[Institution]

A Defense of One-World Quantum Physics

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by

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Contents

0.1 A description of Kent's Interpretation of Quantum Physics

In this section I will provide an account of Kent's interpretation of quantum physics focusing on the ideas Kent presents in his 2014 paper.[1] This section is primarily descriptive. We'll wait until the next section to consider how Kent's interpretation addresses the issues Butterfield raises.

Kent's interpretation of quantum physics has some similarities in common with the pilot wave interpretation. Firstly, there is no quantum state collapse in Kent's interpretation. Secondly, some additional values beyond standard quantum theory (i.e. in addition to the quantum state) are included in Kent's interpretation. And thirdly, Kent's interpretation is a one-world interpretation of quantum physics. I'll consider these three features of Kent's interpretation in some detail as I describe his theory. I'll then present an account of his toy model that provides a simple example of how the ideas of his theory fit together.

0.1.1 The No-collapse Feature of Kent's Interpretation

We first consider the no-collapse feature of Kent's interpretation. This is a feature that belongs both to the many worlds interpretation and to the pilot wave interpretation. In all three interpretations, the quantum state deterministically evolves according to the Schrödinger equation. The Schrödinger equation itself describes how a quantum state evolves over time when there are no outside influences. The precise formula for the Schrödinger equation need not concern us here, but all we need to know is that the Schrödinger equation determines a so-called **unitary operator** $U(t', t)$.

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cbx@1
[1] Adrian Kent, "Lorentzian Quantum Reality: Postulates and Toy Models," 2014, https://doi.org/10.1098/rsta.2014.0241, eprint: arXiv:1411.2957.

What this means is that if a system is in a state $|\psi\rangle$ at time t, then it will be in the state $|\psi'\rangle = U(t',t)\,|\psi\rangle$ at time t'. A unitary operator U has the property that if $|\psi'\rangle = U\,|\psi\rangle$ and $|\chi'\rangle = U\,|\chi\rangle$, then

{unitarycond}
$$\langle\chi'|\psi'\rangle = \langle\chi|\psi\rangle\,.^2 \tag{1}$$

Under the Copenhagen interpretation, a system will evolve unitarily for the most part, but there will typically be a non-unitary change in the state describing the system whenever there is a measurement.[3] However, in non-collapse models such as the pilot wave interpretation, the many-worlds interpretation, and Kent's interpretation, the quantum state always evolves unitarily.

[2]A unitary operator U must also be linear so that for any two states $|\psi\rangle$ and $|\phi\rangle$ and complex numbers α and β, we have

$$U(\alpha\,|\psi\rangle + \beta\,|\phi\rangle) = \alpha U\,|\psi\rangle + \beta U\,|\phi\rangle\,,$$

and furthermore, a unitary operator must have the property that it is invertible: there is a linear operator U^{-1} such that UU^{-1} and $U^{-1}U$ are the identity operator I, i.e. $U^{-1}U\,|\psi\rangle = UU^{-1}\,|\psi\rangle = |\psi\rangle$ for any state $|\psi\rangle$.

[3]Note that to say that the change in a state is non-unitary when a measurement is made is not to say that there is a non-unitary collapse operator that maps the quantum state to an eigenstate of some observable. Such a mapping would not make sense, since the collapse is not deterministic given the initial state. However, one could have a well-defined mapping from a time value t to the quantum state of the system $|\psi(t)\rangle$ at time t. We then say that a system changes unitarily if and only if there is a unitary operator $U(t_1,t_0)$ for any two times t_0 and t_1 such that whenever the state of the system at time t_0 is given by $|\psi(t_0)\rangle$, then the state of the system at time t_1 must be given by $|\psi(t_1)\rangle = U(t_1,t_0)\,|\psi(t_0)\rangle$, and that for an intermediate time t, $U(t_1,t_0) = U(t_1,t)U(t,t_0)$. So to say that the change in a state is non-unitary when a measurement is made is to say that the state $|\psi(t)\rangle$ describing the system does not change unitarily in the process of making a measurement. Now to see why this is the case under the Copenhagen interpretation, we suppose that at time t_0 a system is in the state $|\psi(t_0)\rangle$ and that as long as no measurements are made up until a time $t \geq t_0$, the state evolves to a state $|\psi^{(U)}(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle$ where $U(t,t_0)$ is a unitary operator determined by Schrödinger's equation. Furthermore, we suppose that there is a measurable quantity with which we associate an observable $\hat{\boldsymbol{O}}$ so that whenever the state of the system is an eigenstate of $\hat{\boldsymbol{O}}$, the value of the measurable quantity for the system will be a determinate value and equal to the corresponding eigenvalue of $\hat{\boldsymbol{O}}$. At time t_0, we can express $|\psi(t_0)\rangle$ as a linear combination

$$|\psi(t_0)\rangle = \sum_i c_i\,|s_i(t_0)\rangle$$

where the $|s_i(t_0)\rangle$ are eigenstates of $\hat{\boldsymbol{O}}$ with distinct eigenvalues. As long as no measurement is made, this will evolve as

$$|\psi^{(U)}(t)\rangle = \sum_i c_i U(t,t_0)\,|s_i(t_0)\rangle\,.$$

We assume that as the state $|s_i(t_0)\rangle$ evolves to the state $|s_i(t_1)\rangle$ from time t_0 to t_1, it remains an eigenstates of $\hat{\boldsymbol{O}}$ with approximately the same eigenvalue. This assumption is based on the principle

0.1.2 The Additional Values of Kent's Interpretation

additional

Secondly, like the pilot wave interpretation, some additional values beyond standard quantum theory (i.e. in addition to the quantum state[4]) are included in Kent's interpretation. In the pilot wave interpretation, these additional values are the positions and momenta of all the particles, whereas in Kent's interpretation, the additional values specify the mass-energy density on a three-dimensional distant future hypersurface in spacetime. We refer to this hypersurface as S.

To describe the nature of this three-dimensional hyperspace S, we will need some terminology and notation used in special relativity. A **spacetime location** is a point (x^1, x^2, x^3) in three-dimensional space at a particular instant of time t, and hence described by four numbers (x^0, x^1, x^2, x^3) where $x^0 = ct$ and where c is the speed of light.[5] We will use the convention of boldface type to depict spatial locations,

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that in practice, performing a measurement is not instantaneous, but rather must take place over a time interval, and so the eigenstate and eigenvalue must be stable enough over this time interval so as to specify a definite outcome. We also assume that when the system is already in an eigenstate $|s_i(t_0)\rangle$ of the observable $\hat{\boldsymbol{O}}$, it will evolve unitarily as $|s_i(t)\rangle = U(t, t_0)\,|s_i(t_0)\rangle$ for t between t_0 and t_1, and that performing the measurement corresponding to $\hat{\boldsymbol{O}}$ will have no effect on the system when it is an eigenstate $|s_i(t)\rangle$ of $\hat{\boldsymbol{O}}$ – otherwise we couldn't be sure that whenever we looked at the measurement readout that we weren't changing the value of the quantity we were trying to measure.

Now according to the Copenhagen interpretation, when the measurement corresponding to $\hat{\boldsymbol{O}}$ is made, the system must enter into one of the eigenstates of the observable $\hat{\boldsymbol{O}}$, and at time t_1 shortly after the measurement has been made, the probability the system will be in the $|s_i(t_1)\rangle$-state given that it was in the $|\psi(t_0)\rangle$-state at time t_0 will be $|\left\langle s_i(t_1)\middle|\psi^{(U)}(t_1)\right\rangle|^2$ in accordance with the Born rule. So taking $|\psi(t_1)\rangle$ to be proportional to $|s_i(t_1)\rangle$ for some i, we see that for $j \neq i$, $\langle s_j(t_1)|\psi(t_1)\rangle = 0$. This is because eigenstates of a Hermitian operator that have different eigenvalues must be orthogonal. However, since $U(t_1, t_0)$ is unitary,

$$\langle s_j(t_1)|\psi^{(U)}(t_1)\rangle = \langle s_j(t_0)|\psi^{(U)}(t_0)\rangle = c_j.$$

So we see that $|\psi^{(U)}(t_1)\rangle \neq |\psi(t_1)\rangle$ if $\psi(t_0)$ is not initially in an eigenstate of $\hat{\boldsymbol{O}}$, and hence $|\psi(t)\rangle$ doesn't evolve unitarily up to time t_1 as $|\psi^{(U)}(t)\rangle$ does.

[4]We may wish to think of these additional values as hidden variables, but we are not obliged to since we don't speculate on whether these additional variables are necessarily unknowable. Rather, we just see them as supplementing the quantum state so as to provide a complete description of the system.

[5]Multiplying time by the speed of light means that x^0 is a distance like x^1, x^2, and x^3.

e.g. $\boldsymbol{x} = (x^1, x^2, x^3)$, and non-boldface type to depict a spacetime location, e.g. $x = (x^0, x^1, x^2, x^3)$.

Now a key insight of special relativity is that there is no such thing as absolute time. So for instance, two spacetime locations might seem to be simultaneous from one frame of reference, but another person travelling at a different velocity would judge with equal propriety the same two spacetime locations to be non-simultaneous. But it is not the case that for any two spacetime locations we can always find a frame of reference in which the two spacetime locations are simultaneous – sometimes this is not possible. But we refer to spacetime locations that could be simultaneous in some frame of references as being **spacelike-separated**. For example, the two spacetime locations o and a in figure 1 are spacelike-separated.

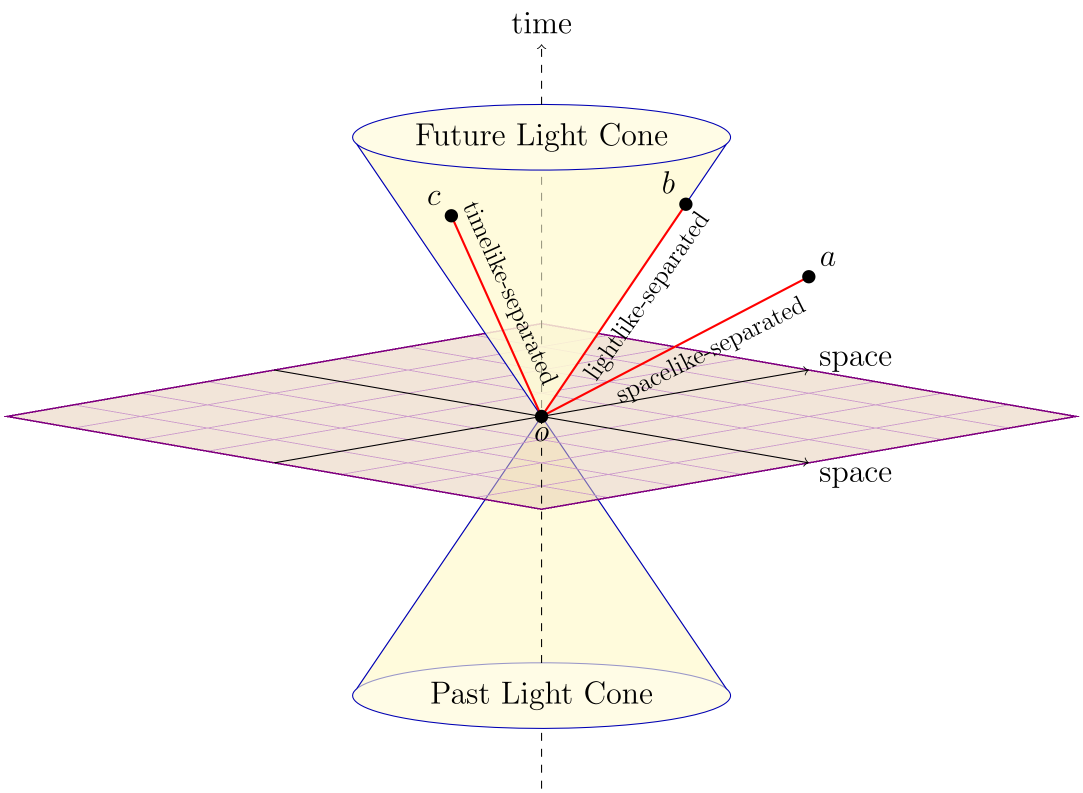

Figure 1: The meaning of spacelike, timelike and lightlike-separation when there are two space dimensions and one time dimension.

`cone`

There are also spacetime locations in spacetime that could be connected by a beam of light such as the two spacetime locations o and b in figure 1. Such spacetime locations are referred to as being **lightlike-separated**. For any given spacetime location, the spacetime locations that are lightlike-separated from it form two cones[6] called the future light cone and the past light cone as shown in figure 1. Because light appears to travel at the same speed no matter what frame of reference one uses, the light cone of a spacetime location remains invariant when one change from one reference frame to another. In other words, if another spacetime location lies on the light cone of a spacetime location in one frame of reference, then it must lie on the light cone of this spacetime location in every frame of reference.

Figure 1 also depicts two spacetime locations o and c that are **timelike-separated**. Such spacetime locations lie within the light cones of each other, and when two spacetime locations are timelike-separated, it is always possible to choose a frame of reference in which the two spacetime locations are located at the same point in space, but with one spacetime location occuring after the other depending on which spacetime location is in the future light cone of the other.

Now a three-dimensional hypersurface S in spacetime is a maximal[7] three-dimensional surface in which all the spacetime locations of S are spacelike-separated. Kent assumes that this hypersurface S is in the distant future of an expanding universe so that nearly all the particles that can decay have already done so, and that all the particles

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[6]Strictly speaking, the set of spacetime locations that are lightlike-separated from a give spacetime location form the surface of a cone rather than a cone (which is a convex object). But among physicists, the terminology light cone has stuck.

[7]That is, it cannot be extended any further along any of its three dimensions, so it is not a small local surface contained within a boundary.

that are not bound together are very far from each other so that the probability of any particle collisions is very small. In other words, all the interesting physics in the universe has played its course before S.

At every spacetime location $x \in S$, there is a quantity $T_S(x)$ called the **mass-energy density**.[8] The important thing to note about $T_S(x)$ is that it does not depend on which frame of reference one is in.[9] This property is in contrast to many physical properties that do depend on which frame of reference one is in. For example, the kinetic energy of an object will depend on the calculated velocity of the object, and this velocity will in turn depend on the frame of reference in which this calculation is done.

Now in order to specify the additional values that Kent's interpretation requires, we need to discuss the Tomonaga-Schwinger picture of relativistic quantum physics.[10] In order to explain their formulation, it is helpful to consider first the distinction between the Heisenberg picture and the Schrödinger picture of quantum mechanics.

In the **Heisenberg picture**, the states describing a system do not change over time. Rather, the observables change over time. So for instance, if there is a time-independent state $|\Phi\rangle$ describing a system and there is some measurable quantity whose expectation value we wish to know at time t given the state $|\Phi\rangle$, then we will need a time dependent observable $\hat{\boldsymbol{O}}(t)$, say, corresponding to the measurable

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[8] The definition of $T_S(x)$ will be discussed in section 0.1.2.

[9] The reason for why this is will be discussed in section 0.1.2.

[10] See Julian Schwinger, "Quantum Electrodynamics. I. A Covariant Formulation," *Physical review* 74, no. 10 (1948): 1439–1461; S. Tomonaga, "On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields," *Progress of theoretical physics* (Tokyo) 1, no. 2 (1946): 27–42

quantity at time t from which we can calculate the expectation value $\langle\Phi|\hat{\boldsymbol{O}}(t)|\Phi\rangle$ at time t given the system is in state $|\Phi\rangle$. In the context of quantum field theory, any observable $\hat{\boldsymbol{O}}(t)$ in the Heisenberg picture will be expressible as a sum (or integral) of observables of the form $\hat{\boldsymbol{O}}(t, \boldsymbol{x})$, where $\hat{\boldsymbol{O}}(t, \boldsymbol{x})$ is an observable of some quantity at a particular time t and spatial location $\boldsymbol{x}$.[11]

The Heisenberg picture is contrasted with the **Schrödinger picture** in which the observables do not change over time, but rather the states change over time. So for instance, if there is a time-dependent state $|\Phi(t)\rangle$ describing a system at a specific time t and there is some measurable quantity whose expectation value we wish to know at time t given the state $|\Phi(t)\rangle$, then we will only require a time-independent observable $\hat{\boldsymbol{O}}$, say, corresponding to the measurable quantity from which we can calculate the expectation value $\langle\Phi(t)|\hat{\boldsymbol{O}}|\Phi(t)\rangle$. As in the Heisenberg picture, we can introduce a spatial dependence into the observables so that any observable $\hat{\boldsymbol{O}}$ is expressible as a sum (or integral) of observables of the form $\hat{\boldsymbol{O}}(\boldsymbol{x})$ where $\hat{\boldsymbol{O}}(\boldsymbol{x})$ is an observable of some quantity at a particular spatial location $\boldsymbol{x}$.

Now despite the Schrödinger and Heisenberg pictures taking these different perspectives, they are nevertheless physically equivalent. This is because in both pictures, there is a unitary operator $U(\Delta t)$ for any time interval Δt such that $U(\Delta t)\,|\Phi(t)\rangle = |\Phi(t + \Delta t)\rangle$,

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[11] For example, in quantum electrodynamics (which is one kind of quantum field theory), the observables will be expressible in terms of fields such as the four-vector potential $A^\mu(x)$ and the bispinor field $\psi(x)$ which are defined at all spacetime locations $(t, \boldsymbol{x}) = (t, x, y, z)$. The four-vector potential $A^\mu(x)$ can be used to determine the electromagnetic field, and the bispinor field $\psi(x)$ can be used to determine the electric current density. In the Heisenberg picture, these fields will have corresponding Hilbert space operators at each spacetime location x from which expectation values can be calculated at the spacetime location x for a given time-independent state.

and $U(\Delta t)\hat{\boldsymbol{O}}(t,\boldsymbol{x})U(\Delta t)^{-1} = \hat{\boldsymbol{O}}(t+\Delta t,\boldsymbol{x})$. Thus, given the Schrödinger picture, to get the Heisenberg picture, all we need to do is the following: firstly, we fix a time t_0 and let all the states of the form $|\Phi(t_0)\rangle$ at time t_0 in the Schrödinger picture be the state space for the Heisenberg picture; then for any Schrödinger picture observable $\hat{\boldsymbol{O}}(\boldsymbol{x})$, we define the corresponding Heisenberg picture time-dependent observable

$$\hat{\boldsymbol{O}}(t,\boldsymbol{x}) = U(t-t_0)\hat{\boldsymbol{O}}(\boldsymbol{x})U(t-t_0)^{-1}.$$

Conversely, to move from the Heisenberg picture to the Schrödinger, we first fix a reference time t_0. Then for any state $|\Phi\rangle$ and observable $\hat{\boldsymbol{O}}(t,\boldsymbol{x})$ in the Heisenberg picture, the corresponding Schrödinger picture time-dependent state at time t will be $U(t-t_0)\,|\Phi\rangle$, and the corresponding Schrödinger picture time-independent observable will be $\hat{\boldsymbol{O}}(t_0,\boldsymbol{x})$.

Now if there is a quantity we wish to measure at time t_0 with corresponding observable $\hat{\boldsymbol{O}}(\boldsymbol{x}) \equiv \hat{\boldsymbol{O}}(t_0,\boldsymbol{x})$, then in both pictures, the expectation value of this measurable quantity given $|\Phi\rangle \equiv |\Phi(t_0)\rangle$ will be

{heisshrodeq}

$$\langle\Phi(t_0)|\hat{\boldsymbol{O}}(\boldsymbol{x})|\Phi(t_0)\rangle = \langle\Phi(t_0)|\hat{\boldsymbol{O}}(t_0,\boldsymbol{x})|\Phi(t_0)\rangle = \langle\Phi|\hat{\boldsymbol{O}}(t_0,\boldsymbol{x})|\Phi\rangle \tag{2}$$

Thus, whatever picture we choose, it will make no difference to the calculated expectation values of observables – in other words, the two pictures are physically equivalent.

Now although it is easy to move between both the Schrödinger and Heisenberg pictures, they both give a privileged status to hypersurfaces of the form $t = \text{const}$. However, according to special relativity, there are no privileged hypersurfaces. One of the advantages of the Tomonaga-Schwinger picture is that it gives no privileged status to any class of hypersurfaces, but rather all hypersurfaces are placed on the same footing. We are going to see that the expectation value $\langle\Phi(t_0)|\hat{\boldsymbol{O}}(t_0,\boldsymbol{x})|\Phi(t_0)\rangle$ of equation (2) is a special case of what Tomonaga and Schwinger consider more generally. We first note that if we can calculate $\langle\Phi(t_0)|\hat{\boldsymbol{O}}(t_0,\boldsymbol{x})|\Phi(t_0)\rangle$ for any t_0 and any $\boldsymbol{x}$, then we can calculate all the expectation values that might interest us. But we also note that in the expectation value $\langle\Phi(t_0)|\hat{\boldsymbol{O}}(t_0,\boldsymbol{x})|\Phi(t_0)\rangle$, the $|\Phi(t_0)\rangle$-state is the state of a hypersurface $t = t_0$, and $(t_0,\boldsymbol{x})$ is a spacetime location on this hypersurface. Now if we are to place all hypersurfaces on the same footing, then in specifying expectation values, we should be just as content in specifying expectation values of the form $\langle\Psi[S]|\hat{O}(x)|\Psi[S]\rangle$, where S is any hypersurface, $|\Psi[S]\rangle$ is any state of this hypersurface,[12] x is any spacetime location on the hypersurface S, and where $\hat{O}(x)$ is any observable of S.[13] The Tomonaga-Schwinger picture thus works with states of the form $|\Psi[S]\rangle$ describing a hypersurface S, and observables of the form $\hat{O}(x)$ acting on the state space of the hypersurface S from which one can calculate the expectation value $\langle\Psi[S]|\hat{O}(x)|\Psi[S]\rangle$.

In order to construct $|\Psi[S]\rangle$ and $\hat{O}(x)$, Schwinger introduces a unitary operator $U[S]$ that maps the $|\Phi\rangle$-state of the Heisenberg picture to the corresponding $|\Psi[S]\rangle$-state that describes the state of the hypersurface S, i.e. $|\Psi[S]\rangle = U[S]\,|\Phi\rangle$. Schwinger then

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[12]The convention of using square brackets such as in $|\Psi[S]\rangle$ indicates that the thing in question is a functional. Functions and functionals are closely related. A function f is a mapping from one set (the domain) to another set (the codomain), such that each input yields a single output. The typical convention is to use round brackets to denote the output, e.g. $f(x)$ where x is the input. A functional g, on the other hand, is a function that maps a space of functions or other mathematical objects (such as surfaces or volumes) to a some value. The typical convention is to use square brackets to denote the output, e.g. $g[y]$ where y is the input function or other mathematical object. So in the present case, $|\Phi[\cdot]\rangle$ is a functional that takes a surface S as input to produce a state to $|\Phi[\cdot]\rangle$ as output.

[13]Here I am following the convention of Schwinger of always using non-boldface type to indicate Tomonaga-Schwinger picture observables, and boldface type to indicate Heisenberg picture observables.

defines the observable

$$\hat{O}(x) = U[S]\hat{\boldsymbol{O}}(x)U[S]^{-1} \tag{3}$$

{tsobservable}

on S where x is any spacetime location on S, and where $\hat{\boldsymbol{O}}(x)$ is any Heisenberg picture observable. Ostensibly, $\hat{O}(x)$ depends on the surface S, but Schwinger shows that under conditions that are readily satisfied, $\hat{O}(x)$ is independent of the hypersurface S.[14] Also, since

$$\langle\Psi[S]|\hat{O}(x)|\Psi[S]\rangle = \langle\Phi|\hat{\boldsymbol{O}}(x)|\Phi\rangle\,,$$

the Tomonaga-Schwinger picture will give the same physics as the Heisenberg and Schrödinger picture. In order to avoid our notation becoming cluttered, we will write $|\Psi\rangle$ instead of $|\Psi[S]\rangle$, and say that $|\Psi\rangle$ is a state of the hypersurface S, and we will speak of the Hilbert space H_S of all such states of the hypersurface S so that we can write $|\Psi\rangle \in H_S$.[15]

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[14]The required condition is that

$$i\hbar\frac{\delta U[S]}{\delta S(x)} = \mathcal{H}(x)U[S]$$

where $\mathcal{H}(x)$ is a Hermitian operator that is a Lorentz invariant function of the field quantities at the spacetime location x and has the dimensions of an energy density, and where the functional derivative $U[S]$ is given by

$$\frac{\delta U[S]}{\delta S(x)} = \lim_{\delta\omega\to 0}\frac{U[S'] - U[S]}{\delta\omega}$$

where S' is a surface that only differs from S in the vicinity of x, and where $\delta\omega$ is the volume enclosed by S and S'. The Hermitian operator

$$\mathcal{H}(x) = -(1/c)j^\mu(x)A_\mu(x)$$

has the desired property where $j^\mu(x)$ is the current density and where $A^\mu(x)$ is the four-vector potential of the electromagnetic field. With this choice for $\hat{H}(x)$, Schwinger shows that $\Box A^\mu(x) = 0$ and $\partial_\mu A^\mu(x)\,|\Psi[S]\rangle = 0$, where $\Box = \partial_\mu\partial^\mu$ is the d'Alembert operator – see Schwinger, "Quantum Electrodynamics. I. A Covariant Formulation," p. 1449-1450.

[15]Though to be clear, the H_S are really identical for all hypersurfaces S since each H_S is the image of the unitary operator $U[S]$ acting on the Heisenberg-picture Hilbert space, and the image of a unitary operator is always equal to the Hilbert space it is acting on.

We are now in a position to come back the question of what the additional values of Kent's interpretation are. As mentioned on page 8, for a given hypersurface S, there will be a mass-energy density $T_S(x)$. Corresponding to this, there will be a Heisenberg picture observable $\hat{\boldsymbol{T}}_S(x)$, and from this we can construct the Tomonaga-Schwinger observable $\hat{T}_S(x) = U[S]\hat{\boldsymbol{T}}_S(x)U[S]^{-1}$.[16] These mass-energy density observables have the property that if x and y are any two spacetime locations of S, then $\hat{T}_S(x)$ and $\hat{T}_S(y)$ will commute. In other words,

$$\hat{T}_S(x)\hat{T}_S(y) = \hat{T}_S(y)\hat{T}_S(x).$$

The commutativity of all the $\hat{T}_S(x)$ for $x \in S$ means that if $|\Psi\rangle \in H_S$ is an eigenstate of $\hat{T}_S(x)$, then for any $y \in S$, $\hat{T}_S(y)\,|\Psi\rangle$ is also an eigenstate of $\hat{T}_S(x)$ with the same eigenvalue as $|\Psi\rangle$. The invariance of any $\hat{T}_S(x)$-eigenspace[17] under the action of $\hat{T}_S(y)$ means that we can create an orthonormal basis of H_S consisting of simultaneous eigenstates of both $\hat{T}_S(x)$ and $\hat{T}_S(y)$, albeit with different eigenvalues.[18] But because x and y are arbitrary points of S, this means we can construct an orthonormal basis $\{\,|\Psi^{(i)}\rangle : i\}$ of H_S such that $\hat{T}_S(x)\,\big|\Psi^{(i)}\big\rangle = \tau_S^{(i)}(x)\,\big|\Psi^{(i)}\big\rangle$ for all $x \in S$, where $\tau_S^{(i)}(x) \geq 0$ is a possible energy-density measurement over the whole of S.[19] simultaneous We will refer to a state $|\Psi\rangle$ as a **simultaneous $\hat{T}_S$-eigenstate**, and a real valued function τ_S defined on

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[16] Note that $\hat{T}_S(x)$ will depend on S. The remark above about $\hat{O}(x)$ not depending on S does not apply here since the independence of $\hat{O}(x)$ from S assumes that the Heisenberg picture observable $\hat{\boldsymbol{O}}(x)$ is independent of S, but this is not the case for $\hat{\boldsymbol{T}}_S(x)$.

[17] An eigenspace of a Hermitian operator $\hat{O}$ acting on a Hilbert space H is just the space of all the eigenstates of $\hat{O}$ in H which have the same eigenvalue.

[18] This is because any $\hat{T}_S(x)$-eigenspace is itself a Hilbert space on which $\hat{T}_S(y)$ acts as a Hermitian operator, so by (**??**), we can find an orthonormal basis of states $\{|\psi_1\rangle, \ldots |\psi_N\rangle\}$ of the $\hat{T}_S(x)$-eigenspace and real numbers $\tau^{(1)}(y), \ldots, \tau^{(N)}(y)$ such that $\hat{T}_S(y)\,|\psi_i\rangle = \tau^{(i)}(y)\,|\psi_i\rangle$ for $i = 1, \ldots, N$. Hence each of the $|\psi_i\rangle$ will be simultaneous eigenstates of both $\hat{T}_S(x)$ and $\hat{T}_S(y)$.

[19] We will gloss over glosssim the details of how to make this rigorous for continuous variables x and continuous indices i. It is sufficient to approximate the continuous variables and indices as discrete variables and indices when thinking about the simultaneous $\hat{T}_S(x)$-eigenspaces, and one can choose the granularity of this approximation to achieve whatever level of accuracy one desires.

the whole of S a **simultaneous $\hat{T}_S$-eigenvalue** if and only if $\hat{T}_S(x)\,|\Psi\rangle = \tau_S(x)\,|\Psi\rangle$ for all $x \in S$.

The additional values beyond standard quantum theory that are included in Kent's interpretation are given by one of these simultaneous $\hat{T}_S$-eigenvalues τ_S that described a possible outcome for an energy-density measurement over the whole of S. But although we speak of the measurement of $T_S(x)$ over S as being $\tau_S(x)$, this is only a notional measurement. Thus, we speak of the measurement of $T_S(x)$ on S only to mean that $T_S(x)$ has a determinate value for every $x \in S$ despite the quantum state of S in general being in a superposition of $\hat{T}_S(x)$-eigenstates for any given $x \in S$. How this determination of $T_S(x)$ comes about is up to one's philosophical preferences. For example, one could suppose that it was simply by divine fiat that this determination of $T_S(x)$ came about.[20]

Nevertheless, the particular density $\tau_S(x)$ which is found to describe S can't be absolutely anything. Rather, we suppose there is a much earlier hypersurface S_0 which is described by a state $|\Psi_0\rangle$ belonging to a Hilbert space H_{S_0} as shown in figure 2. It is assumed that all physics that we wish to describe takes place between these two hypersurfaces S_0 and S. In figure 2, we therefore let y depicts a generic spacetime location that we wish to describe.

If we now define $U_{SS_0} = U[S]U[S_0]^{-1}$, SchwingerUnitaryOP then U_{SS_0} will be a unitary operator that maps states in H_{S_0} such as $|\Psi_0\rangle$ to states in H_S. Then the probability $P(\Psi|\Psi_0)$ that S will be found to be in the state $|\Psi\rangle$ with mass-energy density $\tau_S(x)$ given that S_0 was

[20] I will discuss my philosophical preference in the final chapter.

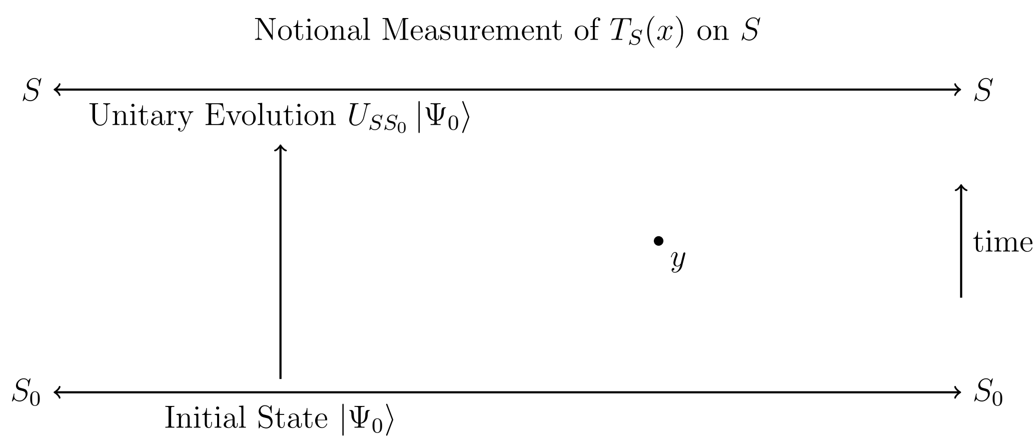

Figure 2: A notional measurement of $T_S(x)$ is made for all $x \in S$. The simultaneous $\hat{T}_S$-eigenstate $|\Psi\rangle$ with $\hat{T}_S(x)\,|\Psi\rangle = \tau_S(x)\,|\Psi\rangle$ is selected with probability $|\langle\Psi|U_{SS_0}|\Psi_0\rangle|^2$. The values $\tau_S(x)$ obtained for $T_S(x)$ are then used to calculate the physical properties at the spacetime location y.

S1

initially in the state $|\Psi_0\rangle$ will be given by the Born Rule (see page **??**):

{bornpsi}

$$P(\Psi|\Psi_0) = |\langle\Psi|U_{SS_0}|\Psi_0\rangle|^2. \tag{4}$$

It's possible that there might be several different states of H_S that have the same mass-energy density $\tau_S(x)$ for all $x \in S$, but one of these states is still realized with probability given by equation (4). But it is the mass-energy density τ_S itself rather than one of the eigenstates with mass-energy density τ_S that constitute the additional values that Kent adds to standard quantum theory.

Also note that if every simultaneous $\hat{T}_S$-eigenstate $|\Psi\rangle$ with simultaneous $\hat{T}_S$-eigenvalue τ_S satisfies $|\langle\Psi|U_{SS_0}|\Psi_0\rangle| = 0$, then by (4), τ_S will not be a possible measurement outcome for T_S given $|\Psi_0\rangle$. For this reason, we can't expect the measurement outcome of T_S on S to be absolutely anything.

0.1.3 The One-World Feature of Kent's Interpretation

The third similarity Kent's interpretation shares with the pilot wave interpretation is that it is a one-world interpretation of quantum physics. It will be helpful to contrast this with the many-worlds interpretation.

Unlike the many-worlds interpretation, there are no superpositions of living and dead cats in Kent's interpretation. Recall that in the many-worlds interpretation, Schrödinger will still only observe his cat to be either dead or alive, and not both dead and alive, but Schrödinger himself goes into a superposition of observing his cat to be alive and his cat to be dead. In the many-worlds interpretation, there is thus a difference between observing something to be so, and something actually being so: the observation is of a particular outcome, but the reality is a superposition of different outcomes.

To capture this distinction between observation and reality, Bell speaks of **beables**. beabledef Bell introduces the term beable when speculating on what would be a more satisfactory physical theory than quantum physics currently has to offer.[21] Bell says that such a theory should be able to say of a system not only that such and such is observed to be so, but that such and such be so. In other words, a more satisfactory theory would be a theory of beables rather than a theory of observables. On the macroscopic level, these beables should be the underlying reality that gives rise to all the familiar things in the world around us, things like cats, laboratories, procedures, and so on. For example proponents of the pilot wave interpretation believe that the beables are

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[21] See cbx@5 J. S. Bell, "Subject and Object," in *Speakable and unspeakable in quantum mechanics*, 2nd ed. (Cambridge: Cambridge University Press, 2004), 40–44

all the particles each with their precise position and momentum. But whatever these beables are, it is because of them that a scientist can observe a physical system to be in such and such a state. Thus, observables are ontologically dependent on beables.

Now the beables in Kent's one world interpretation are expressed in terms of a physical quantity called the **stress-energy tensor** $T^{\mu\nu}(y)$. stressenergy For any spacetime location y, the stress-energy tensor $T^{\mu\nu}(y)$ is an array of 16 values corresponding to each combination of $\mu, \nu = 0, 1, 2,$ or 3. The value $T^{00}(y)$ is the energy density at y divided by c^2,[22] whereas the other values of $T^{\mu\nu}(y)$ indicate how much energy and momentum flow across different surfaces in the neighborhood of y.

It was mentioned in the previous section that for any spacetime location $x \in S$, there is an observable $\hat{T}_S(x)$ acting on H_S. It turns out that for any $\mu, \nu = 0, 1, 2,$ or 3, there is also an observable $\hat{T}^{\mu\nu}(x)$ acting on H_S, such that if $|\Psi\rangle \in H_S$ is a simultaneous eigenstate of $\hat{T}^{\mu\nu}(x)$ with eigenvalue $\tau^{\mu\nu}(x)$ for all $x \in S$, then $|\Psi\rangle$ corresponds to a state of S in which $T^{\mu\nu}(x)$ is $\tau^{\mu\nu}(x)$ for all $x \in S$.[23] Moreover, the observable $\hat{T}_S(x)$ is expressible in terms of the $\hat{T}^{\mu\nu}(x)$-observables.[24]

Now the beables in Kent's interpretation are defined at each spacetime location y that occurs after S_0 and before S. For such a spacetime location y, the beables will be determinate values of the stress-energy tensor $T^{\mu\nu}(y)$, but calculated from the expectation of the observable $\hat{T}^{\mu\nu}(y)$ conditional on the energy-density on S being

---

[22]This is not to be confused with the mass-energy density $T_S(x)$ defined for x on a hypersurface S. As will be shown in section 0.2.1, all 16 elements of $T^{\mu\nu}(x)$ will typically be needed to calculate $T_S(x)$.

[23]Note however, that such a simultaneous eigenstate is only for a fixed choice of μ and ν, since in general, $\hat{T}^{\mu\nu}(x)$ and $\hat{T}^{\mu'\nu'}(x)$ will not commute for $\mu \neq \mu'$ or $\nu \neq \nu'$.

[24]See section 0.2.1 for an explanation for why this is so.

given by $\tau_S(x)$ for all $x \in S$ outside the light cone of y. We will come back to the question of why we can't include any information about $\tau_S(x)$ for $x \in S$ within the light cone of y once we have explained how these conditional expectations are calculated. deadlivecat

So firstly, we recall the definition of expectation in equation (**??**) and the expectation formula (**??**) for an observable. If the beable in question was simply the expectation of $\hat{T}^{\mu\nu}(y)$ without conditioning on the value of the energy-density on S, then the $T^{\mu\nu}(y)$-beable would just be $\langle\Psi'|\hat{T}^{\mu\nu}(y)|\Psi'\rangle$ where $|\Psi'\rangle = U_{S'S_0}|\Psi_0\rangle$ for any hypersurface S' that goes through y.[25] However, such a beable would give a description of reality that was very different from what we observe – for instance, in a Schrödinger cat-like experiment, there would be a stress-energy tensor distribution corresponding to both the cat being alive and the cat being dead in the same world as depicted in figure 3.

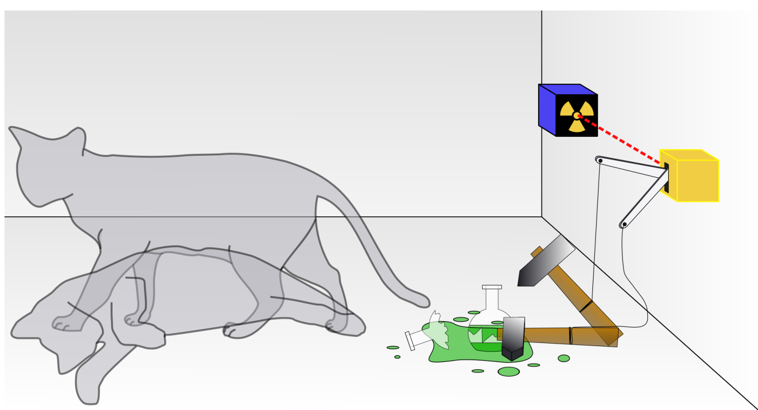


Figure 3: A depiction of Schrödinger's cat being both dead and alive.[26] deadlivecat

[25] This can be done such that $\langle\Psi'|\hat{T}^{\mu\nu}(y)|\Psi'\rangle$ does not depend on the hypersurface S' other than the fact that it contains y. For more details see Schwinger, "Quantum Electrodynamics. I. A Covariant Formulation."

[26] Original by Dhatfield. This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license. Source: https://commons.wikimedia.org/wiki/File:Schrodingers_cat.svg

To overcome this defect, information about the mass-energy density on S is required, specifically the values of $\tau_S(x)$ for $x \in S^1(y)$ where $S^1(y)$ is defined to consist of all the spacetime locations of S outside the light cone of y as depicted in figure 4.

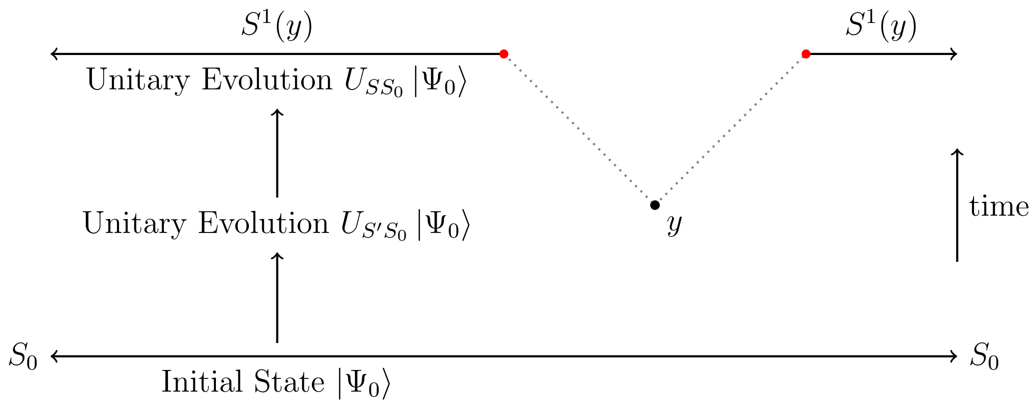


Figure 4: The set $S^1(y)$ consists of all the spacetime locations of S outside the light cone of y. The $T^{\mu\nu}(y)$-beables are calculated using the initial state $|\Psi_0\rangle$ together with the values of $\tau_S(x)$ for $x \in S^1(y)$.

S2

So in the case of Schrödinger's cat, if the cat were dead, light reflecting off the dead cat and going off into outer space would eventually intersect the hypersurface S, and the light distribution on S would register the inanimate status of the cat. On the other hand, if the cat were alive, the light reflecting off the living cat and going off into outer space would also intersect S, but now the light distribution on S would register the different locations the living cat was in as it moved about. Because light travels at a constant speed in a vacuum, the state of the cat at earlier times would be described by light distributions in regions on S that were further away from the cat than those light distributions in regions of S that described the cat in more recent times.

Now if the cat was in a superposition of dead and alive states, the hypersurface S would also enter into a superposition of different states corresponding to these different distributions of light registered on S. But if a notional measurement on S is made that determines which of these distributions is actually realized on S, then this determination will determine which history was actualized, and hence determine whether the cat actually survived Schrödinger's experiment or whether it perished. Thus, by conditioning on one of these two distributions on S being actualized, the conditional expectation of the stress-energy tensor in the vicinity of where Schrödinger's cat might be will not describe a situation like the one depicted in figure 3. Rather, it will either describe a situation like the one depicted in figure 5, or it will describe a situation like the one depicted in figure 6. Which of these two situations will be determined by whether the measurement outcome on S corresponds to a light distribution reflected from a living cat, or to a light distribution reflected from a dead cat.

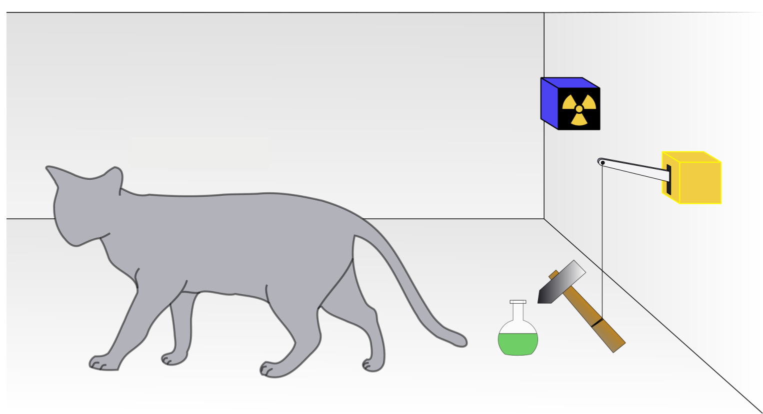


Figure 5: A depiction of Schrödinger's cat being alive.[27] `livecat`

[27] Original by Dhatfield. This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license. Source: https://upload.wikimedia.org/wikipedia/commons/archive/9/91/20080627113554!Schrodingers_cat.svg

[28] Original by Dhatfield. Altered by removing numbers and making into two separate figures. This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license. Source: https://upload.wikimedia.org/wikipedia/commons/archive/9/91/20080627113554!Schrodingers_cat.svg

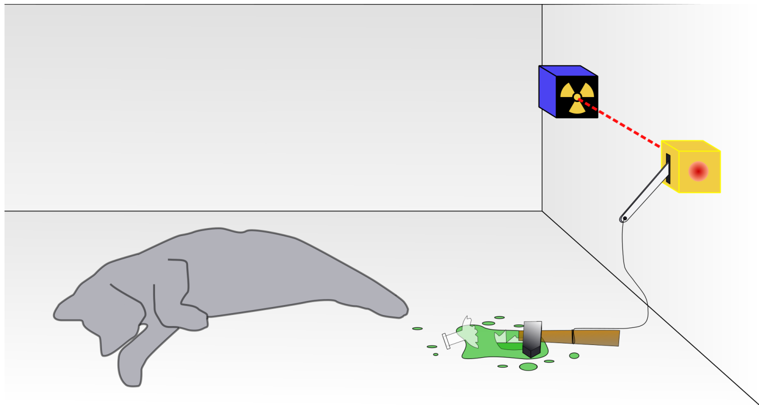


Figure 6: A depiction of Schrödinger's cat being dead.[28] deadcat

The conditional expectation that we need to calculate depends on the notion of **conditional probability**. In probability theory, the conditional probability $P(q|r)$ that a statement q is true given that a statement r is true is given by the formula

{conditionalprobability}

$$P(q|r) = \frac{P(q \,\&\, r)}{P(r)}. \tag{5}$$

If we now define $q(\kappa)$ to be the statement that some quantity T takes the value τ, then the **conditional expectation** of T given r will be given by the formula

{conditionalexpectation}

$$\langle K \rangle_r \stackrel{\text{def}}{=} \sum_\tau P(q(\tau)|r)\tau \tag{6}$$

where the summation is over all the possible values τ that T can take.

We thus define Kent's $T^{\mu\nu}(y)$-beable to be $\langle T^{\mu\nu}(y) \rangle_{\tau_S} \stackrel{\text{def}}{=} \langle T^{\mu\nu}(y) \rangle_{r(\tau_S, y)}$ Kentbeable where $r(\tau_S, y)$ is the statement that $T_S(x) = \tau_S(x)$ for all $x \in S^1(y)$. It is these $T^{\mu\nu}(y)$-beables that give a one-world picture of reality in Kent's interpretation.

Coming back to the question of why we can't include any information about $\tau_S(x)$ for $x \in S$ from within the light cone of y, we need to consider in more detail how we would calculate $\langle T^{\mu\nu}(y) \rangle_{\tau_S}$. From (5) and (6), we will be able to perform this calculation so long as we can calculate $P(q(\tau) \,\&\, r(\tau_S, y))$ and $P(r(\tau_S, y))$. Calculating

$P(r(\tau_S, y))$ is relatively straightforward. As described on page 13, we can find an orthonormal basis $\{\,|\Psi^{(i)}\rangle : i\}$ of H_S consisting of simultaneous $\hat{T}_S$-eigenstates and simultaneous $\hat{T}_S$-eigenvalues $\tau_S^{(i)}$ respectively. The probability $P(r(\tau_S, y))$ will then be

$$P(r(\tau_S, y)) = \sum_{\substack{i \text{ such that} \\ \tau_S^{(i)}(x)=\tau_S(x) \\ \text{for all } x\in S^1(y)}} \left| \left\langle \Psi^{(i)} \middle| U_{SS_0} \middle| \Psi_0 \right\rangle \right|^2$$

where we have used equation (4).

Calculating $P(q(\tau)\,\&\,r(\tau_S, y))$ is a bit more involved because in the Tomonaga-Schwinger picture, the definition of observables via

$$\hat{O}(x) = U[S]\hat{\boldsymbol{O}}(x)U[S]^{-1}. \tag{3 revisted}$$

This means that we can't define $\hat{T}^{\mu\nu}(y)$ according to (3) since $y \notin S$.[29] However, we do not face such restrictions in the Heisenberg picture, so one approach would be to calculate $P(q(\tau)\,\&\,r(\tau_S, y))$ in Heisenberg picture. As we will see shortly, this is not the approach that Kent takes, but nevertheless, in the Heisenberg picture, it is easier to see why we can't include information from S within the light cone (without begging the question) when calculating $P(q(\tau)\,\&\,r(\tau_S, y))$. To see why this is, consider the simpler case of just two measurable quantities F and G which we assume to have a discrete range of possible values and for which we wish to calculate the joint probability $P((F = f)\,\&\,(G = g))$. To do this in the Heisenberg picture, we need an orthonormal basis of the state space $\{\,|\Phi^{(i)}\rangle : i\}$ consisting of simultaneous eigenstate of the observables $\hat{\boldsymbol{F}}$ and $\hat{\boldsymbol{G}}$ with eigenvalues $f^{(i)}$ and $g^{(i)}$ respectively so that $\hat{\boldsymbol{F}}\,|\Phi^{(i)}\rangle = f^{(i)}\,|\Phi^{(i)}\rangle$ and $\hat{\boldsymbol{G}}\,|\Phi^{(i)}\rangle = g^{(i)}\,|\Phi^{(i)}\rangle$. Then, when the system is in the

[29] If we did attempt to use (3) to define $\hat{T}^{\mu\nu}(y) = U[S]\boldsymbol{\hat{T}^{\mu\nu}}(y)U[S]^{-1}$, then $\hat{T}^{\mu\nu}(y)$ would have a (non-local) dependence on S.

state $\{\,|\Phi^{(i)}\rangle : i\}$, the quantity F will have the value $f^{(i)}$, and the quantity G will have the value $g^{(i)}$. Given that the system is in the state $|\Phi\rangle$, the joint probability $P((F = f)\,\&\,(G = g))$ can be calculated using the Born rule to get

$$P((F = f)\,\&\,(G = g)) = \sum_{\substack{i \text{ such that} \\ f^{(i)}=f \text{ and } g^{(i)}=g}} \left|\left\langle \Phi^{(i)} \middle| \Phi \right\rangle\right|^2.$$

But in order for such an orthonormal basis to exist, it is necessary that $\hat{F}$ and $\hat{G}$ commute.[30] This means that if $\hat{F}$ and $\hat{G}$ do not commute, then we cannot define the joint probability $P((F = f)\,\&\,(G = g))$.

Now quantum field theory is so constructed that $\hat{\boldsymbol{T}}^{00}(x)$ and $\hat{\boldsymbol{T}}^{\mu\nu}(y)$ will not commute when x and y are not spacelike separated, but $\hat{\boldsymbol{T}}^{\mu'\nu'}(x)$ and $\hat{\boldsymbol{T}}^{\mu\nu}(y)$ will commute when x and y are spacelike separated.[31] As we will see on page 39, $T_S(x)$ will have a $T^{00}(x)$ component, and so we can only be sure that $\hat{\boldsymbol{T}}_S(x)$ will commute with $\hat{\boldsymbol{T}}^{\mu\nu}(y)$ if x and y are spacelike separated. Extending this argument to multiple $x \in S$ we see that we can only guarantee the conditional expectation of $T^{\mu\nu}(y)$ is definable if we restrict our conditioning on the value of $T_S(x)$ to $x \in S^1(y)$.

---

[30] This is because given such a basis, we have

$$\hat{\boldsymbol{F}}\hat{\boldsymbol{G}}\,|\Phi^{(i)}\rangle = f^{(i)}g^{(i)}\,|\Phi^{(i)}\rangle = g^{(i)}f^{(i)}\,|\Phi^{(i)}\rangle = \hat{\boldsymbol{G}}\hat{\boldsymbol{F}}\,|\Phi^{(i)}\rangle$$

so for any arbitrary state $|\Phi\rangle = \sum_i c_i\,|\Phi^{(i)}\rangle$, we have

$$\hat{\boldsymbol{F}}\hat{\boldsymbol{G}}\,|\Phi\rangle = \sum_i c_i \hat{\boldsymbol{F}}\hat{\boldsymbol{G}}\,|\Phi^{(i)}\rangle = \sum_i c_i \hat{\boldsymbol{G}}\hat{\boldsymbol{F}}\,|\Phi^{(i)}\rangle = \hat{\boldsymbol{G}}\hat{\boldsymbol{F}}\,|\Phi\rangle\,.$$

[31] The proof of this statement need not concern us, but one can see that this is the case by considering the four potential commutation relations and the decomposition of the stress-energy tensors as in terms of the four-potentials – see Schwinger, "Quantum Electrodynamics. I. A Covariant Formulation," p.1443–1444.

toysection

0.1.4 Kent's toy example

To get a feel for how all the elements of Kent's interpretation fit together, it is helpful to consider Kent's toy model example that he discusses in his 2014 paper.[32] In his toy model, Kent considers a system in one spatial dimension which is the superposition of two localized states $\psi_0^{\text{sys}} = c_1\psi_1^{\text{sys}} + c_2\psi_2^{\text{sys}}$ where ψ_1^{sys} is localized at spatial location z_1, ψ_2^{sys} is localized at spatial location z_2, and $|c_1|^2 + |c_2|^2 = 1$. According to the Copenhagen interpretation, a measurement on this system would collapse the wave function of ψ_0^{sys} to the wave function of ψ_1^{sys} with probability $|c_1|^2$, and to the wave function of ψ_2^{sys} with probability $|c_2|^2$. The purpose of Kent's toy model is to show that within his interpretation, there is something analogous to wave function collapse. In order for this "collapse" to happen, one needs to consider how the system interacts with light. Thus, Kent supposes that a photon (which is modelled as a point particle) comes in from the left, and as it interacts with the two states ψ_1^{sys} and ψ_2^{sys}, the photon enters into a superposition of states, corresponding to whether the photon reflects off the localized ψ_1^{sys}-state at time t_1 or the localized ψ_2^{sys}-state at time t_2. The photon in superposition then travels to the left and eventually reaches the one dimensional hypersurface S at locations γ_1 and γ_2 as shown in figure 7.

We now suppose that when the mass-energy density S is "measured", the energy of the photon is found to be at γ_1 rather than at γ_2. We then consider the mass-density at early spacetime locations $y_1^a = (z_1, t_a)$ and $y_2^a = (z_2, t_a)$ as show in figure 8 (a) and (b).

[32]See Kent, "Lorentzian Quantum Reality: Postulates and Toy Models," p.3–4.

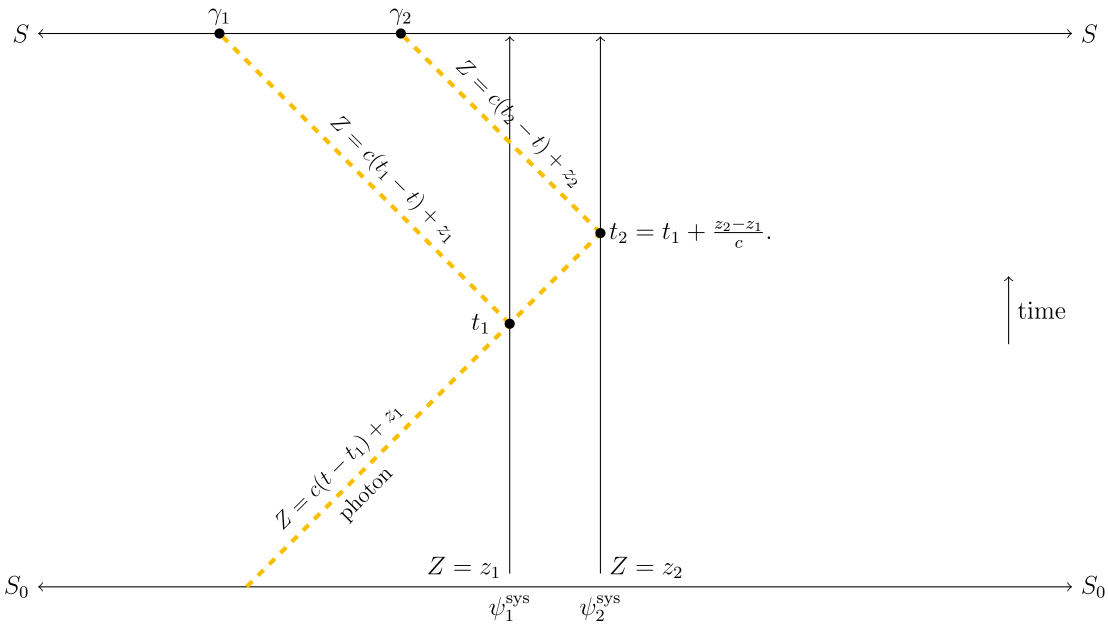


Figure 7: Kent's toy model

TM1

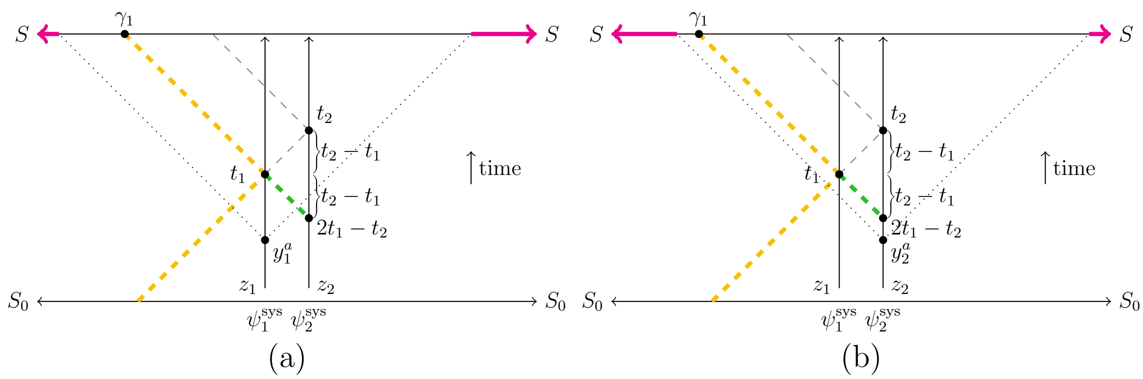


Figure 8: (a) highlights the part of S used to calculate the energy density at y_1^a whose time is less than $2t_1 - t_2$. (b) highlights the part of S used to calculate the energy density at y_2^a whose time is less than $2t_1 - t_2$.

TM2

By early, we mean that $t_a < 2t_1 - t_2$. This will mean that the possible detection locations γ_1 and γ_2 will be outside the forward light cones of y_1^a and y_2^a. Hence, $S^1(y_1^a) \cap S$ and $S^1(y_2^a) \cap S$ contain no additional information beyond standard quantum theory by which we could calculate the conditional expectation values of the energy at y_1^a and y_2^a. Hence, according to Kent's interpretation, the total energy at time t_a will be divided between the two spatial locations with a proportion of $|c_1|^2$ at z_1 and a proportion of $|c_2|^2$ at z_2.

However, the situation is different for two spacetime locations $y_1^b = (z_1, t_b)$ and $y_2^b = (z_2, t_b)$ with t_b slightly after $2t_1 - t_2$ as depicted in figure 9.

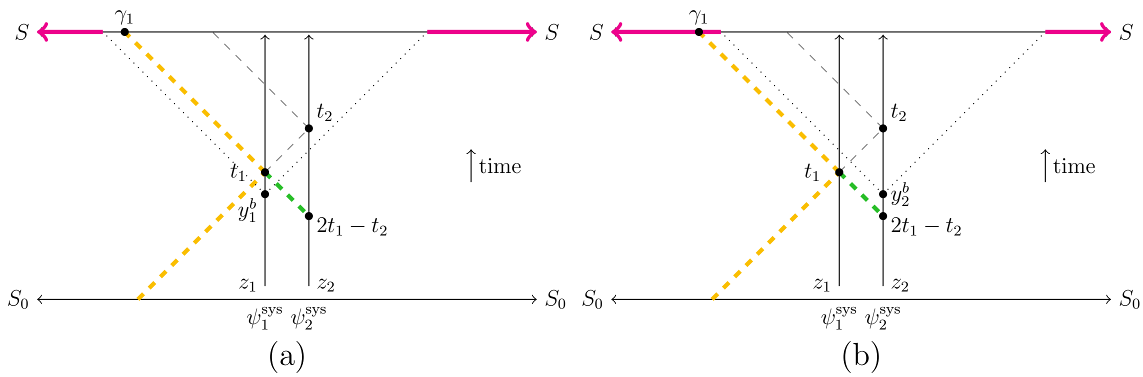
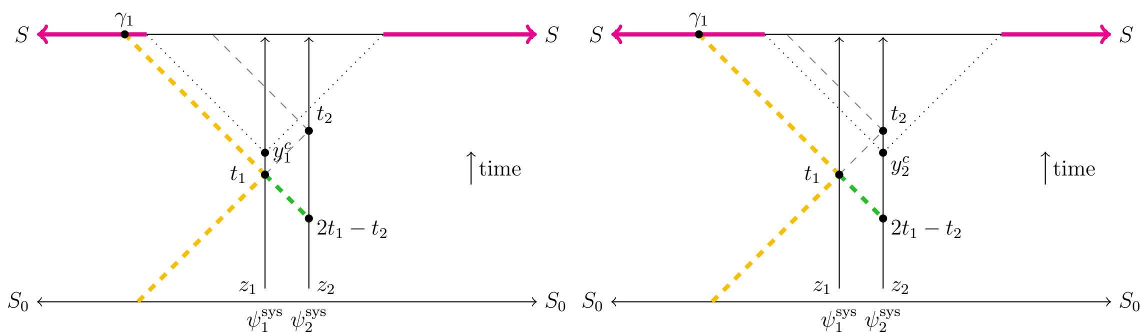


Figure 9: (a) highlights the part of S used to calculate the energy density at y_1^b whose time is greater than $2t_1 - t_2$. (b) highlights the part of S used to calculate the energy density at y_2^b whose time is greater than $2t_1 - t_2$.

TM3

In this situation, when we consider the location y_1^b, there is no additional information in $S^1(y_1^b) \cap S$ beyond standard quantum theory, so there will be a proportion of $|c_1|^2$ of the total initial energy of the system at y_1^b. But at location y_2^b, the information in $S^1(y_2^b) \cap S$ shows that the photon has reflected from the localized ψ_1^{sys}-state, and so this additional information tells us that after time t_b, there is no energy localized at z_2 since from the perspective of y_2^b, the energy is known to be localized at z_1. So it is as though the information of $S^1(y_2^b) \cap S$ has determined that we are in a world in which there is an energy density of zero at y_2^b, and there are no other worlds in which the energy density at y_2^b is non-zero since all worlds have to be consistent with the notional measurement made on S. So for a short time the total energy of the system is reduced by a factor of $|c_1|^2$.

However, as shown in figure 10, for times t_c greater than t_1, the total energy of the system is once again restored to the initial energy the system had when in the state ψ_0^{sys}.



(a) (b)

Figure 10: (a) highlights the part of S used to calculate the energy density at y_1^c whose time is greater than t_1. (b) highlights the part of S used to calculate the energy density at y_2^c whose time is greater than t_1.

TM4

In this situation, there is now information in $S^1(y_1^c) \cap S$ that determines that the photon reflected off the localized ψ_1^{sys}-state. This means that when the conditional expectation of the energy density of y_1^c is calculated, the extra information in $S^1(y_1^c) \cap S$ determines that all the energy of the system is located at location z_1 for times t_c greater than t_1, and the energy is equal to the initial energy of the system so that energy is conserved.

0.2 Evaluating Kent's Interpretation

In this section I will consider how well Kent's interpretation allows for peaceful coexistence between standard quantum theory and special relativity. I'll begin by showing that Kent's interpretation is consistent with standard quantum theory. I'll then show that Kent's interpretation is Lorentz invariant. The problems of outcomes will be addressed in a subsection that considers how Kent's interpretation ties in with decoherence theory and d'Espagnat's objection about improper mixtures. I'll then consider PI in Kent's interpretation and the consistency of Kent's interpretation with Colbeck and Renner's theorem. Finally, I will raise some issues about the nature of Kent's beables.

0.2.1 Consistency of Kent's interpretation with Standard Quantum Physics*

LorentzInvarianceSection

If we are to take Kent's interpretation seriously, it had better not contradict empirical observations. Standard quantum physics is a firmly established scientific theory, and so far, it has not been contradicted by any experimental observations. Thus, standard quantum theory is empirically adequate in its domain of applicability. Thus, if we can show that Kent's interpretation is consistent with standard quantum theory, then it too will be empirically adequate to the same degree.

In order to show that Kent's interpretation is consistent with standard quantum theory and does not contradict it, we will need to express $\langle T^{\mu\nu}(y)\rangle_{\tau_S}$ in terms of the observable $\hat{T}^{\mu\nu}(y)$ and the initial state $|\Psi_0\rangle$. To find such an expression, we would ideally like to find a hypersurface S' that contains both $S^1(y)$ and y. Then we could

*As mentioned in the introduction on page **??**, sections marked with an asterisk may be challenging to readers who do not have a mathematics or physics background.

consider how the observables $\hat{T}_S(x)$ and $\hat{T}^{\mu\nu}(y)$ act on the state $|\Psi'\rangle = U_{S'S_0}|\Psi_0\rangle$ and use this action to determine the probabilities $P(q(\tau), r)$ and $P(r)$ needed to define the conditional probability $P(q(\kappa)|r)$ as defined in (5). However, since by definition, a hypersurface must be continuous with any two locations on it being spacelike-separated, it is going to be impossible to find a hypersurface S' with the desired properties of containing both $S^1(y)$ and y. Nevertheless, what we can do is find a sequence of hypersurfaces $S_n(y)$ such that siydef $S_n(y) \subset S_{n'}(y)$ for $n < n'$, and such that for any $x \in S^1(y)$, there exists n such that $x \in S_n(y)$. An example of one such $S_n(y)$ is shown in figure 11. When there is no ambiguity, we will drop the y and write S_n instead of $S_n(y)$.

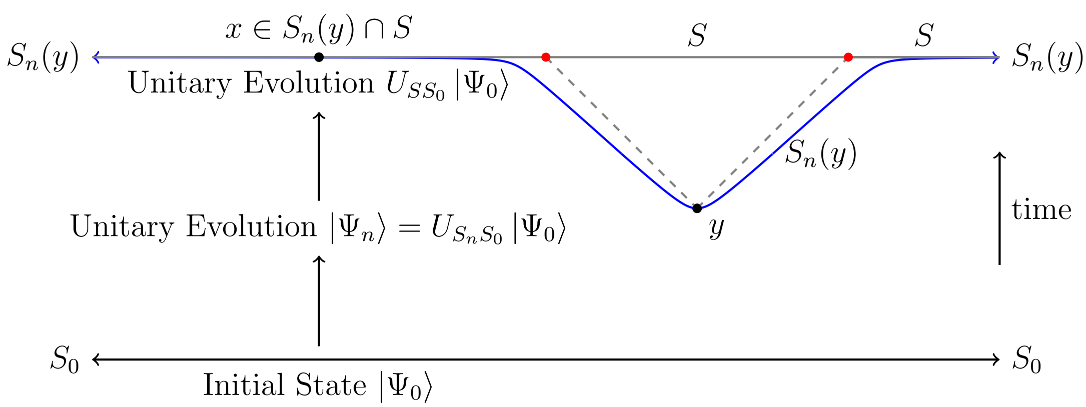


Figure 11: $S_n \stackrel{\text{def}}{=} S_n(y)$ is a hypersurface containing y and all of $S^1(y)$ in the limit as $n \to \infty$.

S3

Now if r_n is the statement that $T_S(x) = \tau_S(x)$ for all $x \in S_n(y) \cap S$, then so long as $\tau_S(x)$ is chosen by the Born Rule so that $P(r) \neq 0$, it will follow that

$$P(q(\tau)|r) = \lim_{n\to\infty} P(q(\tau)|r_n). \tag{7}$$

Therefore, from the definition of the beable $\langle T^{\mu\nu}(y)\rangle_{\tau_S}$ given on page 21 together with the definition of conditional expectation given in equation (6), we have

{beable1}

$$\langle T^{\mu\nu}(y)\rangle_{\tau_S} = \lim_{n\to\infty} \sum_{\tau} \frac{P(q(\tau), r_n)\tau}{P(r_n)}. \tag{8}$$

To calculate $P(q(\tau)|r_n)$, we note that since S_n is a hypersurface, there will exist a unitary operator $U_{S_nS_0}$ which maps the Hilbert space of states H_{S_0} describing S_0 to the Hilbert space of states H_{S_n} HSidef describing S_n in accord with how the states of H_{S_0} evolve over time. Now let $H_{S_n,\tau_S} \subset H_{S_n}$ be the subspace of states $|\xi\rangle$ for which $\hat{T}_S(x)\,|\xi\rangle = \tau_S(x)\,|\xi\rangle$ for all $x \in S_n \cap S$, and let $\{|\xi_1\rangle\,, |\xi_2\rangle\,, \ldots\}$ be an orthonormal basis of H_{S_n,τ_S}. Given that the initial state of the world is $|\Psi_0\rangle$, the probability $P(r_n)$ of "measuring" the value of $T_S(x)$ on $S_n \cap S$ to be $\tau_S(x)$ will be

{Pri}

$$P(r_n) = \sum_j |\langle\xi_j|\Psi_n\rangle|^2, \tag{9}$$

where $|\Psi_n\rangle = U_{S_nS_0}\,|\Psi_0\rangle$, and this probability will be independent of the particular orthonormal basis $\{|\xi_j\rangle : j\}$ of H_{S_n,τ_S}.[33] If we define

{tauprojection}

$$\pi_n = \sum_j |\xi_j\rangle\langle\xi_j|\,, \tag{10}$$

then it is easy to see that

{Prn}

$$P(r_n) = \langle\Psi_n|\pi_n|\Psi_n\rangle\,. \tag{11}$$

We also see that π_n is Hermitian (i.e. has real eigenvalues) and that $\pi_n\pi_n = \pi_n$. Any Hermitian operator π with $\pi^2 = \pi$ is called a **projection**. We thus see that π_n is a projection.

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[33] To see why this is, we note that we can extend the orthonormal set $\{|\xi_1\rangle\,, |\xi_2\rangle\,, \ldots\}$ to an orthonormal basis $\{|\xi_1\rangle\,, |\xi_2\rangle\,, \ldots\} \cup \{|\zeta_1\rangle\,, |\zeta_2\rangle\,, \ldots\}$ of H_{S_n} which consists entirely of $\hat{T}_S$-eigenstates. We can think of each of the states of this orthonormal basis as the possible measurement outcomes when making the notional measurement of $T_S(x)$ on $S_n \cap S$. By the Born Rule, it therefore follows that $P(r_n) = \sum_j |\langle\xi_j|\Psi_n\rangle|^2$. But to see that this probability is independent of the particular basis, we can uniquely write $|\Psi_n\rangle$ as a sum $|\Psi_n\rangle = |\xi\rangle + |\zeta\rangle$ where $|\xi\rangle$ belongs to the span of $\{|\xi_j\rangle : j\}$ and $|\zeta\rangle$

Turning to the calculation of $P(q(\tau), r_n)$, note that for the Tomonaga-Schwinger formulation of relativistic quantum physics, the operators $\hat{T}_S(x)$ and $\hat{T}^{\mu\nu}(y)$ for fixed μ, ν commute when x and y are spacelike-separated. It therefore follows that we can express any state of H_{S_n} as a superposition of simultaneous eigenstates of $\hat{T}^{\mu\nu}(y)$ and $\hat{T}_S(x)$ for $x \in S_n \cap S$.[34] For a particular choice of μ, ν, we can then form an orthonormal basis $\{|\eta_j\rangle : j\}$ of H_{S_n} consisting of simultaneous $\hat{T}^{\mu\nu}(y)$, $\hat{T}_S(x)$-eigenstates so that $\hat{T}^{\mu\nu}(y)\,|\eta_j\rangle = \tau^{(j)}\,|\eta_j\rangle$ and $\hat{T}_S(x)\,|\eta_j\rangle = \tau_S^{(j)}(x)\,|\eta_j\rangle$ for $x \in S_n \cap S$, where $\tau^{(j)}$ and $\tau_S^{(j)}(x)$ are the corresponding eigenstates. If we define $\pi_{n,\tau} = \sum_j |\chi_{j,\tau}\rangle\langle\chi_{j,\tau}|$ where $\{|\chi_{j,\tau}\rangle : j\}$ is the subset of $\{|\eta_j\rangle : j\}$ such that $\hat{T}^{\mu\nu}(y)\,|\chi_{j,\tau}\rangle = \tau\,|\chi_{j,\tau}\rangle$ and $\hat{T}_S(x)\,|\chi_{j,\tau}\rangle = \tau_S(x)\,|\chi_{j,\tau}\rangle$ for all $x \in S_n \cap S$, then

{pqtauri}
$$P(q(\tau), r_n) = \sum_j |\langle\chi_{j,\tau}|\Psi_n\rangle|^2 = \langle\Psi_n|\pi_{n,\tau}|\Psi_n\rangle\,.^{35} \tag{12}$$

But if we define $\pi_\tau = \sum_j |\eta_{j,\tau}\rangle\langle\eta_{j,\tau}|$ where $\{|\eta_{j,\tau}\rangle : j\}$ is the subset of $\{|\eta_j\rangle : j\}$ with $\hat{T}^{\mu\nu}(y)\,|\eta_{j,\tau}\rangle = \tau\,|\eta_{j,\tau}\rangle$, then we also have $\pi_{n,\tau} = \pi_n\pi_\tau$.[36] Hence,

{pqtauri2}
$$P(q(\tau), r_n) = \langle\Psi_n|\pi_n\pi_\tau|\Psi_n\rangle\,. \tag{13}$$

But clearly $\hat{T}^{\mu\nu}(y) = \sum_\tau \tau\pi_\tau$. Therefore, combining (8), (12), and (13), we have

{kentconsistency0}
$$\langle T^{\mu\nu}(y)\rangle_{\tau_S} = \lim_{n\to\infty} \frac{\sum_\tau \langle\Psi_n|\pi_n\pi_\tau|\Psi_n\rangle\,\tau}{\langle\Psi_n|\pi_n|\Psi_n\rangle} = \lim_{n\to\infty} \frac{\langle\Psi_n|\pi_n\hat{T}^{\mu\nu}(y)|\Psi_n\rangle}{\langle\Psi_n|\pi_n|\Psi_n\rangle}. \tag{14}$$

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belongs to the span of $\{|\zeta_j\rangle : j\}$. Then since $|\xi\rangle = \sum_j \langle\xi_j|\Psi_n\rangle\,|\xi_j\rangle$, it follows that

$$\langle\xi|\xi\rangle = \sum_j |\langle\xi_j|\Psi_n\rangle|^2 = P(r_n).$$

Therefore, since $\langle\xi|\xi\rangle$ is independent of the particular basis chosen of H_{S_n,τ_S}, then so is $P(r_n)$. priproof

[34] We make the same approximation as mentioned on page 13 in footnote 19.

[35] The proof of this is very similar to the proof given in footnote 33.

[36] To see why this is, we first show that $\pi_n = \sum_j |h_{n,j}\rangle\langle h_{n,j}|$ where $\{|h_{n,j}\rangle : j\}$ is the subset of $\{|\eta_j\rangle : j\}$ for which $|h_{n,j}\rangle \in H_{S_n,\tau_S}$. Note that $\pi_n\,|h_{n,j}\rangle = |h_{n,j}\rangle$ since $\{|\xi_j\rangle : j\}$ is a basis for H_{S_n,τ_S} and $|h_{n,j}\rangle \in H_{S_n,\tau_S}$. Therefore, $\pi_n\pi_{n,h} = \pi_{n,h}$ where $\pi_{n,h} = \sum_j |h_{n,j}\rangle\langle h_{n,j}|$. But $\pi_{n,h}\,|\xi_j\rangle = |\xi_j\rangle$ since $\{|h_{n,j}\rangle : j\}$ is a basis for H_{S_n,τ_S} and $|\xi_j\rangle \in H_{S_n,\tau_S}$. Therefore, $\pi_{n,h}\pi_n = \pi_n$. But $\pi_{n,h}\pi_n = \pi_n\pi_{n,h}$ since π_n and $\pi_{n,h}$ are Hermitian. Hence, $\pi_n = \pi_{n,h}$. Now the summands of $\pi_n\pi_\tau$ are only going to

We are now in a position to show that Kent's theory is consistent with standard quantum theory. First let us consider what we need to show.

In the pilot wave interpretation, its consistency with standard quantum theory requires that if one averages the expectation values of an observable over the hidden variables (i.e. the positions and the momenta of all the particles) then one obtains the expectation value of the observable given by standard quantum theory as indicated in equation (**??**).

Now in Kent's interpretation, the hidden variables on which his beables $\langle T^{\mu\nu}(y)\rangle_{\tau_S}$ depend are the values $\tau_S(x)$ of $T_S(x)$ for $x \in S^1(y) \cap S$. The operator π_n in equation (14) in the limit as $n \to \infty$ encapsulates this hidden information. To remind ourselves of π_n's dependency on τ_S restricted to $S_n \cap S$, we will now write $\pi_n(\tau_{S_n \cap S})$ for π_n where $\tau_{S_n \cap S}$ is the function τ_S restricted to $S_n \cap S$. Likewise, we will write $r_n(\tau_{S_n \cap S})$ for r_n, the statement that $T_S(x) = \tau_S(x)$ for all $x \in S_n(y) \cap S$. If we let j index all possible functions $\tau^{(j)}_{S_n \cap S}$ taking real values on $S_n \cap S$, then the analogue of (**??**) requires us to show that

$$\langle \hat{T}^{\mu\nu}(y)\rangle = \lim_{n\to\infty} \sum_j P\Big(r_n(\tau^{(j)}_{S_n \cap S})\Big) \left\langle T^{\mu\nu}(y)\right\rangle_{\tau^{(j)}_{S_n \cap S}} \qquad \texttt{\{kentconsistency\}} \quad (15)$$

for all y lying between S_0 and S, where the left-hand side of (15) is just the expectation value of $\hat{T}^{\mu\nu}(y)$ predicted by standard quantum theory. Equation (15) is sufficient to establish consistency with standard quantum theory because ultimately, all observables are going to be reducible to expressions dependent on $\hat{T}^{\mu\nu}(y)$, since once we know what to expect for $\hat{T}^{\mu\nu}(y)$, we will know what to expect for the energy and momentum

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consist of those $|\eta_j\rangle\langle\eta_j|$ for which $\hat{T}^{\mu\nu}(y)\,|\eta_j\rangle = \tau\,|\eta_j\rangle$ and for which $\hat{T}_S(x)\,|\eta_j\rangle = \tau_S(x)\,|\eta_j\rangle$ for all $x \in S_n \cap S$, and these are just the $|\chi_{j,\tau}\rangle\langle\chi_{j,\tau}|$ which are the summands of $\pi_{n,\tau}$. Hence, $\pi_n\pi_\tau = \pi_{n,\tau}$.

densities for all measuring apparatus readouts etc. and hence what to expect for all measurement outcomes. But from (11) and (14), we have

{kentconsistency1}

$$\lim_{n\to\infty} \sum_j P\big(r_n(\tau^{(j)}_{S_n\cap S})\big) \langle T^{\mu\nu}(y)\rangle_{\tau^{(j)}_{S_n\cap S}} = \lim_{n\to\infty} \sum_j \langle \Psi_n|\pi_n(\tau^{(j)}_{S_n\cap S}) T^{\mu\nu}(y)|\Psi_n\rangle \tag{16}$$

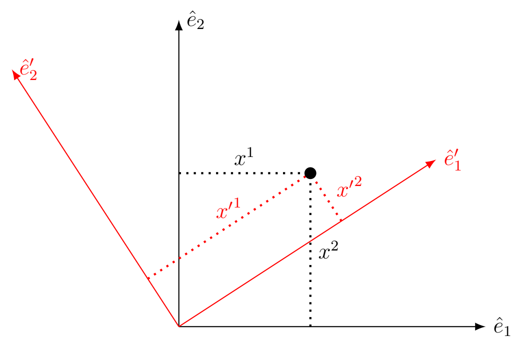
Since there is an orthonormal basis $\{|\eta_j\rangle : j\}$ of H_{S_n} consisting of simultaneous $\hat{T}_S(x)$-eigenstates so that $\hat{T}_S(x)\,|\eta_j\rangle = \tau^{(j)}_{S_n\cap S}(x)\,|\eta_j\rangle$ for all $x \in S_n \cap S$, it follows that $\sum_j \pi_n(\tau^{(j)}_{S_n\cap S}) = I$. Therefore, equation (15) follows from (16) which is what we were aiming to show for standard quantum consistency to hold.

0.2.2 Kent's interpretation and Lorentz Invariance LorentzInvariance

In order to explain what it means for Kent's interpretation to be Lorentz invariant, we first need to explain how spacetime coordinates look to different observers.

A spacetime location is represented by a four-tuple (x^0, x^1, x^2, x^3) where $(x^i)^3_{n=1}$ are spatial coordinates, and where $x^0 = ct$ with c being equal to the speed of light and t being the time. If $(1, 0, 0, 0)$ corresponds to the spacetime location $\hat{e}_0$, and $(0, 1, 0, 0)$ corresponds to the spacetime location $\hat{e}_1$, etc., then we can express any other spacetime location as a sum $\sum_{\mu=0}^3 x^\mu \hat{e}_\mu$. We will use the so-called **Einstein summation convention** Einsteinsum of dropping the summation sign and implicitly assuming that there is a summation whenever an upper index and a lower index are the same so that we can write $x^\mu \hat{e}_\mu$ instead of $\sum_{\mu=0}^3 x^\mu \hat{e}_\mu$.

Now suppose an observer O expresses spacetime locations in terms of $\{\hat{e}_\mu : \mu = 0, \ldots, 3\}$ and hence uses the coordinates (x^0, x^1, x^2, x^3) to describe various spacetime locations. For another observer O', it may be more natural to express spacetime locations in terms of a different set of spacetime locations $\{\hat{e}'_\mu : \mu = 0, \ldots, 3\}$ so that the location described by O as (x^0, x^1, x^2, x^3) would be described by O' as (x'^0, x'^1, x'^2, x'^3) where $x'^\mu \hat{e}'_\mu = x^\mu \hat{e}_\mu$. For instance if O and O' are moving with respect to each other, they may both want to use coordinates in which their own spatial coordinates are fixed and in which the spatial coordinates of the other observer are changing. As another example, figure 12 shows how the (x^1, x^2)-coordinates transform under a spatial rotation.



(a)

Figure 12: Shows how a location (marked as •) can be expressed either in coordinates (x^1, x^2) with respect to the basis $\{\hat{e}_1, \hat{e}_2\}$ or in coordinates (x'^1, x'^2) with respect to the basis $\{\hat{e}'_1, \hat{e}'_2\}$.

`rotfigure`

Now the key fact about all observers is that they must always observe light in a vacuum to have a constant speed c. Thus, for a photon that goes through the spacetime locations $(0, 0, 0, 0)$ and (x^0, x^1, x^2, x^3) in the coordinates of O, we must have $(x^0, x^1, x^2, x^3) = (ct, tv^1, tv^2, tv^3)$ where

$$\sqrt{(v^1)^2 + (v^2)^2 + (v^3)^2} = c.$$

But if $(0, 0, 0, 0)$ and (x^0, x^1, x^2, x^3) corresponds to $(0, 0, 0, 0)$ and (x'^0, x'^1, x'^2, x'^3) respectively in the coordinates of another observer O', then we must also have

$(x'^0, x'^1, x'^2, x'^3) = (ct', t'v'^1, t'v'^2, t'v'^3)$ where

$$\sqrt{(v'^1)^2 + (v'^2)^2 + (v'^3)^2} = c.$$

In either case, we must have

{invariant}

$$(x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = (x'^0)^2 - (x'^1)^2 - (x'^2)^2 - (x'^3)^2 = 0. \tag{17}$$

If we define $\eta_{00} = 1$, $\eta_{ni} = -1$ for $i = 1, 2, 3$ and $\eta_{\mu\nu} = 0$ for $\mu \neq \nu$, then using the Einstein summation convention as well as the convention of lowering indices so that we define $x_\mu \stackrel{\text{def}}{=} \eta_{\mu\nu} x^\nu$, then (17) is equivalent to

$$x_\mu x^\mu = {x'}_\mu x'^\mu = 0.$$

Thus, for any coordinate transformation $x \to x'$ such that $x_\mu x^\mu = {x'}_\mu x'^\mu$, if the speed of light is c in the x-coordinates, then the speed of light is also guaranteed to be c in the x'-coordinates. A **Lorentz Transformation** Λ is any coordinate transformation of the form $x'^\mu = \Lambda^\mu{}_\nu x^\nu$ such that $x_\mu x^\mu = {x'}_\mu x'^\mu$. Since a Lorentz transformation must satisfy

$$x_\mu x^\mu = \eta_{\mu\rho} \Lambda^\rho{}_\sigma x^\sigma \Lambda^\mu{}_\nu x^\nu$$

for all x, it follows that

{lorentztrans}

$$\Lambda^\rho{}_\mu \eta_{\rho\sigma} \Lambda^\sigma{}_\nu = \eta_{\mu\nu}.\text{[37]} \tag{18}$$

Having considered how the coordinates of a spacetime location viewed by one observer relate to the coordinates of the same spacetime location viewed by a different observer, we can now consider how physical quantities viewed by different observers relate to each

[37] To see why this is, note that if $x_\mu x^\mu = {x'}_\mu x'^\mu$ for all x, then for any other spacetime location y, we have $(x+y)_\mu (x+y)^\mu = (x'+y')_\mu (x'+y')^\mu$. If we expand this out and cancel $x_\mu x^\mu$ with ${x'}_\mu x'^\mu$ and cancel $y_\mu y^\mu$ with ${y'}_\mu y'^\mu$, and using the fact that $y_\mu x^\mu = x_\mu y^\mu$, etc. we find that $x_\mu y^\mu = {x'}_\mu y'^\mu$ for all x and y. Hence,

$$\eta_{\nu\mu} x^\mu y^\nu = x_\mu y^\mu = \eta_{\sigma\rho} \Lambda^\rho{}_\mu \Lambda^\sigma{}_\nu x^\mu y^\nu.$$

other. The simplest kind of physical quantity is called a **scalar**. A scalar defined at a particular spacetime location has the same value no matter what frame of reference an observer uses. One example of a scalar is an object's **rest mass** which is the mass an object would have if it had no velocity. There is still a transformation rule for scalars since the spacetime location at which the scalar is measured is usually expressed in terms of an observer's coordinate system, and the coordinates of such a location will differ for different observers. Thus, if $\phi(x) \stackrel{\text{def}}{=} \phi(x^0, x^1, x^2, x^3)$ is the value of a scalar defined at the spacetime location (x^0, x^1, x^2, x^3) as described by an observer O, then another observer O' using a different set of coordinate (x'^0, x'^1, x'^2, x'^3) to describe the location (x^0, x^1, x^2, x^3) will describe this same scalar as $\phi'(x') \stackrel{\text{def}}{=} \phi'(x'^0, x'^1, x'^2, x'^3)$ where $\phi'(x') = \phi(x)$. Since ϕ' is just a function of the four numbers x'^0, x'^1, x'^2, and x'^3, we can rename these numbers x^0, x^1, x^2, and x^3, and then

{lorentzscalar}

$$\phi'(x) = \phi(\Lambda^{-1}x) \tag{19}$$

where Λ^{-1} is the inverse Lorentz transformation that takes the coordinates $x' = (x'^0, x'^1, x'^2, x'^3)$ of a location to the coordinates $x = (x^0, x^1, x^2, x^3)$ describing that location. Thus, equation (19) shows us how a scalar transforms under a Lorentz transformation Λ.

Many physical quantities, however, are not scalars and so will look different to different observers. For instance, the energy of an object has a kinetic component that depends on the velocity the object has relative to an observer. However, it turns out that if

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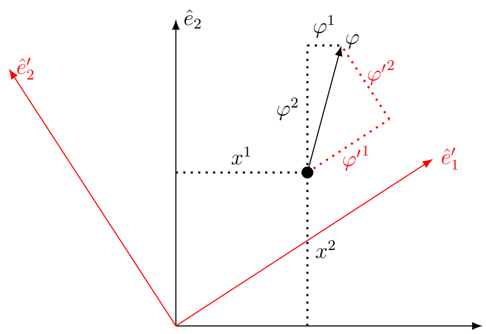
Since we can choose x such that $x^\mu = 1$ and $x^\alpha = 0$ for $\alpha \neq \mu$, and can choose y such that $y^\nu = 1$ and $y^\beta = 0$ for $\beta \neq \nu$. Then we get

$$\eta_{\mu\nu} = \eta_{\sigma\rho}\Lambda^\rho{}_\mu\Lambda^\sigma{}_\nu,$$

and hence the result follows.

an observer O considers an object's energy E together with its three components of momentum p^1, p^2, and p^3 (in the directions $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$ respectively) to form the four-tuple $p \stackrel{\text{def}}{=} (E/c, p^1, p^2, p^3)$ known as the object's **four-momentum**, then p transforms in the same way as spacetime coordinates transform between different observers. In other words, a different observer O' whose coordinates are given by $x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu}$ would observe the object's four-momentum to be $p'^{\mu} = \Lambda^{\mu}{}_{\nu} p^{\nu}$.[38] More generally, any list of four physical quantities $(\varphi^0, \varphi^1, \varphi^2, \varphi^3)$ that transforms as $\varphi \to \varphi'$ with $\varphi'^{\mu} = \Lambda^{\mu}{}_{\nu} \varphi^{\nu}$ is called a **four-vector**. Figure 13 shows how (two of) the components of a four-vector



(b)

Figure 13: Shows how a four-vector φ (of which only two components are shown) defined at a spacetime location (indicated by $\bullet$) can be expressed either as (φ^1, φ^2) with respect to the basis $\{\hat{e}_1, \hat{e}_2\}$ or as (φ'^1, φ'^2) with respect to the basis $\{\hat{e}'_1, \hat{e}'_2\}$. rotfigure2

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[38] In order for p to transform in this way, we have to redefine what we mean by energy and momentum. In classical mechanics, the momentum of an object is the product of the object's mass and its velocity. In the context of special relativity, however, the four-momentum of an object is defined to be the product of its rest mass m_0 and its **four-velocity** where the four velocity of an object is a four-tuple (u^0, u^1, u^2, u^3) with $u_\mu u^\mu = c^2$ such that the object's velocity (in the classical sense) is the vector $(c\frac{u^1}{u^0}, c\frac{u^2}{u^0}, c\frac{u^3}{u^0})$. The motivation for this definition can be seen by considering an object whose classical velocity is $\mathbf{v} = (v^1, v^2, v^3)$ that goes through $(0,0,0,0)$. It will have a spacetime trajectory $x(t) = (ct, v^1 t, v^2 t, v^3 t)$. u is just the four-vector proportional to $x(1)$ with $u_\mu u^\mu = c^2$. We can easily work out the four-velocity u of an object whose classical velocity is $\mathbf{v}$. For we must have $u^i = \frac{v^i u^0}{c}$, for $i = 1$ to 3. Therefore, since $u_\mu u^\mu = c^2$, we must have $(u^0)^2\big(1 - \frac{v^2}{c^2}\big) = c^2$ where $v = \sqrt{(v^1)^2 + (v^2)^2 + (v^3)^2}$. Thus, if we define $\beta = v/c$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, then $u^0 = c\gamma$ and $u^i = \gamma v^i$ for $i = 1$ to 3, and hence the four-velocity of the object must be $u = \gamma(c, v^1, v^2, v^3)$. From this, we see that the object's four-momentum will be $\gamma m_0(c, v^1, v^2, v^3)$. If the object's velocity is very small compared to the speed of light, then $\gamma \approx 1 + \frac{v^2}{2c^2}$, and hence the object's four-momentum $(E/c, p^1, p^2, p^3)$ will be approximately $(m_0 c + \frac{1}{2} m_0 v^2/c, m_0 v^1, m_0 v^2, m_0 v^3)$. Therefore, (p^1, p^2, p^3) is approximately equal to the classical momentum. However, the energy is now $E = m_0 c^2 + \frac{1}{2} m_0 v^2$. Thus, in addition to the kinetic energy term $\frac{1}{2} m_0 v^2$, there is a rest mass energy $m_0 c^2$. If we define the **relativistic mass** $m = \gamma m_0$, then we obtain Einstein's famous formula $E = mc^2$.

φ at a particular location will differ for different observers under a spatial rotation of the coordinates. A four-vector $\varphi^\mu(x)$ defined at every spacetime location x is called a **four-vector field**. If O observes this vector-field $\varphi^\mu(x)$, and O' is another observer whose coordinates are related to the coordinates O via the Lorentz transformation Λ, then O' will describe this vector-field as $\varphi'^\mu(x') \stackrel{\text{def}}{=} \Lambda^\mu{}_\nu \varphi^\nu(x)$. Hence, under the Lorentz transformation Λ, a vector field $\varphi^\mu(x)$ transforms as $\varphi^\mu(x) \to \varphi'^\mu(x')$ where

{lorentzvector}

$$\varphi'^\mu(x) = \Lambda^\mu{}_\nu \varphi^\nu(\Lambda^{-1}x). \tag{20}$$

From a four-vector φ^μ, we can also define the so-called **four-covector**:

{covector}

$$\varphi_\mu \stackrel{\text{def}}{=} \eta_{\mu\nu}\varphi^\nu. \tag{21}$$

To see how four-covectors transform under a Lorentz transformation Λ, it will be helpful to define

{colambda}

$$\Lambda_\mu{}^\nu \stackrel{\text{def}}{=} \eta_{\mu\rho}\eta^{\nu\sigma}\Lambda^\rho{}_\sigma \tag{22}$$

where $\eta^{\nu\sigma} = \eta_{\nu\sigma}$. If we also define the **Kronecker-delta** δ^ν_μ such that $\delta^\nu_\mu = 1$ when $\mu = \nu$ and $\delta^\nu_\mu = 0$ otherwise, then using the fact that $\eta_{\mu\rho}\eta^{\nu\rho} = \delta^\nu_\mu$ together with equation (18), we have

{lambdainverse}

$$\Lambda^\rho{}_\mu \Lambda_\rho{}^\nu = \delta^\nu_\mu. \tag{23}$$

Since by definition, the inverse of Λ^{-1} satisfies $(\Lambda^{-1})^\nu{}_\rho \Lambda^\rho{}_\mu = \delta^\nu_\mu$, we have $(\Lambda^{-1})^\nu{}_\rho = \Lambda_\rho{}^\nu$. From (20), (21), and (22), we therefore see that under a Lorentz transformation Λ, a four-covector field $\varphi_\mu(x)$ transforms as $\varphi_\mu(x) \to \varphi'_\mu(x')$ where

{lorentzcovector}

$$\varphi'_\mu(x) = \Lambda_\mu{}^\nu \varphi_\nu(\Lambda^{-1}x) \tag{24}$$

Besides scalars, four-vectors, and four-covectors, we also need to consider physical quantities called rank-two tensors. The stress-energy tensor $T^{\mu\nu}$ mentioned on page 17 is an example of a rank-two tensor. The defining property of a rank-two tensor field $\varphi^{\mu\nu}(x)$ is that under a Lorentz transformation Λ, it transforms as $\varphi^{\mu\nu}(x) \to \varphi'^{\mu\nu}(x')$ where

{lorentztensor}

$$\varphi'^{\mu\nu}(x) = \Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma}\varphi^{\rho\sigma}(\Lambda^{-1}x). \tag{25}$$

On page 8, we introduced the mass-energy density $T_S(x)$ on a hypersurface S. As explained in section 0.1.2, the values of $T_S(x)$ are the additional values that Kent uses to supplement standard quantum theory. It was mentioned in passing that $T_S(x)$ does not depend on which frame of reference one is in. In other words, $T_S(x)$ is a scalar. I will now explain why this is so.

We first need to consider the precise definition of $T_S(x)$. At each spacetime location on the hypersurface S which an observer O describes as having coordinates $x = (x^\mu)_{\mu=0}^3$, we define $\eta^\mu(x)$ to be the future-directed unit four-vector at x that is orthogonal to S. In other words, $\eta^0(x) > 0$, $\eta_\mu(x)\eta^\mu(x) = 1$, and if $y \in S$ is very close to x, then $\frac{(x-y)_\mu \eta^\mu(x)}{\sqrt{(x-y)_\nu (x-y)^\nu}} \approx 0$. $T_S(x)$ is then given by the formula

{TSdef}

$$T_S(x) = T^{\mu\nu}(x)\eta_\mu(x)\eta_\nu(x). \tag{26}$$

For example, if S was the hypersurface consisting of all spacetime locations $x = (0, x^1, x^2, x^3)$, then $\big(\eta^0(x), \eta^1(x), \eta^3(x), \eta^3(x)\big) = (1, 0, 0, 0)$, and hence $T_S(x) = T^{00}(x)$ which is the density of relativistic mass at x, i.e. the energy density at x divided by c^2.

To see why $T_S(x)$ is a scalar, suppose that Λ is a Lorentz transformation such that $\Lambda^0{}_\mu \eta^\mu > 0$ for any future-directed unit four-vector vector η^μ. We refer to a Λ with this property as an **orthochronous** Lorentz transformation. Also, suppose that O and O' are two observers such that spacetime locations that observer O describes as having coordinates $x = (x^\mu)_{\mu=0}^3$ are described by O' as having coordinates $x' = (\Lambda^\mu{}_\nu x^\nu)_{\mu=0}^3$. Then since $x'_\mu y'^\mu = x_\mu y^\mu$, it follows that the future-directed unit four-vector orthogonal to S at x which O describes as $\eta^\mu(x)$ will be described by O' as $\eta'^\mu(x') = \Lambda^\mu{}_\nu \eta^\nu(x)$. Thus, for any location in S that O' describes as having coordinates x' with corresponding future-directed S-orthogonal unit four-vector $\eta'^\mu(x')$, O' can construct a function $T'_S(x')$ with

{TSprimedef}

$$T'_S(x') = T'^{\mu\nu}(x')\eta'_\mu(x')\eta'_\nu(x'). \tag{27}$$

Then using (24) and (25) on the right-hand side of (27), we have

{invariantTS1}

$$\begin{aligned} T'_S(x') &= \Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma T^{\rho\sigma}(x) \Lambda_\mu{}^\alpha \eta_\alpha(x) \Lambda_\nu{}^\beta \eta_\beta(x) \\ &= \Lambda^\mu{}_\rho \Lambda_\mu{}^\alpha \Lambda^\nu{}_\sigma \Lambda_\nu{}^\beta T^{\rho\sigma}(x)\eta_\alpha(x)\eta_\beta(x) \\ &= \delta_\rho^\alpha \delta_\sigma^\beta T^{\rho\sigma}(x)\eta_\alpha(x)\eta_\beta(x) \\ &= T^{\alpha\beta}(x)\eta_\alpha(x)\eta_\beta(x) \\ &= T_S(x) \end{aligned} \tag{28}$$

where on the third line we have used (23), and on the last line we have used (26). To obtain (28), we assumed that Λ is orthochronous, but if Λ is non-orthochronous, we would need to take the negations of $\eta'^\mu(x')$ to get the future-directed S-orthogonal unit four-vector. But clearly this will not affect the equality in (28), so (28) holds for all Lorentz transformations, whether they are orthochronous or non-orthochronous. We thus see that $T_S(x)$ is a scalar.

Let us now consider the Hilbert space H_{S_n} as defined on page 30 for a hypersurface S_n. Given that $\hat{T}^{\mu\nu}(x)$ is the observable whose eigenstates with eigenvalues τ are the states of S_n for which an observer O observes the stress-energy tensor $T^{\mu\nu}(x)$ to take the value τ at x, it follows from (26) that

{TShat}

$$\hat{T}_S(x) \stackrel{\text{def}}{=} \hat{T}^{\mu\nu}(x)\eta_\mu(x)\eta_\nu(x) \tag{29}$$

will be the observable whose eigenstates with eigenvalues $\tau_S(x)$ are the states of S_n for which an observer O observes $T_S(x)$ to take the value $\tau_S(x)$ at x. Two observers O and O' will typically assign different physical states to S_n based on their frame of reference. E.g. if O and O' are traveling at different speeds, they will attribute different energy levels and momenta to the spacetime locations of S_n. For the Lorentz transformation that relates the coordinates of O' to the coordinates of O, i.e. $x' = \Lambda x$, there will then be a unitary operator $U(\Lambda) : H_{S_n} \to H_{S_n}$ such that if O observes S_n to be in the state $|\psi_n\rangle \in H_{S_n}$, then O' will observe S_n to be in the state $U(\Lambda)\,|\psi_n\rangle$. But in order for $U(\Lambda)\,|\psi_n\rangle$ to be meaningful, we need to specify how the Hermitian operators that act on $U(\Lambda)\,|\psi_n\rangle$ correspond to the physical quantities that O' observes. So we specify that the Hermitian operator

{Thatprime}

$$\hat{T}'^{\mu\nu}(x') = \hat{T}^{\mu\nu}(x') \tag{30}$$

will be the observable whose eigenstates with eigenvalues τ' are the states of S_n for which O' observes the stress-energy tensor $T'^{\mu\nu}(x')$ to take the value τ' at x'. Since $T^{\mu\nu}(x)$ transforms according to (25), it will follow that

{TUrelation}

$$U(\Lambda)^{-1}\hat{T}^{\mu\nu}(x)U(\Lambda) = \Lambda^\mu{}_\rho\Lambda^\nu_\sigma\hat{T}^{\rho\sigma}(\Lambda^{-1}x). \tag{31}$$

We also insist that $U(\Lambda)$ is unitary because this means that if O calculates the probability S_n transitions from state $|\psi_n\rangle$ to state $|\chi_n\rangle$, then O' would calculate the same probability for the corresponding transition from the state $|\psi'_n\rangle = U(\Lambda)\,|\psi_n\rangle$ to the state $|\chi'_n\rangle = U(\Lambda)\,|\chi_n\rangle$.[39]

Now to say that Kent's model is Lorentz invariant, is to say that (14) defines a rank-two tensor, for then this quantity and the quantities on which it depends will transform in the way that physical quantities should transform under a Lorentz transformation. Thus, in order to show that Kent's model is Lorentz invariant, we need to show that if $\{|\xi_j\rangle : j\}$ is an orthonormal basis of the Hilbert space of states H_{S_n,τ_S} for which O observes $T_S(x)$ to be $\tau_S(x)$ for all $x \in S_n(y) \cap S$, and if $\{|\xi'_j\rangle : j\}$ is an orthonormal basis of the Hilbert space of states H_{S_n,τ'_S} for which O' observes $T'_S(x')$ to be $\tau'_S(x')$ for all $x' \in S_n(y') \cap S$, then

$$\lim_{n\to\infty} \frac{\langle\Psi'_n|\pi'_n\hat{T}^{\mu\nu}(y')|\Psi'_n\rangle}{\langle\Psi'_n|\pi'_n|\Psi'_n\rangle} = \Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma} \lim_{n\to\infty} \frac{\langle\Psi_n|\pi_n\hat{T}^{\rho\sigma}(y)|\Psi_n\rangle}{\langle\Psi_n|\pi_n|\Psi_n\rangle} \quad \text{\{kentlorentz\}} \tag{32}$$

where $\pi_n = \sum_j |\xi_j\rangle\langle\xi_j|$, $\pi'_n = \sum_j |\xi'_j\rangle\langle\xi'_j|$, and $|\Psi'_n\rangle = U(\Lambda)\,|\Psi_n\rangle$.

To see why (32) holds, we first recall that π'_n will be independent of which orthonormal basis we choose for H_{S_n,τ'_S}.[40] Therefore, if we can show that $\{|\xi'_j\rangle \stackrel{\text{def}}{=} U(\Lambda)\,|\xi_j\rangle : j\}$ is an orthonormal basis of H_{S_n,τ'_S}, it will follow that $\pi'_n = U(\Lambda)\pi_n U(\Lambda)^{-1}$.

That the elements of $\{U(\Lambda)\,|\xi_j\rangle : j\}$ are orthonormal follows from the unitarity of $U(\Lambda)$ together with the orthonormality of $\{|\xi_j\rangle : j\}$. Since $\hat{T}^{\mu\nu}(x')$ is the observable whose eigenstates with eigenvalue τ' are the states of $S_n(y')$ for which O' observes the

[39]This follows from (1) which implies $|\langle\chi'_n|\psi'_n\rangle|^2 = |\langle\chi_n|\psi_n\rangle|^2$, together with the Born Rule given on page **??**.

[40]We showed this was the case for π_n in footnote 33 on page 31.

stress-energy tensor $T'^{\mu\nu}(x')$ to take the value τ' at x', it follows from (27) and (30) that

{TShatprime}

$$\hat{T}'_S(x') \stackrel{\text{def}}{=} \eta'_\mu(x')\eta'_\nu(x')\hat{T}^{\mu\nu}(x') \tag{33}$$

will be the observable whose eigenstates with eigenvalue τ'_S are the states of $S_n(y')$ for which O' observes $T'_S(x')$ to take the value τ'_S at x', where as usual, $\eta'^\mu(x')$ is the unit four-vector orthogonal to $S_n(y')$ at x'. Now if $x' \in S_n(y') \cap S$ in the coordinates of O', then $x = \Lambda^{-1}x' \in S_n(y) \cap S$ in the coordinates of O. Using the same calculation as in (28) together with (24), we have

{TSLambda}

$$\begin{aligned} \hat{T}_S(x) &\stackrel{\text{def}}{=} \eta_\mu(x)\eta_\nu(x)\hat{T}^{\mu\nu}(x) \\ &= \eta'_\mu(x')\eta'_\nu(x')\Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma\hat{T}^{\rho\sigma}(x). \end{aligned} \tag{34}$$

By (31) we have

$$U(\Lambda)^{-1}\hat{T}^{\mu\nu}(x') = \Lambda^\mu{}_\rho\Lambda^\nu_\sigma\hat{T}^{\rho\sigma}(x)U(\Lambda)^{-1}, \tag{35}$$

so using this with (34) and (33), we have

{TSU}

$$\begin{aligned} \hat{T}_S(x)U(\Lambda)^{-1} &= \eta'_\mu(x')\eta'_\nu(x')\Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma\hat{T}^{\rho\sigma}(x)U(\Lambda)^{-1} \\ &= U(\Lambda)^{-1}\eta'_\mu(x')\eta'_\nu(x')\hat{T}^{\mu\nu}(x') \\ &= U(\Lambda)^{-1}\hat{T}'_S(x'). \end{aligned} \tag{36}$$

Now suppose that $|\xi'\rangle$ is a state for which O' observes $T'_S(x')$ to be $\tau'_S(x')$ for all $x' \in S_n(y') \cap S$. Then $\hat{T}'_S(x')\,|\xi'\rangle = \tau'_S(x')$, and so by (36),

{TSUxi}

$$\begin{aligned} \hat{T}_S(x)U(\Lambda)^{-1}\,|\xi'\rangle &= U(\Lambda)^{-1}\hat{T}'_S(x')\,|\xi'\rangle \\ &= \tau'_S(x')U(\Lambda)^{-1}\,|\xi'\rangle \\ &= \tau_S(x)U(\Lambda)^{-1}\,|\xi'\rangle \end{aligned} \tag{37}$$

where on the last line we have used the fact that $T_S(x)$ is a scalar. Therefore, $U(\Lambda)^{-1}\left|\xi'\right\rangle$ can be expressed as a linear combination of the basis elements $\{|\xi_j\rangle : j\}$ of H_{S_n,τ_S}, and hence $|\xi'\rangle$ can be expressed as a linear combination of $\{U(\Lambda)\,|\xi_j\rangle : j\}$. From (36) we also see that $U(\Lambda)\hat{T}_S(x) = \hat{T}'_S(x')U(\Lambda)$, so

$$\hat{T}'_S(x')U(\Lambda)\,|\xi_j\rangle = U(\Lambda)\hat{T}_S(x)\,|\xi_j\rangle = \tau_S(x)U(\Lambda)\,|\xi_j\rangle = \tau'_S(x')U(\Lambda)\,|\xi_j\rangle$$

for all $x' \in S_n(y') \cap S$. Therefore, $U(\Lambda)\,|\xi_j\rangle \in H_{S_n,\tau'_S}$. Since $\{\left|\xi'_j\right\rangle \stackrel{\text{def}}{=} U(\Lambda)\,|\xi_j\rangle : j\}$ is a spanning orthonormal subset of H_{S_n,τ'_S}, it must therefore be an orthonormal basis of H_{S_n,τ'_S}. From this it follows that $\pi'_n = U(\Lambda)\pi_n U(\Lambda)^{-1}$. Therefore,

$$\begin{aligned}
\frac{\langle\Psi'_n|\pi'_n\hat{T}^{\mu\nu}(y')|\Psi'_n\rangle}{\langle\Psi'_n|\pi'_n|\Psi'_n\rangle} &= \frac{\langle\Psi_n|U(\Lambda)^{-1}U(\Lambda)\pi_n U(\Lambda)^{-1}\hat{T}^{\mu\nu}(y')U(\Lambda)|\Psi_n\rangle}{\langle\Psi_n|U(\Lambda)^{-1}U(\Lambda)\pi_n U(\Lambda)^{-1}U(\Lambda)|\Psi_n\rangle} \\
&= \frac{\langle\Psi_n|\pi_n U(\Lambda)^{-1}\hat{T}^{\mu\nu}(y')U(\Lambda)|\Psi_n\rangle}{\langle\Psi_n|\pi_n|\Psi_n\rangle} \qquad \text{\{kentlorentz2\}} \\
&= \frac{\langle\Psi_n|\pi_n\Lambda^{\mu}{}_{\rho}\Lambda^{\nu}_{\sigma}\hat{T}^{\rho\sigma}(y)|\Psi_n\rangle}{\langle\Psi_n|\pi_n|\Psi_n\rangle}
\end{aligned} \tag{38}$$

where on the last line we have used (31). Thus, equation (32) holds, and hence Kent's model is Lorentz invariant.

Note that in this proof of Lorentz invariance, we don't need to take the limit of S_n as $n \to \infty$. That is, we could remove the $\lim_{n\to\infty}$ from equation (14) and consider a particular S_n, and the corresponding $\langle T^{\mu\nu}(y)\rangle_{\tau_S}$ would still be a rank-two tensor. Butterfield tells us that Kent's interpretation is Lorentz invariant because his algorithm respects the light cone structure of y.[41] However, this statement could be slightly misleading because we don't need to consider the subset $S^1(y) \subset S$ of locations outside the light cone of y in order to obtain a Lorentz invariant model. Doing the calculation on any Tomonaga-Schwinger hypersurface is sufficient to guarantee Lorentz invariance

cbx@9
[41] See Jeremy Butterfield, "Peaceful Coexistence: Examining Kent's Relativistic Solution to the Quantum Measurement Problem," 2017, 30, eprint: arXiv:1710.07844.

since any such hypersurface (e.g. S_n) is not altered at all by a Lorentz transformation – only its coordinate description changes under a Lorentz transformation, and so the additional information of the scalar $\tau_S(x)$ on $S_n \cap S$ is Lorentz invariant. The only reason we need to consider the limit $\lim_{n\to\infty} S_n$ and hence $S^1(y) = \lim_{n\to\infty} S_n \cap S$ is that it is only in the limit that we use all the available information in $\tau_S(x)$ to calculate $\langle T^{\mu\nu}(y)\rangle_{\tau_S}$.

0.2.3 Kent's Interpretation and Decoherence Theory Kentdecoherencesection *

In section ?? we saw that decoherence theory by itself does not offer a solution to the problem of outcomes. In this section, we consider how the additional information in Kent's interpretation is sufficient to address this problem. We will explain this by again considering Kent's toy model discussed in section 0.1.4.

We thus suppose that a system is in a superposition $\psi_0^{\text{sys}} = c_1\psi_1^{\text{sys}} + c_2\psi_2^{\text{sys}}$ of two local states ψ_1^{sys} and ψ_2^{sys} where $|c_1|^2 + |c_2|^2 = 1$, and that there is a photon coming in from the left that interacts with the system. We also suppose that y_1 is a spacetime location with spatial location z_1 between the two hypersurfaces S_0 and S, and we consider a hypersurface $S_n = S_n(y_1)$ in a sequence of hypersurfaces that each contain y_1 as described on page 29.

In order to obtain a sufficiently simple description of the state $|\Psi_n\rangle \in H_{S_n}$ of S_n for which we can use the formula (14) to calculate Kent's beable, we will use a coarse-grained model so that S_n is treated as a mesh of tiny cells labeled by a sequence $(y_k)_{k=1}^{\infty}$. Thus, for each cell y_k there will be a Hilbert space H_k describing the state of that cell. We can think of each of these y_k as systems that can become entangled with one another, but we will assume that y_1 is entangled with only a finite number M of the other y_k. What this means is that the most general expression for $|\Psi_n\rangle$ will be of the form

$$|\Psi_n\rangle = \Big(\sum_j \sum_{n\in\mathbb{N}^M} c_{j,n} |\xi_{1,j}\rangle \prod_{l=1}^{M} |\xi_{k_l,n_l}\rangle\Big)\Xi. \tag{39}$$

{Sistate}

In this expression, $\{|\xi_{1,j}\rangle : j\}$ is an orthonormal basis of H_1, $\mathbb{N}^M$ means the set of all lists $(n_1,\ldots,n_M)$ with each $n_l \in \mathbb{N}$ where $\mathbb{N}$ is the set of positive integers greater than 0. The set of states $\{|\xi_{k_l,n_l}\rangle : n_l \in \mathbb{N}\}$ form an orthonormal basis of H_{k_l} for each k_l, and the k_l are all distinct from each other and from 1. Also, M is chosen to be as small as possible so that any common factors of $|\Psi_n\rangle$ belong to Ξ which is a sum of states of the form $\prod_l |\xi_{\kappa_l}\rangle$ where the states $|\xi_{\kappa_l}\rangle \in H_{\kappa_l}$ ranging over all the cells of S_n not included in the set $\{k_l : l = 1,\ldots,M\}$. We also assume that each summand $c_{j,n}|\xi_{1,j}\rangle \prod_{l=1}^M |\xi_{k_l,n_l}\rangle \Xi$ of Ψ_n contains a state in each H_k for every cell k of S_n. In other words if $k \neq 1$ and does not belong to the set $\{k_l : l = 1,\ldots,M\}$ then k belongs to the set $\{\kappa_l : l\}$. Also, we will give H_{S_n} an inner product so that if

$$|\Psi'_n\rangle = \Big(\sum_j \sum_{n\in\mathbb{N}^M} c'_{j,n} |\xi_{1,j}\rangle \prod_{l=1}^{M} |\xi_{k_l,n_l}\rangle\Big)\Xi',$$

then

$$\langle\Psi'_n|\Psi_n\rangle = \Big(\sum_j \sum_{n\in\mathbb{N}^M} \overline{c'_{j,n}} c_{j,n}\Big) \langle\Xi'|\Xi\rangle$$

where $\langle\Xi'|\Xi\rangle$ is defined in the obvious way. With this inner product, we will assume that $|\Psi_n\rangle$ is appropriately normalized so that $\langle\Psi_n|\Psi_n\rangle = 1$. If we also assume that $\langle\Xi|\Xi\rangle = 1$, it will follow that $\sum_j \sum_{n\in\mathbb{N}^M} |c_{j,n}|^2 = 1$.

We now consider several scenarios from Kent's toy model. In each scenario, we will use the decomposition (39) of $|\Psi_n\rangle$ to calculate the partial trace encapsulating all the information needed to calculate expectation values at different spacetime locations.

First, consider Figure 14 which depicts the hypersurface $S_n(y_1^a)$ for a spacetime location y_1^a that occurs before the photon has interacted with the system.

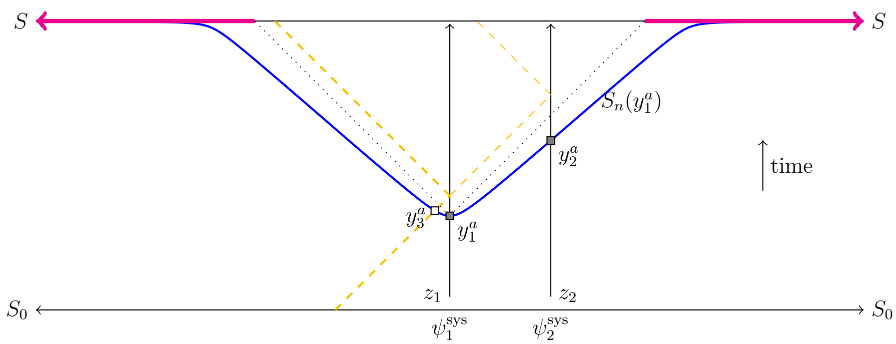

Figure 14: Depiction of a superposition of two local states at z_1 and z_2 before the photon has interacted with them. The gray squares indicate cells in $S^1(y_1^a)$ whose states are among the summands in (39) rather than in Ξ. The white square indicates a cell in $S_n(y_1^a)$ whose state is a factor in Ξ.

kentdeco1

The gray squares correspond to the summands that appear in (39). If the system were in the ψ_1^{sys}-state, then the state describing $S_n(y_1^a)$ would have a factor $|\psi_1^{\text{sys}}\rangle \in H_1$ indicating that there is a non-zero mass at the y_1^a-cell, and there would also be a factor $|0_2\rangle \in H_2$ which we use to indicate that there is zero mass/energy at y_2^a. There is also an incoming photon at the y_3^a-cell, and so we use $|\gamma_3\rangle$ to indicate that there is a photon there. Thus, if the system were in the ψ_1^{sys}-state, we would write the state of $S_n(y_1^a)$ as $|\Psi_n\rangle = |\psi_1^{\text{sys}}\rangle\,|0_2\rangle\,|\gamma_3\rangle\,\Xi'$, where Ξ' describes the states of all the other cells of $S_n(y_1^a)$. In this very simple scenario, $\Xi' = \sum_{k\neq 1,2,3} |0_k\rangle$ indicating that there is zero mass/energy at all the other y_k.

Alternatively, if the system were in the state ψ_2^{sys}, then the state describing $S_n(y_1^a)$ would have a factor $|\psi_2^{\text{sys}}\rangle \in H_2$ indicating that there is a non-zero mass at the y_2^a-cell, and there would also be a factor $|0_1\rangle \in H_1$ which we use to indicate that there is zero mass at y_1^a, and again the y_3^a-cell would be in the $|\gamma_3\rangle$, and every other cell would be described by Ξ' just as if the system had been in the ψ_1^{sys}-state. Therefore, when the system is in the state ψ_2^{sys}, we would write the state of $S_n(y_1^a)$ as $|\Psi_n\rangle = |0_1\rangle\,|\psi_2^{\text{sys}}\rangle\,|\gamma_3\rangle\,\Xi'$.

Now since the system is actually in a supposition $\psi_0^{\text{sys}} = c_1\psi_1^{\text{sys}} + c_2\psi_2^{\text{sys}}$, the state of $S_n(y_1^a)$ will be

$$|\Psi_n\rangle = \big(c_1\,|\psi_1^{\text{sys}}\rangle\,|0_2\rangle + c_2\,|0_1\rangle\,|\psi_2^{\text{sys}}\rangle\,\big)\,|\gamma_3\rangle\,\Xi' = \big(c_1\,|\psi_1^{\text{sys}}\rangle\,|0_2\rangle + c_2\,|0_1\rangle\,|\psi_2^{\text{sys}}\rangle\,\big)\Xi$$

where we have absorbed the $|\gamma_3\rangle$-state into Ξ (i.e. $\Xi = |\gamma_3\rangle\,\Xi'$).

Now as it stands, the state $|\Psi_n\rangle$ describing $S_n(y_1^a)$ has a definite mass-energy density $\tau_S(x)$ for $x \in S_n(y_1^a)\cap S$, namely 0. Thus, if π_n is the operator featuring in (14) that corresponds to this definite mass-energy density, then $\pi_n\,|\Psi_n\rangle = |\Psi_n\rangle$. Therefore, equation (14) for Kent's beables tells us that

$$\langle T^{\mu\nu}(y_1^a)\rangle_{\tau_S} = \langle\Psi_n|\hat{T}^{\mu\nu}(y_1^a)|\Psi_n\rangle\,,$$

where we have also used the fact that $\langle\Psi_n|\Psi_n\rangle = 1$.

Now as we saw in section **??**, if we are interested only in the expectation values of observables for a system $\mathcal{S}$ contained within a universe $\mathcal{U} = \mathcal{S}+\mathcal{E}$, then the information needed to do this can be encapsulated in the reduced density matrix for $\mathcal{S}$. Thus, if the universe is described by a state $|\Psi\rangle = \sum_j c_j\,|\psi_j\rangle_{\mathcal{S}}\,|E_j\rangle$ with corresponding density matrix $\hat{\rho} = |\Psi\rangle\langle\Psi| \in M(H_{\mathcal{U}})$, then the reduced density matrix $\hat{\rho}_{\mathcal{S}} \in M(H_{\mathcal{S}})$ is the Hermitian operator acting on the state space $H_{\mathcal{S}}$ with the property that

$$\langle \hat{O}_{\mathcal{U}} \rangle_{\rho} = \mathrm{Tr}_{\mathcal{S}}(\hat{\rho}_{\mathcal{S}} \hat{O}_{\mathcal{S}}) \qquad \text{(\textbf{??} revisited)}$$

where $\hat{O}_{\mathcal{S}}$ is an observable on $H_{\mathcal{S}}$, and $\hat{O}_{\mathcal{U}}$ is the corresponding observable on $H_{\mathcal{U}}$. Furthermore, we also have

$$\hat{\rho}_{\mathcal{S}} = \sum_j |c_j|^2 |\psi_j\rangle\langle\psi_j| + \sum_{j\neq k} c_j \overline{c_k} \langle E_k|E_j\rangle |\psi_j\rangle\langle\psi_k| .^{42} \qquad \texttt{\{reduced2\}} \quad (40)$$

We can thus apply this to the situation at hand by taking S_n to be our universe $\mathcal{U}$ and y_1^a to be the system $\mathcal{S}$, and $S_n \setminus \{y_1^a\}$ to be the environment $\mathcal{E}$. If we assume that $\langle 0_2|\psi_2^{\mathrm{sys}}\rangle = 0$, then by (40), the corresponding reduced density matrix $\hat{\rho}_{y_1^a}$ takes the form of an improper mixture

$$\hat{\rho}_{y_1^a} = |c_1|^2 |\psi_1^{\mathrm{sys}}\rangle\langle\psi_1^{\mathrm{sys}}| + |c_2|^2 |0_1\rangle\langle 0_1| . \qquad \texttt{\{kentred\}} \quad (41)$$

Kent's beables at y_1^a will thus take the form

$$\begin{aligned} \langle T^{\mu\nu}(y_1^a)\rangle_{\tau_S} &= \mathrm{Tr}_{y_1^a}(\hat{\rho}_{y_1^a} \hat{T}^{\mu\nu}(y_1^a)) \\ &= |c_1|^2 \langle\psi_1^{\mathrm{sys}}|\hat{T}^{\mu\nu}(y_1^a)|\psi_1^{\mathrm{sys}}\rangle + |c_2|^2 \langle 0_1|\hat{T}^{\mu\nu}(y_1^a)|0_1\rangle . \end{aligned} \qquad \texttt{\{kentbe\}} \quad (42)$$

Let us now consider Kent's beables at the spacetime location y_1^b depicted in figure 15.

The state of $S_n(y_1^b)$ will then be

$$|\Psi_n\rangle = \big(c_1 |\psi_1^{\mathrm{sys}}\rangle |0_2\rangle |\gamma_3\rangle |0_4\rangle + c_2 |0_1\rangle |\psi_2^{\mathrm{sys}}\rangle |0_3\rangle |\gamma_4\rangle \big)\Xi$$

where the notation is analogous to that in the previous example. Since no photon detections are registered on $S_n(y_1^b) \cap S$, we again have $\pi_n |\Psi_n\rangle = |\Psi_n\rangle$ so that the reduced density matrix $\hat{\rho}_{y_1^b}$ will again be given by (41) with y_1^a replaced by y_1^b. However,

[42] cf. (**??**)

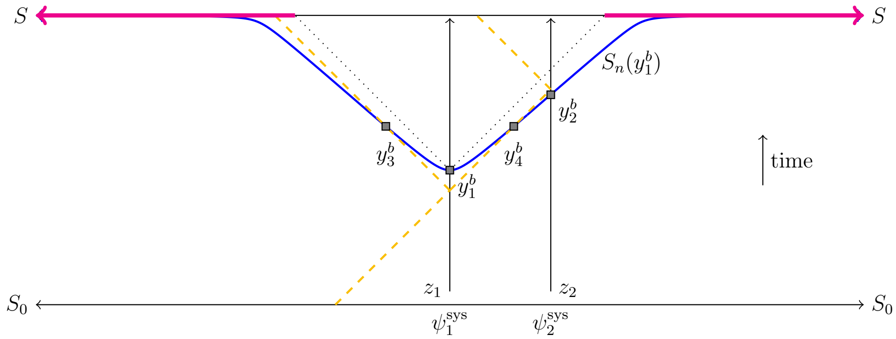

Figure 15: Depiction of a superposition of two local states at z_1 and z_2 with $S_n(y_1^b)$ being after the photon has interacted without the photon intersecting $S_n(y_1^b) \cap S$. The gray squares indicate cells in $S^1(y_1^b)$ whose states are among the summands in (39). kentdecoh2

in this case, Kent's beables $\langle T^{\mu\nu}(y_1^b)\rangle_{\tau_S}$ will not be given by (42) because in the limit as $n \to \infty$, the photon *will* be registered on $S_n(y_1^b) \cap S$.

To deal with the case when a photon is registered on $S_n(y_1^b) \cap S$, we consider a third example as depicted in figure 16.

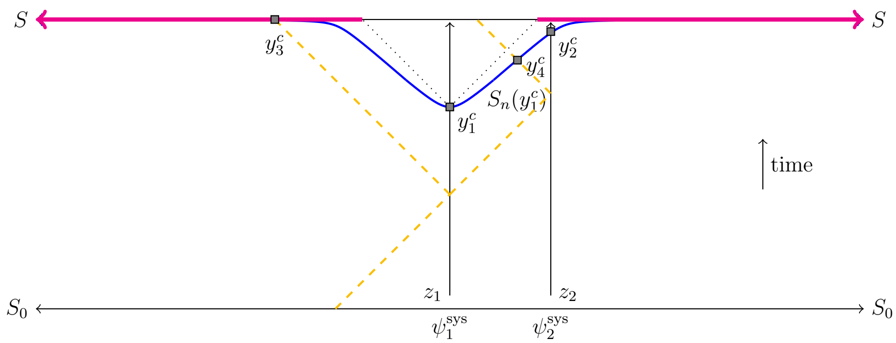

Figure 16: Depiction of a superposition of two local states at z_1 and z_2 with y_1^c sufficiently late that the photon intersects $S_n(y_1^c) \cap S$. The gray squares indicate cells in $S^1(y_1^c)$ whose states are among the summands in (39) kentdecoh3

In this case, the state of $S_n(y_1^c)$ will be

$$|\Psi_n\rangle = \big(c_1 \, |\psi_1^{\text{sys}}\rangle \, |0_2\rangle \, |\gamma_3\rangle \, |0_4\rangle + c_2 \, |0_1\rangle \, |\psi_2^{\text{sys}}\rangle \, |0_3\rangle \, |\gamma_4\rangle \big)\Xi$$

but now we have to consider the fact that the photon intersects $S_n(y_1^c) \cap S$. There are two possible (notional) measurement outcomes that can occur on $S_n(y_1^c) \cap S$: either $T_S = \tau_{S,1}$ where $\tau_{S,1}(y_3^c) \neq 0$, or $T_S = \tau_{S,2}$ where $\tau_{S,2}(y_3^c) = 0$.

The case $T_S = \tau_{S,1}$ indicates that there is a photon detection at y_3^c so that the local state at the y_3^c-cell is $|\gamma_3\rangle$. Therefore, if we write $\pi_{n,1}$ for the operator π_n, we have

$$\pi_{n,1} |\Psi_n\rangle = c_1 |\psi_1^{\text{sys}}\rangle |0_2\rangle |\gamma_3\rangle |0_4\rangle \, \Xi.$$

Therefore, $\langle \Psi_n | \pi_{n,1} \hat{T}^{\mu\nu}(y_1^c) | \Psi_n \rangle = |c_1|^2 \langle \psi_1^{\text{sys}} | \hat{T}^{\mu\nu}(y_1^c) | \psi_1^{\text{sys}} \rangle$ and $\langle \Psi_n | \pi_{n,1} | \Psi_n \rangle = |c_1|^2$. Hence, by (14), Kent's beables at y_1^c will be

$$\langle T^{\mu\nu}(y_1^c) \rangle_{\tau_{S,1}} = \langle \psi_1^{\text{sys}} | \hat{T}^{\mu\nu}(y_1^c) | \psi_1^{\text{sys}} \rangle \, .$$

From this, it follows that the reduced density matrix at y_1^c will take the form of a pure state:

{purerho}

$$\hat{\rho}_{y_1^c} = |\psi_1^{\text{sys}}\rangle\langle\psi_1^{\text{sys}}| \, . \tag{43}$$

On the other hand, for the case when $T_S = \tau_{S,2}$, this indicates that there is no photon detection at y_3^c, so that the local state at the y_3^c-cell will be $|0_3\rangle$. So if we now write $\pi_{n,2}$ for the operator π_n, we have

$$\pi_{n,2} |\Psi_n\rangle = c_2 |0_1\rangle |\psi_2^{\text{sys}}\rangle |0_3\rangle |\gamma_4\rangle \, \Xi.$$

Therefore, $\langle \Psi_n | \pi_{n,2} \hat{T}^{\mu\nu}(y_1^c) | \Psi_n \rangle = |c_2|^2 \langle 0_1 | \hat{T}^{\mu\nu}(y_1^c) | 0_1 \rangle$ and $\langle \Psi_n | \pi_{n,2} | \Psi_n \rangle = |c_2|^2$, and so by (14), Kent's beables at y_1^c will be

$$\langle T^{\mu\nu}(y_1^c) \rangle_{\tau_{S,2}} = \langle 0_1 | \hat{T}^{\mu\nu}(y_1^c) | 0_1 \rangle \, .$$

In this case, the reduced density matrix at y_1^c will be

$$\hat{\rho}_{y_1^c} = |0_1\rangle\langle 0_1| \, , \tag{44}$$

which is again a pure state.

In these examples we have therefore seen how the additional information concerning photon detection on $S_n(y_1) \cap S$ is able to determine whether the reduced density matrix at y_1 is a pure state or an improper mixture. Hence, Kent's interpretation offers an answer to d'Espagnat's problem of outcomes. As mentioned in section **??**, d'Espagnat noticed that with decoherence theory alone, we are not entitled to give an ignorance interpretation to the reduced density matrix for a system that is an improper mixture, and thus we are not able to conclude from the reduced density matrix alone that an outcome has occurred. However, if the reduced density matrix of a system goes from being an improper mixture to a pure state of the form $|\psi\rangle\langle\psi|$ as it does when Kent's additional information is taken into account, then we can say that an outcome has occurred, namely the outcome of the system being in the state $|\psi\rangle$.

0.2.4 Butterfield's Analysis of Outcome Independence in Kent's interpretation

Let us now consider Kent's interpretation in the light of Shimony's notion of Outcome Independence (OI) as defined in section **??**.

Butterfield[43] tries to answer the question of whether OI holds in Kent's interpretation by considering an example that builds on Kent's toy model. Butterfield's example is designed to capture the salient features of a Bell experiment where two spatially separated observers always observe opposite outcomes of some measurement. Following Kent, Butterfield thus considers a universe in one spatial dimensional. In this universe,

[43]See Butterfield, "Peaceful Coexistence: Examining Kent's Relativistic Solution to the Quantum Measurement Problem," 30–32

there are two entangled systems, a left-system and a right-system as depicted in figure 17.

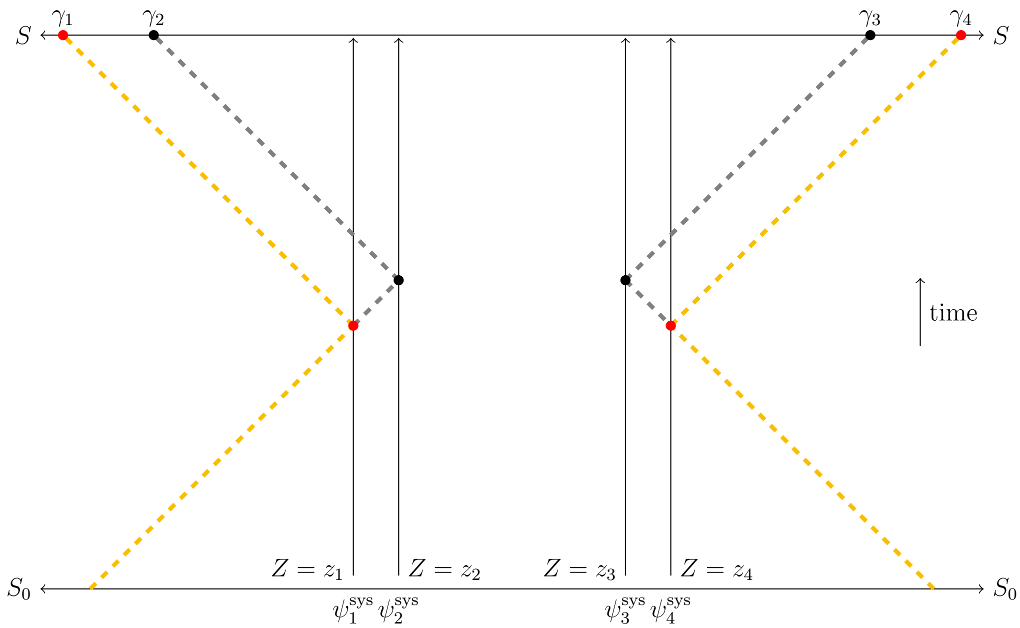

Figure 17: Butterfield's thought experiment for analyzing QI ButterfieldOI

Two locations z_1 and z_2 with $z_2 > z_1$ belong to a left-system, and there are two possible outcomes for a measurement on the left-system: either all the mass/energy of the left-system is localized at z_1 or all the mass/energy of the left-system is localized at z_2. These two possibilities are analogous to a spin up or a spin down measurement outcome in a Stern-Gerlach statement. Likewise, two locations z_3 and z_4 with $z_3 < z_4$ and $z_3 \gg z_2$ belong to a right-system, and again, there are two possible measurement outcomes: either all the mass/energy of the right-system is localized at z_3 or all the mass/energy of the right-system is localized at z_4.

The initial joint state of the two systems is $a\psi_1\psi_4 + b\psi_2\psi_3$. This means that the left-system will be found to be localized at z_1 with probability $|a|^2$, and at z_2 with probability $|b|^2$, and if the left-system is localized at z_1, the right system must be localized at z_4, whereas if the left-system is localized at z_2, then the right system must be localized at z_3.

Now Butterfield supposes that there are two photons, one coming in from the left that interacts with the left system, and one coming in from the right that interacts with the right system. As in Kent's toy model, there is a late time hypersurface S, on which the photons are "measured". Since the joint state of the two systems is in superposition, there will be two possible measurement outcomes for the two photons that arrive at S. Either the left-photon is measured at γ_1 and the right-photon is measured at γ_4, or the left-photon is measured at γ_2 and the right photon is measured at γ_3. Thus, if we suppose that the (notional) measurement for $T_S(x)$ yields an energy distribution $\tau_S(x)$ that is nonzero at γ_1 and γ_4, but is zero at γ_2 and γ_3, then we can say that the outcome of the measurement on the two systems is that the left system is localized at z_1 and the right system is localized at z_4. Moreover, the probability of this outcome is 1 given that the (notional) measurement of $T_S(x)$ on S is $\tau_S(x)$. In other words, this model is deterministic. But as we saw on page **??**, if a model is deterministic, then OI must hold. This is the conclusion that Butterfield draws.

Now if Kent's interpretation is to be consistent with special relativity, OI being satisfied might initially seem concerning. Indeed, we saw in section **??** that OI implies the negation of PI, and the negation of PI is not consistent with special relativity. However, there is one salient feature of a Bell experiment that is not captured in Butterfield's scenario, namely, in a Bell experiment, one can perform different measurements. PI and its negation only make sense when there are parameters that can be changed.

Furthermore, in the proof that OI implies the negation of PI,[44] it is assumed that the choice of parameter is not determined by the hidden variable λ. If the choice of parameters did depend on λ, then for $\hat{a} \neq \hat{b}$, at least one of the probabilities $P_{\lambda,\hat{\boldsymbol{a}},\hat{\boldsymbol{c}}}(\hat{\mathbf{a}}+;\hat{\mathbf{c}}+)$, $P_{\lambda,\hat{\boldsymbol{c}},\hat{\boldsymbol{b}}}(\hat{\mathbf{c}}+;\hat{\mathbf{b}}+)$ or $P_{\lambda,\hat{\boldsymbol{a}},\hat{\boldsymbol{b}}}(\hat{\mathbf{a}}+;\hat{\mathbf{b}}+)$ would not be well-defined.[45] Even though Butterfield is only considering OI in his thought experiment, a proper analysis of OI shouldn't be undertaken without considering an experiment with parameters (e.g. knob settings that correspond to measurement axes of a Stern-Gerlach experiment). This is because the determination of whether OI holds will depend on what one counts as being the hidden variable of a system, and we need the hidden variable of a system to be such that the notion of PI is well-defined. Otherwise, one's verdict on OI will be irrelevant to Shimony's analysis of why Bell's inequality fails to hold.

0.2.5 Hidden variables and the Colbeck-Renner theorem

Butterfield assumes that the hidden variables in Kent's interpretation consist in the outcome $\tau_S(x)$ of $T_S(x)$ over the whole of S, and so far, I haven't questioned this assumption. However, this assumption is going to cause difficulties in the context of Shimony's analysis. This is because in Kent's interpretation, the information in $\tau_S(x)$ over the whole of S clearly would determine which parameters are chosen in a Bell experiment, for this information would determine where a silver atom coming out of a Stern-Gerlach apparatus would be detected on a detection screen (as depicted in figure **??**), and from the position of this detection, one could determine the orientation of the magnetic field used in the Stern-Gerlach experiment. So if we stipulated that

[44] The proof that determinism implies the negation of PI (on pages **??** to **??**), also assumes that the choice of parameter is not determined by the hidden variable λ.

[45] For example, if we thought of $P_{\lambda,\hat{\boldsymbol{a}},\hat{\boldsymbol{b}}}(X;Y)$ as a conditional probability $P(X;Y|\lambda,\hat{\boldsymbol{a}},\hat{\boldsymbol{b}})$ and the probability $P(\lambda,\hat{\boldsymbol{a}},\hat{\boldsymbol{b}}) = 0$, then according to the definition of conditional probability, $P(X;Y|\lambda,\hat{\boldsymbol{a}},\hat{\boldsymbol{b}}) = \frac{0}{0}$.

$\lambda = \tau_S$ is the hidden variable of every system in Kent's interpretation, then Kent's interpretation wouldn't satisfy the preconditions necessary for defining OI and PI. This would make Kent's Interpretation radically different from the pilot wave interpretation where one can define OI and PI because the hidden variables, being the positions and momenta of the particles, are independent of the measurement choices. An unfortunate consequence of not being able to define OI and PI is that we wouldn't be able to evaluate Kent's interpretation in the light of Shimony's analysis of why Bell's inequality fails to hold.

But it is not obvious that we should stipulate that $\lambda = \tau_S$ is the hidden variable of every system in Kent's interpretation. Just because we give τ_S a single label λ, it doesn't follow that τ_S is a single piece of information. There is typically going to be a huge amount of information in τ_S, and for a given system $\mathcal{S}$, we should discern carefully what collection of information in τ_S should be stipulated as being the hidden variable λ of $\mathcal{S}$. The criteria on which we should make such a decision should at least include the following:

1. all the information of λ is about $\mathcal{S}$ so that a change in λ corresponds to a change in the system $\mathcal{S}$. hidden1

In the pilot wave interpretation, the positions and momenta of the particles that constitute a system would fulfil this criterion. On the other hand, all the information in τ_S of Kent's interpretation would not fulfil this criterion unless of course $\mathcal{S}$ was the whole universe.

Note, however, that we don't insist that a difference in $\mathcal{S}$ entails a difference in λ. This is because a hidden-variables theory is envisaged as augmenting standard quantum theory. So in the case when $\mathcal{S}$ is not entangled with any other system, there will be a quantum state describing $\mathcal{S}$, and this quantum state can be other than it is (indicating that $\mathcal{S}$ can be in a different physical state) whilst the hidden variable remains the same. We thus impose a second criterion for a hidden-variables theory: `hidden3`

2. If λ is the hidden variable of a system $\mathcal{S}$ and if $|\phi\rangle$ is the quantum state of $\mathcal{S}$ or of some composite system $\mathcal{U}$ that contains $\mathcal{S}$ as a subsystem, then it is possible for there to be a different quantum state $|\phi'\rangle$ of $\mathcal{S}$ (or $\mathcal{U}$) while the hidden variable λ remains unchanged, and it is possible for there to be a different hidden variable λ while $|\phi\rangle$ remains unchanged.

This criterion is satisfied in the pilot wave interpretation, since the quantum state is the pilot wave itself. The pilot could be other than it is without any of the positions and momenta of the particles changing, but changing the pilot wave would result in a physical change of the system since the pilot wave governs how the positions and the momenta of the particles subsequently evolve over time.

Another criterion for a collection of information λ to constitute the hidden variable of a system $\mathcal{S}$ is the following: `hidden2`

3. it should be possible to change the measurement parameters when measuring $\mathcal{S}$ without this having any affect on λ.

If this criterion doesn't hold, we cannot even begin to consider whether PI holds in a given theory. In the pilot wave interpretation, the positions and momenta of the particles that constitute a system would fulfil this criterion, whereas all the information in τ_S of Kent's interpretation would not. We used this criterion when showing that OI implies the negation of PI.

Closely related to criterion 3 is the following criterion: `hidden5`

4. If p_λ is the probability that a system $\mathcal{S}$ has hidden variable λ, then p_λ must be independent of any choice of measurement made on $\mathcal{S}$.

We are thus assuming there is a whole range of possibilities for the hidden variable λ, but because we don't know what the hidden variable λ is, we can only assign it a probability. Knowledge of the quantum state of the system may help us assign such a probability, but this probability cannot depend on the choice of any future measurement we might make on the system. Butterfield refers to criterion 4 as the 'no-conspiracy' assumption, though he adds that this is a rather unfair label since there wouldn't necessarily be anything conspiratorial if this assumption was violated.[46]

We should also state explicitly a fifth criterion: `hidden4`

5. Suppose $\mathcal{A}$ is any system that is entangled with $\mathcal{S}$, and that the quantum state of the composite system $\mathcal{S}+\mathcal{A}$ is $|\phi\rangle_{\mathcal{S}+\mathcal{A}}$. Then for any measurement $O_{\mathcal{S}}$ on $\mathcal{S}$ and $O_{\mathcal{A}}$ on $\mathcal{A}$, there is a probability $P_\lambda^{|\phi\rangle_{\mathcal{S}+\mathcal{A}}}(O_{\mathcal{S}}=o_{\mathcal{S}}, O_{\mathcal{A}}=o_{\mathcal{A}})$

[46]See Butterfield, "Peaceful Coexistence: Examining Kent's Relativistic Solution to the Quantum Measurement Problem," 34.

for the joint measurement of $O_{\mathcal{S}}$ and $O_{\mathcal{A}}$ on $\mathcal{S}+\mathcal{A}$ that is a function of λ despite λ only referring to the system $\mathcal{S}$.

In addition to these five criteria for a hidden variable λ of a system $\mathcal{S}$, it is also desirable for a hidden-variables theory to satisfy PI and empirical adequacy. We defined PI for a two-outcome measurement on page **??**, but it is easy to generalize the definition of PI for measurements with more than two outcomes. Thus, using the notation of criterion 5 and letting $O'_{\mathcal{A}}$ denote a second choice of measurement on $\mathcal{A}$, PI states that

$$\sum_{o_{\mathcal{A}}} P_{\lambda}^{|\phi\rangle_{\mathcal{S}+\mathcal{A}}}(O_{\mathcal{S}}=o_{\mathcal{S}}, O_{\mathcal{A}}=o_{\mathcal{A}}) = \sum_{o'_{\mathcal{A}}} P_{\lambda}^{|\phi\rangle_{\mathcal{S}+\mathcal{A}}}(O_{\mathcal{S}}=o_{\mathcal{S}}, O'_{\mathcal{A}}=o'_{\mathcal{A}}) \tag{PI}$$

where the summations on both sides are over all the possible measurement outcomes of $O_{\mathcal{A}}$ and $O'_{\mathcal{A}}$ respectively.

As for the definition of **empirical adequacy** (EA), using the notation of criteria 4 and 5, this states that

{adeq}

$$\sum_{\lambda\in\Lambda} p_{\lambda} P_{\lambda}^{|\phi\rangle_{\mathcal{S}+\mathcal{A}}}(O_{\mathcal{S}}=o_{\mathcal{S}}, O_{\mathcal{A}}=o_{\mathcal{A}}) = P^{|\phi\rangle_{\mathcal{S}+\mathcal{A}}}(O_{\mathcal{S}}=o_{\mathcal{S}}, O_{\mathcal{A}}=o_{\mathcal{A}}) \tag{EA}$$

where Λ is the set of all hidden variables so that $\sum_{\lambda\in\Lambda} p_{\lambda}=1$, and where

$$P^{|\phi\rangle_{\mathcal{S}+\mathcal{A}}}(O_{\mathcal{S}}=o_{\mathcal{S}}, O_{\mathcal{A}}=o_{\mathcal{A}})$$

is the standard probability calculated using the Born Rule with the eigenstates of the observables $\hat{O}_{\mathcal{S}}$ and $\hat{O}_{\mathcal{A}}$ and the quantum state $|\phi\rangle_{\mathcal{S}+\mathcal{A}}$. EA is essentially the same as equation (**??**). It also has some similarities with (15), though the main difference is the range of the summation – the index of the summands of (15) does not parametrize hidden variables that satisfy criteria 1 to 5 above.

Now it turns out that criteria 1 to 5 together with the conditions of PI and EA are very restrictive. In his 2016 paper, Leegwater proves a version of the Colbeck-Renner theorem.[47] Leegwater's version takes the following form: if one defines hidden variables according to criteria 1 to 5, then in any hidden-variables theory for which PI and EA hold, the hidden variables are redundant. In other words, in the notation of criterion 5,

{colbeckrenner}

$$P_{\lambda}^{|\phi\rangle_{\mathcal{S}+\mathcal{A}}}(O_{\mathcal{S}} = o_{\mathcal{S}}, O_{\mathcal{A}} = o_{\mathcal{A}}) = P^{|\phi\rangle_{\mathcal{S}+\mathcal{A}}}(O_{\mathcal{S}} = o_{\mathcal{S}}, O_{\mathcal{A}} = o_{\mathcal{A}}) \tag{45}$$

for any measurement $O_{\mathcal{S}}$ on $\mathcal{S}$ and $O_{\mathcal{A}}$ on $\mathcal{A}$.[48]

Thus, the Colbeck-Renner theorem means that we cannot hope to make Kent's interpretation into a hidden-variables theory that satisfies PI and AE by simply defining more carefully what the hidden variables should be, for the information in Kent's interpretation is clearly non-redundant.

But nevertheless, it still seems that we should be able to make some kind of sense of PI and AE in Kent's interpretation and that we should be able to evaluate Kent's interpretation on the basis of whether these notions of PI and EA are true in this context. To achieve this aim, one strategy would be to relax one of the five criteria for a hidden variable. Since we still want to be able to make sense of PI and AE, a process of elimination suggests that the most obvious hidden variable criterion to drop would be criterion 2. In other words, instead of thinking of τ_S as an augmentation of

cbx@12

[47] See Gijs Leegwater, "An impossibility theorem for parameter independent hidden-variable theories," *Studies in History and Philosophy of Modern Physics* 54 (2016): 18–34.

[48] Strictly speaking, we should say that equation (45) holds for almost all λ, but we need not concern ourselves here with the details of measure theory that would be needed to make sense of this qualification.

standard quantum theory, we could instead think of τ_S as a rather elaborate way of stipulating the initial quantum states of experiments as well as the quantum states of measurement outcomes. The information of τ_S would then be non-redundant. Moreover, if we could appropriately partition the information in $\tau_S(x)$ on the basis of whether it determined the quantum state of the particle, or the quantum state of the apparatus, or the quantum state of the rest of the universe, we could then consider whether Kent's interpretation gave the same predictions as standard quantum theory. If it did, then PI and AE would hold in Kent's interpretation, since these both hold in standard quantum theory. And since Kent's interpretation is formulated in the Lorentz invariant setting of Schwinger and Tomonaga, this would mean that Kent's interpretation is a solution to the measurement problem!

0.2.6 Kent's interpretation and standard quantum theory*

In this section, I will show that Kent's interpretation does indeed give the same predictions as standard quantum theory in the case of an experimental apparatus $\mathcal{A}$ measuring the properties of a particle $\mathcal{S}$.

Let's assume that the apparatus has already interacted with many photons during its existence up until the time t_i. Likewise, let's assume the particle has interacted with many photons up until this time. Now suppose that the hypersurface S has energy density $\tau_S(x)$ indicating that some photons have been "measured" on S to be in a state $|\gamma_i^{(\mathcal{A})}\rangle$ which is correlated with the apparatus being in a state $|a\rangle$ shortly before time t_i and in the vicinity of spatial location z_1 as depicted in figure 18. Similarly, we suppose that $\tau_S(x)$ also indicates that some photons have been "measured" on

S to be in a state $|\gamma_i^{(\mathcal{S})}\rangle$ which is correlated with the particle being in a state $|s\rangle$ also shortly before time t_i and in the vicinity of spatial location z_1. We assume that the time t_i is just before the particle enters the apparatus (given the measurement of $|\gamma_i^{(\mathcal{S})}\rangle$ and $|\gamma_i^{(\mathcal{A})}\rangle$ on S) and that no more photons are measured on S that have become entangled with particle or the apparatus until the particle emerges from the apparatus at time t_f. Then some more photons interact with the apparatus $\mathcal{A}$ and get entangled with it shortly before time t_m, and these photons are detected on S to be in a state $|\gamma_f''\rangle$, and this state is correlated with the apparatus now being in a state $|a_f\rangle$, and hence the particle being in state $|s_f\rangle$.

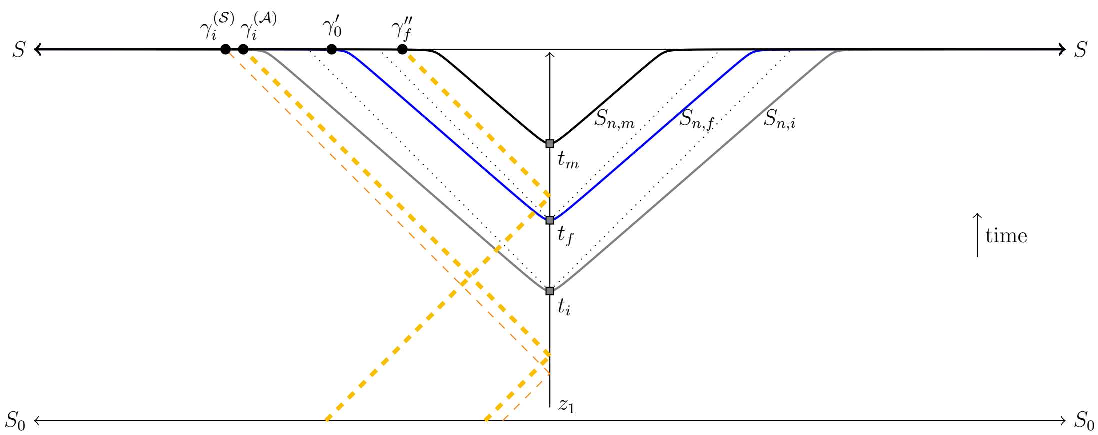

Figure 18: Depicts an experiment where the state of some photons $\gamma_i^{(\mathcal{S})}$ and $\gamma_i^{(\mathcal{A})}$ on the hypersurface S determines the initial conditions of an experimental setup of a particle $\mathcal{S}$ and apparatus $\mathcal{A}$ in the vicinity of the spacetime location (z_1, t_i). The state of the photons γ_f'' on the hypersurface S determines the final state of the apparatus after the particle has left it at time t_f so that the apparatus at time t_m displays a definite measurement outcome. It is assumed that no incoming photons have become entangled with the experiment after the $\gamma_i^{(\mathcal{S})}$ and $\gamma_i^{(\mathcal{A})}$ photons and before the γ_f'' photons have become entangled with the experiment.

`pisolution`

We aim to show that within Kent's interpretation, we can calculate the probability the particle emerges from the measuring apparatus $\mathcal{A}$ in state $|s_f\rangle$ given that it enters $\mathcal{A}$ in state $|s\rangle$, and that this probability is the same as if one ignored S and just applied the Born Rule to $|s\rangle$ and $|s_f\rangle$.

In order to show this, let us choose a sequence of hypersurfaces $S_{n,i}$ which go through the spacetime location $y_i = (t_i, z_1)$ such that $\lim_{n\to\infty} S_{n,i} \cap S = S^1(y_i)$, where as usual, $S^1(y_i)$ consists of all the spacetime locations of S outside the light cone of y_i. Let us assume that n is sufficiently large so that the photons described by $|\gamma_i^{(\mathcal{S})}\rangle$ and $|\gamma_i^{(\mathcal{A})}\rangle$ belong to $S_{n,i}$. The hypersurface $S_{n,i}$ and the photons being reflected from the vicinity of z_1 just before time t_i are also depicted in figure 18.

Typically, the quantum state $|\Psi_{n,i}\rangle = U_{S_{n,i},S_0}\,|\Psi_0\rangle$ of the hypersurface $S_{n,i}$ (where $U_{S_{n,i},S_0}$ is the unitary operator relating the states of two hypersurfaces as discussed on page 14) will also include photon correlations with $\mathcal{S}$ and $\mathcal{A}$ corresponding to other possible "measurements" of $T_S(x)$ besides $\tau_S(x)$. So in general, we would expect the state of $S_{n,i}$ to be of the form

$$|\Psi_{n,i}\rangle = \sum_{j,k} c_{j,k}\,|\sigma_j\rangle\,|\alpha_k\rangle\;|\gamma_j^{(\mathcal{S})}\rangle\;|\gamma_k^{(\mathcal{A})}\rangle\,,$$

where $\{|\sigma_j\rangle : j\}$ is an orthonormal basis of states for the particle $\mathcal{S}$ with $|s\rangle \in \{|\sigma_j\rangle : j\}$, $\{|\alpha_k\rangle : k\}$ is an orthonormal basis of states describing the apparatus $\mathcal{A}$ with $|a\rangle \in \{|\alpha_k\rangle : k\}$, $\{\,|\gamma_j^{(\mathcal{S})}\rangle : j\}$ are normalized states of photons in $S_{n,i} \cap S$ that are entangled with the particle $\mathcal{S}$ such that $\langle\gamma_j^{(\mathcal{S})}|\gamma_{j'}^{(\mathcal{S})}\rangle \approx 0$ for $j \neq j'$, $\{\,|\gamma_k^{(\mathcal{A})}\rangle : k\}$ are normalized states of photons in $S_{n,i} \cap S$ that are entangled with the apparatus $\mathcal{A}$ such that $\langle\gamma_k^{(\mathcal{A})}|\gamma_{k'}^{(\mathcal{A})}\rangle \approx 0$ for $k \neq k'$, and for clarity, we have absorbed any other environmental information into the states $|\alpha_k\rangle$.

If we now define the projection $\pi_{n,i}$ corresponding to the "measurement outcome" $\tau_S(x)$ on $S_{n,i} \cap S$ as in equation (10), and if we also assume that the bases are indexed so that $|s\rangle = |\sigma_i\rangle$ and $|a\rangle = |\alpha_i\rangle$ then

$$\pi_{n,i}\,|\Psi_{n,i}\rangle \approx c\,|s\rangle\,|a\rangle\,\left|\gamma_i^{(\mathcal{S})}\right\rangle\,\left|\gamma_i^{(\mathcal{A})}\right\rangle \tag{46}$$

{piphi}

where $c = c_{i,i}$. For convenience, we will omit the reference to n and write S_i for $S_{n,i}$ and π_i for $\pi_{n,i}$. We will also write $|\Phi_i\rangle$ for the normalized state of $\pi_i\,|\Psi_i\rangle$ so that

$$|\Phi_i\rangle \approx |s\rangle\,|a\rangle\,\left|\gamma_i^{(\mathcal{S})}\right\rangle\,\left|\gamma_i^{(\mathcal{A})}\right\rangle. \tag{47}$$

{phii}

We now suppose that at time t_i, $|a\rangle$ is the ready state of the apparatus with pointer states $\{|a_j\rangle : j\}$ so that if $|s\rangle = \sum_i c_j\,|s_j\rangle$, then under Schrödinger evolution from time t_i to t_f,

$$|s\rangle\,|a\rangle \to \sum_j c_j\,|s_j\rangle\,|a_j\rangle\,.$$

We assume that before time t_f, no photons have had a chance to get entangled with $\mathcal{S} + \mathcal{A}$. It is only by time t_m that we assume a measurement of photons in state $\left|\gamma_f''\right\rangle$ on S outside the light cone of (t_m, z_1) is able to determine that the apparatus is in state $|a_f\rangle$ and hence that the particle is in state $|s_f\rangle$. Although the measurement outcome $\tau_S(x)$ on the whole of S determines with probability 1 that the apparatus and the particle will be in the states $|a_f\rangle$ and $|s_f\rangle$ respectively at time t_m, if we consider the probability $P(f|\,|\Phi_i\rangle)$ that this outcome occurs based just on the state $|\Phi_i\rangle$, then typically this probability is going to be less than 1.

To calculate this probability, we first consider the evolution of $|\Phi_i\rangle$ to $U_{S_f,S_i}\,|\Phi_i\rangle$. It's possible that there may be photons "measured" on S between the hypersurfaces S_i and S_f as indicated in figure 18 (i.e. on $(S \cap S_f) \setminus (S \cap S_i)$), but we are assuming that they do not get entangled with the different pointer states of the apparatus. In other words, $|\Phi_i\rangle$ will evolve to a state of the form

$$U_{S_f,S_i}|\Phi_i\rangle \approx \sum_j c_j |s_j\rangle |a_j\rangle \; |\gamma_i^{(\mathcal{S})}\rangle \; |\gamma_i^{(\mathcal{A})}\rangle \sum_k g_k |\gamma'_k\rangle \,, \tag{48}$$

{USfievolve1}

where $|\gamma'_k\rangle$ correspond to the possible measurements of $T_S(x)$ on $(S \cap S_f) \setminus (S \cap S_i)$, and $\sum_k |g_k|^2 = 1$.

But from time t_f to t_m, we assume that the apparatus does get entangled with photons which are measured on $S \cap S_m$. Thus, if $\{|\gamma''_j\rangle : j\}$ are the normalized states representing the possible measurements outcomes of these photons such that $\langle \gamma''_j | \gamma''_k \rangle \approx 0$ for $j \neq k$, then

$$U_{S_m,S_f} U_{S_f,S_i}|\Phi_i\rangle \approx \sum_j c_j |s_j\rangle |a_j\rangle \; |\gamma_i^{(\mathcal{S})}\rangle \; |\gamma_i^{(\mathcal{A})}\rangle \sum_k g_k |\gamma'_k\rangle \left|\gamma''_j\right\rangle .$$

Since we are assuming that at time t_m a measurement of photons on $S \cap S_m$ is able to determine that the apparatus is in state $|a_f\rangle$, this can only happen if $U_{S_m,S_f} U_{S_f,S_i}|\Phi_i\rangle$ is found to be in one of the states $|\Phi_{k,f}\rangle$ for some k where

$$|\Phi_{k,j}\rangle = |s_j\rangle |a_j\rangle \; |\gamma_i^{(\mathcal{S})}\rangle \; |\gamma_i^{(\mathcal{A})}\rangle |\gamma'_k\rangle \left|\gamma''_j\right\rangle .$$

By the Born Rule, the probability $|\Phi_i\rangle$ will be found to be in state $|\Phi_{k,j}\rangle$ will be

$$|\langle \Phi_{k,j} | \Phi_i \rangle| = |c_j|^2 |g_k|^2.$$

Therefore,

$$P(f|\,|\Phi_i\rangle) = \sum_k |\langle \Phi_{k,f} | \Phi_i \rangle| = |c_f|^2 = |\langle s_f | s \rangle|^2.$$

Hence, the probability that a complete measurement of $T_S(x)$ on S will give a measurement outcome of the particle being in state $|s_f\rangle$ given the partial measurement of $T_S(x)$ on $S_i \cap S$ determines the particle to be initially in the state $|s\rangle$ will be the same as the standard Born Rule probability $|\langle s_f|s\rangle|^2$ of $|s\rangle$ being found to be in state $|s_f\rangle$.

We can also recover this probability using Kent's conditional expectation. To do this, we recall that in standard quantum theory, if for some state $|\psi\rangle$ of a system we define the operator $[\psi] = |\psi\rangle\langle\psi|$, then when the system is in some initial state $|\chi\rangle$, the Born Rule implies that $\langle\chi|[\psi]|\chi\rangle = P(\psi|\chi)$, where $P(\psi|\chi)$ is the probability that the system will be found to be in state $|\psi\rangle$ given that it was initially in state $|\chi\rangle$. But by (**??**), $\langle\chi|[\psi]|\chi\rangle$ is just the expectation $\langle\psi\rangle_\chi$ of $[\psi]$ when $[\psi]$ is treated as an observable.

Now in equation (14), we saw how to calculate the expectation value $\langle T^{\mu\nu}(y)\rangle_{\tau_S}$ of the observable $\hat{T}^{\mu\nu}(y)$ given the notional measurement τ_S on S outside the light cone of y. This suggests that the expectation value of any observable $\hat{O}$ defined at spacetime location (t_i, z_1) given the notional measurement τ_S on S outside the light cone of (t_i, z_1) is going to be

$$\langle\hat{O}\rangle_{\tau_S} = \frac{\langle\Psi_i|\pi_i\hat{O}|\Psi_i\rangle}{\langle\Psi_i|\pi_i|\Psi_i\rangle}.$$

By (46), $\langle\Psi_i|\pi_i|\Psi_i\rangle = |c|^2$, and so taking $\hat{O}$ to be $[s_f]$ we have

$$\langle[s_f]\rangle_{\tau_S} = \frac{|c|^2|\langle s_f|s\rangle|^2}{|c|^2} = |\langle s_f|s\rangle|^2.$$

Thus, Kent's conditional expectation $\langle[s_f]\rangle_{\tau_S}$ gives us the same probability $|\langle s_f|s\rangle|^2$ for a particle transitioning from state $|s\rangle$ to state $|s_f\rangle$ as in standard quantum theory.

Also note that we can typically expect the $|\gamma_i^{(\mathcal{S})}\rangle$-state to be independent of the $|\gamma_i^{(\mathcal{A})}\rangle$-state. Therefore, since $|\gamma_i^{(\mathcal{A})}\rangle$ will determine the measurement choice, and since $|\gamma_i^{(\mathcal{S})}\rangle$ determines the initial state of the particle, we can expect the state of the particle to be independent of the measurement choice in Kent's interpretation. Thus, we can fulfil one of the necessary criteria (i.e. criterion 3) for PI to be a well-defined notion.

0.2.7 Kent's Interpretation and Parameter Independence kentpi

In addition to criterion 3 being satisfied, criterion 5 must also be true if PI is to be a well-defined notion. In the previous section, we saw how we can generalize Kent's beable $\langle \hat{T}^{\mu\nu}(y)\rangle_{\tau_S}$ to calculate conditional expectations $\langle \hat{O}\rangle_{\tau_S}$ for any observable $\hat{O}$ defined at a particular spacetime location (t_i, z_1). Calculating the probability for two measurements requires calculating the conditional expectation of an observable that depends on two spacetime locations. In order to do this, we need to make a further adaption to Kent's interpretation. In this section, we will describe this adaption and show that with it, Kent's interpretation allows us to calculate probabilities for Bell-type experiments, and that these probabilities are the same as in standard quantum theory. Since PI holds in standard quantum theory, a consequence of Kent's interpretation agreeing with standard quantum theory is that PI will also hold in Kent's interpretation.

So let's consider figure 19 which depicts a one-dimensional view of a Bell-type experiment. There is a left wing of the experiment located in the vicinity of z_L, and a right wing of the experiment located in the vicinity of z_R. Shortly before time t_i, photons interact with a Stern-Gerlach apparatus on the left wing and a Stern-Gerlach apparatus on the right wing, with the result that the photons being measured on a hypersurface $S_{n,i} \cap S$ to be in states $|\gamma_i^{(\mathcal{A}_L)}\rangle$ and $|\gamma_i^{(\mathcal{A}_R)}\rangle$ determine the measurement

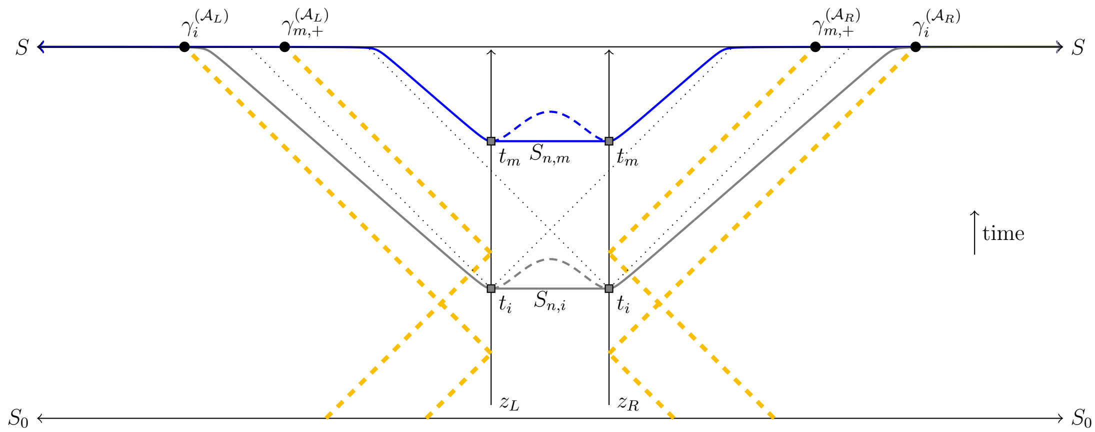

Figure 19: Depicts a Bell-type experiment where the state of some photons $\gamma_i^{(\mathcal{A}_L)}$ and $\gamma_i^{(\mathcal{A}_R)}$ on the hypersurface S determines the choice of measurement parameters of the left wing and right wing of the experiment respectively, and some photons $\gamma_{m,+}^{(\mathcal{A}_L)}$ and $\gamma_{m,+}^{(\mathcal{A}_R)}$ on the hypersurface S determine the measurement outcome of the experiment on the left wing and the right wing respectively. The dashed lines on the hypersurfaces $S_{n,m}$ and $S_{n,i}$ indicate other choices for the hypersurfaces, but they still lead to the same probability being calculated. `bellsolution`

parameters of the apparatuses on the left wing and the right wing of the experiment respectively.

We need to adapt Kent's sequences of hypersurfaces in order to proceed. Sequences of hypersurfaces $S_{n,i}$ are chosen so that they all contain the spacetime locations $y_L = (t_i, z_L)$ and $y_R = (t_i, z_R)$, and that in the limit, $\lim_{n\to\infty} S_{n,i}$ contains as much of $S^1(y_L)$ and $S^1(y_R)$ as possible, where as usual, $S^1(y)$ denotes the subset of S lying outside the light cone of y. Ultimately, this limit (unlike the limit of Kent's hypersurfaces) will not contain the whole of $S^1(y_L)$ or $S^1(y_R)$, but only serves to guarantee that we use as much of the information in S as possible in calculating the expectation values of observables at (t_i, z_L) and (t_i, z_R). There will be some degree of freedom in what we choose for the hypersurface between (t_i, z_L) and (t_i, z_R) as depicted by the dashed line in the figure. However, such freedom will have no effect on the probabilities calculated, because under the assumption that the hypersurface is very far into the future, there will be no choice of hypersurface in this region that would give us more information in S to condition on. Also, we recall that the stress-energy operators in the Tomonaga-Schwinger formulation of relativistic quantum physics are chosen so that they are invariant under perturbations of the hypersurface, so under the assumption that all physical observables will be ultimately expressible in terms of the stress-energy operators, the arbitrary choice of the hypersurfaces in regions that can't intersect with S will have no effect of the probabilities calculated.

On the hypersurface $S_{n,i}$, we assume that there are some photons "measured" on it to be in the states $|\gamma_i^{(\mathcal{A}_L)}\rangle$ and $|\gamma_i^{(\mathcal{A}_R)}\rangle$ that determine the choice of measurement axes for the left and right wings of the experiment respectively. We assume that the axis of orientation of the right wing Stern-Gerlach apparatus makes an angle θ with the axis of the left wing apparatus.

We also assume that there are two particles that together form a Bell-state

$$\frac{1}{\sqrt{2}}(\,|\hat{\mathbf{s}}+\rangle_L\,|\hat{\mathbf{s}}-\rangle_R - |\hat{\mathbf{s}}-\rangle_L\,|\hat{\mathbf{s}}+\rangle_R). \qquad \texttt{\{bellstatePI\}} \quad (49)$$

We saw in footnote **??** on page **??** that a Bell state does not depend on the orientation of $\hat{\mathbf{s}}$, so without loss of generality, we can suppose that the $|\hat{\mathbf{s}}+\rangle_L$ and $|\hat{\mathbf{s}}-\rangle_L$ are pointer states for the apparatus on the left-wing of the experiment. This means there will be a ready state $|a\rangle_L$ as well as two states $|a+\rangle_L$ and $|a-\rangle_L$ of the left wing apparatus such that

$$|\hat{\mathbf{s}}\pm\rangle_L\,|a\rangle_L \rightarrow |\hat{\mathbf{s}}\pm\rangle_L\,|a\pm\rangle_L\,.$$

As for the right wing of the experiment, we let $|\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+\rangle_R$ and $|\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}-\rangle_R$ be pointer states for the apparatus so that there is a ready state $|a\rangle_R$ as well as two states $|a_\theta+\rangle_R$ and $|a_\theta-\rangle_R$ of the right wing apparatus such that

$$|\hat{\boldsymbol{s}}_\theta\pm\rangle_R\, |a\rangle_R \to |\hat{\boldsymbol{s}}_\theta\pm\rangle_R\, |a_\theta\pm\rangle_R\,.$$

As in approximation (47), the detections of the photons on $S_{n,i}\cap S$ being in state $|\gamma_i^{(\mathcal{A}_L)}\rangle$ and $|\gamma_i^{(\mathcal{A}_R)}\rangle$ determine the two particles and the apparatuses on both wings of the experiment to be in the state

{bellstatePI2}

$$|\Phi_i\rangle \approx \frac{1}{\sqrt{2}}(\,|\hat{\mathrm{s}}+\rangle_L\; |\hat{\mathrm{s}}-\rangle_R - \;|\hat{\mathrm{s}}-\rangle_L\; |\hat{\mathrm{s}}+\rangle_R)\,|a\rangle_L\,|a\rangle_R\; |\gamma_i^{(\mathcal{A}_L)}\rangle\; |\gamma_i^{(\mathcal{A}_R)}\rangle\,. \tag{50}$$

As in equations (**??**) and (**??**), we have

$$|\hat{\mathrm{s}}+\rangle_R = \alpha_\theta\, |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+\rangle_R + \beta_\theta\, |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}-\rangle_R\,,$$

$$|\hat{\mathrm{s}}-\rangle_R = \alpha_\theta\, |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}-\rangle_R - \beta_\theta\, |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+\rangle_R\,,$$

where $\alpha_\theta = \cos(\theta/2)$, and $\beta_\theta = \sin(\theta/2)$. Substituting this into (50), we can express the state of the two particles as

{bellstatePI3}

$$\begin{aligned}|\Phi_{n,i}\rangle \approx \frac{1}{\sqrt{2}}(&\alpha_\theta\, |\hat{\mathrm{s}}+\rangle_L\; |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}-\rangle_R - \beta_\theta\, |\hat{\mathrm{s}}+\rangle_L\; |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+\rangle_R \\ &- \alpha_\theta\, |\hat{\mathrm{s}}-\rangle_L\; |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+\rangle_R - \beta_\theta\, |\hat{\mathrm{s}}-\rangle_L\; |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}-\rangle_R)\, |a\rangle_L\, |a\rangle_R\; |\gamma_i^{(\mathcal{A}_L)}\rangle\; |\gamma_i^{(\mathcal{A}_R)}\rangle\,.\end{aligned} \tag{51}$$

If we apply the unitary operator $U_{S_{n,m},S_{n,i}}$ to each of the terms of (51), we get

$$\begin{aligned}&U_{S_{n,m},S_{n,i}}\, |\hat{\mathrm{s}}\pm\rangle_L\; |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}\pm'\rangle_R\, |a\rangle_L\, |a\rangle_R\; |\gamma_i^{(\mathcal{A}_L)}\rangle\; |\gamma_i^{(\mathcal{A}_R)}\rangle \\ &\qquad = \;|\hat{\mathrm{s}}\pm\rangle_L\; |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}\pm'\rangle_R\, |a\pm\rangle_L\, |a_\theta\pm'\rangle_R\; |\gamma_{m,\pm}^{(\mathcal{A}_L)}\rangle\; |\gamma_{m,\pm'}^{(\mathcal{A}_R)}\rangle\end{aligned} \tag{52}$$

where $|\gamma_{m,\pm}^{(\mathcal{A}_L)}\rangle$ are the states of possible detections of photons on $S_{n,m}$ that would determine the left wing apparatus to be in the state $|a\pm\rangle_L$, and where $|\gamma_{m,\pm}^{(\mathcal{A}_R)}\rangle$ are the states of possible detections of photons on $S_{n,m}$ that would determine the right wing apparatus to be in the state $|a_\theta\pm'\rangle_R$. Using the Born Rule, we therefore see that given a measurement of $T_S(x)$ determines the state of the hypersurface $S_{n,i}$ to be in state $|\Phi_{n,i}\rangle$, the probability that the hypersurface $S_{n,m}$ will be found to be in the state

$$|\hat{\mathbf{s}}+\rangle_L \; |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+\rangle_R \, |a+\rangle_L \, |a_\theta+\rangle_R \; |\gamma_{m,+}^{(\mathcal{A}_L)}\rangle \; |\gamma_{m,+}^{(\mathcal{A}_R)}\rangle$$

will be

$$\frac{1}{2}|\beta_\theta|^2 = \frac{1}{2}\sin^2(\theta/2)$$

From this it follows that the probability that the left wing particle will be in state $|\hat{\mathbf{s}}+\rangle_L$ and that the right wing particle will be in state $|\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+\rangle_R$ given the initial conditions will also be $\frac{1}{2}\sin^2(\theta/2)$. This is the same probability as that given by standard quantum theory on page **??**.

Also note that if we define the observable $[\hat{\mathbf{s}}+]_L = |\hat{\mathbf{s}}+\rangle_{L\,L}\langle\hat{\mathbf{s}}+|$ that depends on spacetime location (t_1, z_L), and the observable $[\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+]_R = |\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+\rangle_{R\,R}\langle\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+|$ that depends on spacetime location (t_1, z_R), then we can construct the observable $[\hat{\mathbf{s}}+]_L[\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+]_R$, and with the adapted sequence $S_{n,i}$ of hypersurfaces, we can calculate the conditional expectation

$$\langle[\hat{\boldsymbol{s}}+]_L[\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+]_R\rangle_{\tau_S} = \lim_{n\to\infty} \frac{\langle\Psi_{n,i}|\pi_{n,i}[\hat{\boldsymbol{s}}+]_L[\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+]_R|\Psi_{n,i}\rangle}{\langle\Psi_{n,i}|\pi_{n,i}|\Psi_{n,i}\rangle}.$$

With this adaption and the notional measurement of $T_S(x)$ on S described in this section, it is easy to see that

$$\langle[\hat{\boldsymbol{s}}+]_L[\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+]_R\rangle_{\tau_S} = \frac{1}{2}\sin^2(\theta/2)$$

which is the joint probability for finding the left wing particle in state $|\hat{\mathbf{s}}+\rangle_L$ and the right wing particle in state $|\hat{\boldsymbol{s}}_{\boldsymbol{\theta}}+\rangle_R$. Thus, we can adapt Kent's model so that criterion

5 of page 58 is satisfied and such that Kent's model gives the same probabilities as standard quantum theory. Hence, PI holds in Kent's interpretation.kentpiend

0.2.8 An Alternative to Kent's Beables

A final question to consider in this chapter is the nature of Kent's beables. As discussed on page 16, Bell hoped for a more satisfactory theory than standard quantum theory. This more satisfactory theory was to be a theory of beables rather than a theory of observables, and these beables would form the underlying reality that gives rise to all the familiar things in the world around us.

Now in the context of Kent's interpretation, the definite mass/energy density $\tau_S(x)$ at a location $x \in S$ seems like a good candidate to be a beable. But what is less obvious is what the beables should be for a spacetime locations prior to S. Kent seems to be saying that the beables at a spacetime location y prior to S would consist in the stress-energy tensor whose values were given by the conditional expectation value $\langle \hat{T}^{\mu\nu}(y) \rangle_{\tau_S}$ for all the different combinations of μ and ν.

In some situations, the definite values of $\tau_S(x)$ on S will give rise to $\hat{T}^{\mu\nu}(y)$-eigenstates at y, in which case the conditional expectation $\langle \hat{T}^{\mu\nu}(y) \rangle_{\tau_S}$ will be identical to a definite value for $T^{\mu\nu}(y)$ as normally understood in standard quantum theory. But there will inevitably be situations in which the definite values of $\tau_S(x)$ on S will not give rise to $\hat{T}^{\mu\nu}(y)$-eigenstates at y. In ohter words, in the notation defined in section 0.2.1 on page 30, we may find that the state $\pi_n |\Psi_n\rangle$ at y is in a superposition of eigenstates of $\hat{T}^{\mu\nu}(y)$. But in that case, it seems very dubious to claim that the expectation value $\langle \hat{T}^{\mu\nu}(y) \rangle_{\tau_S}$ is the true state of reality and hence the beable at y. As an analogy, it seems a bit like saying if the throw of a six-sided dice was described quantum mechanically, then the dice could yield a beable of 3.5 for its outcome since 3.5 is the expectation value for the throw of a six-sided dice. In Kent's interpretation, there would of course nearly always be sufficient information in τ_S on the hypersurface S to determine that the dice had an integer outcome between 1 and 6. But this still allows for the remote possibility that this information might not be sufficient. It nevertheless seems unnecessary to insist that physical reality is so definite that we have to allow for the possibility of a six-sided dice having 3.5 as an outcome.

One way to avoid such a possibility would be to suppose that there are degrees of definiteness in physical reality. In the context of Kent's interpretation, one could still suppose that the mass/energy density $\tau_S(x)$ on S was perfectly definite, and that some definite facts about physical reality would flow from the definite facts about S. But there would also be much indefiniteness about physical reality. This would occur whenever the information in $\tau_S(x)$ was insufficient to determine whether the state $\pi_n |\Psi_n\rangle$ was an eigenstate of some observable. If light were to interact with the location y in a different way, then this might be able to settle the question of which eigenstate $\pi_n |\Psi_n\rangle$ was in, but this definite outcome would then result in y being indefinite with respect to other observables. But even though there would inevitably be many physical quantities of a system that lacked definiteness, we would still be able to make claims about how mass/energy *might* have been measured on S and hence how likely a physical quantity might have had a particular value corresponding to this possible measurement on S.

Another issue that should concern us is the question of what should be the subject of the beables. Any beable presupposes a subject as well as a property that belongs to the subject. So in the case of Kent's beables, the spacetime location y would be the subject and $\langle \hat{T}^{\mu\nu}(y) \rangle_{\tau_S}$ would be a property of the subject. However, in section 0.2.7 (pages 67–72), we found that in order to deal with joint probabilities in Bell-type experiments, it was necessary to calculate expectation values for observables that depended on two spacetime locations rather than just one spacetime location.

Now perhaps some people would argue that joint probabilities are not beables and they might therefore conclude that the need to use multiple spacetime locations in determining joint probabilities is irrelevant when it comes to deciding what the subject of a beable should be. But in defense of their relevance, it does seem that joint probabilities are saying something about reality. Moreover, in the absence of hidden variables, these joint probabilities can't be reduced to some more basic reality whose components only depend on single spacetime locations. It therefore doesn't seem unreasonable to attribute beability to probabilities, and if a probability could count as being a beable, then when that probability depended on more than one location, we would obviously have a beable whose subject consisted in more than one location.

But there is another reason for allowing beables whose subject consists in multiple locations, for by allowing this, we can avoid some of the strange consequences that occur if we only allow a single location to be the subject of a beable. For consider figure 20. We suppose that initially the state of S_0 is

$$|\Psi_0\rangle = a\,|\psi\rangle_L\,|0\rangle_R + b\,|0\rangle_L\,|\psi\rangle_R$$

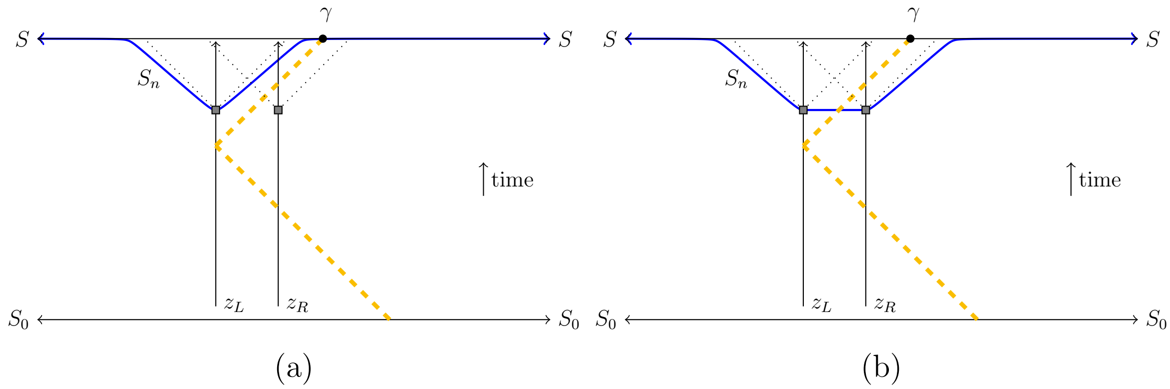

Figure 20: (a) and (b) show how conditional expectations are going to depend on the subject. In (a) the subject consists of spacetime locations (t, z_L) and (t, z_R), whereas in (b) the subject is just (t, z_L). `strangesubject`

with $|a|^2 + |b|^2 = 1$ (and where we have omitted all other terms corresponding to other spacetime locations). A photon then interacts with the joint system and is "measured" on S to be in state $|\gamma\rangle$ thus determining the mass of the composite system to be located at z_L. Now we consider the observable $[\psi]_L$ and suppose its corresponding beable has as its subject a single spacetime location (t, z_L), so that the expectation of $[\psi]_L$ is conditioned on the information in $S_n \cap S$ (taking the limit as $n \to \infty$ as usual) as depicted in figure 20 (a). Then as soon as the photon has interacted with the mass centered at z_L, we can say that the system has collapsed to this mass center (since the photon will be detected on $S_n \cap S$ when we take the limit as $n \to \infty$). But if the photon registers at a location on S that is within the light cone of (t, z_R), and we consider the observable $[\psi]_R$ and its corresponding (t, z_R)-beable, then there is nothing to condition on in S when calculating the expectation value of $[\psi]_R$, and so we will obtain a non-zero answer for this (t, z_R)-beable suggesting that there is some matter/energy at z_R at time t, even though according to the (t, z_L)-beable all the mass/energy is located at z_L at time t. This seems very strange.

However, if we consider the beable whose subject consists of both (t, z_L) and (t, z_R), then because we have to choose a hypersurface that contains both (t, z_L) and (t, z_R) as depicted in figure 20 (b), the system won't collapse to the mass center at z_L as soon as the photon interacts with it. Rather, it will only collapse to this mass center once the photon registered on S is outside the light cones of both (t', z_L) and (t', z_R) for some $t' > t$. Once this time t' has occurred, z_L and the z_R will no longer be entangled, and so we would be able to consider beables whose subjects were the single spacetime locations at (t', z_L) and (t', z_R).

This suggests we should take all the spacetime locations in an entangled system to be the subject of a beable where entanglement is determined by the information in τ_S as well as the initial state $|\Psi_0\rangle$. As the initial state evolves, particle interactions and particle spreading will tend to induce entanglement between spacetime locations, but "measurement" on S will tend to disentangle spacetime locations. For example, if we have an entangled state of the form

$$a\left|\psi_1^a\right\rangle_L\left|\psi_2^a\right\rangle_R\left|\gamma_a\right\rangle + b\left|\psi_1^b\right\rangle_L\left|\psi_2^b\right\rangle_R\left|\gamma_b\right\rangle$$

then once the $|\gamma\rangle$-state is measured on S outside the light cone of these two locations, then the state will disentangle to either

$$\left|\psi_1^a\right\rangle_L\left|\psi_2^a\right\rangle_R\left|\gamma_a\right\rangle$$

or

$$\left|\psi_1^b\right\rangle_L\left|\psi_2^b\right\rangle_R\left|\gamma_b\right\rangle.$$

So with all this entanglement and disentanglement going on, we can expect there to be many different beable-subjects.

In dealing with subjects that contain multiple spacetime locations, the issue of simultaneity needs to be considered carefully, for if all the spacetime locations are simultaneous in one frame of reference, they are not going to be simultaneous in another frame of reference. Moreover, what we deem to be simultaneous in a subject is going to affect which hypersurfaces S_n contain the subject, and these hypersurfaces will in turn determine the state $\pi_n |\Psi_n\rangle$ and hence affect what spacetime locations are entangled. Since entanglement is being proposed as the criterion for spacetime locations to belong to the same subject, we therefore need to consider carefully what is the most appropriate frame of reference for the subject such that its spacetime locations are simultaneous.

A natural candidate for such a frame of reference would be one in which the expectation value of the center of mass of the subject had zero velocity. We still need to be careful, since for greater n, the intersection between S and the hypersurface S_n that contains the subject is going to increase, and so there is typically going to be more information in τ_S available to condition on. This additional information could then possibly lead to disentanglement of regions that were previously entangled for smaller n. If such disentanglement occurred, we would then need to choose one of these new (and smaller) entangled regions and choose a frame of reference in which the new entangled region was simultaneous and in which the center of mass of the entangled region had zero velocity. So we would have to proceed in an iterative manner, but one would hope that eventually we would obtain a set of entangled locations that could serve as the natural subject of a beable. Hopefully, the corresponding beables of this entangled system would be Lorentz invariant in a way analogous to the rest mass of a particle being Lorentz invariant.

In addition to specifying what the subject of a beable should be, we would also need to consider what the beable itself should be. A guiding principle in specifying what the beables should be is to think about what the beables would be in the many-worlds interpretation, and then consider in this light what the most natural beables would be when one introduces a hypersurface S with its "measured" value of T_S.

Now proponents of the many-worlds interpretation often speak as though there is nothing more to reality that the universal state $|\Psi\rangle$ of the universe. If this state were to factorize as

{PhiBeables}

$$|\Psi\rangle = \prod_j |\Psi_j\rangle \tag{53}$$

where $|\Psi_j\rangle$ cannot be factorized further, one might be inclined to say that the beables would be the states $|\Psi_j\rangle$ since these states could be specified independently of each other. But as soon as the systems these states described interacted with each other, they would become entangled, and since there is no principle of disentanglement in the many-worlds interpretation, eventually all systems would get entangled with one another. It would thus seem natural to say there was only one beable in the many-worlds interpretation, namely $|\Psi\rangle$, and that its subject was the whole of physical reality. But once we have a principle by which disentanglement can occur, then we can expect a factorization of $|\Psi\rangle$ as in equation (53), and we could then deem the $|\Psi_j\rangle$ to be the beables, and the subject of each $|\Psi_j\rangle$ would be the entangled region in spacetime that it described.

In the context of Kent's interpretation, I am thus proposing that the beables would be of two sorts. Firstly, there would be the beables at every spacetime location x of S, and the value of each beable would be the "measured" value $\tau_S(x)$ of $T_S(x)$. Here, I am in agreement with Kent.

Secondly, there would be a beable for every region of spacetime B that satisfied the following conditions:

1. the spacetime locations of B are all spacelike separated from one another,

2. there is a Lorentz transformation under which all the members of B become simultaneous,

3. if S_B is any hypersurface that contains B, and S_B is tiled with equally sized cells that can serve as a course-graining for S_B, then B will consist of a finite number of cells,

4. given a course-graining of S_B in which B consists of M cells, there will be a sequence of hypersurfaces S_n containing B and corresponding projections π_n (as described on page 30) such that for sufficiently large n, $\pi_n |\Psi_n\rangle$ will have an irreducible factor of the form

$$|\Psi_B\rangle = \sum_{n \in \mathbb{N}^M} c_n \prod_{l=1}^{M} |\xi_{k_l, n_l}\rangle$$

where the set $\{k_l : l\}$ indexes the cells of B and where we use the notation of page 46,

5. when the Lorentz transformation is applied that makes all the members of B simultaneous, the center of mass of the region B calculated using $|\Psi_B\rangle$ will have zero velocity.

If these conditions hold, then I propose that $|\Psi_B\rangle$ is the beable with B as its subject. This in contrast to Kent's beable $\langle\hat{T}^{\mu\nu}(y)\rangle$ whose subject is the single location y.

Given the first kind of beable $\tau_S(x)$, the second kind of beable, $|\Psi_B\rangle$, will not be superfluous, since $|\Psi_B\rangle$ will say something about the manner in which $\tau_S(x)$ could have been other than it is. For if other photons had interacted with B at different times, the manner in which they interacted with B would depend on $|\Psi_B\rangle$, and from $|\Psi_B\rangle$ we would be able to calculate other possible "measurements" of $T_S(x)$ (and their probability) that could have made on S.

Despite the possible shortcomings of Kent's account of beables that have been mentioned in this section, Kent nevertheless provides a very valuable interpretation of quantum physics: it is Lorentz invariant, it makes predictions consistent with standard quantum theory, it is an interpretation in which parameter independence holds, and it is a one world interpretation. Since parameter independence holds, Kent's interpretation is superior to the pilot wave interpretation, and since it is a one-world interpretation, it avoids the absurdities of the many-worlds interpretation. Kent's interpretation therefore deserves to be taken very seriously as a possible solution to the measurement problem.

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