[Institution]

A Defense of One-World Quantum Physics

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by

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Contents

Chapter 1

Introduction

Many ideas in quantum physics are expressed in mathematical terms. I will do my best to avoid unnecessary mathematical jargon, but in order to explain the ideas of this thesis, a certain amount of mathematics is unavoidable. I will endeavor to explain all the mathematical terminology as I go along. However, there will be some sections which may be very challenging to readers who do not have a mathematics or physics background. These sections will be marked with and asterisk *. asteriskmeaning There is also a lot of terminology from theoretical physics that we will need to invoke. I will use the convention of putting terminology in bold typeface whenever the terminology is first defined.

Chapter 2

The Challenge of One-World Quantum Physics

In recent times, it has become increasingly common for popularizers of quantum physics to tell us that we need to let go of our naïve common sense understanding of reality. We're told we must replace this common sense understanding with something that at first seems very bizarre and counter-intuitive: a many-worlds interpretation of reality. This is the idea that whenever there is quantum indeterminacy among several possibilities, then all these possibilities are realized, and the actualization of these possibilities can be extrapolated up to the macroscopic level. Thus, many-worlds advocates, when reflecting on the famous Schrödinger's Cat thought experiment do not question the foundations of quantum mechanics on which the thought experiment is based, but rather they embrace the seemingly absurd conclusion of Schrödinger that a cat could be both dead and alive. They thus speak of the cat being dead in one world and the cat being alive in another world that is just as real as the first. We will be examining the many-worlds interpretation of quantum physics in chapter **??**, but in this chapter, we will consider why some people are so keen to reject a one-world interpretation of quantum physics.

The central challenge that one-world interpretations of quantum physics must deal with is how to make sense of the experimental violation of Bell's inequality in a way that is consistent with Einstein's theory of special relativity. In this chapter, no prior knowledge of quantum theory will be assumed. We will therefore need to describe some key ideas of quantum theory, and this we will do in the context of the

Stern-Gerlach experiment. We will then describe the EPR paradox and the difficulty the traditional Copenhagen interpretation of quantum physics has in dealing with this paradox. A seemingly natural way to overcome this paradox is to supplement standard quantum theory with hidden variables. We will thus describe one way in which this can be done, and we will show how this leads to the remarkable inequality first derived by Bell. However, Bell's inequality is known to be experimentally violated. This means there must be something wrong with Bell's assumptions. We will therefore consider Shimony's analysis of the proof of Bell's inequality in which Shimony draws a distinction between parameter independence and outcome independence. Finally, we will consider the Colbeck-Renner theorem which states that there is no satisfactory[1] way of supplementing quantum theory with hidden variables.

[1]By satisfactory, I mean that the supplemented theory agrees with experimental observations, and that the parameter independence condition is satisfied.

2.1 Quantum Physics Basics and the Stern-Gerlach Experiment

Some of the key features of quantum physics are exhibited in the Stern-Gerlach experiment (see figure 2.1). In this experiment, silver atoms are heated in a furnace which randomly emerge from the furnace with various velocities. By aligning two plates with circular holes near the furnace, it is possible to select a subset of the emerging silver atoms having (approximately) the same momentum to form a beam in one direction, the other silver atoms having been absorbed by the two plates. This beam of silver atoms is then directed between two magnets with the north pole of one magnet being aligned toward the south pole of the other magnet as shown in figure 2.1. Now silver atoms have a property somewhat analogous to the classical notion of

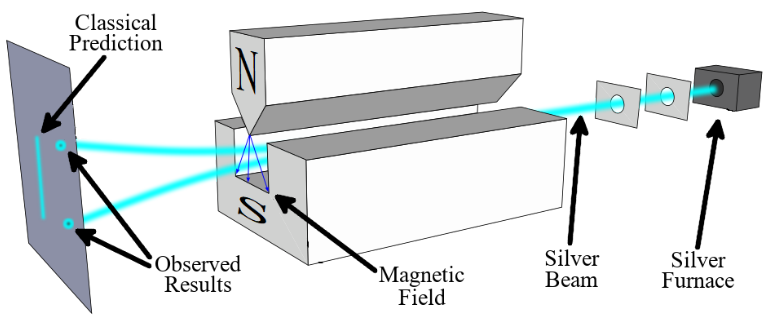

Figure 2.1: The Stern-Gerlach Experiment.[2] `stern`

angular momentum. For instance, a spinning top has angular momentum as shown in figure 2.2. Angular momentum is a vector, so it has direction and magnitude. In the case of a spinning top, the direction of the angular momentum would be parallel to the axis of rotation, pointing one way or the other depending on whether the rotation was clockwise or counterclockwise. The magnitude of the angular momentum would then be proportional to the angular velocity of the spinning top.

[2]Original diagram drawn by Theresa Knott. Labeling was modified for use in this dissertation. This image is licensed under the Creative Commons Attribution-Share Alike 4.0 International license. Source: https://commons.wikimedia.org/wiki/File:Stern-Gerlach_experiment_svg.svg

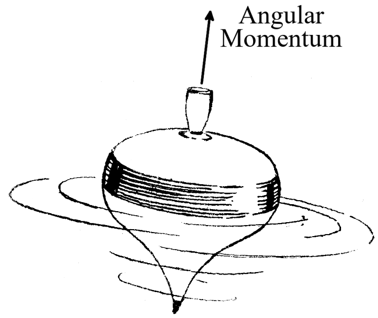

Figure 2.2: Angular Momentum of a Spinning Top.[3] spintop

Now if we tried to understand the angular momentum of a silver atom classically, we would expect the magnetic field of the two magnets to interact with the silver atom in a way that was determined by the relative direction of the silver atom's magnetic momentum compared to the direction of the magnetic field. Since we would expect the silver atom to have an entirely random angular momentum, we would expect it to be deflected by varying degrees either up or down in the direction of the magnetic field. Thus, if a detection screen were placed beyond the two magnets which the silver atoms would hit, we would expect there to be a whole continuum of possible locations where the silver atoms would be detected. However, in reality, it is found that there are precisely two locations where the silver atoms hit the screen. It is as though the particles can spin either clockwise or anticlockwise, but that there is absolutely no variance in the angular speed at which they rotate. This is surprising. The angular momentum appears to be **quantized** in one of two directions, either parallel to the magnetic field or antiparallel to it.[4] Corresponding to this quantization of angular

[3]Drawing by Pearson Scott Foresman, Public domain, via Wikimedia Commons. Labeling was added for use in this dissertation. Original: https://commons.wikimedia.org/wiki/File:Top_(PSF).png.

[4]See figure 2.3 for what is meant be antiparallel.

momentum, we say that the atom is either in the spin up state or the spin down state with respect to the direction of the magnetic field.

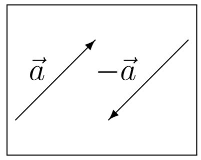

Figure 2.3: Meaning of antiparallel: the arrows in opposite directions are said to be antiparallel to one another.

`antiparallel`

If the direction of the magnetic field is implicitly understood, we write $|+\rangle$ and $|-\rangle$ for the spin up and spin down states of the atom respectively. We refer to the symbols $|+\rangle$ and $|-\rangle$ as **ket-vectors**, or simply as kets. We can think of the ket $|+\rangle$ for instance as shorthand for the proposition "the particle is in the spin up state." If we knew this proposition to be true, we would know which of the two locations on the detection screen the particle would end up if it were to travel between the two magnets of the Stern-Gerlach apparatus. If we need to specify the spin with respect to a particular direction of the magnetic field, say in the $\hat{\mathbf{a}}$-direction, we write the corresponding spin up and down states as $|\hat{\mathbf{a}}+\rangle$ and $|\hat{\mathbf{a}}-\rangle$. For convenience, we write $\hat{\mathbf{a}}+$ and $\hat{\mathbf{a}}-$ respectively for the location that the particle would hit the detection screen.

The question then arises as to what happens when we rotate the magnetic field around the axis of the particle beam in the Stern-Gerlach experimental setup. It turns out that when we do this, the atoms are again detected in only one of two locations (see figure 2.4).

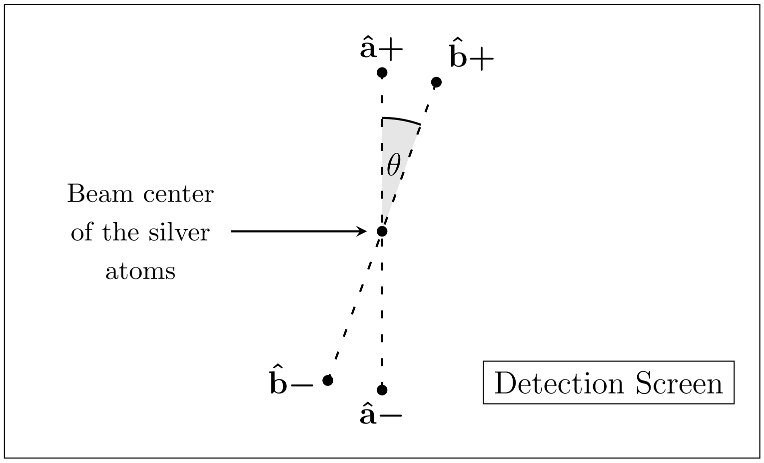

Figure 2.4: Locations of detections before and after rotating the magnetic field by an angle θ. Before rotation, the particles can be detected at either location $\hat{\mathbf{a}}+$ or location $\hat{\mathbf{a}}-$. After the rotation, particles can be detected at either location $\hat{\mathbf{b}}+$ or $\hat{\mathbf{b}}-$.

`rotate`

So suppose we knew the particle was in a spin state such that it was on course to arrive at location $\hat{\mathbf{a}}+$ because we had previously directed it through another magnetic field in the $\hat{\mathbf{a}}$-direction. For example, see figure 2.5 for how this might be done. In

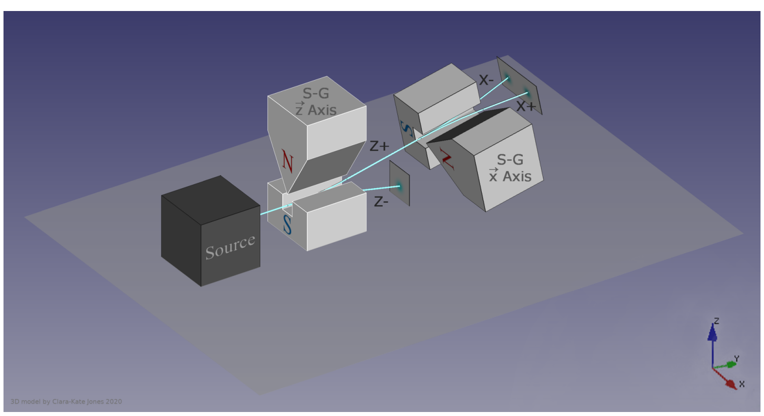

Figure 2.5: Two Stern-Gerlach experiements in sequence. By directing the beam of particles through one magnetic field first, the particles emerging in one of the two beams will be in a known spin state before they enter the second magnetic field.[5]

`knownspin`

this experimental setup, the second magnetic field has been rotated by an angle of

[5]Diagram by MJasK. This file is licensed under the Creative Commons Attribution-Share Alike 4.0 International license. Source: https://commons.wikimedia.org/wiki/File:Stern-Gerlach_Analyzer_Sequential_Series_E2.png.

$90°$ with respect to the first magnetic field. But suppose we just rotated the second magnetic field by a very small angle θ with respect to the first magnetic field. Then we would expect the particle now to arrive at a location $\hat{\mathbf{b}}+$ close by to $\hat{\mathbf{a}}+$ as shown in figure 2.4. And this is what we notice experimentally for the most part. However, occasionally, the particle will arrive at location $\hat{\mathbf{b}}-$. The frequency of this occurrence becomes less and less the less and less the magnetic field is rotated (i.e. the smaller θ is), so that if the magnetic field is not rotated at all, i.e. $\theta = 0$, the particle will always arrive at location $\hat{\mathbf{a}}+$. To capture the probabilistic nature of these outcomes, we use the bra-ket notation. Thus, if $|\psi\rangle$ stands for either the $|\hat{\mathbf{a}}+\rangle$ or the $|\hat{\mathbf{a}}-\rangle$-state, and $|\chi\rangle$ stands for either the $|\hat{\mathbf{b}}+\rangle$ or the $|\hat{\mathbf{b}}+\rangle$-state, then we define the bra-ket $\langle\psi|\chi\rangle \in \mathbb{C}$ to be a complex number[6] that satisfies the **Born Rule**, bornrule namely $|\langle\psi|\chi\rangle|^2$ is the probability that the particle will be found to be in state $|\psi\rangle$ given that we know that the particle is in state $|\chi\rangle$. For example, if $|\langle\psi|\chi\rangle|^2 = \frac{1}{4}$, and we performed the experiment a 1000 times with the particle initially prepared in the $|\chi\rangle$-state, then we would expect the particle to be found in the $|\psi\rangle$-state in around 250 runs of the experiment. We would thus expect $|\langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}+\rangle|^2$ to be 0, from which it will follow that $\langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}+\rangle$ has to be 0. The Born Rule also implies that $|\langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}+\rangle|^2 = 1$. We will also insist that $\langle\psi|\chi\rangle = \overline{\langle\chi|\psi\rangle}$[7], from which it will follow that $\langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}+\rangle$ is a real number of

[6]With regards to the set of complex numbers $\mathbb{C}$, we will use the notation $i = \sqrt{-1}$. Complex conjugation will be denoted by an overline so that $\overline{x+iy} = x - iy$ for real numbers x and y. The modulus of a complex number $z = x + iy$ will then be given by $|z| = \sqrt{z\overline{z}} = \sqrt{x^2 + y^2}$. Since the defining property of $\langle\psi|\chi\rangle$ is that $|\langle\psi|\chi\rangle|^2$ is the probability that the particle will be found to be in state $|\psi\rangle$ given that we know that the particle is in state $|\chi\rangle$, we have to choose an arbitrary phase to fully determine $\langle\psi|\chi\rangle$.

[7]Note that this conditions implies time symmetry: the probability a particle transitions from a state $|\chi\rangle$ to a state $|\psi\rangle$ will be the same as the probability a particle transitions from the state $|\psi\rangle$ to the state $|\chi\rangle$. This is in accord with the observation that closed quantum systems are symmetric on time-reversal. This might at first seem surprising in the light of the fact that phenomena such as radioactive decay are not obviously time-symmetric. However, it turns out that this time asymmetry results from the quantum system not being closed. For more details, see cbx@1 Saverio Pascazio, "All you

modulus 1 (i.e. $+1$ or -1). By convention, we choose $\langle\psi|\chi\rangle$ such that $\langle\psi|\psi\rangle$ is a real and positive number, in which case we must have $\langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}+\rangle = 1$. If we now rotate the magnetic field by an angle θ as indicated in figure 2.4, the particle will be detected either at location $\hat{\mathbf{b}}+$ or location $\hat{\mathbf{b}}-$. We can then ask the question "given that the particle is in state $|\hat{\mathbf{a}}+\rangle$, what is the probability that the particle will be found to be in state $|\hat{\mathbf{b}}+\rangle$?" According to the notation discussed above, this probability will be $|\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle|^2$ where $\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle$ is a complex number such that $\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle = 1$ when $\theta = 0$ and $\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle = 0$ when $\theta = 180°$. We would likewise expect $\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle = 0$ when $\theta = 0$ and $\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle = 1$ when $\theta = 180°$. Since $\cos 0° = \sin 90° = 1$ and $\cos 90° = \sin 0° = 0$, we might guess that in general $|\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle| = |\cos(\theta/2)|$ and $|\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle| = |\sin(\theta/2)|$. Experimentation shows us that this guess is correct. This suggests that we can express the state $|\hat{\mathbf{b}}+\rangle$ in terms of the states $|\hat{\mathbf{a}}+\rangle$ and $|\hat{\mathbf{a}}-\rangle$. We thus suppose that vectoradd

{vectoradd1}
$$|\hat{\mathbf{b}}+\rangle = \alpha\,|\hat{\mathbf{a}}+\rangle + \beta\,|\hat{\mathbf{a}}-\rangle \tag{2.1a}$$

{vectoradd2}
$$|\hat{\mathbf{b}}-\rangle = \bar{\alpha}\,|\hat{\mathbf{a}}-\rangle - \bar{\beta}\,|\hat{\mathbf{a}}+\rangle \tag{2.1b}$$

for complex numbers $\alpha, \beta \in \mathbb{C}$ such that $|\alpha|^2 + |\beta|^2 = 1$, and we suppose that the bra-ket has the **linearity** property so that $\langle\psi|\hat{\mathbf{b}}+\rangle = \alpha\,\langle\psi|\hat{\mathbf{a}}+\rangle + \beta\,\langle\psi|\hat{\mathbf{a}}-\rangle$ and $\langle\psi|\hat{\mathbf{b}}-\rangle = \bar{\alpha}\,\langle\psi|\hat{\mathbf{a}}-\rangle - \bar{\beta}\,\langle\psi|\hat{\mathbf{a}}+\rangle$ for any state $|\psi\rangle$. Then it will follow that $\langle\hat{\mathbf{b}}+|\hat{\mathbf{b}}-\rangle = 0$,[8] and that $\langle\hat{\mathbf{b}}+|\hat{\mathbf{b}}+\rangle = \langle\hat{\mathbf{b}}-|\hat{\mathbf{b}}-\rangle = 1$.[9] If we then put $\alpha = \cos(\theta/2)$ and $\beta = \sin(\theta/2)$, it

ever wanted to know about the quantum Zeno effect in 70 minutes," *44th Symposium on Mathematical Physics on New Developments in the Theory of Open Quantum Systems*, 2013, https://doi.org/10.1142/S1230161214400071, eprint: arXiv:1311.6645v1[quant-ph].

[8]To see this, by putting $|\psi\rangle = |\hat{\mathbf{b}}+\rangle$, we will have $\langle\hat{\mathbf{b}}+|\hat{\mathbf{b}}-\rangle = \bar{\alpha}\,\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle - \bar{\beta}\,\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle$ by equation (2.1b). Since $\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle = \overline{\langle\hat{\mathbf{a}}-|\hat{\mathbf{b}}+\rangle}$ we have $\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle = \overline{\beta}$ by equation (2.1a), and likewise, since $\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle = \overline{\langle\hat{\mathbf{a}}+|\hat{\mathbf{b}}+\rangle}$, we have $\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle = \bar{\alpha}$. Therefore $\langle\hat{\mathbf{b}}+|\hat{\mathbf{b}}-\rangle = \overline{\alpha}\overline{\beta} - \overline{\beta}\overline{\alpha} = 0$.

[9]To see this, by putting $|\psi\rangle = |\hat{\mathbf{b}}+\rangle$ and using equation (2.1a), we will have $\langle\hat{\mathbf{b}}+|\hat{\mathbf{b}}+\rangle = \alpha\,\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle + \beta\,\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle = \alpha\overline{\alpha} + \beta\overline{\beta} = |\alpha|^2 + |\beta|^2 = 1$. By a similar calculation, we also see $\langle\hat{\mathbf{b}}+|\hat{\mathbf{b}}+\rangle = 1$.

will follow that $|\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle| = |\cos(\theta/2)|$ and $|\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle| = |\sin(\theta/2)|$,[10] and so with these values for α and β we will have spintrans

{spintrans1}
$$|\hat{\mathbf{b}}+\rangle = \cos(\theta/2)\,|\hat{\mathbf{a}}+\rangle + \sin(\theta/2)\,|\hat{\mathbf{a}}-\rangle\,, \tag{2.2a}$$

{spintrans2}
$$|\hat{\mathbf{b}}-\rangle = \cos(\theta/2)\,|\hat{\mathbf{a}}-\rangle - \sin(\theta/2)\,|\hat{\mathbf{a}}+\rangle\,. \tag{2.2b}$$

[10] To satisfy these criteria, α and β are only determined up to rotation by a complex number. Rotating a complex number $z \in \mathbb{C}$ just means multiplying it by a complex number λ of modulus 1 (i.e. $|\lambda| = 1$) to get λz. We would need to take into account this rotation factor if we considered the three-dimensional situation. Then, without loss of generality, $\alpha = \cos(\theta/2)$ and $\beta = e^{i\phi}\sin(\theta/2)$ where θ and ϕ are the polar and azimuthal angles respectively.

2.2 The Copenhagen Interpretation and the EPR-Bohm Paradox

Given the equations (2.2a) and (2.2b) that relate the states $|\hat{\mathbf{a}}\pm\rangle$ and $|\hat{\mathbf{b}}\pm\rangle$ to each other, we can calculate probabilities such as the probability a particle will be measured to be in the $|\hat{\mathbf{a}}+\rangle$-state given that it is in the $|\hat{\mathbf{b}}+\rangle$-state. There however arises the question of what the physical meaning of one of these states is. Clearly, the $|\hat{\mathbf{b}}+\rangle$-state says something about the spin of a particle; but is this a complete description of the particle's spin state? For the $|\hat{\mathbf{b}}+\rangle$-state only tells us what the outcome of a spin measurement would be along one particular axis $\hat{\mathbf{b}}$. For a spin measurement along another axis $\hat{\mathbf{a}} \neq \pm\hat{\mathbf{b}}$, $|\hat{\mathbf{b}}+\rangle$ only tells us the probabilities (via equation (2.2a)) that the measurement outcome would be $|\hat{\mathbf{a}}+\rangle$ or $|\hat{\mathbf{a}}+\rangle$ – the state doesn't determine either of these outcomes. So is this indetermination of the measurement outcome along the $\hat{\mathbf{a}}$-axis merely a reflection of our lack of knowledge of a more complete specification of the particle's spin state? Or is the $|\hat{\mathbf{b}}+\rangle$-state a complete description of the spin state of the particle so that there is no fact of the matter about what spin state the particle would be found to be in along the $\hat{\mathbf{a}}$-axis until a measurement of spin along the $\hat{\mathbf{a}}$-axis is made.

Now Bohr and Heisenberg believed the latter to be the case. This was because their mathematical formalism of quantum physics implied that there were physical quantities of particles that couldn't be simultaneously determined. For example, their mathematical formalism is incapable of representing a particle which has a definite spin in both the $\hat{\mathbf{a}}$-direction and the $\hat{\mathbf{b}}$-direction when $\hat{\mathbf{a}} \neq \pm\hat{\mathbf{b}}$. So when a particle that is in the $|\hat{\mathbf{b}}+\rangle$ is measured along the $\hat{\mathbf{a}}$-axis and is found to be in the $|\hat{\mathbf{a}}+\rangle$-state, there is a so-called collapse of the $|\hat{\mathbf{b}}+\rangle$-state:

$$|\hat{\mathbf{b}}+\rangle = \cos(\theta/2)\,|\hat{\mathbf{a}}+\rangle + \sin(\theta/2)\,|\hat{\mathbf{a}}-\rangle \xrightarrow{\text{Collapse!!}} |\hat{\mathbf{a}}+\rangle$$

so that after the measurement, the particle is no longer in the $|\hat{\mathbf{b}}+\rangle$-state. This interpretation of the quantum state where the state collapses to another state upon measurement is known as the **Copenhagen Interpretation**.

Einstein, Podolsky, and Rosen, however, strongly objected to the Copenhagen Interpretation, and introduced their EPR paradox to explain what troubled them.[11] The EPR paradox was originally expressed in terms of the position and momentum of a particle rather than its spin, but Bohm translated the EPR paradox to the context of spin,[12] and this is the version we will consider here.

The EPR-Bohm paradox arises in the context of particle pairs known as spin-singlets. A **spin singlet** describes the state of two particles which a single particle of zero spin has decayed into. For example, a high energy **photon**, that is, a particle of light, can decay into a negatively charged electron, and a positively charged position (where a **positron** is a fundamental particle like an electron but of opposite charge). Since spin is a conserved physical quantity, the spin of the two particles q_A and q_B of a spin singlet state must be equal and opposite when measured along the same axis, no matter what direction this axis happens to point in. The existence of spin singlet states thus raises the question of what the physical mechanism or principle is that ensures two experimenters, Alice and Bob say, will always obtain opposite spin

[11] cbx@2 See A. Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," *Physical review* 47, no. 10 (1935): 777–780.

[12] cbx@3 e.g. see D. Bohm, *Quantum Theory* (Englewood Cliffs: Prentice-Hall, 1951), p. 29, Ch. 5 sec. 3, and Ch. 22 sec. 19.

measurement results if Alice measures the spin of particle q_A, and Bob measures the spin of particle q_B along the same axis.

Naively, one would expect that if the spin of q_A were to be measured, then this would have no effect on any spin-measurement of q_B. This assumption is a special case of **Einstein's locality principle**: For two spatially separated systems S_1 and S_2, the real factual situation of the system S_2 should be independent of what is done to the system S_1.[13] If Einstein's locality principle holds we would be able to attribute a state $|\psi\rangle_A$ to particle q_A, and a state $|\chi\rangle_B$ to particle q_B, so that if Alice were to perform a Stern-Gerlach experiment on particle q_A in which one of the possible outcomes was a spin state $|\psi'\rangle_A$, then by the Born rule, the probability Alice would find q_A to be in state $|\psi'\rangle_A$ would be $|\langle\psi'|\psi\rangle_A|^2$. Likewise, if Bob were to perform a Stern-Gerlach experiment on particle q_B in which one of the possible outcomes was a spin state $|\chi'\rangle$, then the probability Bob would find q_B to be in state $|\chi'\rangle_B$ would be $|\langle\chi'|\chi\rangle_B|^2$.

Now in probability theory, we say that two events X and Y are **independent** if and only if

{indep}

$$P(X,Y) = P(X)P(Y) \tag{2.3}$$

where $P(X)$ is the probability that X occurs, $P(Y)$ is the probability that Y occurs, and $P(X,Y)$ is the probability that both X and Y both occur. Then the independence of Alice and Bob's measurement outcomes would imply that the joint probability of

[13]Einstein expressed this locality principle in his autobiographical notes: "But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former." cbx@4 Albert Einstein, *Albert Einstein, Philosopher Scientist*, ed. P. A. Schilp (Evanston, Illinois: Library of Living Philosophers, 1949), p. 85.

Alice finding q_A to be in state $|\psi'\rangle_A$, and Bob finding q_B to be in state $|\chi'\rangle_B$, would be

$$P_{A,B}(\psi', \chi'|\psi, \chi) = |\, \langle\psi'|\psi\rangle_A \,|^2 \times |\, \langle\chi'|\chi\rangle_B \,|^2 = |\, \langle\psi'|\psi\rangle_A \, \langle\chi'|\chi\rangle_B \,|^2.$$

This suggests that if we write $|\psi\rangle_A \, |\chi\rangle_B$ for the state of the composite system of both particles, then the bra-ket of $|\psi'\rangle_A \, |\chi'\rangle_B$ and $|\psi\rangle_A \, |\chi\rangle_B$ would be

$${}_B\langle\chi'| \, {}_A\langle\psi'|\psi\rangle_A \, |\chi\rangle_B = \langle\psi'|\psi\rangle_A \, \langle\chi'|\chi\rangle_B \, .$$

However, it turns out that there are physical situations in which Alice and Bob's measurements will not be independent. An important example of this phenomenon occurs when a particle known as a meson decays into two particles called muons.[14] **Mesons** are particles composed of two quarks, where **quarks** are the fundamental particles out of which protons, neutrons, and mesons are made. Both protons and neutrons are made up of three quarks. **Muons**, on the other hand, are fundamental particles similar to electrons having negative charge, but muons have a much greater mass. Both muons and electrons have spin so like silver atoms, they will be deflected in one of two directions when travelling through a Stern-Gerlach apparatus. The spin of one of these muons can then be measured by Alice, and the spin of the other muon can be measured by Bob. But remarkably, when Alice and Bob measure the spin of their respective muons along the same axis, they will always get the opposite results from one another.

Now muons, electrons, and protons, are all examples of particles know as fermions. **Fermions** are particles that have the property that no two fermions in the same system can be in exactly the same state. Thus, when a meson decays into two muons,

[14]See cbx@5 Jim J. Napolitano and J. J. Sakurai, *Modern Quantum Mechanics* (Pearson Education, 2013), p. 242.

the two muons form a single system, and so since mesons are fermions, they cannot have the same spin.

Fermions are contrasted with **bosons** which are particles which can coexist in the same system and be in exactly the same state. **Photons** which are particles of light are examples of bosons, as are mesons.

Now since the outcomes of Alice and Bob's measurements of their respective muons would be independent if the state of the composite system was of the form $|\psi\rangle_A |\chi\rangle_B$, it follows that it is not possible to represent the composite state of the two muons in this form. However, if we knew that Alice had found her particle to be in spin state $|\hat{\mathbf{a}}\pm\rangle_A$, then we would know that Bob would find his particle to be in the state $|\hat{\mathbf{a}}\mp\rangle_B$ if he were to measure his particle along the same $\hat{\mathbf{a}}$-axis as Alice made her measurement. Although neither the state $|\hat{\mathbf{a}}+\rangle_A\ |\hat{\mathbf{a}}-\rangle_B$ or $|\hat{\mathbf{a}}-\rangle_A\ |\hat{\mathbf{a}}+\rangle_B$ can describe the composite system of the two particles, it turns out that the summation of states:

{bell}

$$|\Psi_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}}(\ |\hat{\mathbf{a}}+\rangle_A\ |\hat{\mathbf{a}}-\rangle_B - |\hat{\mathbf{a}}-\rangle_A\ |\hat{\mathbf{a}}+\rangle_B). \tag{2.4}$$

can describe this composite system. We refer to the state (2.4) as a **Bell state**.[15] If the composite system is in the Bell state $|\Psi_{\text{Bell}}\rangle$, then according to the Born Rule, the probability that Alice measures her particle to be in state $|\psi\rangle$ and Bob measures his particle to be in the state $|\chi\rangle$ will be:

$$\begin{aligned} P_{A,B}(\psi,\chi|\Psi_{\text{Bell}}) &= |\ {}_B\langle\chi|\ {}_A\langle\psi|\Psi_{\text{Bell}}\rangle|^2 \\ &= \frac{1}{2}|\ \langle\psi|\hat{\mathbf{a}}+\rangle_A\ \langle\chi|\hat{\mathbf{a}}-\rangle_B - \langle\psi|\hat{\mathbf{a}}-\rangle_A\ \langle\chi|\hat{\mathbf{a}}+\rangle_B\,|^2 \end{aligned} \tag{2.5}$$

[15]By convention, the states $\frac{1}{\sqrt{2}}(\ |\hat{\mathbf{a}}+\rangle_A\ |\hat{\mathbf{a}}-\rangle_B + |\hat{\mathbf{a}}-\rangle_A\ |\hat{\mathbf{a}}+\rangle_B)$, $\frac{1}{\sqrt{2}}(\ |\hat{\mathbf{a}}+\rangle_A\ |\hat{\mathbf{a}}+\rangle_B - |\hat{\mathbf{a}}-\rangle_A\ |\hat{\mathbf{a}}-\rangle_B)$, and $\frac{1}{\sqrt{2}}(\ |\hat{\mathbf{a}}+\rangle_A\ |\hat{\mathbf{a}}+\rangle_B + |\hat{\mathbf{a}}-\rangle_A\ |\hat{\mathbf{a}}-\rangle_B)$ are also referred to as Bell states.

This means that whatever axis Bob decides to measure along, if Alice measures her particle along the $\hat{\mathbf{a}}$-axis, then the Born rule predicts that she will measure the particle to be in either the $|\hat{\mathbf{a}}+\rangle_A$-state or the $|\hat{\mathbf{a}}-\rangle_A$-state, each with probability of $\frac{1}{2}$.[16] But also, the Born rule implies that if both Alice and Bob measure their respective particles along the same $\hat{\mathbf{a}}$-axis, then the probability Bob will measure his particle to have the same spin as Alice's particle will be zero, and the probability that Bob measures his particle to have the opposite spin from Alice's particle will be one.[17] These probabilities predicted by the Born rule using the Bell state $|\Psi_{\text{Bell}}\rangle$ correspond

[16]To see this, suppose Bob performs his measurement along an arbitrary axis $\hat{\mathbf{b}}$. Then the probability Alice measures her particle to be in the $|\hat{\mathbf{a}}+\rangle$-state will be

$$\begin{aligned} P_{A,B}(\hat{\mathbf{a}}+,\hat{\mathbf{b}}+|\Psi_{\text{Bell}}) + P_{A,B}(\hat{\mathbf{a}}+,\hat{\mathbf{b}}-|\Psi_{\text{Bell}}) = \\ =&\frac{1}{2}\Big|\langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}+\rangle_A\ \langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle_B - \langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}-\rangle_A\ \langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}+\rangle_B\Big|^2 \\ &+\frac{1}{2}\Big|\langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}+\rangle_A\ \langle\hat{\mathbf{b}}-|\hat{\mathbf{a}}-\rangle_B - \langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}-\rangle_A\ \langle\hat{\mathbf{b}}-|\hat{\mathbf{a}}+\rangle_B\Big|^2 \\ =&\frac{1}{2}\big|\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle_B\big|^2 + \frac{1}{2}\big|\langle\hat{\mathbf{b}}-|\hat{\mathbf{a}}-\rangle_B\big|^2 = \frac{1}{2} \end{aligned} \tag{2.6}$$

where on the last line we have used the fact that

$$|\hat{\mathbf{a}}-\rangle_B = |\hat{\mathbf{b}}+\rangle_B\ \langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle_B + |\hat{\mathbf{b}}-\rangle\ \langle\hat{\mathbf{b}}-|\hat{\mathbf{a}}-\rangle_B$$

and

$$\langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}-\rangle_B = 1, \quad \langle\hat{\mathbf{b}}\pm|\hat{\mathbf{b}}\pm\rangle_B = 1, \quad \text{and} \quad \langle\hat{\mathbf{b}}\pm|\hat{\mathbf{b}}\mp\rangle_B = 0$$

so that

$$|\langle\hat{\mathbf{b}}+|\hat{\mathbf{a}}-\rangle_B|^2 + \langle\hat{\mathbf{b}}-|\hat{\mathbf{a}}-\rangle_B|^2 = 1.$$

[17]This is because the probability Alice and Bob will measure their particles to have the same spin will be

$$\begin{aligned} P_{A,B}(\hat{\mathbf{a}}+,\hat{\mathbf{a}}+|\Psi_{\text{Bell}}) + P_{A,B}(\hat{\mathbf{a}}-,\hat{\mathbf{a}}-|\Psi_{\text{Bell}}) = \\ =&\frac{1}{2}\Big|\langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}+\rangle_A\ \langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}-\rangle_B - \langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}-\rangle_A\ \langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}+\rangle_B\Big|^2 \\ &+\frac{1}{2}\Big|\langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}+\rangle_A\ \langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}-\rangle_B - \langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}-\rangle_A\ \langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}+\rangle_B\Big|^2 = 0, \end{aligned} \tag{2.7}$$

and the probability Alice and Bob will measure their particles to have different spins will be

$$\begin{aligned} P_{A,B}(\hat{\mathbf{a}}+,\hat{\mathbf{a}}-|\Psi_{\text{Bell}}) + P_{A,B}(\hat{\mathbf{a}}-,\hat{\mathbf{a}}+|\Psi_{\text{Bell}}) = \\ =&\frac{1}{2}\Big|\langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}+\rangle_A\ \langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}-\rangle_B - \langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}-\rangle_A\ \langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}+\rangle_B\Big|^2 \\ &+\frac{1}{2}\Big|\langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}+\rangle_A\ \langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}-\rangle_B - \langle\hat{\mathbf{a}}-|\hat{\mathbf{a}}-\rangle_A\ \langle\hat{\mathbf{a}}+|\hat{\mathbf{a}}+\rangle_B\Big|^2 = 1. \end{aligned} \tag{2.8}$$

to the probabilities observed experimentally. Also note that despite the of appearance the formula (2.4), $|\Psi_{\text{Bell}}\rangle$ is independent of the axis $\hat{\mathbf{a}}$. That is, for any other direction $\hat{\mathbf{b}}$ it can be shown that,

{bellstate2}

$$\frac{1}{\sqrt{2}}(\,|\hat{\mathbf{a}}+\rangle_A\,|\hat{\mathbf{a}}-\rangle_B - |\hat{\mathbf{a}}-\rangle_A\,|\hat{\mathbf{a}}+\rangle_B) = \frac{1}{\sqrt{2}}(\,|\hat{\mathbf{b}}+\rangle_A\,|\hat{\mathbf{b}}-\rangle_B - |\hat{\mathbf{b}}-\rangle_A\,|\hat{\mathbf{b}}+\rangle_B).^{18} \quad (2.9)$$

Therefore, if Alice had chosen to measure her particle along the $\hat{\mathbf{b}}$-axis rather than the $\hat{\mathbf{a}}$-axis, she would still obtain equal probabilities for finding her particle to be in either the state $|\hat{\mathbf{b}}+\rangle_A$ or $|\hat{\mathbf{b}}-\rangle_A$, and the same equal probabilities for Bob's measurement outcomes hold as well.

Now according to the independence criterion given by (2.3), it is clearly the case that Alice and Bob's measurement outcomes are not independent,[19] However, it is not immediately obvious that the non-independence of Alice and Bob's measurement outcome is paradoxical. That there might be something paradoxical going on

The EPR paradox, named after Einstein, Rosen, and P

[18] bellstate2pf To see this, using the transformation rules given in equation (2.2) we have

$$\begin{aligned}
&\frac{1}{\sqrt{2}}(\,|\hat{\mathbf{b}}+\rangle\,|\hat{\mathbf{b}}-\rangle - |\hat{\mathbf{b}}-\rangle\,|\hat{\mathbf{b}}+\rangle)\\
&\qquad= \frac{1}{\sqrt{2}}((\cos(\theta/2)\,|\hat{\mathbf{a}}+\rangle + \sin(\theta/2)\,|\hat{\mathbf{a}}-\rangle)(\cos(\theta/2)\,|\hat{\mathbf{a}}-\rangle - \sin(\theta/2)\,|\hat{\mathbf{a}}+\rangle)\\
&\qquad\quad - (\cos(\theta/2)\,|\hat{\mathbf{a}}-\rangle - \sin(\theta/2)\,|\hat{\mathbf{a}}+\rangle)(\cos(\theta/2)\,|\hat{\mathbf{a}}+\rangle + \sin(\theta/2)\,|\hat{\mathbf{a}}-\rangle))\\
&\qquad= \frac{1}{\sqrt{2}}(\cos(\theta/2)\,|\hat{\mathbf{a}}+\rangle\cos(\theta/2)\,|\hat{\mathbf{a}}-\rangle - \cos(\theta/2)\,|\hat{\mathbf{a}}+\rangle\sin(\theta/2)\,|\hat{\mathbf{a}}+\rangle\\
&\qquad\quad + \sin(\theta/2)\,|\hat{\mathbf{a}}-\rangle\cos(\theta/2)\,|\hat{\mathbf{a}}-\rangle - \sin(\theta/2)\,|\hat{\mathbf{a}}-\rangle\sin(\theta/2)\,|\hat{\mathbf{a}}+\rangle\\
&\qquad\quad - \cos(\theta/2)\,|\hat{\mathbf{a}}-\rangle\cos(\theta/2)\,|\hat{\mathbf{a}}+\rangle - \cos(\theta/2)\,|\hat{\mathbf{a}}-\rangle\sin(\theta/2)\,|\hat{\mathbf{a}}-\rangle\\
&\qquad\quad + \sin(\theta/2)\,|\hat{\mathbf{a}}+\rangle\cos(\theta/2)\,|\hat{\mathbf{a}}+\rangle + \sin(\theta/2)\,|\hat{\mathbf{a}}+\rangle\sin(\theta/2)\,|\hat{\mathbf{a}}-\rangle)\\
&\qquad= \frac{1}{\sqrt{2}}((\cos^2(\theta/2) + \sin^2(\theta/2))\,|\hat{\mathbf{a}}+\rangle\,|\hat{\mathbf{a}}-\rangle - (\cos^2(\theta/2) + \sin^2(\theta/2))\,|\hat{\mathbf{a}}-\rangle\,|\hat{\mathbf{a}}+\rangle)\\
&\qquad= \frac{1}{\sqrt{2}}(\,|\hat{\mathbf{a}}+\rangle\,|\hat{\mathbf{a}}-\rangle - |\hat{\mathbf{a}}-\rangle\,|\hat{\mathbf{a}}+\rangle).
\end{aligned}$$

[19] e.g. $P_{A,B}(\hat{\mathbf{a}}+,\hat{\mathbf{a}}+) = 0$, but $P_A(\hat{\mathbf{a}}+) = P_B(\hat{\mathbf{a}}+) = \frac{1}{2}$, and so the independence criterion $P_{A,B}(\hat{\mathbf{a}}+,\hat{\mathbf{a}}+) = P_A(\hat{\mathbf{a}}+)P_B(\hat{\mathbf{a}}+)$ fails to hold.

2.3 The Copenhagen Interpretation

The experiment described in the previous section implies that the behavior of Alice's and Bob's particles can't be explained in terms of local hidden variables. But this experiment also calls into question the Copenhagen interpretation of quantum physics. To explain the Copenhagen interpretation and what is problematic about it, suppose Alice has a measurement device which we will denote by $\Lambda_{\hat{\mathbf{a}}+}$ Lambdaa and which outputs the number 1 when her particle is detected at location $\hat{\mathbf{a}}+$, and outputs 0 when her particle is detected at location $\hat{\mathbf{a}}-$. Given that Alice knows that the state of both particles together is given by equation (2.4), she can work out the expectation value of her measurement $\langle\Lambda_{\hat{\mathbf{a}}+}\rangle$ by summing up the product of each probability measurement outcome with the value of each measurement. This will give an expectation value of $\langle\Lambda_{\hat{\mathbf{a}}+}\rangle = \frac{1}{2}\times 1 + \frac{1}{2}\times 0 = \frac{1}{2}$. More generally, if Alice had a measuring device Λ with N measurement outcome values $o_1, \ldots, o_N$ and with respective probabilities $p_1, \ldots, p_N$ so that $\sum_{i=1}^{N} p_i = 1$, then the expectation $\langle\Lambda\rangle$ would be given by the formula

{expectation}

$$\langle\Lambda\rangle = \sum_{i=1}^{N} p_i o_i. \tag{2.10}$$

Now given that there are no hidden variables and that equation (2.4) encodes everything that can be said about the spins of the two particle system, it is tempting to suppose that the expectation value $\langle\Lambda_{\hat{\mathbf{a}}+}\rangle$ tells us something objective about the system rather than just something about Alice's state of knowledge about the system. Given this assumption, there then arises the question of what happens when a measurement is made. According to the Copenhagen interpretation, when Bob makes his measurement, the quantum state collapses to a component of the quantum state corresponding to the measurement Bob makes. Thus, if Bob's measurement device $\Lambda_{\hat{\mathbf{a}}-}$ outputs the number

1, (i.e. Bob's particle is detected at location $\hat{\mathbf{a}}-$), then the state of the combined system would change accordingly as:

$$\frac{1}{\sqrt{2}}\big(\,|\hat{\mathbf{a}}+\rangle_A\;|\hat{\mathbf{a}}-\rangle_B - |\hat{\mathbf{a}}-\rangle_A\;|\hat{\mathbf{a}}+\rangle_B\big) \xrightarrow{\text{Collapse!!}} |\hat{\mathbf{a}}+\rangle_A\;|\hat{\mathbf{a}}-\rangle_B$$

If Bob makes his measurement first with his measurement device $\Lambda_{\hat{\mathbf{a}}-}$ outputting 1, then with probability 1 Alice's measurement device $\Lambda_{\hat{\mathbf{a}}+}$ will output 1. Hence, once Bob has made this measurement, then the expectation value for Alice's measurement will be $\langle\Lambda_{\hat{\mathbf{a}}+}\rangle = 1$. Thus, the expectation value for Alice's measurement device changes from $\frac{1}{2}$ to 1 when Bob makes his measurement.

Copenhagenproblem

Now the problem with this change in expectation value for Alice's measurement is that it will depend on whether Bob performs his measurement first or whether Alice performs her measurement first. But according to Einstein's theory of relativity, who performs their measurement first will depend on which inertial frame of reference one is in.[20] Thus, if we are moving at one velocity, it may appear that Alice makes her measurement first, whereas if we are moving at another velocity, it may appear that Bob makes his measurement first. This suggests the expectation values for Alice and Bob's measuring devices will depend on which frame of reference we are in. However, Einstein's theory of relativity tells us that scalar quantities such as the expectation values for Alice and Bob's measuring devices should be independent of which inertial frame we are in.

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[20] In special relativity, an inertial frame of reference is a spacetime coordinate system (t, x, y, z) in which all objects which have no forces acting on them have trajectories that are straight lines. Thus, we can move to another inertial frame by moving to a reference frame with constant velocity $\mathbf{v}$ with respect to the first reference frame. In the case when $\mathbf{v} = (v, 0, 0)$, Einstein's theory of special relativity tells us that under such a "boost", spacetime coordinates will transform as $(t, \mathbf{x}) \to (t', x', y', z') = (\gamma\big(t - \frac{vx}{c^2}\big), \gamma(x - vt), y, z)$ where c is the speed of light and $\gamma = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)^{-1}$.

Now many physicists would be loath to reject Einstein's theory of relativity. At the same time, many physicists are also convinced by the violation of Bell's inequality that there are no hidden variables for the spin states of particles, and hence they are convinced that the Bell state is not just a description of someone's epistemic state: rather it is a complete physical description of two coupled fermionic particles with regard to their spins. The way many physicists seek to resolve this tension between Einstein's theory of relativity and the violation of Bell's inequality is to deny the Copenhagen interpretation of quantum physics so that there is no quantum state collapse. But if one denies that there is any quantum state collapse and denies that there are any hidden variables, the question then arises of how is one meant to interpret the quantum state?

2.4 Hidden Variables

Now it is tempting to suppose that the expression of $|\hat{\mathbf{b}}+\rangle$ in terms of $|\hat{\mathbf{a}}+\rangle$ and $|\hat{\mathbf{a}}-\rangle$ merely represents our knowledge of the true spin state of a particle along the $\hat{\mathbf{a}}$ axis given our knowledge that it would be detected at location $\hat{\mathbf{b}}+$ with probability 1 should we decide to measure the particle's state along the $\hat{\mathbf{b}}$-axis. If we were to make this supposition, there would be a fact of the matter, albeit unknown to us, concerning what spin state the particle would be found to be in were we to measure its spin along the $\hat{\mathbf{a}}$-axis. And even though we might decide not to measure the spin of the particle along the $\hat{\mathbf{a}}$-axis, there would still be this hidden fact about the particle's spin in this direction. And given this supposition, since there would be no reason to suppose there was anything special about the $\hat{\mathbf{a}}$-axis, it would then be reasonable to suppose that there were hidden facts about what spin direction the particle would be found to be in for every possible axis orientation. This would mean that a complete description of the particle's spin state would require an infinite list of outcomes for all the possible orientations we could configure the magnetic field of our Stern-Gerlach apparatus. Given this assumption, as well as the assumption that it is already known that the particle would be detected at $\hat{\mathbf{b}}+$, a complete description of the particle's state could be depicted as $|\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \ldots\rangle$ or $|\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \ldots\rangle$, etc. where the ellipses would range over one of the two possible measurement outcomes for every other magnetic field orientation. However, because we would never in practice be able to perform all these experiments, and since only one such experiment would be needed to alter this infinite list,[21] nearly all of the entries in this infinite list would remain forever hidden. Hence, this would be an example of a **hidden variables** interpretation of quantum theory.

[21] In other words, it is assumed that directly measuring the particle will involve perturbing it so that its state will change.

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