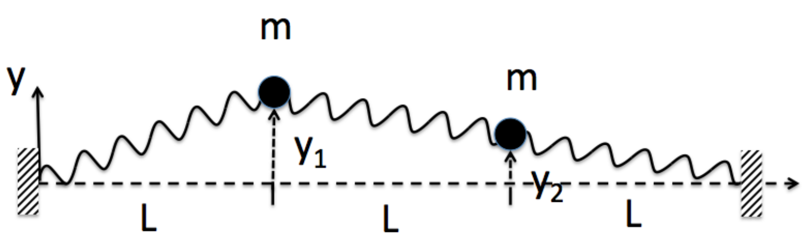
QP-1 String Normal Modes

Two masses are connected by springs as shown in the picture and constrained to only make vertical (y) displacements. Note that the vertical displacements are SMALL, i.e., y_i<<L, which means that the length of the spring and the resulting force does not change. We can therefor write the force on each mass as a tension T_0. What matters is the component of the tension in the y-direction.



a. Show that the y-component of the force on mass 1 is $F_1 = T_0 \frac{y_1}{L} + T_0 \left(\frac{y_1 - y_2}{L} \right)$.

b. Write down the equations of motion in terms of y_1, y_2, and $\omega_0^2 = \frac{T_0}{L}$.

c. From symmetry sketch the two normal modes and argue which one has the larger frequency. For example, you can use our usual notation showing the maximum amplitude vs. oscillator number for each mode.

d. Substitute a complex exponential solution into b. and find the algebraic equation set that results.

e. Find the normal mode frequencies in terms of ω_0 .

f. Find the normal mode amplitudes (to within an arbitrary constant) and write down the two normal modes (you can use the compact vector format if you like).

g. Write down the most general solution.
h. Now write down a solution which pertains if mass 1 is started at zero velocity and position y=A, while mass 2 is started with position 0 and velocity 0, i.e.,
$y_1(t=0) = A, \quad \dot{y}_1(t=0) = 0, \quad y_2(t=0) = 0, \quad \dot{y}_2(t=0) = 0$