ML notes

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1 Second order accurateness SpicePy

The source can be a Kronecker-delta impulse such that:

$$\hat{V}(t) = -\frac{2}{\Delta t} A \cdot \mathrm{rect}(\frac{2}{\Delta t}(t - \frac{\Delta t}{2}))\,.$$

The cut-off for spectral content is $\omega_{\max} \sim 1/\Delta t$. The smallest simulated wavelengths are thus

$$\lambda_{\min} = \frac{v_{\min}}{f_{\max}}\,.$$

The spatial discritization step Δz is thus

$$\Delta z = \frac{\lambda_{\min}}{\mathrm{cpw}}\,,$$

where cpw is the amounts of cells per wavelength. Now using the Courant limit

$$\Delta t = \frac{\alpha \Delta z}{\max(v_n)} \quad , \quad 0 < \alpha \leq 1\,,$$

we finally obtain that Δt sets Δz but this also sets Δt such that

$$\Delta z \sim \Delta t^2\,.$$

Let there be m space discretizations of size Δz. Assuming second order accuracy:

$$u(i\Delta z) - u_{exact}(i\Delta z) \sim \mathcal{O}\left(\Delta z^2\right)\,,$$

$$\mathrm{mse} = \sqrt{\frac{\sum_{i=1}^{m-1}\left[u(i\Delta z) - u_{exact}(i\Delta z)\right]^2}{m}} \approx \sqrt{\frac{m}{m}\int_0^1 \mathcal{O}\left(\Delta z^4\right) dx} \sim \mathcal{O}\left(\Delta z^2\right) \sim \mathcal{O}\left(\Delta t^4\right)\,,$$

which is what we experimentally observed. The second order accuracy is thus correct.