rather must take place over a time interval, and so the eigenstate and eigenvalue must be stable enough over this time interval so as to specify a definite outcome. We also assume that when the system is already in an eigenstate $|s_i(t_0)\rangle$ of the observable $\hat{O}$, it will evolve unitarily as $|s_i(t)\rangle = U(t, t_0)\,|s_i(t_0)\rangle$ for $t$ between $t_0$ and $t_1$, and that performing the measurement corresponding to $\hat{O}$ will have no effect on the system when it is an eigenstate $|s_i(t)\rangle$ of $\hat{O}$ – otherwise we couldn't be sure that whenever we looked at the measurement readout that we weren't changing the value of the quantity we were trying to measure.

Now according to the Copenhagen interpretation, when the measurement corresponding to $\hat{O}$ is made, the system must enter into one of the eigenstates of the observable $\hat{O}$, and at time $t_1$ shortly after the measurement has been made, the probability the system will be in the $|s_i(t_1)\rangle$-state given that it was in the $|\psi(t_0)\rangle$-state at time $t_0$ will be $|\left\langle s_i(t_1)\middle|\psi^{(U)}(t_1)\right\rangle|^2$ in accordance with the Born rule. So taking $|\psi(t_1)\rangle$ to be proportional to $|s_i(t_1)\rangle$ for some $i$, we see that for $j \neq i$, $\langle s_j(t_1)|\psi(t_1)\rangle = 0$. This is because eigenstates of a Hermitian operator that have different eigenvalues must be orthogonal. However, since $U(t_1, t_0)$ is unitary,

$$\langle s_j(t_1)|\psi^{(U)}(t_1)\rangle = \langle s_j(t_0)|\psi^{(U)}(t_0)\rangle = c_j.$$

So we see that $|\psi^{(U)}(t_1)\rangle \neq |\psi(t_1)\rangle$ if $\psi(t_0)$ is not initially in an eigenstate of $\hat{O}$, and hence $|\psi(t)\rangle$ doesn't evolve unitarily up to time $t_1$ as $|\psi^{(U)}(t)\rangle$ does.

[4]We may wish to think of these additional values as hidden variables, but we are not obliged to since we don't speculate on whether these additional variables are necessarily unknowable. Rather, we just see them as supplementing the quantum state so as to provide a complete description of the system.

[5]Multiplying time by the speed of light means that $x^0$ is a distance like $x^1, x^2$, and $x^3$.

e.g. $\boldsymbol{x} = (x^1, x^2, x^3)$, and non-boldface type to depict a spacetime location, e.g. $x = (x^0, x^1, x^2, x^3)$.

Now a key insight of special relativity is that there is no such thing as absolute time. So for instance, two spacetime locations might seem to be simultaneous from one frame of reference, but another person travelling at a different velocity would judge with equal propriety the same two spacetime locations to be non-simultaneous. But it is not the case that for any two spacetime locations we can always find a frame of reference in which the two spacetime locations are simultaneous – sometimes this is not possible. But we refer to spacetime locations that could be simultaneous in some frame of references as being **spacelike-separated**. For example, the two spacetime locations $O$ and $A$ in figure 1 are spacelike-separated.

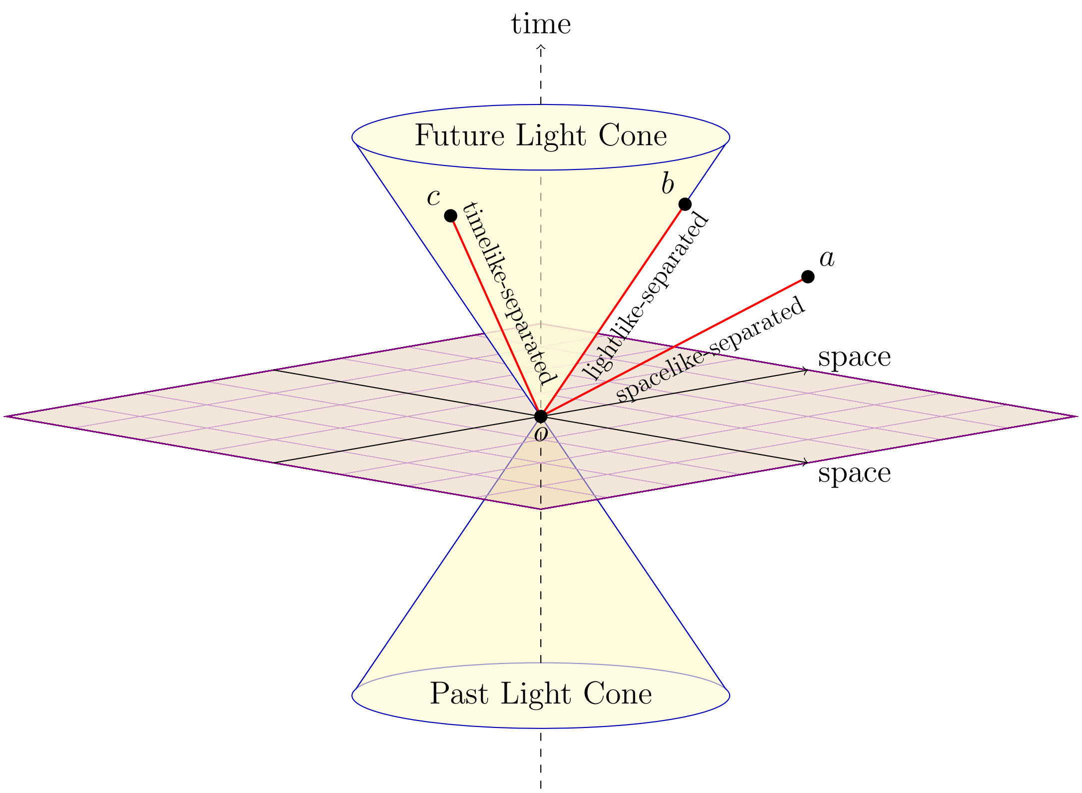


Figure 1: The meaning of spacelike, timelike and lightlike-separation when there are two space dimensions and one time dimension.

cone

There are also spacetime locations in spacetime that could be connected by a beam of light such as the two spacetime locations $o$ and $b$ in figure 1. Such spacetime locations are referred to as being **lightlike-separated**. For any given spacetime location, the spacetime locations that are lightlike-separated from it form two cones[6] called the future light cone and the past light cone as shown in figure 1. Because light appears to travel at the same speed no matter what frame of reference one uses, the light cone of a spacetime location remains invariant when one change from one reference frame to another. In other words, if another spacetime location lies on the light cone of a spacetime location in one frame of reference, then it must lie on the light cone of this spacetime location in every frame of reference.

Figure 1 also depicts two spacetime locations $o$ and $c$ that are **timelike-separated**. Such spacetime locations lie within the light cones of each other, and when two spacetime locations are timelike-separated, it is always possible to choose a frame of reference in which the two spacetime locations are located at the same point in space, but with one spacetime location occuring after the other depending on which spacetime location is in the future light cone of the other.

Now a three-dimensional hypersurface $S$ in spacetime is a maximal[7] three-dimensional surface in which all the spacetime locations of $S$ are spacelike-separated. Kent assumes that this hypersurface $S$ is in the distant future of an expanding universe so that nearly all the particles that can decay have already done so, and that all the particles

[6]Strictly speaking, the set of spacetime locations that are lightlike-separated from a give spacetime location form the surface of a cone rather than a cone (which is a convex object). But among physicists, the terminology light cone has stuck.

[7]That is, it cannot be extended any further along any of its three dimensions, so it is not a small local surface contained within a boundary.

that are not bound together are very far from each other so that the probability of any particle collisions is very small. In other words, all the interesting physics in the universe has played its course before $S$.

massenergydensity

At every spacetime location $x \in S$, there is a quantity $T_S(x)$ called the **mass-energy density**.[8] The important thing to note about $T_S(x)$ is that it does not depend on which frame of reference one is in.[9] This property is in contrast to many physical properties that do depend on which frame of reference one is in. For example, the kinetic energy of an object will depend on the calculated velocity of the object, and this velocity will in turn depend on the frame of reference in which this calculation is done.

Now in order to specify the additional values that Kent's interpretation requires, we need to discuss the Tomonaga-Schwinger picture of relativistic quantum physics.[10] In order to explain their formulation, it is helpful to consider the distinction between the Heisenberg picture and the Schrödinger picture of quantum mechanics. In the **Heisenberg picture**, the states describing a system do not change over time. Rather, the observables change over time. So for instance, if there is a time-independent state $|\Phi\rangle$ describing a system and there is some measurable quantity whose expectation value we wish to know at time $t$ given the state $|\Phi\rangle$, then we will need a time dependent observable $\hat{O}(t)$, say, corresponding to the measurable quantity at time $t$ from which

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[8]The definition of $T_S(x)$ will be discussed in section 0.1.2.

[9]The reason for why this is will be discussed in section 0.1.2.

cbx@2 cbx@3

[10]See Julian Schwinger, "Quantum Electrodynamics. I. A Covariant Formulation," *Physical review* 74, no. 10 (1948): 1439–1461; S. Tomonaga, "On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields," *Progress of theoretical physics* (Tokyo) 1, no. 2 (1946): 27–42

we can calculate the expectation value $\langle\Phi|\hat{O}(t)|\Phi\rangle$ at time $t$ given the system is in state $|\Phi\rangle$.

The Heisenberg picture is contrasted with the **Schrödinger picture** in which the observables do not change over time, but rather the states change over time. So for instance, if there is a time-dependent state $|\Phi(t)\rangle$ describing a system at a specific time $t$ and there is some measurable quantity whose expectation value we wish to know at time $t$ given the state $|\Phi(t)\rangle$, then we will only require a time-independent observable $\hat{O}$, say, corresponding to the measurable quantity from which we can calculate the expectation value $\langle\Phi(t)|\hat{O}|\Phi(t)\rangle$.

Now despite the Schrödinger and Heisenberg pictures taking different perspectives, they are nevertheless physically equivalent. This is because in both pictures, there is a unitary operator $U(\Delta t)$ for any time interval $\Delta t$ such that $U(\Delta t)\,|\Phi(t)\rangle = |\Phi(t+\Delta t)\rangle$, and $U(\Delta t)\hat{O}(t)U(\Delta t)^{-1} = \hat{O}(t+\Delta t)$. Thus, given the Schrödinger picture, to get the Heisenberg picture, all we need to do is the following: firstly, we fix a time $t_0$ and let all the states of the form $|\Phi(t_0)\rangle$ at time $t_0$ in the Schrödinger picture be the state space for the Heisenberg picture; then for any Schrödinger picture observable $\hat{O}$, we define the corresponding Heisenberg picture time-dependent observable

$$\hat{O}(t) = U(t-t_0)\hat{O}U(t-t_0)^{-1}.$$

Conversely, to move from the Heisenberg picture to the Schrödinger, we first fix a reference time $t_0$. Then for any state $|\Phi\rangle$ and observable $\hat{O}(t)$ in the Heisenberg picture, the corresponding Schrödinger picture time-dependent state at time $t$ will be

$U(t - t_0) |\Phi\rangle$, and the corresponding Schrödinger picture time-independent observable will be $\hat{O}(t_0)$.

Now if there is a quantity we wish to measure at time $t_0$ with corresponding observable $\hat{O} \equiv \hat{O}(t_0)$, then in both pictures, the expectation value of this measurable quantity given $|\Phi\rangle \equiv |\Phi(t_0)\rangle$ will be

$$\langle\Phi(t_0)|\hat{O}|\Phi(t_0)\rangle = \langle\Phi(t_0)|\hat{O}(t_0)|\Phi(t_0)\rangle = \langle\Phi|\hat{O}(t_0)|\Phi\rangle \quad \text{\{heisshrodeq\}} \tag{2}$$

Thus, whatever picture we choose, it will make no difference to the calculated expectation values of observables – in other words, the two pictures are physically equivalent.

Now although it is easy to move between both the Schrödinger and Heisenberg pictures, they both give a privileged status to hypersurfaces of the form $t = \text{const}$. However, according to special relativity, there are no privileged hypersurfaces. One of the great advantages of the Tomonaga-Schwinger picture is that it gives no privileged status to any class of hypersurfaces, but rather all hypersurfaces are placed on the same footing.

We almost ready to describe the Tomonaga-Schwinger picture, but before we do so, it will be helpful to specify the observables in our pictures more precisely. What is currently missing from the observables we've mentioned so far is any specification of the spatial location of the quantity that is being measured. We thus note that in the Heisenberg picture, any observable $\hat{O}(t)$ can be expressed as a sum (or integral) of observables of the form $\hat{O}(t, \boldsymbol{x})$, where $\hat{O}(t, \boldsymbol{x})$ is an observable of some quantity at a particular time $t$ and spatial location $\boldsymbol{x}$. Similarly, in the Schrödinger picture, any observable $\hat{O}$ can be expressed as a sum (or integral) of observables of the form $\hat{O}(\boldsymbol{x})$ where $\hat{O}(\boldsymbol{x})$ is an observable of some quantity at a particular spatial location $\boldsymbol{x}$. Plugging the observables $\hat{O}(t_0, \boldsymbol{x})$ and $\hat{O}(\boldsymbol{x})$ into (2), we see that if we can calculate $\langle\Phi(t_0)|\hat{O}(t_0, \boldsymbol{x})|\Phi(t_0)\rangle$ for any $t_0$ and any $\boldsymbol{x}$, then we can calculate all the expectation values that might interest us. But now note that in the expectation value $\langle\Phi(t_0)|\hat{O}(t_0, \boldsymbol{x})|\Phi(t_0)\rangle$, the $|\Phi(t_0)\rangle$-state is the state of a hypersurface $t = t_0$, and $(t_0, \boldsymbol{x})$ is a spacetime location on this hypersurface. Now if we are to place all hypersurfaces on the same footing, then in specifying expectation values, we should be just as content in specifying expectation values of the form $\langle\Phi(S)|\hat{O}_S(x)|\Phi(S)\rangle$, where $S$ is any hypersurface, $|\Phi(S)\rangle$ is any state of this hypersurface, $x$ is any spacetime location on the hypersurface $S$, and where $\hat{O}_S(x)$ is any observable of $S$. The Tomonaga-Schwinger picture thus works with states of the form $|\Phi(S)\rangle$ describing a hypersurface $S$, and observables of the form $\hat{O}_S(x)$ acting on the state space of the hypersurface $S$ from which one can calculate the expectation value $\langle\Phi(S)|\hat{O}_S(x)|\Phi(S)\rangle$.

In order to construct $|\Phi(S)\rangle$ and $\hat{O}_S(x)$, Schwinger introduces a unitary operator $U[S]$ that maps the $|\Phi\rangle$-state of the Heisenberg picture to the corresponding $|\Phi(S)\rangle$-state describing the state of the hypersurface $S$. Schwinger then defines the observable

$$\hat{O}_S(x) = U[S]\hat{O}(x)\hat{U}[S]^{-1}$$

on $S$ where $x$ is any spacetime location on $S$, and where $\hat{O}(x)$ is any Heisenberg picture observable. The specification of $|\Phi(S)\rangle$ and $\hat{O}_S(x)$ Clearly,

$$\langle\Phi(S)|\hat{O}_S(x)|\Phi(S)\rangle = \langle\Phi|\hat{O}(x)|\Phi\rangle$$

and so the Tomonaga-Schwinger picture gives the same physics as the Heisenberg and Schrödinger picture. Moreover, Schwinger show that under conditions that are readily satisfied conditions, $\hat{O}_S(x)$ is independent of the hypersurface $S$.[11]

Thus, we note that when we are interested in making a measurement of some quantity, the is made, there will

To translate the Schrödinger/Heisenberg pictures to the Tomonaga-Schwinger picture, we note that given a space of states $H_{t_0}$ at time $t_0$ in the Schrödinger picture (which we are identifying to be the space of states for the Heisenberg picture), we can think of the unitary operator $U(\Delta t)$ mapping the space of states from the $t = t_0$ hypersurface to the space of states on the $t = t_0 + \Delta t$ hypersurface $H_{t_0+\Delta t}$. Since a unitary operator maps a Hilbert space of states to itself and preserves the inner product, we have $H_{t_0+\Delta t} = H_{t_0}$ – we are just interpreting the states of $H_{t_0}$ and $H_{t_0+\Delta t}$ differently. Now the Tomonaga-Schwinger picture relies on there being a more generalized unitary operator $U[S]$ for any hypersurface $S$.[12] Taking $|\Phi\rangle \in H_{t_0}$, we define $|\Psi[S]\rangle = U[S]\,|\Phi\rangle$ . As for the observables, we note that in the Heisenberg

[11]The required condition is that

$$i\hbar\frac{\delta U[S]}{\delta S(x)} = \mathcal{H}(x)U[S]$$

where $\mathcal{H}(x)$ is a Hermitian operator that is a Lorentz invariant function of the field quantities at the spacetime location $x$ and has the dimensions of an energy density, and where the functional derivative $U[S]$ is given by

$$\frac{\delta F[S]}{\delta S(x)} = \lim_{\delta\omega\to 0}\frac{F[S'] - F[S]}{\delta\omega}$$

where $S'$ is a surface that only differs from $S$ in the vicinity of $x$, and where $\delta\omega$ is the volume enclosed by $S$ and $S'$. The Hermitian operator

$$\mathcal{H}(x) = -(1/c)j^{\mu}(x)A_{\mu}(x)$$

has the desired property where $j^{\mu}(x)$ is the current density and where $A^{\mu}(x)$ is the four-vector potential of the electromagnetic field.

[12]For a proof of the existence of $U[S]$, see Schwinger, "Quantum Electrodynamics. I. A Covariant Formulation," p. 1445.

picture, an observable $\hat{O}(t)$ can be expressed in terms of observables of the form $\hat{O}(t, \boldsymbol{x})$, that is in terms of observables corresponding to a particular time $t$ and spatial location $\boldsymbol{x}$. Now we can generalize this would be to construct a unitary operator $U[S]$ for any hypersurface The Tomonaga-Schwinger picture overcomes this limitation by thi

there is a Hilbert space $H_S$ such that for any hypersurface $S$ and any spacetime location $x \in S$, there is an observable $\hat{T}_S(x)$ acting on $H_S$ such that if $|\Psi\rangle \in H$ is an eigenstate of $\hat{T}_S(x)$ with eigenvalue $\tau(x)$, then $|\Psi\rangle$ corresponds to a state in which the energy-density at $x$ is $\tau(x)$.[13] This can be done in such a way that $\hat{T}_S(x)$ only depends on $x$ rather than on the hypersurface $S$ that contains $x$. Furthermore, if $x$ and $y$ are

[13]We can understand this Hilbert space in terms of the **Heisenberg picture**.
Now the Heisenberg picture is better suited for the formulation of quantum field theory. Fields such as the vector potential $A_\mu(t, \boldsymbol{x})$ which are defined at all spacetime locations $(t, \boldsymbol{x}) = (t, x, y, z)$ and which can be used to determine the electromagnetic field have corresponding Hilbert space operators $\hat{A}_\mu(t, \boldsymbol{x})$ from which the observables for the electromagnetic can be calculated. The index $\mu$ is an integer between 0 and 3 with 0 being the time index, and the numbers 1 to 3 being spatial indices. As in the Heisenberg picture, the operators $\hat{A}_\mu(t, \boldsymbol{x})$ have a time dependency. By analogy with the canonical commutation relation from quantum mechanics in which $[\hat{x}, \hat{p}] \stackrel{\text{def}}{=} \hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$ for position operator $\hat{x}$ and momentum operator $\hat{p}$, in the quantum field theory setting, we require that the operators $\hat{A}_\mu(t, \boldsymbol{x})$ and $\frac{1}{c}\frac{\partial \hat{A}_\nu(t, \boldsymbol{y})}{\partial t}$ satisfy the commutation relation

$$\int [\hat{A}_\mu(t, \boldsymbol{x}), \frac{1}{c}\frac{\partial \hat{A}_\nu(t, \boldsymbol{y})}{\partial t}] \mathrm{d}V(t, \boldsymbol{y}) = i\hbar c \delta_{\mu\nu}$$

where the integral is taken over all space at a particular time using the volume element $\mathrm{d}V(t, \boldsymbol{y})$, and where $\delta_{\mu\nu} = 1$ if and only if $\mu = \nu$. We note however that this commutation relation gives a privileged status to the hypersurface $t = \text{const}$, whereas according to special relativity, there are no privileged hypersurfaces. We therefore need to generalize this commutation relation so that it applies to any hypersurface $S$. As shown by Schwinger (e.g. ), the generalized version of this commutation relation takes the form

$$\int_S [\hat{A}_\mu(x), \frac{1}{c}\frac{\partial \hat{A}_\nu(y)}{\partial y_\lambda}] \mathrm{d}S_\lambda(y) = \frac{\hbar c}{i}\delta_{\mu\nu}$$

where $S$ is any hypersurface, $x \in S$, and where the integral is taken over the hypersurface $S$ using the hypersurface volume element $\mathrm{d}S_\lambda(y)$ which is normal to the hypersurface at every point $y \in S$. Also, we need to generalize the notion of an observable given in the Heisenberg picture which are dependent on a privileged time coordinate $t$. In the Heisenberg picture, there is a unitary operator $U(\Delta t)$ for any time increment $\Delta t$ such that for an observable $\hat{O}(t)$ at time $t$ becomes $\hat{O}(t + \Delta t) = U(\Delta t)\hat{O}(t)U(\Delta t)^{-1}$ at time $t + \Delta t$.
, in the Tomonaga-Schwinger picture, we specify and arbtrary hypersurface

any two spacetime locations of $S$, then the observables $\hat{T}_S(x)$ and $\hat{T}_S(y)$ commute. In other words,

$$\hat{T}_S(x)\hat{T}_S(y) = \hat{T}_S(y)\hat{T}_S(x).$$

The commutativity of all the $\hat{T}_S(x)$ for $x \in S$ means that if $|\Psi\rangle$ is an eigenstate of $\hat{T}_S(x)$, then for any $y \in S$, $\hat{T}_S(y)\,|\Psi\rangle$ is also an eigenstate of $\hat{T}_S(x)$ with the same eigenvalue as $|\Psi\rangle$. The invariance of any $\hat{T}_S(x)$-eigenspace[14] under the action of $\hat{T}_S(y)$ means that we can create simultaneous eigenstates for both $\hat{T}_S(x)$ and $\hat{T}_S(y)$, albeit with different eigenvalues.[15] But because $x$ and $y$ are arbitrary points of $S$, this means that we can express any state $H$ as a superposition of simultaneous $\hat{T}_S$-eigenstates of the form $\left|\Psi^{(i)}\right\rangle$ where $\hat{T}_S(x)\left|\Psi^{(i)}\right\rangle = \tau_S^{(i)}(x)\left|\Psi^{(i)}\right\rangle$ for all $x \in S$, where $\tau_S^{(i)}(x) \geq 0$ is a possible energy-density measurement over the whole of $S$.[16] simultaneous

The additional values beyond standard quantum theory that are included in Kent's interpretation are given by one of these possible outcomes for an energy-density measurement over the whole of $S$. We will denote this outcome as $\tau_S(x)$, and we will let $|\Psi\rangle$ be the state such that $\hat{T}_S(x)\,|\Psi\rangle = \tau_S(x)\,|\Psi\rangle$ for all $x \in S$. But although we speak of the measurement of $T_S(x)$ over $S$ as being $\tau_S(x)$, this is only a notional measurement. Thus, we speak of the measurement of $T_S(x)$ on $S$ only to mean that $T_S(x)$ has a determinate value on $S$ despite the quantum state of $S$ in general

[14] An eigenspace of a Hermitian operator $\hat{O}$ acting on a Hilbert space $H$ is just the space of all the eigenstates of $\hat{O}$ in $H$ which have the same eigenvalue.

[15] This is because any $\hat{T}_S(x)$-eigenspace is itself a Hilbert space on which $\hat{T}_S(y)$ acts as a Hermitian operator, so by (??), we can find a basis of states $\{|\psi_1\rangle, \ldots |\psi_N\rangle\}$ of the $\hat{T}_S(x)$-eigenspace and real numbers $\tau^{(1)}(y), \ldots, \tau^{(N)}(y)$ such that $\hat{T}_S(y)\,|\psi_i\rangle = \tau^{(i)}(y)\,|\psi_i\rangle$ for $i = 1, \ldots, N$. Hence each of the $|\psi_i\rangle$ will be simultaneous eigenstates of both $\hat{T}_S(x)$ and $\hat{T}_S(y)$.

[16] We will gloss over glosssim the details of how to make this rigorous for continuous variables $x$ and continuous indices $i$. It is sufficient to approximate the continuous variables and indices as discrete variables and indices when thinking about the simultaneous $\hat{T}_S(x)$-eigenspaces, and one can choose the granularity of this approximation to achieve whatever level of accuracy one desires.

being in a superposition of simultaneous $\hat{T}_S(x)$-eigenstates for every $x \in S$. How this determination of $T_S(x)$ comes about is up to one's philosophical preferences. For example, one could suppose that it was simply by divine fiat that this determination of $T_S(x)$ came about.[17]

Nevertheless, the particular density $\tau_S(x)$ which is found to describe $S$ can't be absolutely anything. Rather, we suppose there is a much earlier hypersurface $S_0$ which is described by a state $|\Psi_0\rangle$ belonging to a Hilbert space $H_{S_0}$ as shown in figure 2. It is assumed that all physics that we wish to describe takes place between these two hypersurfaces $S_0$ and $S$. In figure 2, we therefore let $y$ depicts a generic spacetime location that we wish to describe.

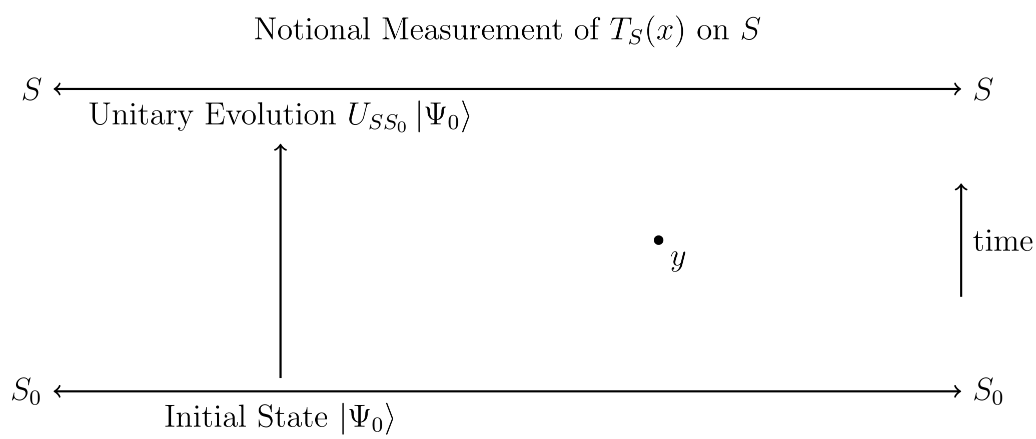


Figure 2: A notional measurement of $T_S(x)$ is made for all $x \in S$. The simultaneous $\hat{T}_S$-eigenstate $|\Psi\rangle$ with $\hat{T}_S(x)\,|\Psi\rangle = \tau_S(x)\,|\Psi\rangle$ is selected with probability $|\langle\Psi|U_{SS_0}|\Psi_0\rangle|^2$. The values $\tau_S(x)$ obtained for $T_S(x)$ are then used to calculate the physical properties at the spacetime location $y$.
S1

According to Schwinger,[18] there is a unitary operator SchwingerUnitaryOP $U_{SS_0}$ that maps states in $H_{S_0}$ such as $|\Psi_0\rangle$ to states in $H$. Then the probability $P(\Psi|\Psi_0)$ that $S$ will be found to be in the state $|\Psi\rangle$ with mass-energy density $\tau_S(x)$ given that $S_0$ was initially in the

[17] I will discuss my philosophical preference in the final chapter.

[18] Schwinger, "Quantum Electrodynamics. I. A Covariant Formulation," p.1459

state $|\Psi_0\rangle$ will be given by the Born Rule (see page **??**):

$$P(\Psi|\Psi_0) = |\langle\Psi|U_{SS_0}|\Psi_0\rangle|^2. \tag{3}$$ {bornpsi}

It's possible that there might be several different states of $H$ that have the same mass-energy density $\tau_S(x)$, but one of these states is still realized with probability given by equation (3). But it is $\tau_S(x)$ rather than one of the eigenstates with mass-energy density $\tau_S(x)$ that constitute the additional values that Kent adds to standard quantum theory.

### 0.1.3 The One-World Feature of Kent's Interpretation

The third similarity Kent's interpretation shares with the pilot wave interpretation is that it is a one-world interpretation of quantum physics. It will be helpful to contrast this with the many-worlds interpretation.

Unlike the many-worlds interpretation, there are no superpositions of living and dead cats in Kent's interpretation. Recall that in the many-worlds interpretation, Schrödinger will still only observe his cat to be either dead or alive, and not both dead and alive, but Schrödinger himself goes into a superposition of observing his cat to be alive and his cat to be dead. In the many-worlds interpretation, there is thus a difference between observing something to be so, and something actually being so: the observation is of a particular outcome, but the reality is a superposition of different outcomes.

To capture this distinction between observation and reality, Bell speaks of **beables**. beabledef

Bell introduces the term beable when speculating on what would be a more satisfactory physical theory than quantum physics currently has to offer.[19] Bell says that such a theory should be able to say of a system not only that such and such is observed to be so, but that such and such be so. In other words, a more satisfactory theory would be a theory of beables rather than a theory of observables. On the macroscopic level, these beables should be the underlying reality that gives rise to all the familiar things in the world around us, things like cats, laboratories, procedures, and so on. For example proponents of the pilot wave interpretation believe that the beables are all the particles with their precise position and momentum. But whatever these beables are, it is because of them that a scientist can observe a physical system to be in such and such a state. Thus, observables are ontologically dependent on beables.

Now the beables in Kent's one world interpretation are expressed in terms of a physical quantity called the **stress-energy tensor** $T^{\mu\nu}(y)$. stressenergy For any spacetime location $y$, the stress-energy tensor $T^{\mu\nu}(y)$ is an array of 16 values corresponding to each combination of $\mu, \nu = 0, 1, 2,$ or 3. The value $T^{00}(y)$ is the energy density at $y$ divided by $c^2$,[20] whereas the other values of $T^{\mu\nu}(y)$ indicate how much energy and momentum flow across different surfaces in the neighborhood of $y$.

It was mentioned in the previous section that for any spacetime location $x \in S$, there is an observable $\hat{T}_S(x)$ acting on $H$. It turns out that for any $\mu, \nu = 0, 1, 2,$ or 3, there is also an observable $\hat{T}^{\mu\nu}(x)$ acting on $H$, such that if $|\Psi\rangle \in H$ is a simultaneous eigenstate of $\hat{T}^{\mu\nu}(x)$ with eigenvalue $\tau^{\mu\nu}(x)$ for all $x \in S$, then $|\Psi\rangle$ corresponds to a

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[19] See cbx@6 J. S. Bell, "Subject and Object," in *Speakable and unspeakable in quantum mechanics*, 2nd ed. (Cambridge: Cambridge University Press, 2004), 40–44

[20] This is not to be confused with the mass-energy density $T_S(x)$ defined for $x$ on a hypersurface $S$. As will be shown in section **??**, all 16 elements of $T^{\mu\nu}(x)$ will typically be needed to calculate $T_S(x)$.

state of $S$ in which $T^{\mu\nu}(x)$ is $\tau^{\mu\nu}(x)$ for all $x \in S$.[21] Moreover, the observable $\hat{T}_S(x)$ is expressible in terms of the $\hat{T}^{\mu\nu}(x)$-observables.[22]

Now the beables in Kent's interpretation are defined at each spacetime location $y$ that occurs after $S_0$ and before $S$. For such a spacetime location $y$, the beables will be determinate values of the stress-energy tensor $T^{\mu\nu}(y)$, but calculated from the expectation of the observable $\hat{T}^{\mu\nu}(y)$ conditional on the energy-density on $S$ being given by $\tau_S(x)$ for all $x$ outside the light cone of $y$.

To explain what this means in more detail, recall the definition of expectation in equation (**??**) and the expectation formula (**??**) for an observable. If the beable in question was simply the expectation of $\hat{T}^{\mu\nu}(y)$ without conditioning on the value of the energy-density on $S$, then the $T^{\mu\nu}(y)$-beable would just be $\langle\Psi'|\hat{T}^{\mu\nu}(y)|\Psi'\rangle$ where $|\Psi'\rangle = U_{S'S_0}|\Psi_0\rangle$ for any hypersurface $S'$ that goes through $y$.[23] However, such a beable would give a description of reality that was very different from what we observe – for instance, in a Schrödinger cat-like experiment, there would be energy-densities corresponding to both the cat being alive and the cat being dead in the same world. To overcome this defect, information about the mass-energy density on $S$ is required, specifically the values of $\tau_S(x)$ for $x \in S^1(y)$ where $S^1(y)$ is defined to consist of all the spacetime locations of $S$ outside the light cone of $y$ as depicted in figure 3.

[21]Note however, that such a simultaneous eigenstate is only for a fixed choice of $\mu$ and $\nu$, since in general, $\hat{T}^{\mu\nu}(x)$ and $\hat{T}^{\mu'\nu'}(x)$ will not commute for $\mu \neq \mu'$ or $\nu \neq \nu'$.

[22]See section **??** for an explanation for why this is so.

[23]This can be done such that $\langle\Psi'|\hat{T}^{\mu\nu}(y)|\Psi'\rangle$ does not depend on the hypersurface $S'$ other than the fact that it contains $y$. For more details see Schwinger, "Quantum Electrodynamics. I. A Covariant Formulation."

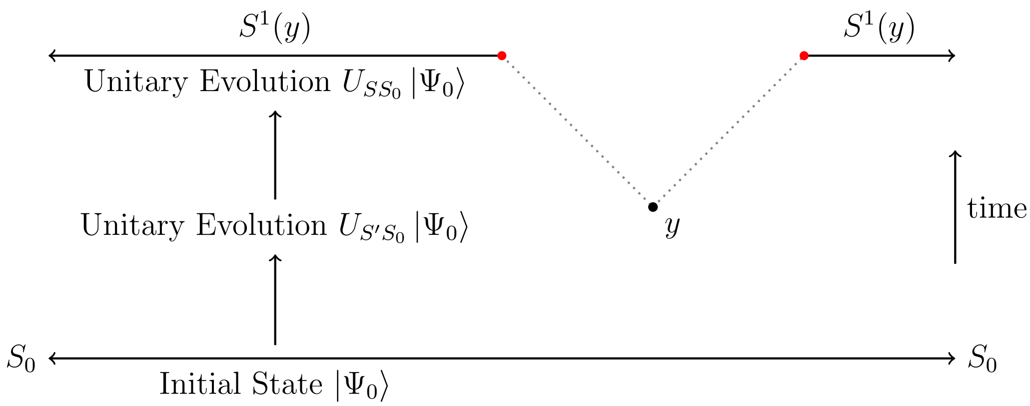


Figure 3: The set $S^1(y)$ consists of all the spacetime locations of $S$ outside the light cone of $y$. The $T^{\mu\nu}(y)$-beables are calculated using the initial state $|\Psi_0\rangle$ together with the values of $\tau_S(x)$ for $x \in S^1(y)$.

S2

The conditional expectation that we need to calculate depends on the notion of **conditional probability**. In probability theory, the conditional probability $P(q|r)$ that a statement $q$ is true given that a statement $r$ is true is given by the formula

$$P(q|r) = \frac{P(q\&r)}{P(r)}. \qquad \text{\{conditionalprobability\}} \tag{4}$$

If we now define $q(\kappa)$ to be the statement that some quantity $K$ takes the value $\kappa$, then the **conditional expectation** of $K$ given $r$ will be given by the formula

$$\langle K \rangle_r \stackrel{\text{def}}{=} \sum_{\kappa} \frac{P(q(\kappa), r)\kappa}{P(r)} \qquad \text{\{conditionalexpectation\}} \tag{5}$$

where the summation is over all the possible values $\kappa$ that $K$ can take.

If we let $\langle T^{\mu\nu}(y) \rangle_{\tau_S}$ stand Kentbeable for Kent's $T^{\mu\nu}(y)$-beable, then this can be calculated from (5) by taking $r$ to be the statement that $T_S(x) = \tau_S(x)$ for all $x \in S^1(y)$, and $q(\tau)$ to be the statement that the universe is found to be in a quantum eigenstate of the observable $\hat{T}^{\mu\nu}(y)$ with eigenvalue $\tau$. It is these $T^{\mu\nu}(y)$-beables that give a one-world picture of reality in Kent's interpretation.

toysection

#### 0.1.4 Kent's toy example

To get a feel for how all the elements of Kent's interpretation fit together, it is helpful to consider Kent's toy model example that he discusses in his 2014 paper.[24] In his toy model, Kent considers a system in one spatial dimension which is the superposition of two localized states $\psi_0^{\text{sys}} = c_1\psi_1^{\text{sys}} + c_2\psi_2^{\text{sys}}$ where $\psi_1^{\text{sys}}$ is localized at spatial location $z_1$, $\psi_2^{\text{sys}}$ is localized at spatial location $z_2$, and $|c_1|^2 + |c_2|^2 = 1$. According to the Copenhagen interpretation, a measurement on this system would collapse the wave function of $\psi_0^{\text{sys}}$ to the wave function of $\psi_1^{\text{sys}}$ with probability $|c_1|^2$, and to the wave function of $\psi_2^{\text{sys}}$ with probability $|c_2|^2$. The purpose of Kent's toy model is to show that within his interpretation, there is something analogous to wave function collapse. In order for this "collapse" to happen, one needs to consider how the system interacts with light. Thus, Kent supposes that a photon (which is modelled as a point particle) comes in from the left, and as it interacts with the two states $\psi_1^{\text{sys}}$ and $\psi_2^{\text{sys}}$, the photon enters into a superposition of states, corresponding to whether the photon reflects off the localized $\psi_1^{\text{sys}}$-state at time $t_1$ or the localized $\psi_2^{\text{sys}}$-state at time $t_2$. The photon in superposition then travels to the left and eventually reaches the one dimensional hypersurface $S$ at locations $\gamma_1$ and $\gamma_2$ as shown in figure 4.

We now suppose that when the mass-energy density $S$ is "measured", the energy of the photon is found to be at $\gamma_1$ rather than at $\gamma_2$. We then consider the mass-density at early spacetime locations $y_1^a = (z_1, t_a)$ and $y_2^a = (z_2, t_a)$ as show in figure 5 (a) and (b).

[24]See Kent, "Lorentzian Quantum Reality: Postulates and Toy Models," p.3–4.

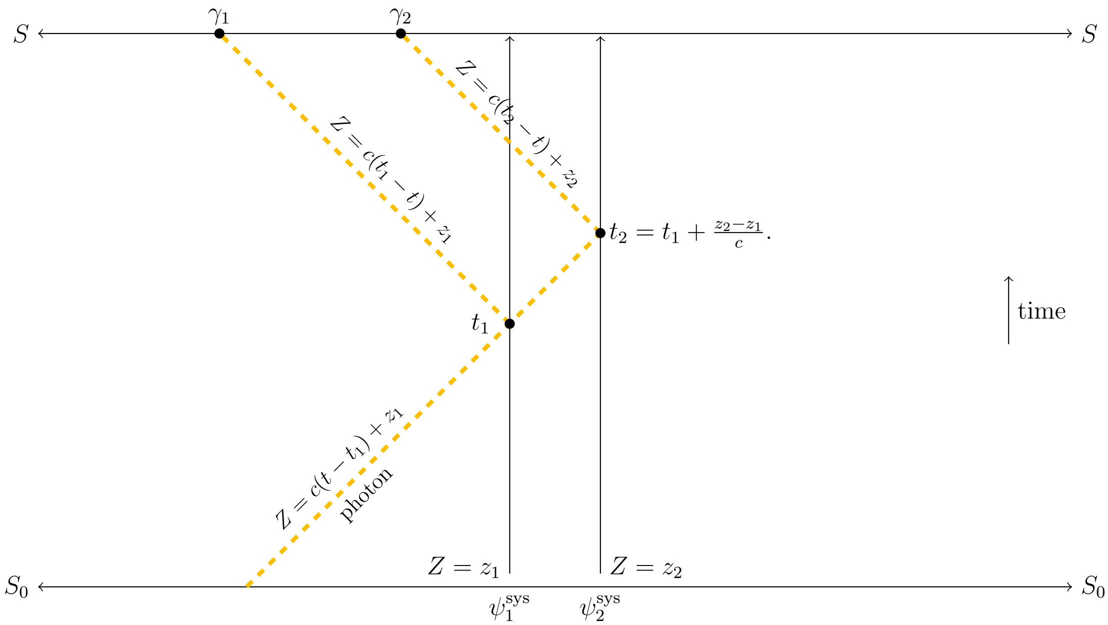


Figure 4: Kent's toy model

TM1

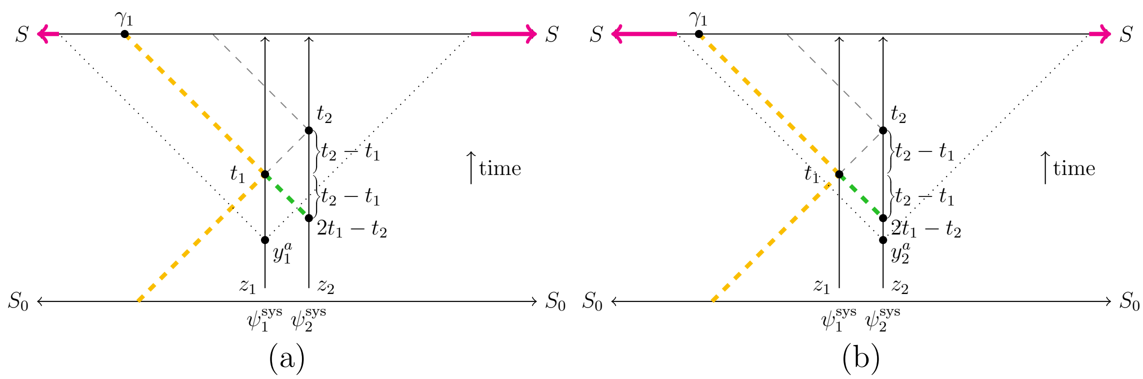


Figure 5: (a) highlights the part of $S$ used to calculate the energy density at $y_1^a$ whose time is less than $2t_1 - t_2$. (b) highlights the part of $S$ used to calculate the energy density at $y_2^a$ whose time is less than $2t_1 - t_2$.

TM2

By early, we mean that $t_a < 2t_1 - t_2$. This will mean that the possible detection locations $\gamma_1$ and $\gamma_2$ will be outside the forward light cones of $y_1^a$ and $y_2^a$. Hence, $S^1(y_1^a) \cap S$ and $S^1(y_2^a) \cap S$ contain no additional information beyond standard quantum theory by which we could calculate the conditional expectation values of the energy at $y_1^a$ and $y_2^a$. Hence, according to Kent's interpretation, the total energy at time $t_a$ will be divided between the two spatial locations with a proportion of $|c_1|^2$ at $z_1$ and a proportion of $|c_2|^2$ at $z_2$.

However, the situation is different for two spacetime locations $y_1^b = (z_1, t_b)$ and $y_2^b = (z_2, t_b)$ with $t_b$ slightly after $2t_1 - t_2$ as depicted in figure 6.

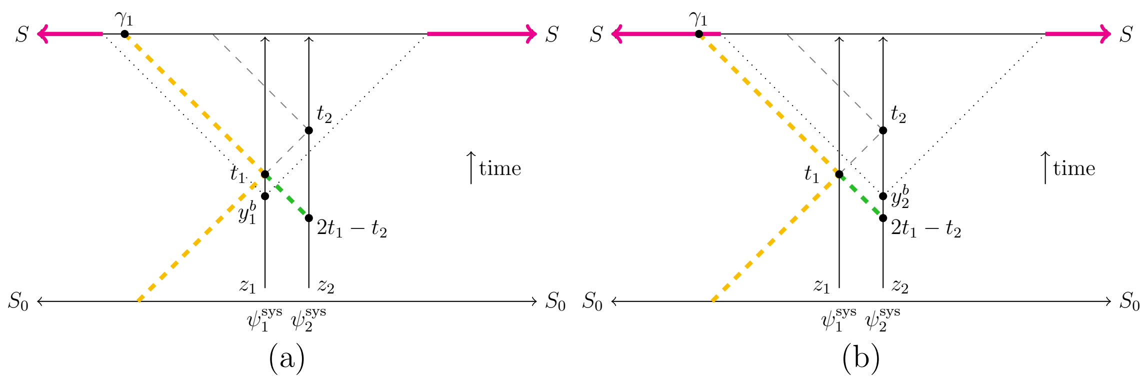


Figure 6: (a) highlights the part of $S$ used to calculate the energy density at $y_1^b$ whose time is greater than $2t_1 - t_2$. (b) highlights the part of $S$ used to calculate the energy density at $y_2^b$ whose time is greater than $2t_1 - t_2$.

TM3

In this situation, when we consider the location $y_1^b$, there is no additional information in $S^1(y_1^b) \cap S$ beyond standard quantum theory, so there will be a proportion of $|c_1|^2$ of the total initial energy of the system at $y_1^b$. But at location $y_2^b$, the information in $S^1(y_2^b) \cap S$ shows that the photon has reflected from the localized $\psi_1^{\mathrm{sys}}$-state, and so this additional information tells us that after time $t_b$, there is no energy localized at $z_2$ since from the perspective of $y_2^b$, the energy is known to be localized at $z_1$. So it is as though the information of $S^1(y_2^b) \cap S$ has determined that we are in a world in which there is an energy density of zero at $y_2^b$, and there are no other worlds in which the energy density at $y_2^b$ is non-zero since all worlds have to be consistent with the notional measurement made on $S$. So for a short time the total energy of the system is reduced by a factor of $|c_1|^2$.

However, as shown in figure 7, for times $t_c$ greater than $t_1$, the total energy of the system is once again restored to the initial energy the system had when in the state $\psi_0^{\text{sys}}$.

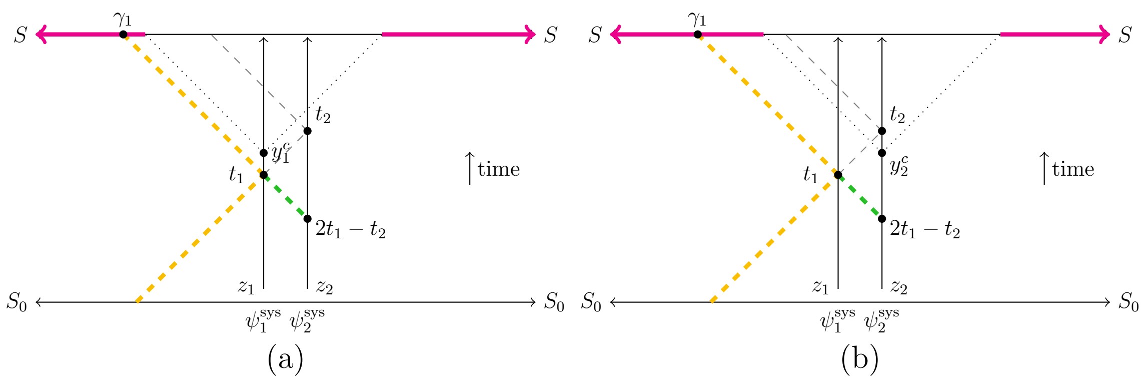


Figure 7: (a) highlights the part of $S$ used to calculate the energy density at $y_1^c$ whose time is greater than $t_1$. (b) highlights the part of $S$ used to calculate the energy density at $y_2^c$ whose time is greater than $t_1$.

`TM4`

In this situation, there is now information in $S^1(y_1^c) \cap S$ that determines that the photon reflected off the localized $\psi_1^{\text{sys}}$-state. This means that when the conditional expectation of the energy density of $y_1^c$ is calculated, the extra information in $S^1(y_1^c) \cap S$ determines that all the energy of the system is located at location $z_1$ for times $t_c$ greater than $t_1$, and the energy is equal to the initial energy of the system so that energy is conserved.

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