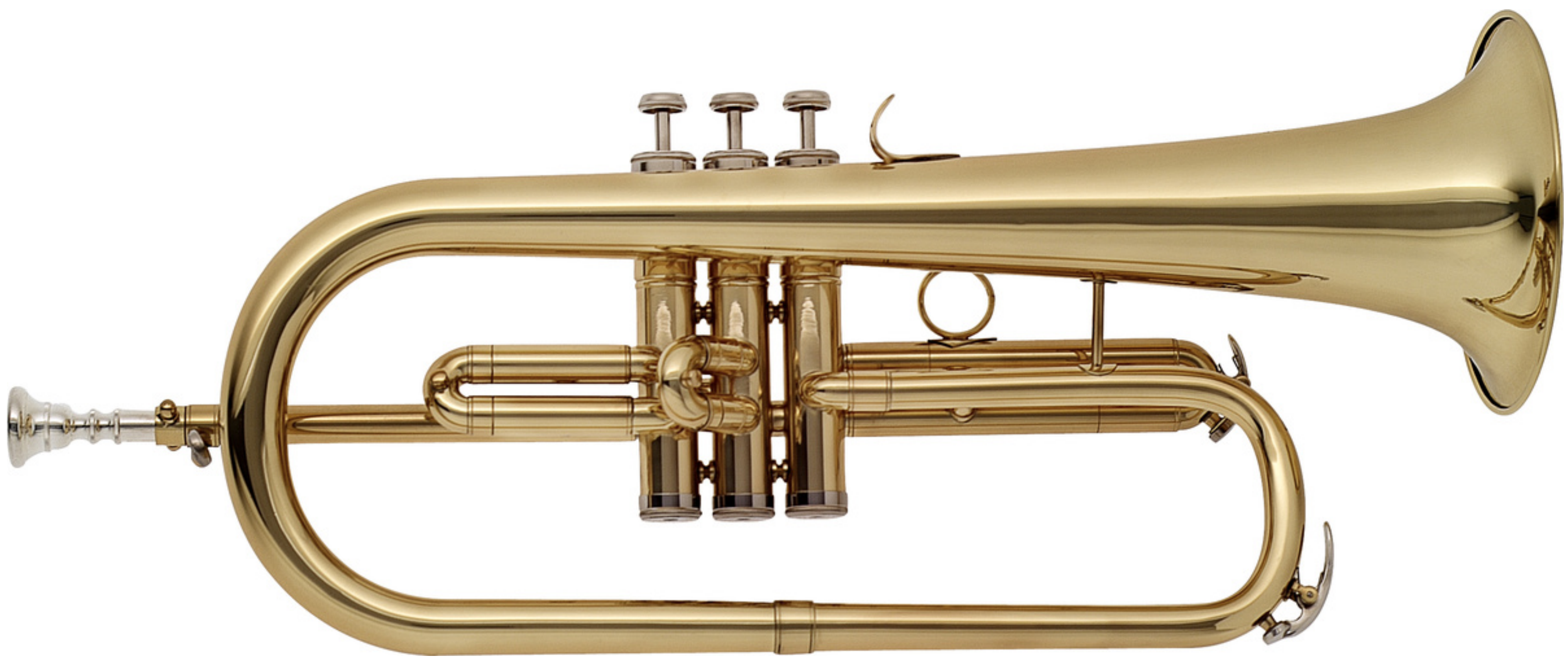
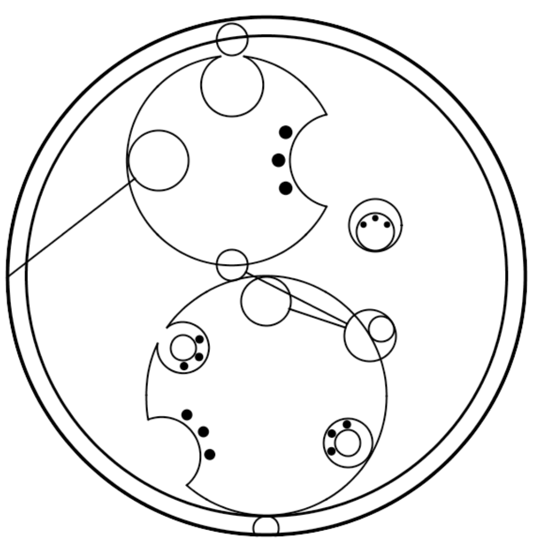
vibrations 'n waves

lecture 12 (+/- 1)





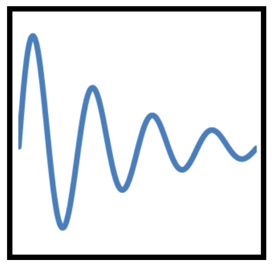
Alex Urban

(in for Prof. Rana Adhikari)

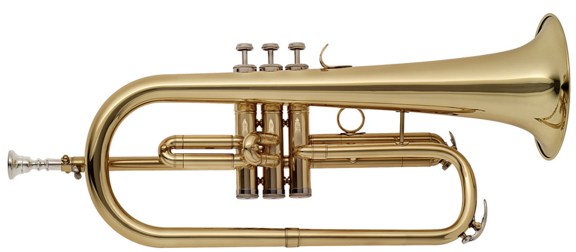
California Institute of Technology

2 November 2017

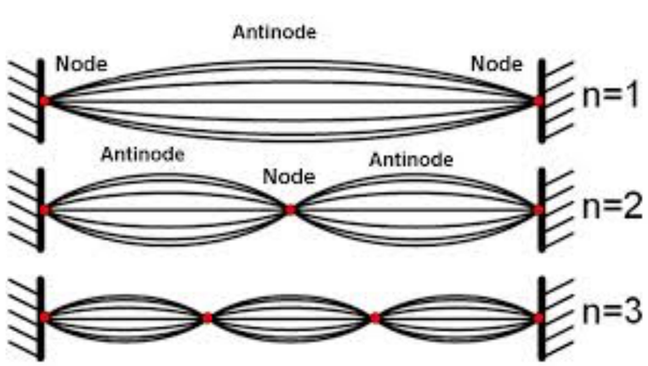
What are we going to look at today?



REFRESHER: FOURIER SERIES



MUSIC: BRASS INSTRUMENTS, TIMBRE, BEAT PHENOMENA



PROBLEMS: 2-D TRAVELING WAVES, CHLADNI WAVES

How does this work, again?

$$h(x) = \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{m=1}^{\infty} b_m \sin(mx)$$

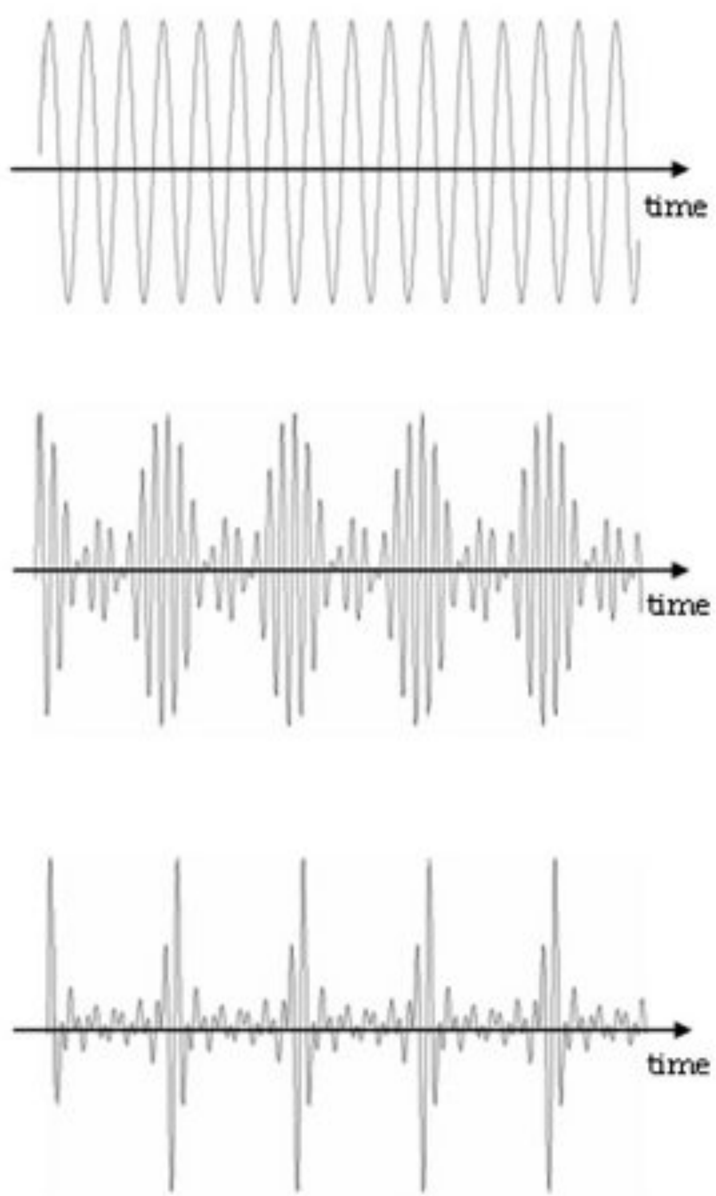
$$a_n = \frac{1}{\pi} \int_{-\infty}^{\infty} h(x) \cos(nx)\ dx$$

$$b_m = \frac{1}{\pi} \int_{-\infty}^{\infty} h(x) \sin(mx)\ dx$$

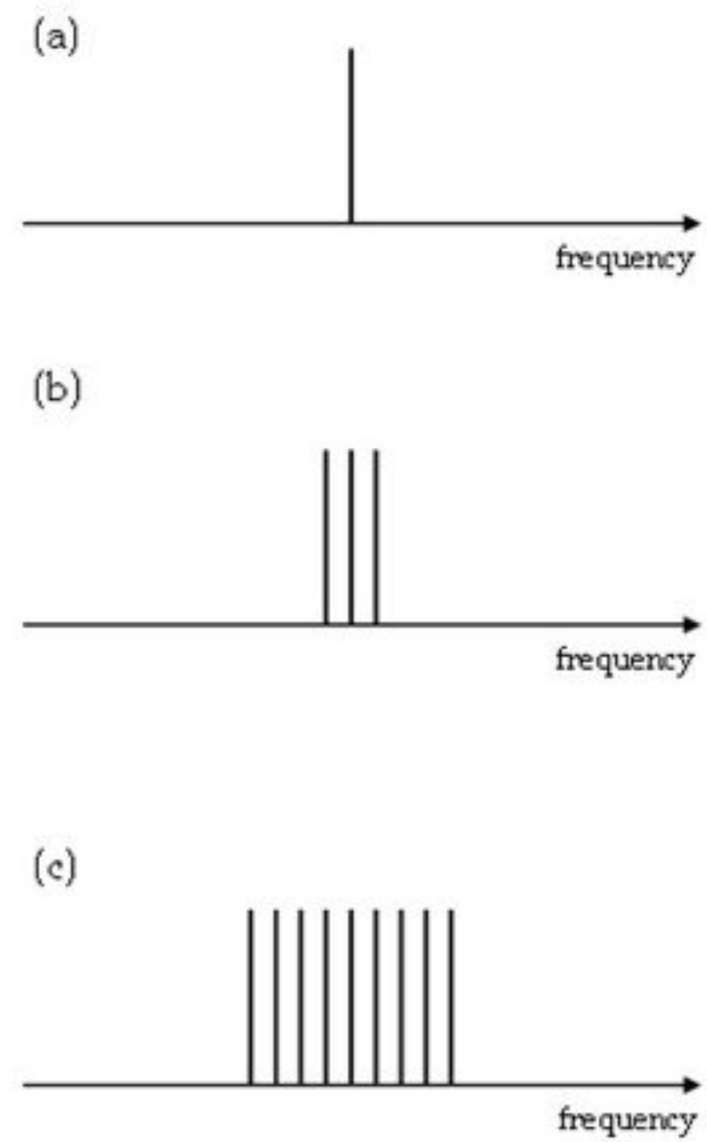
Fourier series: a representation of periodic functions, written as sines and cosines of different frequencies

How does this work, again?

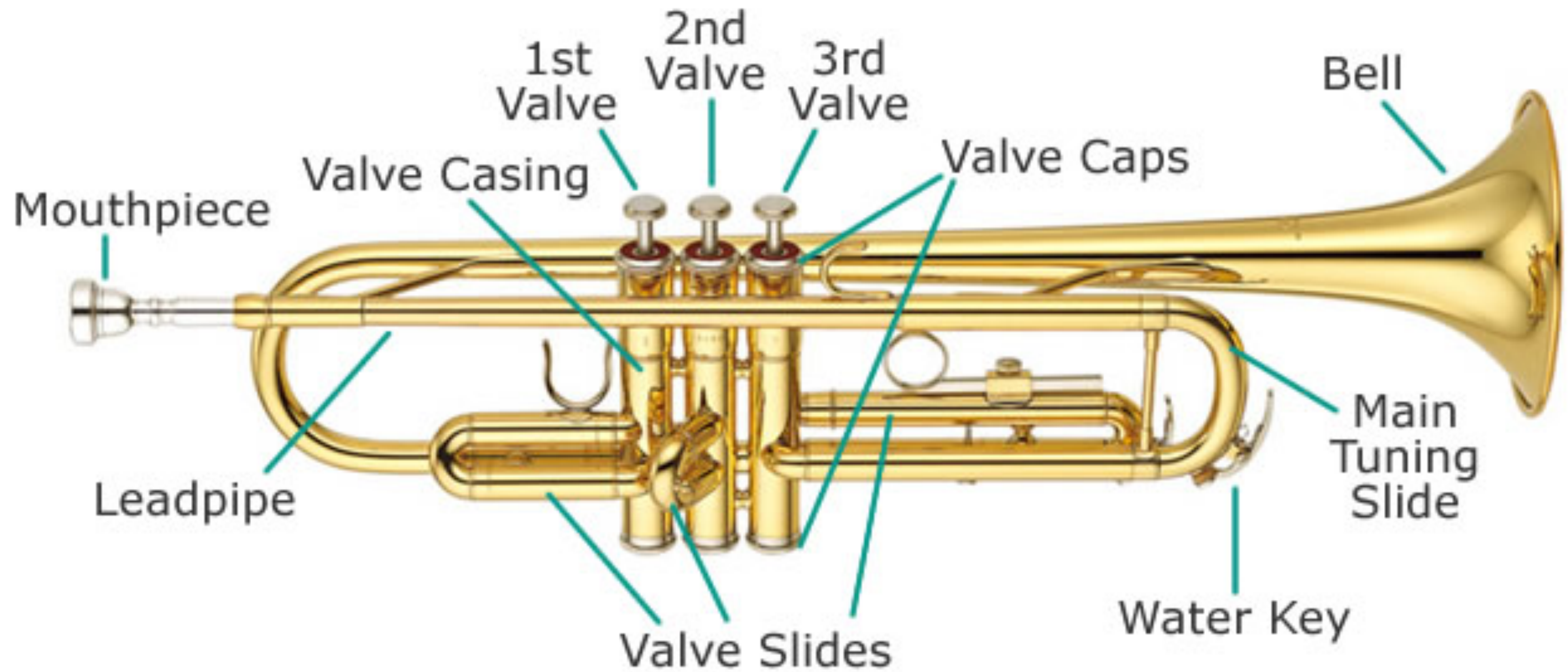
Time



Frequency



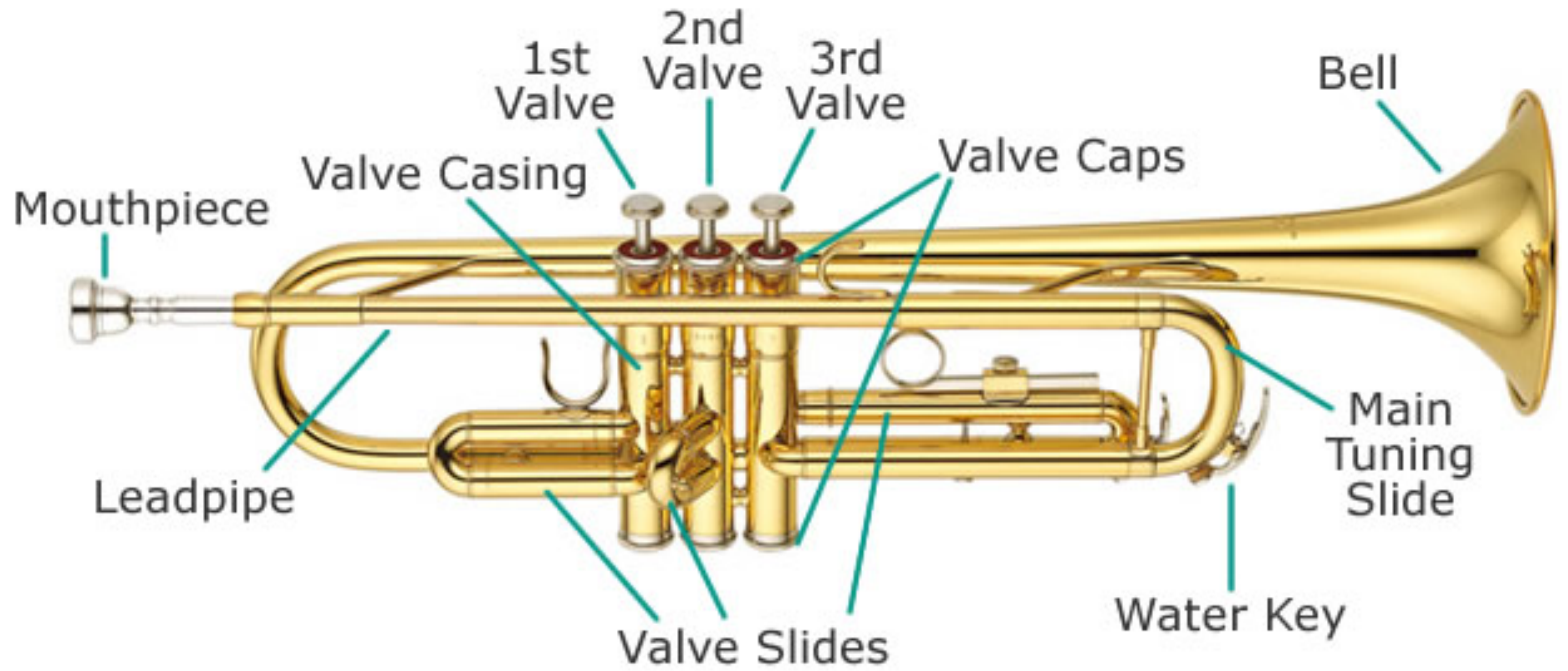
example: brass instruments

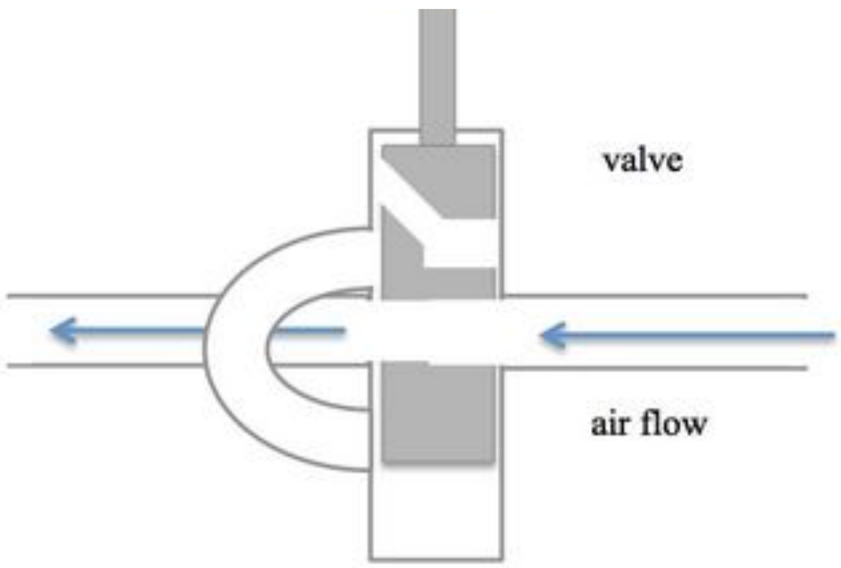


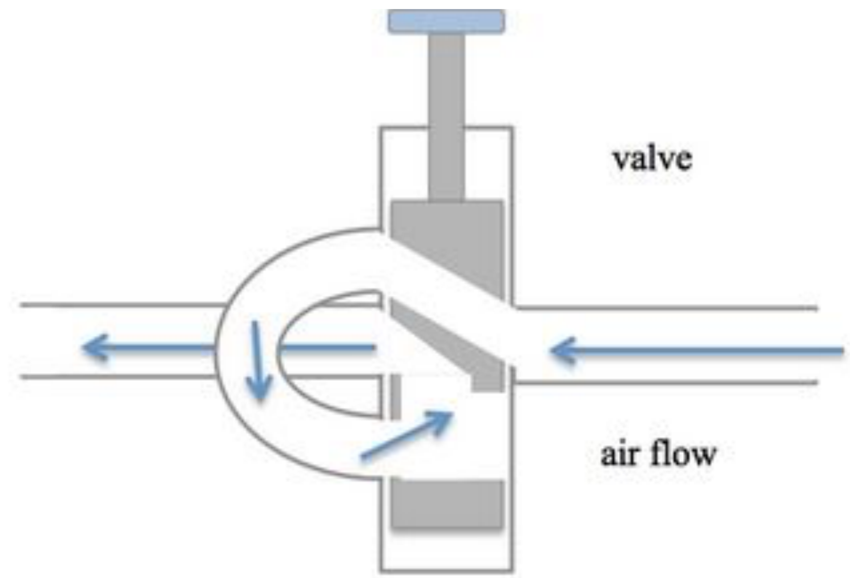
Sympathetic resonance: a system responding to external vibrations that share harmonics

let's watch: https://youtu.be/sxRkOQmzLgo?t=1m31s

example: brass instruments

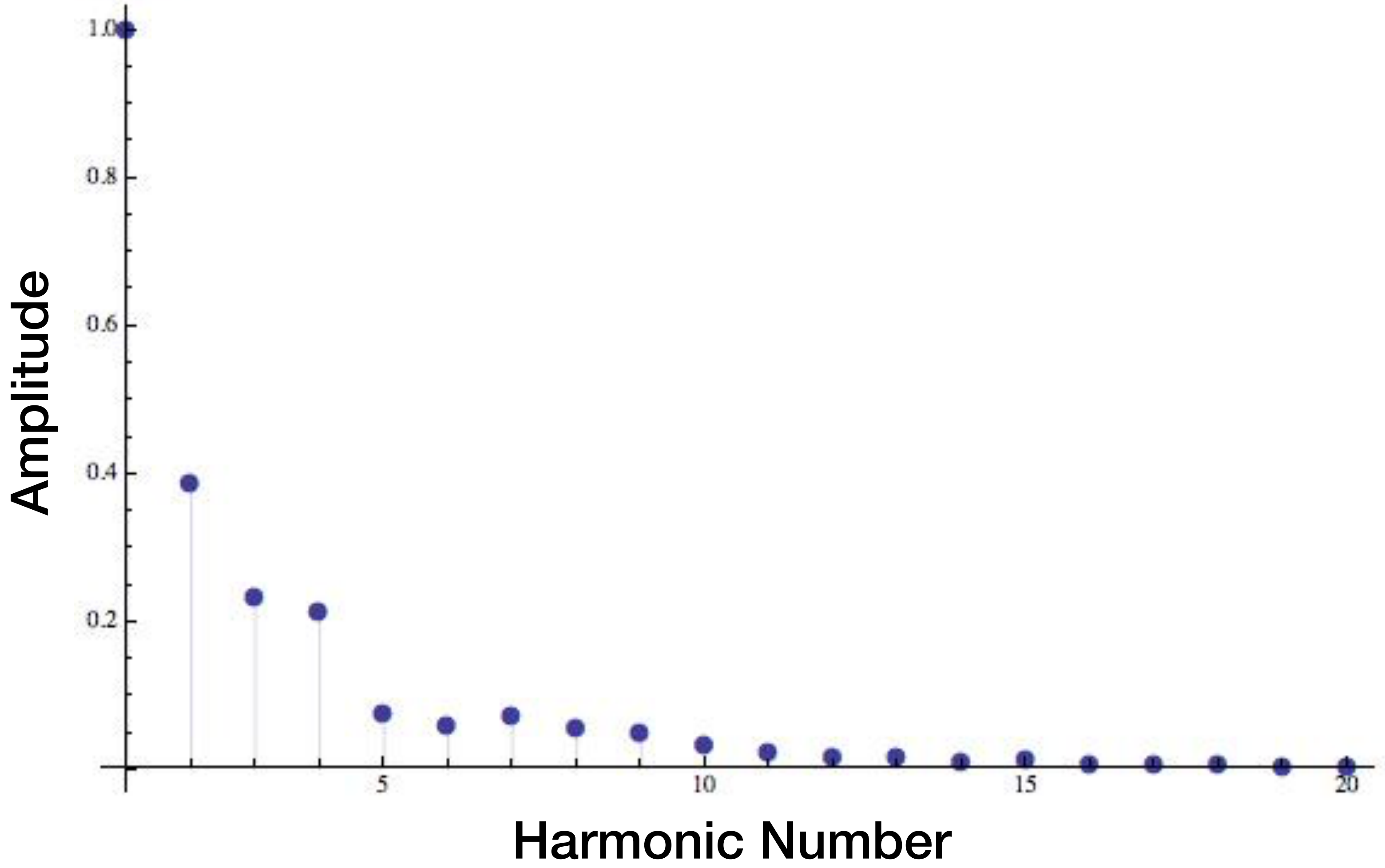




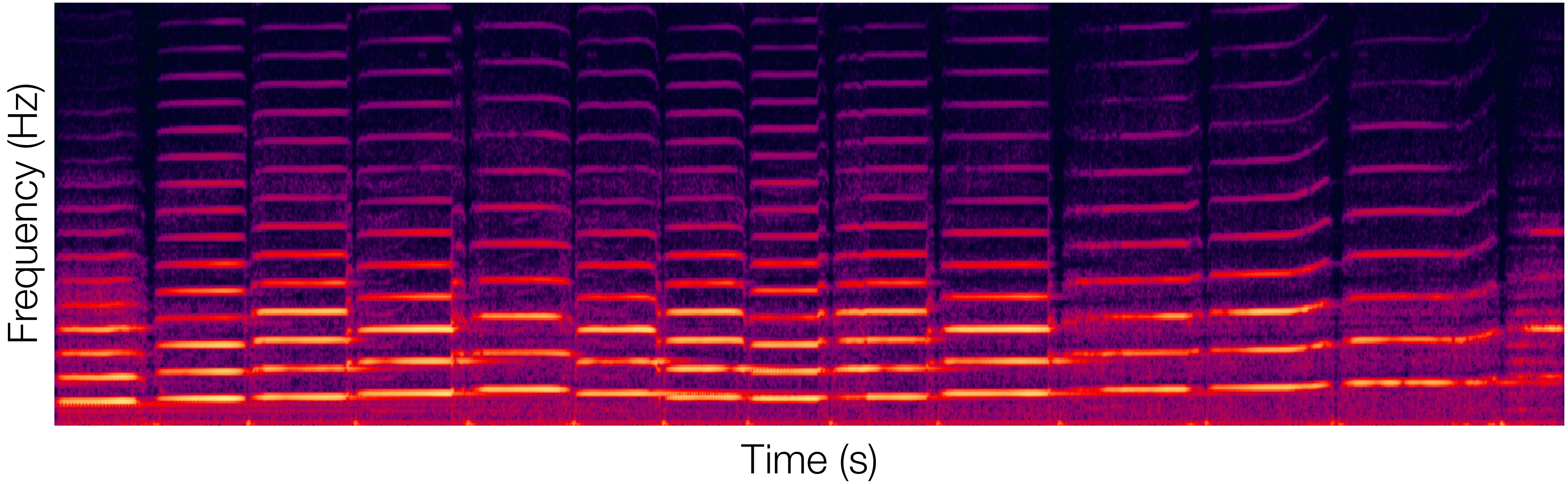


slides and valves used to change the length of tubing; embouchure, lip tension, air flow control specific harmonics

example: brass instruments

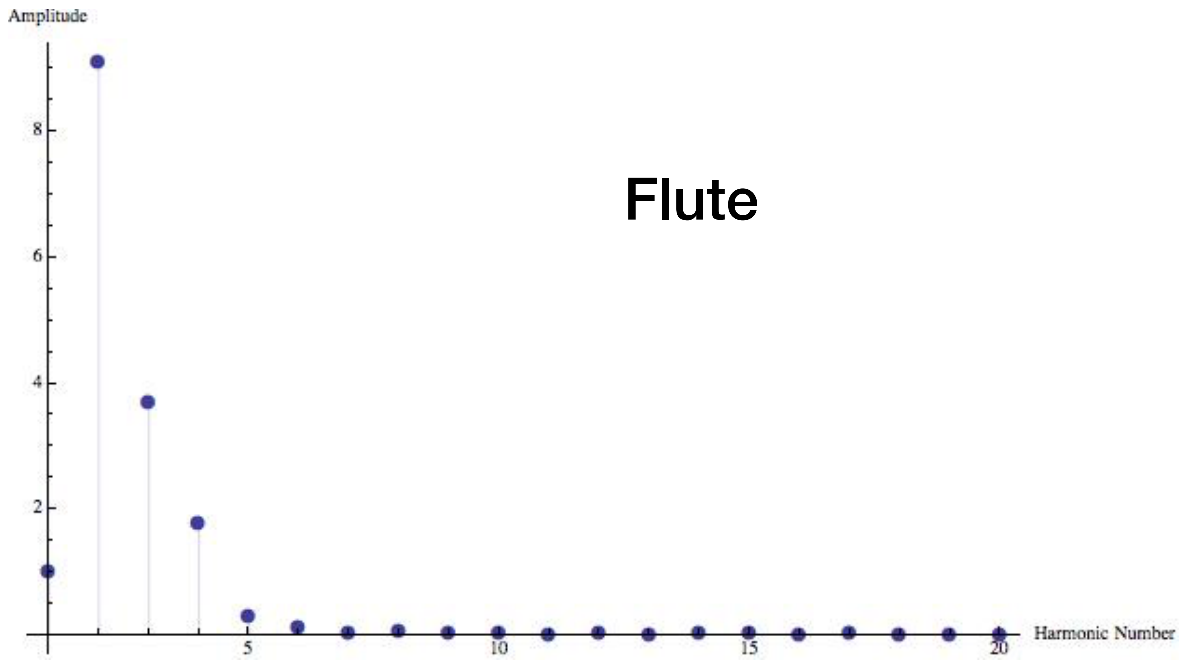


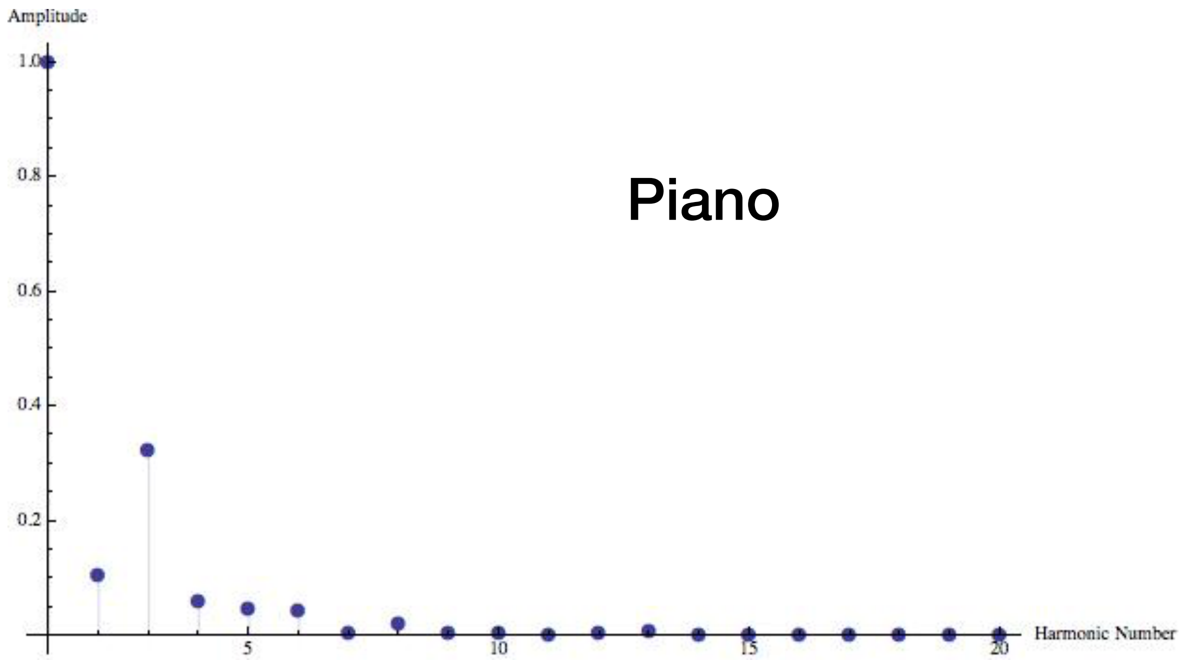
example: string instruments



why do separate instruments playing the same tone sound so different?

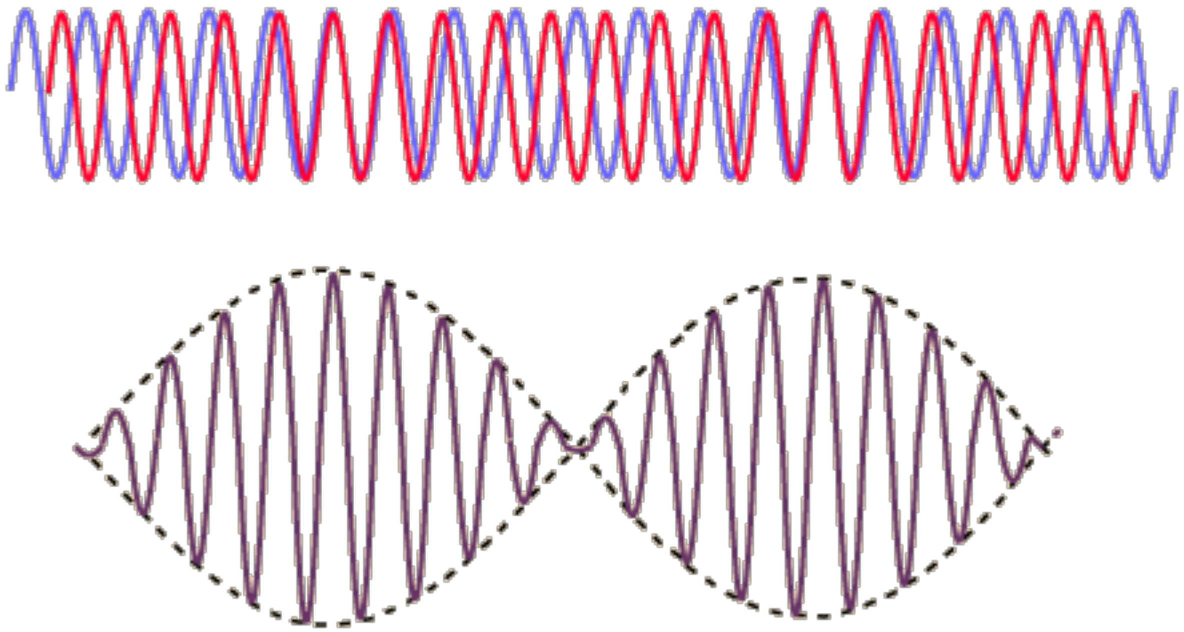
I’M YELLING ‘TIMBRE!’





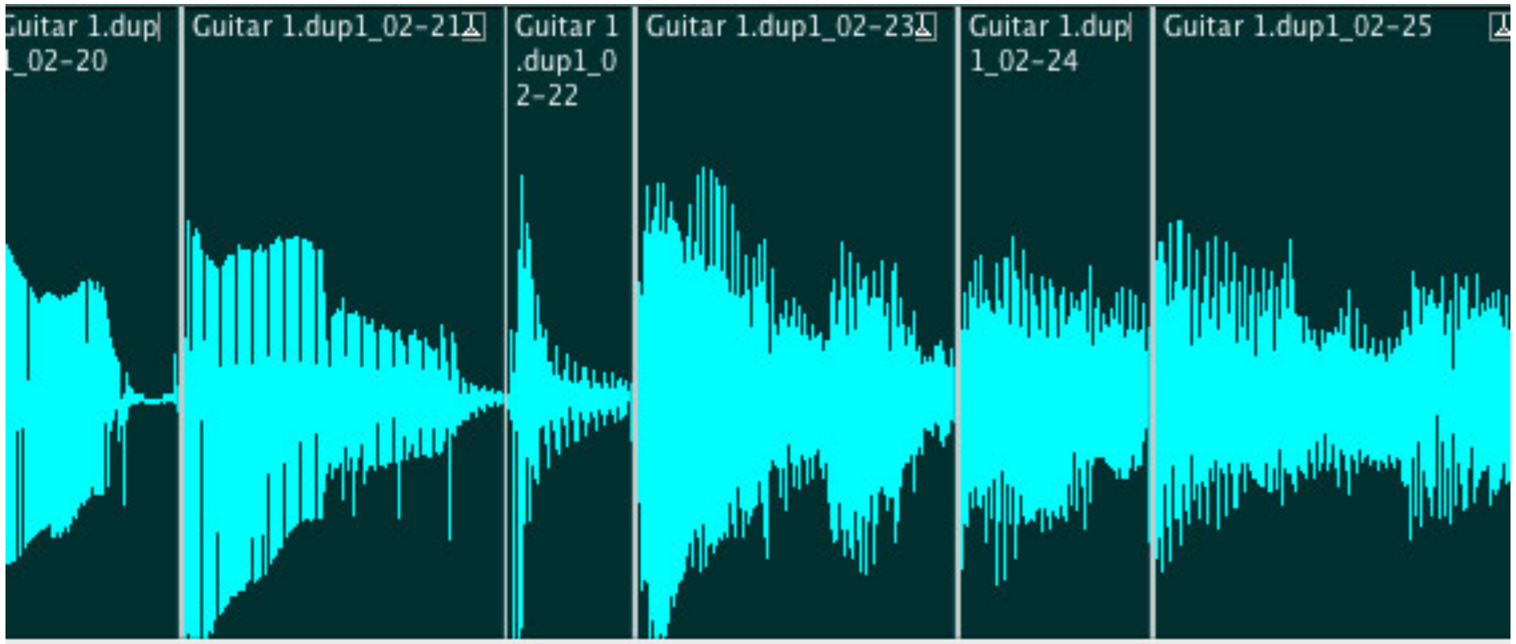
another example
(play for sound)

example: beat notes



$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

guitar melody



2-D traveling waves

$$\frac{1}{v^2}\frac{\partial^2 h}{\partial t^2} = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}$$

2-D wave equation

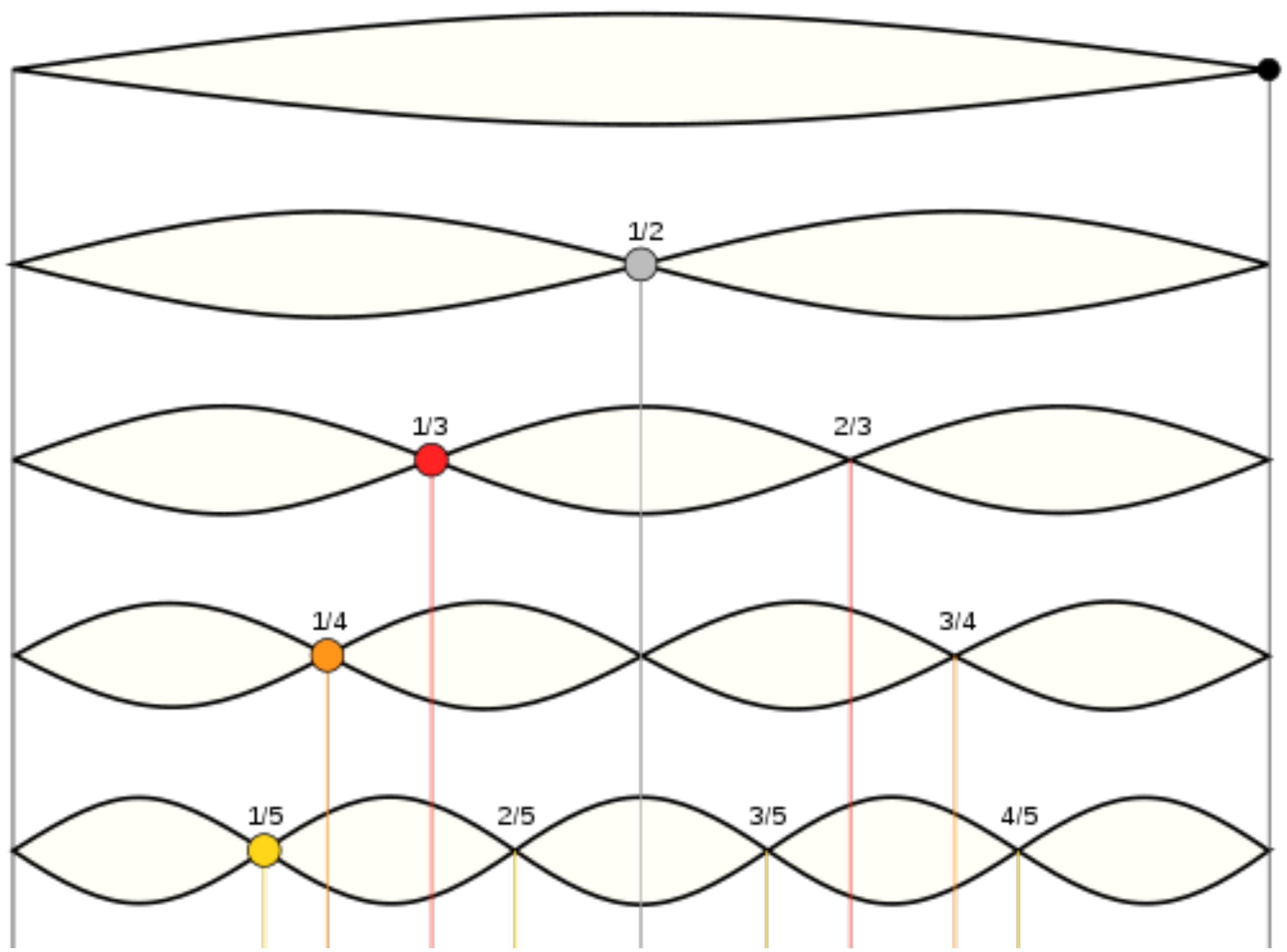
$h = h(x, y, t)$

SEPARATION OF VARIABLES

Let $h(x, y, t) = X(x)Y(y)T(t)$

$$\frac{1}{v^2 T}\frac{d^2 T}{dt^2} = \frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} \quad \cdots$$

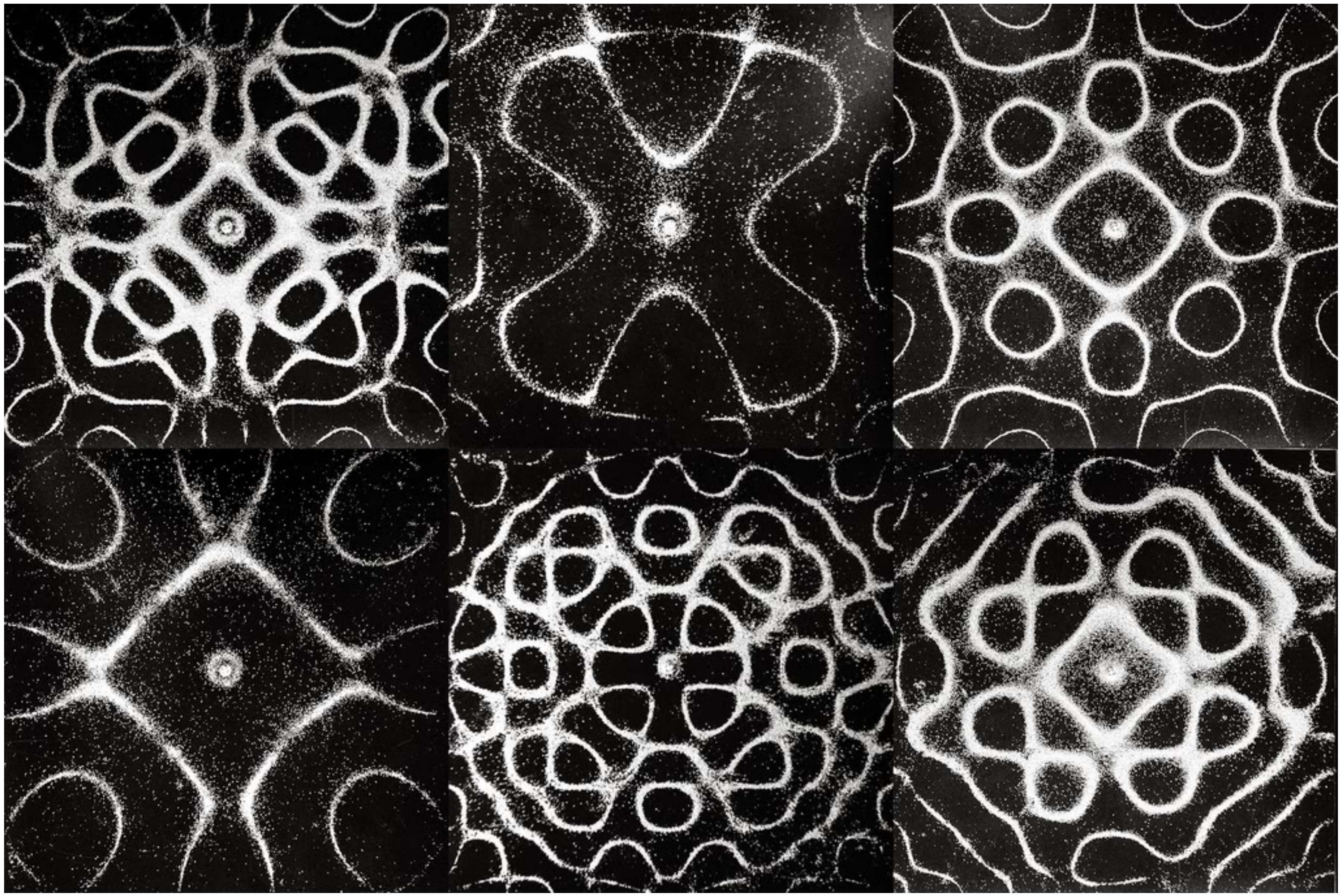
standing waves



superposition of two traveling waves such that the combined wave just sorta…. stands in place

(here is a gif)

2-D example: Chladni waves



standing waves on the surface of a vibrating plate, driven by a heavy speaker or subwoofer

2-D example: Chladni waves

video: https://youtu.be/wYoxOJDrZzw

simulation: https://www.desmos.com/calculator/rdpbran7og

we can estimate how the resonance frequency of Chladni waves scales with wavenumber

2-D example: Chladni waves

standing wave

$$h(x, y, t) = \sin(2\pi f t)\, s(x, y)$$

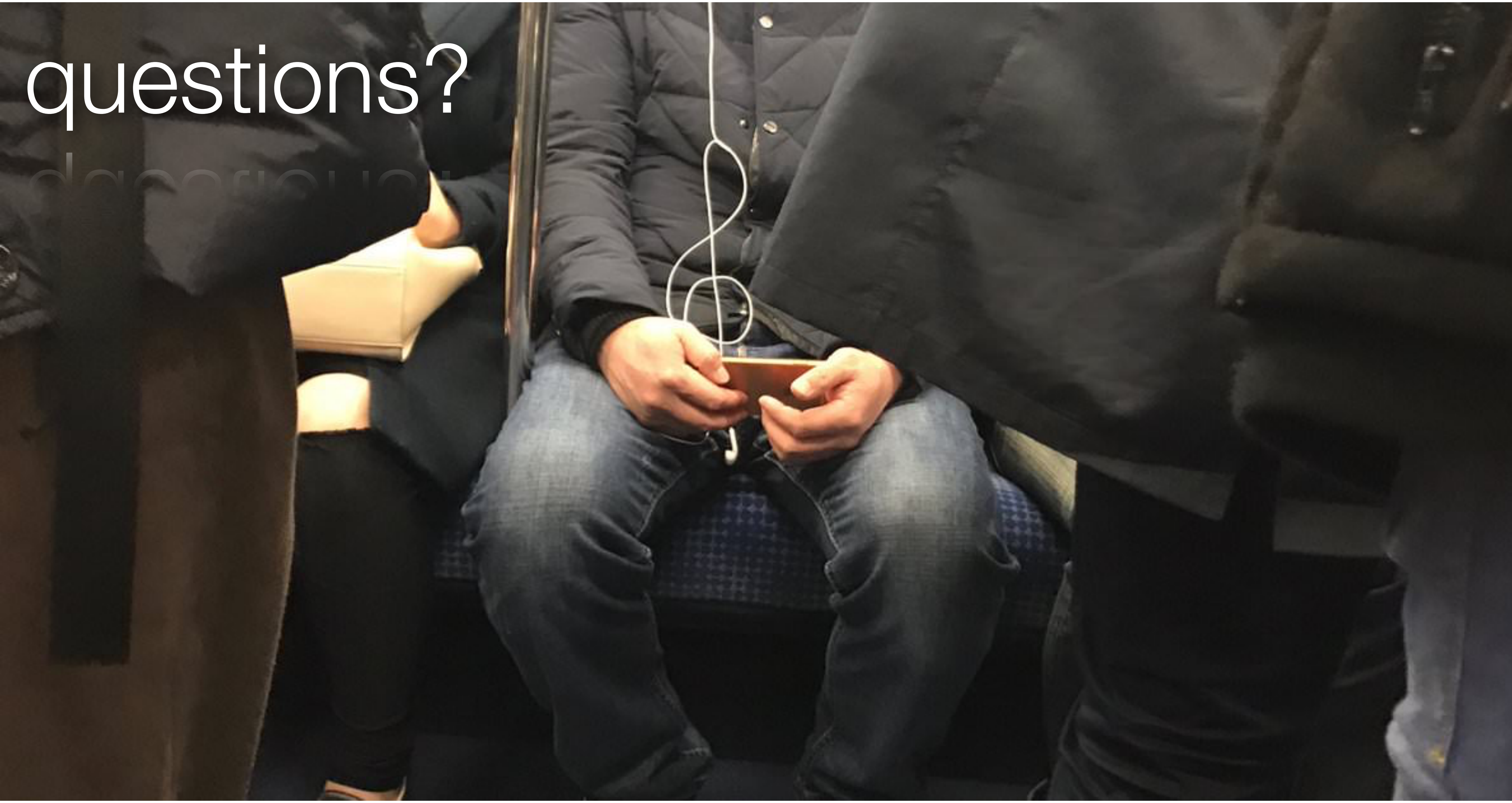
(simplified) boundary conditions

$$s(0, 0) = 0$$
$$s(\pm L, y) = 0$$
$$s(x, \pm L) = 0$$

we can estimate how the resonance frequency of Chladni waves scales with wavenumber

dispersion: a phenomenon where a traveling wave's phase velocity depends on frequency

transmission and reflection: what happens when a traveling wavefront strikes some physical boundary



“every human being has, like Socrates, an attendant spirit; and wise are they who obey its signals.”

lydia m. child

philothea: a grecian romance