vibrations ’n waves

lecture 13

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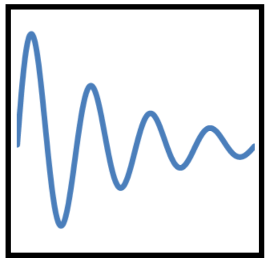
(in for Prof. Rana Adhikari)

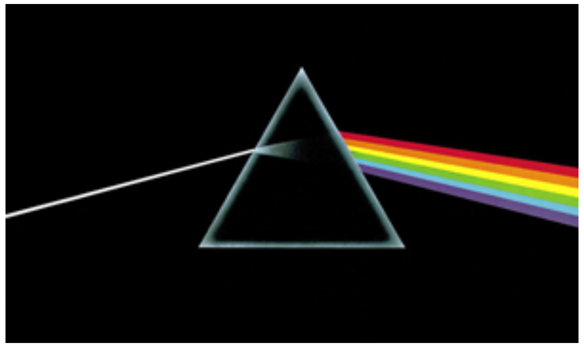
California Institute of Technology

7 November 2017

What are we going to look at today?



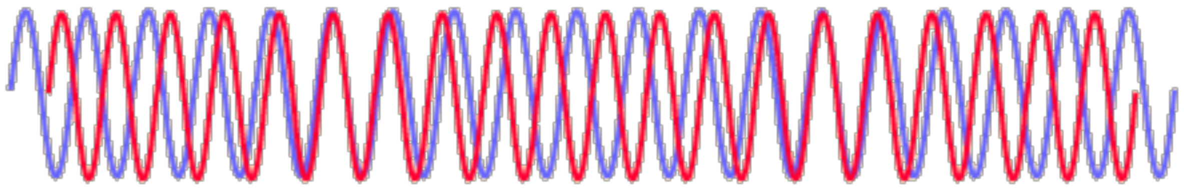
REFRESHER: FOURIER SERIES

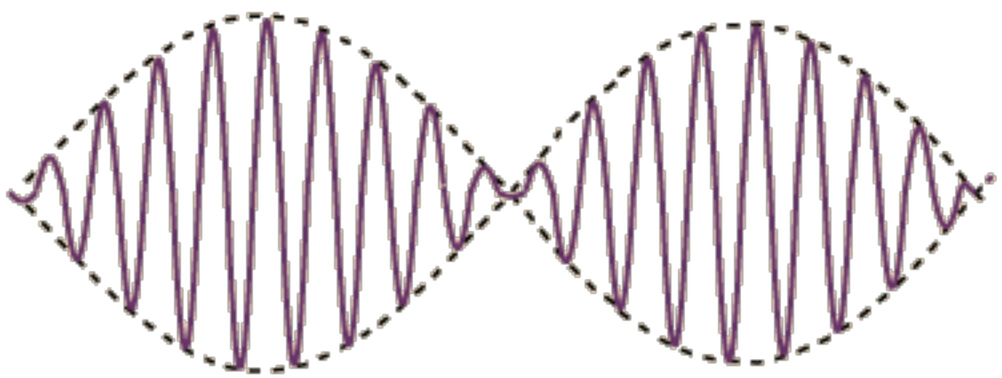


TRAVELING WAVES: DISPERSION

WAVES @ A BOUNDARY:
TRANSMISSION & REFLECTION

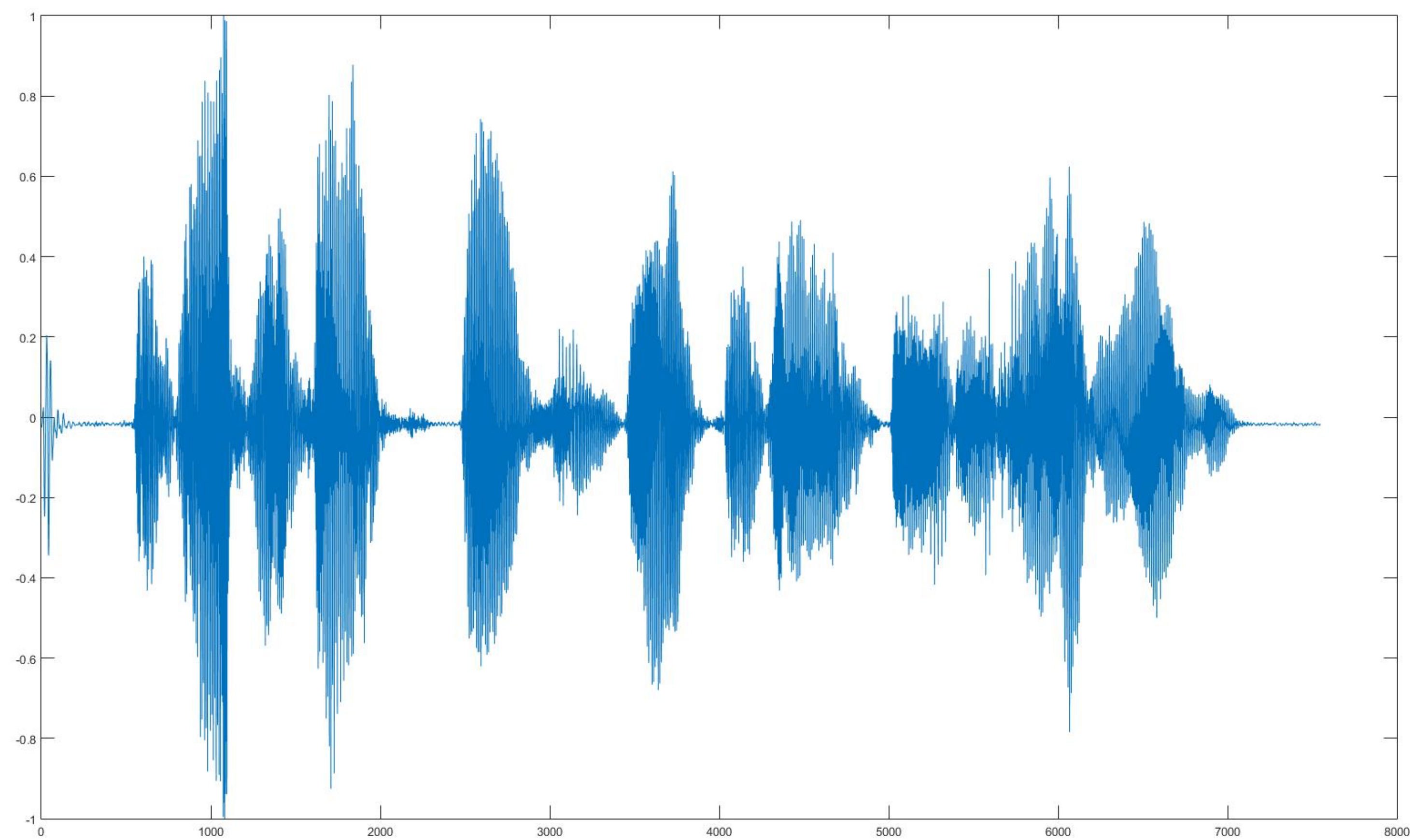
foreword: beat notes





$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

music signal



refresher: traveling waves

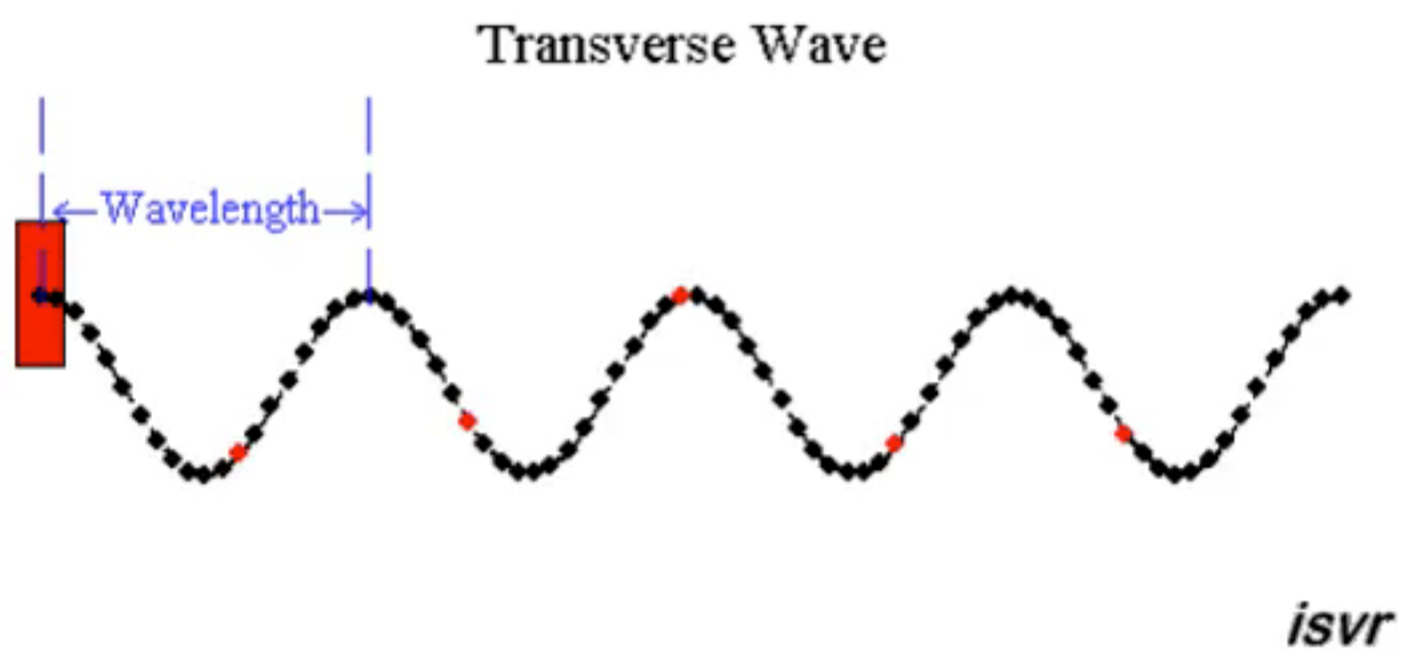
$$\frac{1}{v_p^2}\frac{\partial^2 h}{\partial t^2} = \frac{\partial^2 h}{\partial x^2}$$

1-D (vacuum) wave equation

$$h = h(x,t)$$

v_p : **phase velocity** — propagation speed of a wave held at fixed wavelength

measures how fast a wave is at constant phase



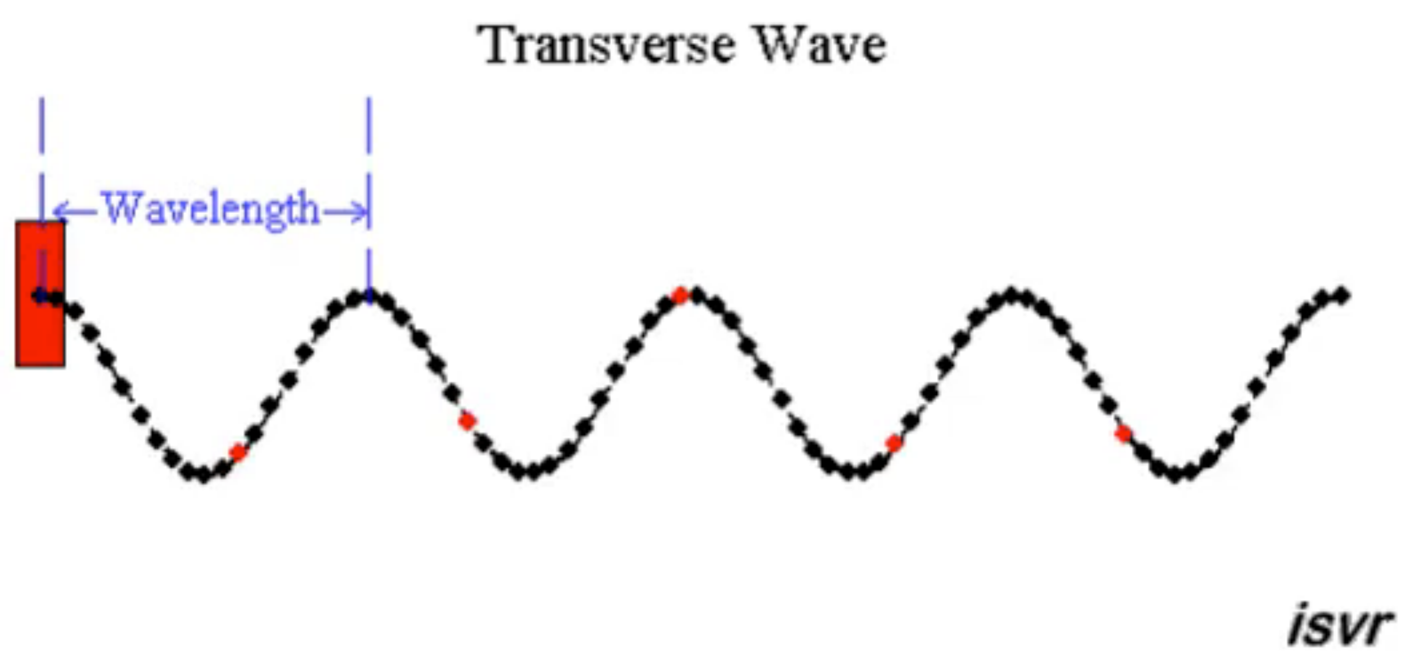
refresher: traveling waves

if we look at a wave solution such as sin(*kx* - *ωt*), we see that:

$$\frac{1}{v_p^2}\frac{\partial^2 h}{\partial t^2} = \frac{\partial^2 h}{\partial x^2} \qquad \Rightarrow \qquad v_p^2 = \omega^2/k^2$$

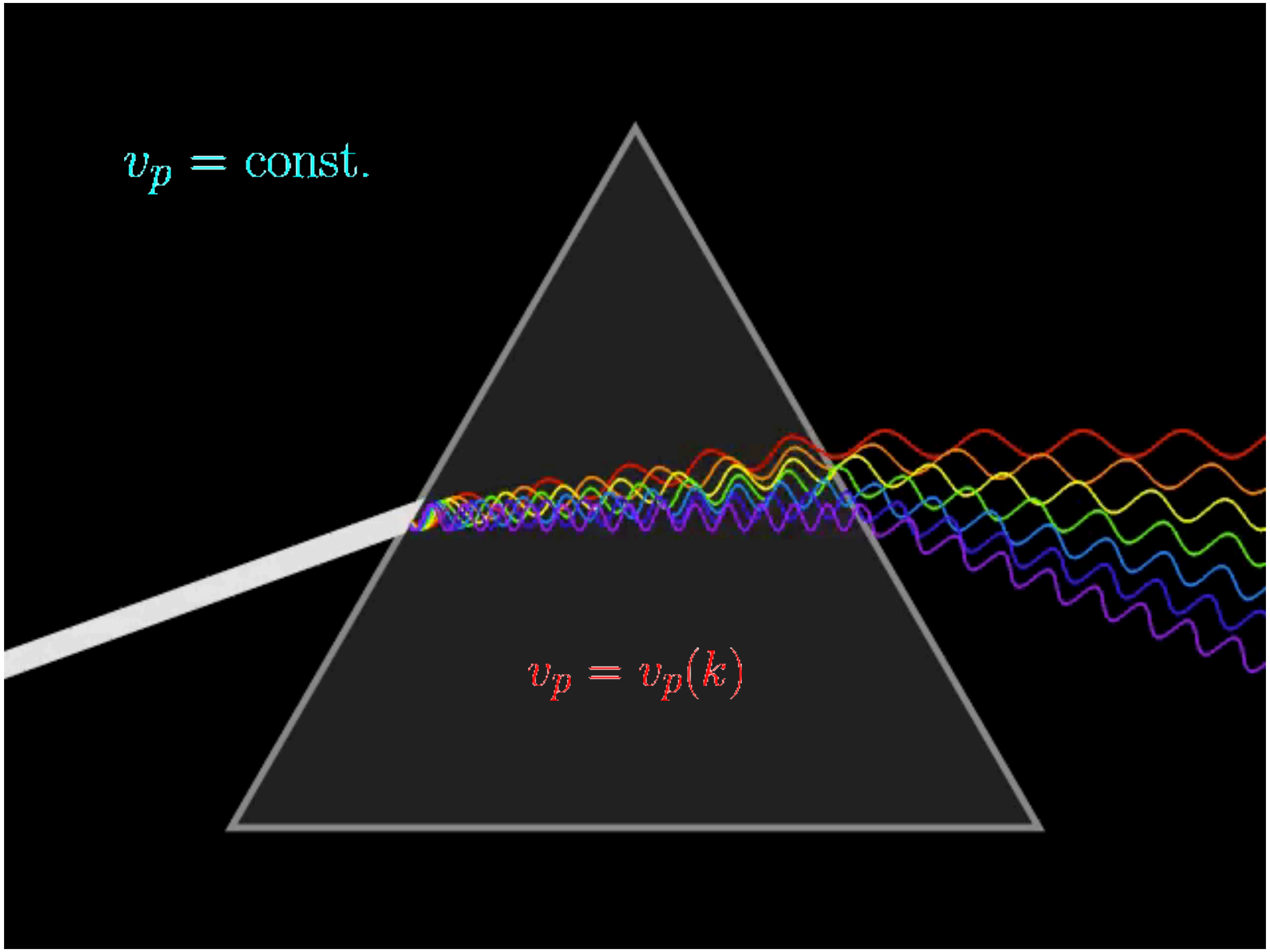
or, to put it another way….

$$kx - \omega t = \text{const.} \qquad \Rightarrow \qquad dx/dt = \omega/k = v_p$$



dispersion

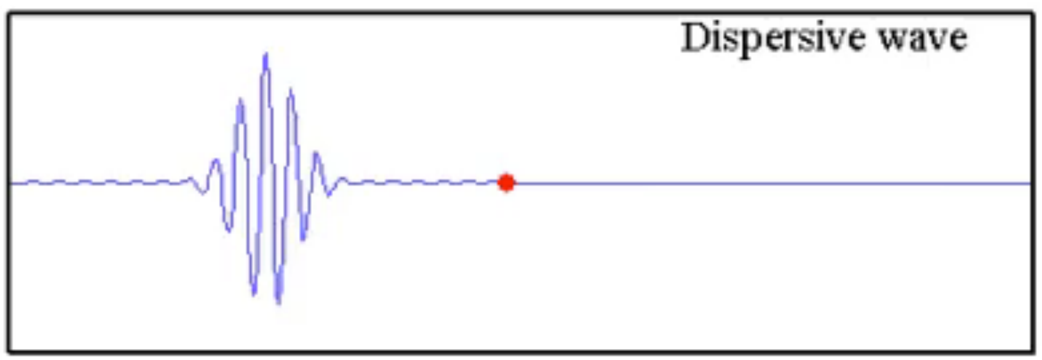
dispersion: a general phenomenon in which waves of different frequencies travel at different speeds



dispersion relation

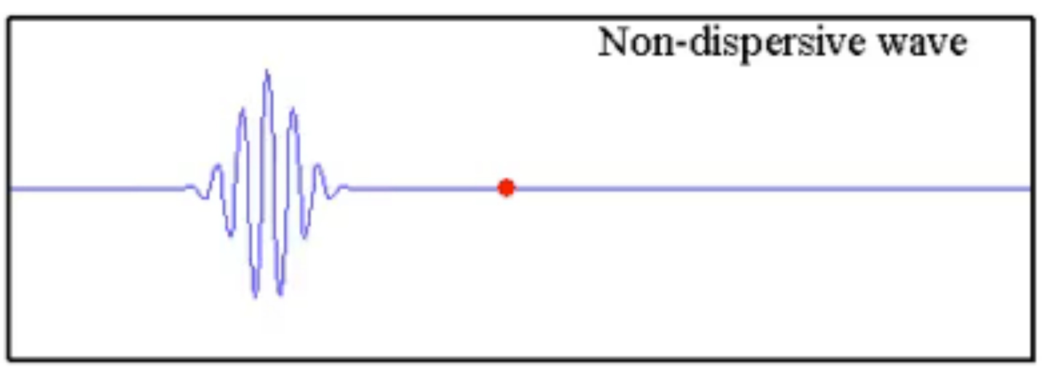
v_p : **phase velocity** — propagation speed of a wave with fixed wavelength $\qquad \omega/k$

v_g : **group velocity** — propagation speed of a "group" of waves, all at different frequencies $\qquad d\omega/dk$



dispersion

$$v_g \neq v_p$$
$$\omega = \omega(k)$$



isvr

no dispersion (boring)

$$v_g = v_p$$
$$\omega \propto k$$

example: ocean waves

v_p : **phase velocity** — propagation speed of a wave with fixed wavelength ω / k

v_g : **group velocity** — propagation speed of a "group" of waves, all at different frequencies $d\omega / dk$

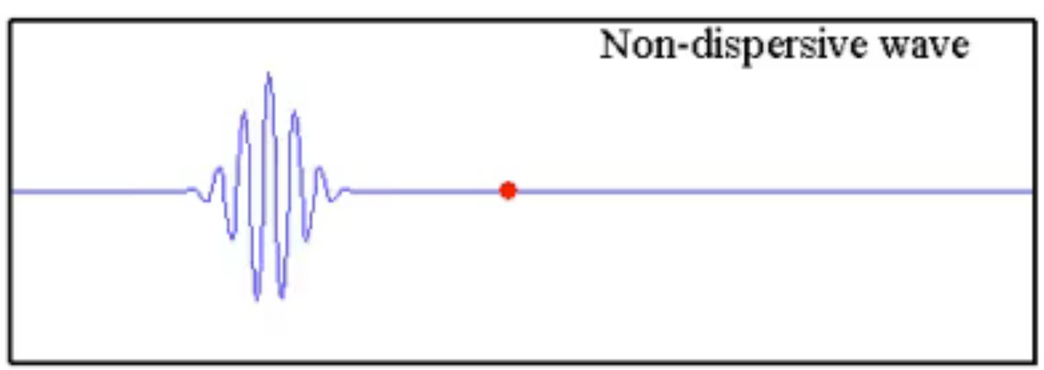
deep water

$$v_g = v_p = \sqrt{gh}$$

$$\omega \propto \sqrt{k}$$

$$2 \text{ cm} \ll \lambda \ll \text{water depth}$$

$$\text{water depth} \lesssim \lambda$$



isvr

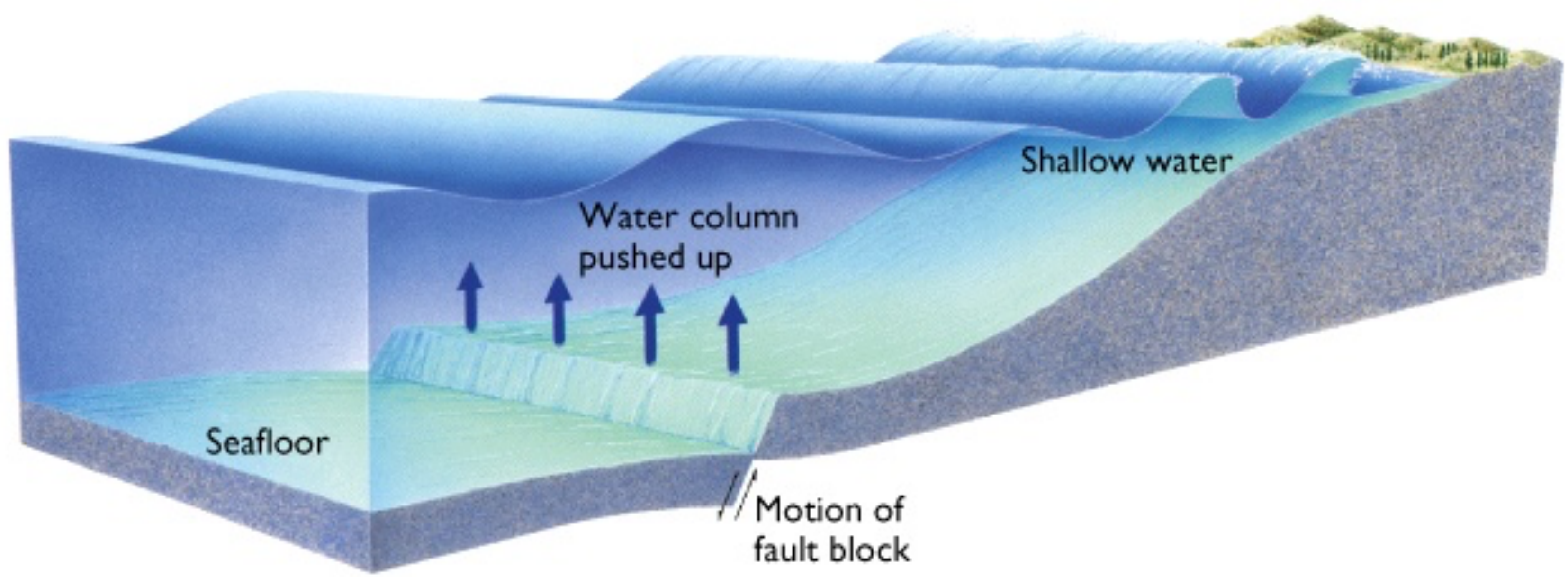
shallow water

$$v_g = v_p$$

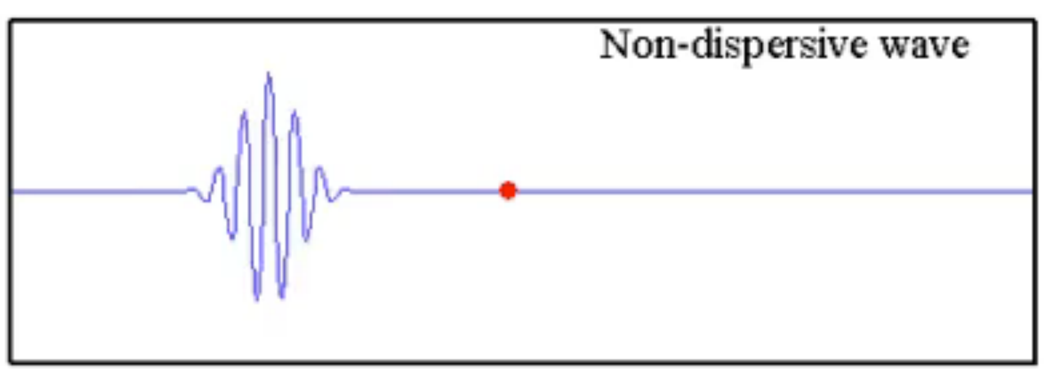
$$\omega \propto k$$

example: tsunamis

long wavelength (~10 mi) compared to typical ocean depth (~several mi) produces **non-dispersive waves**



$$\text{water depth} \lesssim \lambda$$



isvr

shallow water

$$v_g = v_p$$

$$\omega \propto k$$

refresher: traveling waves

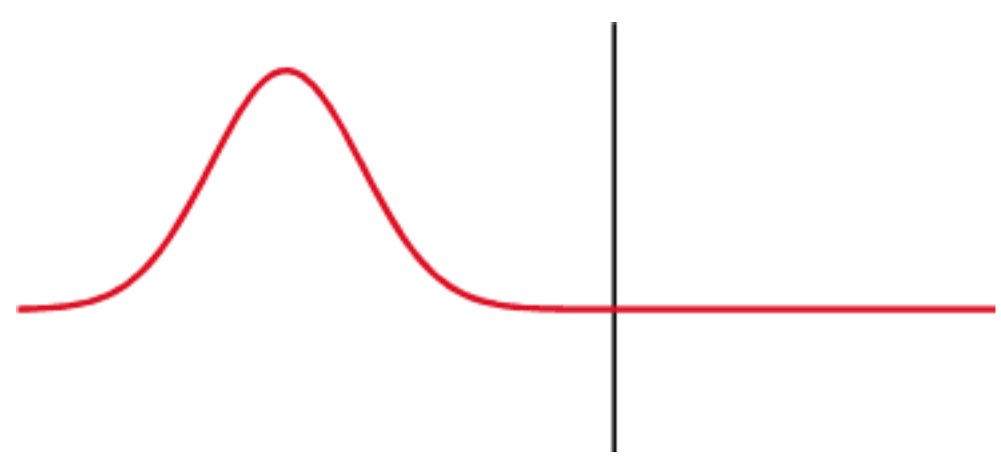
$$\frac{1}{v_p^2}\frac{\partial^2 h}{\partial t^2} = \frac{\partial^2 h}{\partial x^2}$$

1-D wave equation

$$h = h(x,t)$$

boundary conditions: must specify the value at points on the boundary to get a unique solution

what happens when a wave in motion strikes one of the boundaries?



transmission and reflection

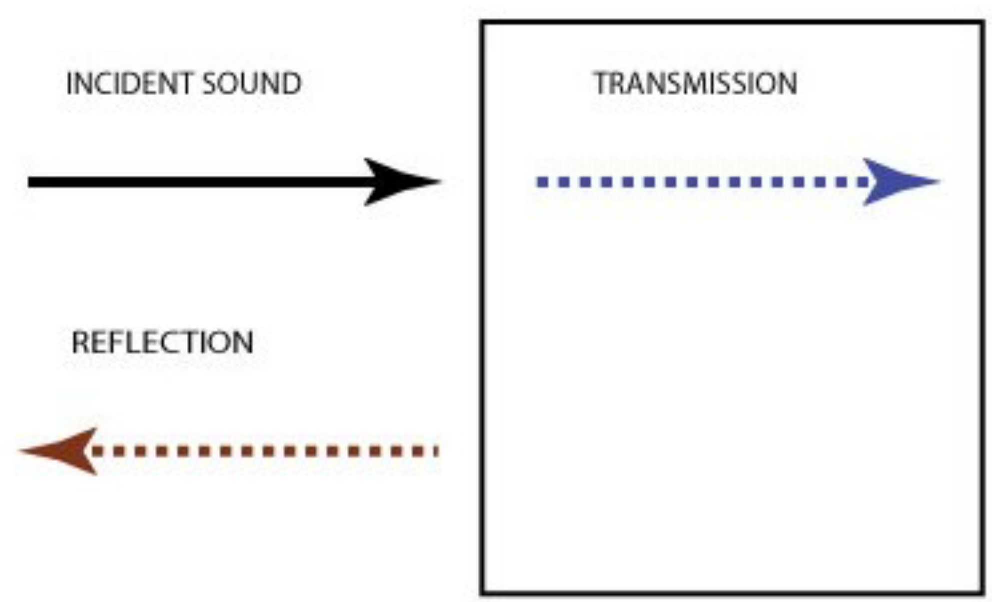
$$\frac{1}{v_p^2}\frac{\partial^2 h}{\partial t^2} = \frac{\partial^2 h}{\partial x^2}$$

1-D wave equation

$$h = h(x,t)$$

transmission: at the boundary between two media — say, air and water — part of the original wave passes through

reflection: the other part of the wave bounces back off of the boundary



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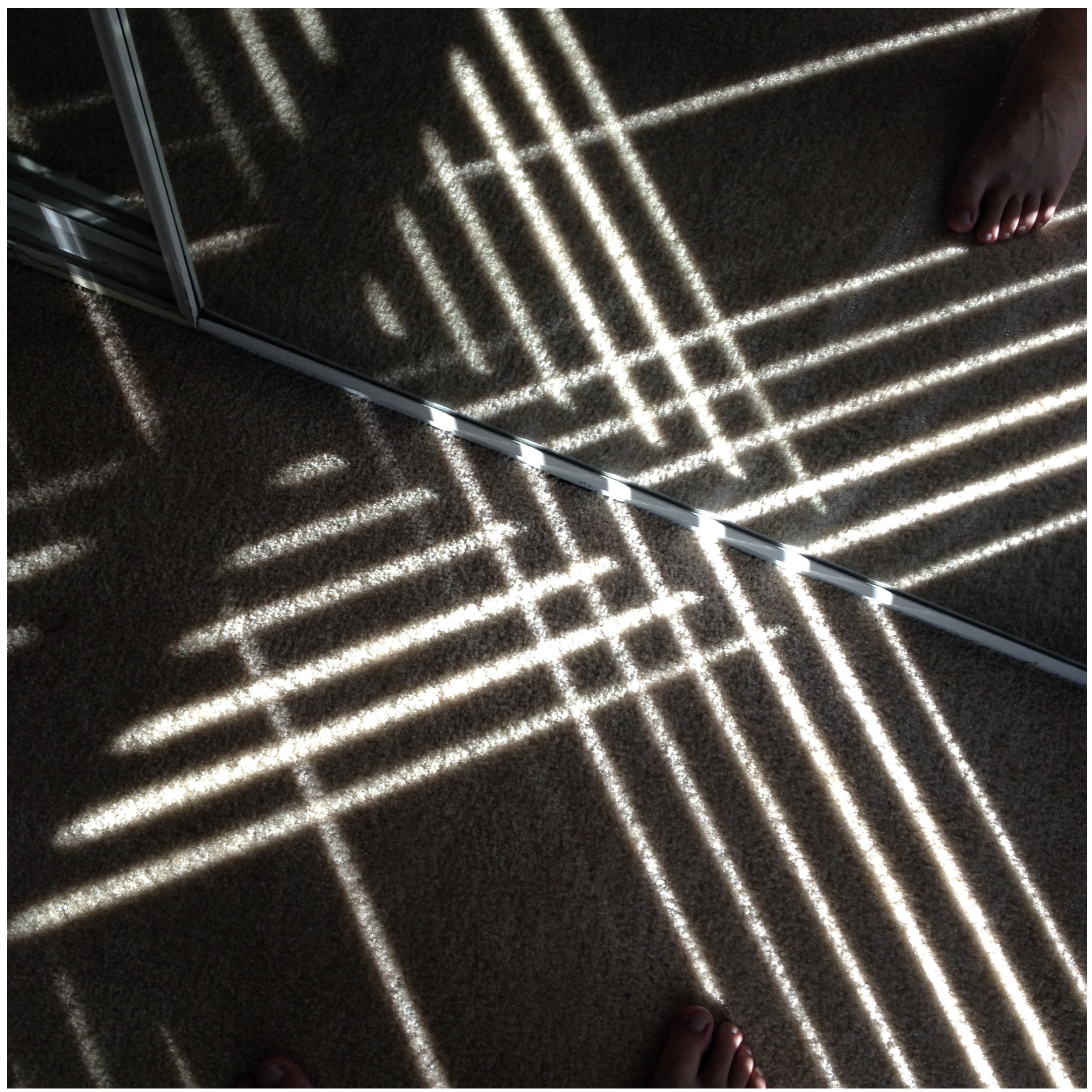
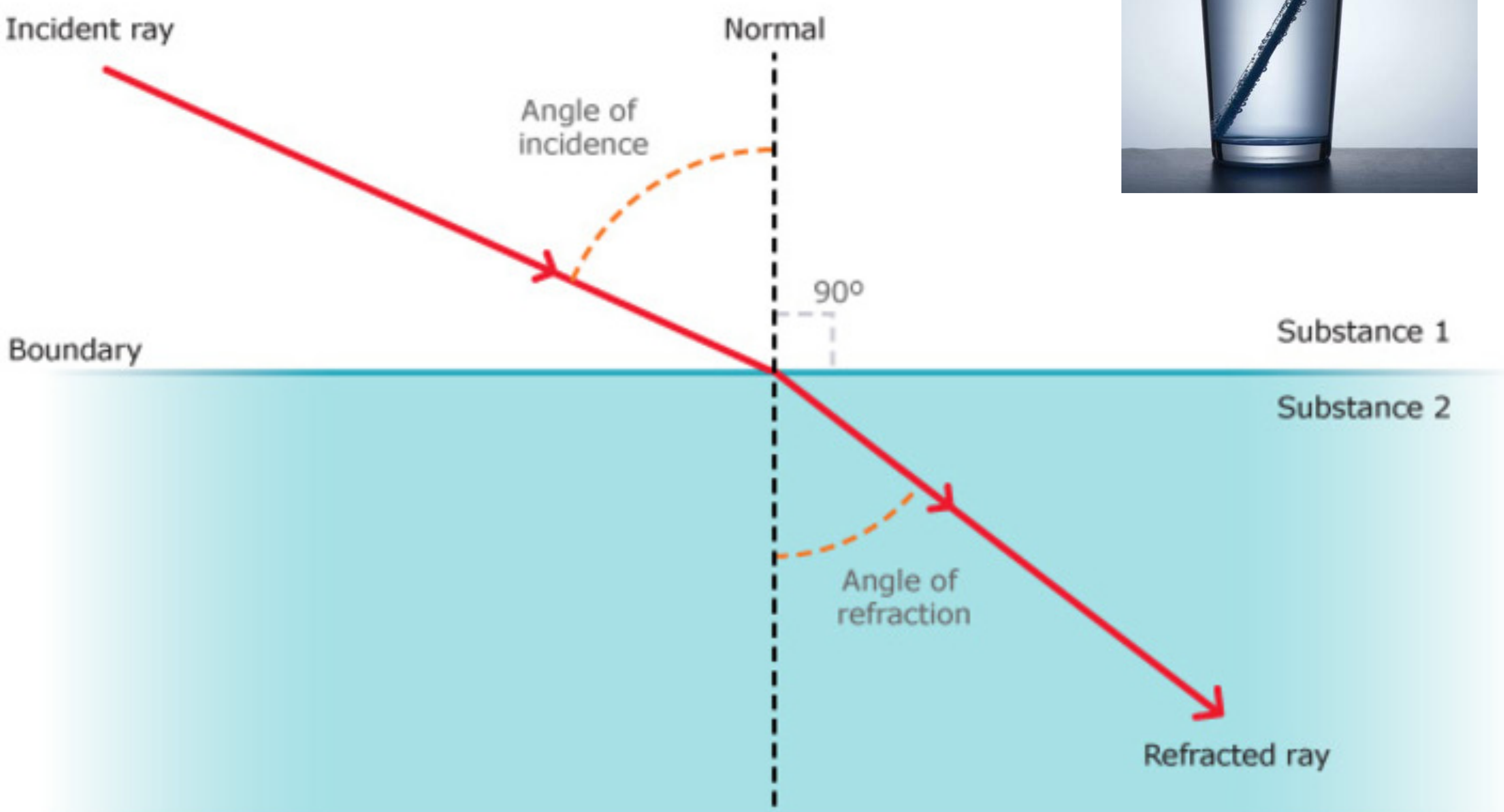
reflection: the other part of the wave bounces back off of the boundary



refraction

when a wave — for example, light — changes speed after crossing a boundary between two media



questions?

How does this work, again?

$$h(x) = \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{m=1}^{\infty} b_m \sin(mx)$$

Fourier series: a representation of periodic functions, written as sines and cosines of different frequencies

How does this work, again?

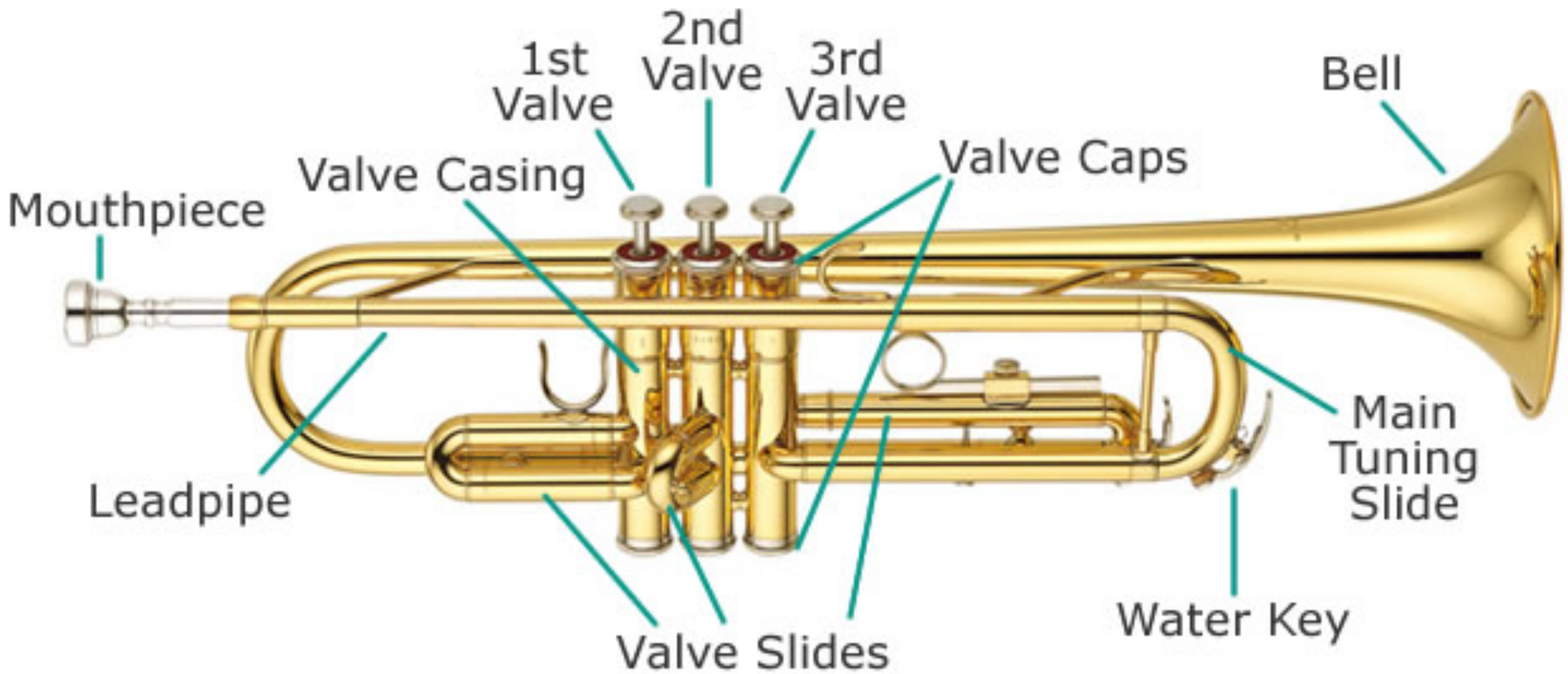
$$h(x) = \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{m=1}^{\infty} b_m \sin(mx)$$

$$a_n = \frac{1}{\pi} \int_{-\infty}^{\infty} h(x) \cos(nx)\ dx$$

$$b_m = \frac{1}{\pi} \int_{-\infty}^{\infty} h(x) \sin(mx)\ dx$$

Fourier series: a representation of periodic functions, written as sines and cosines of different frequencies

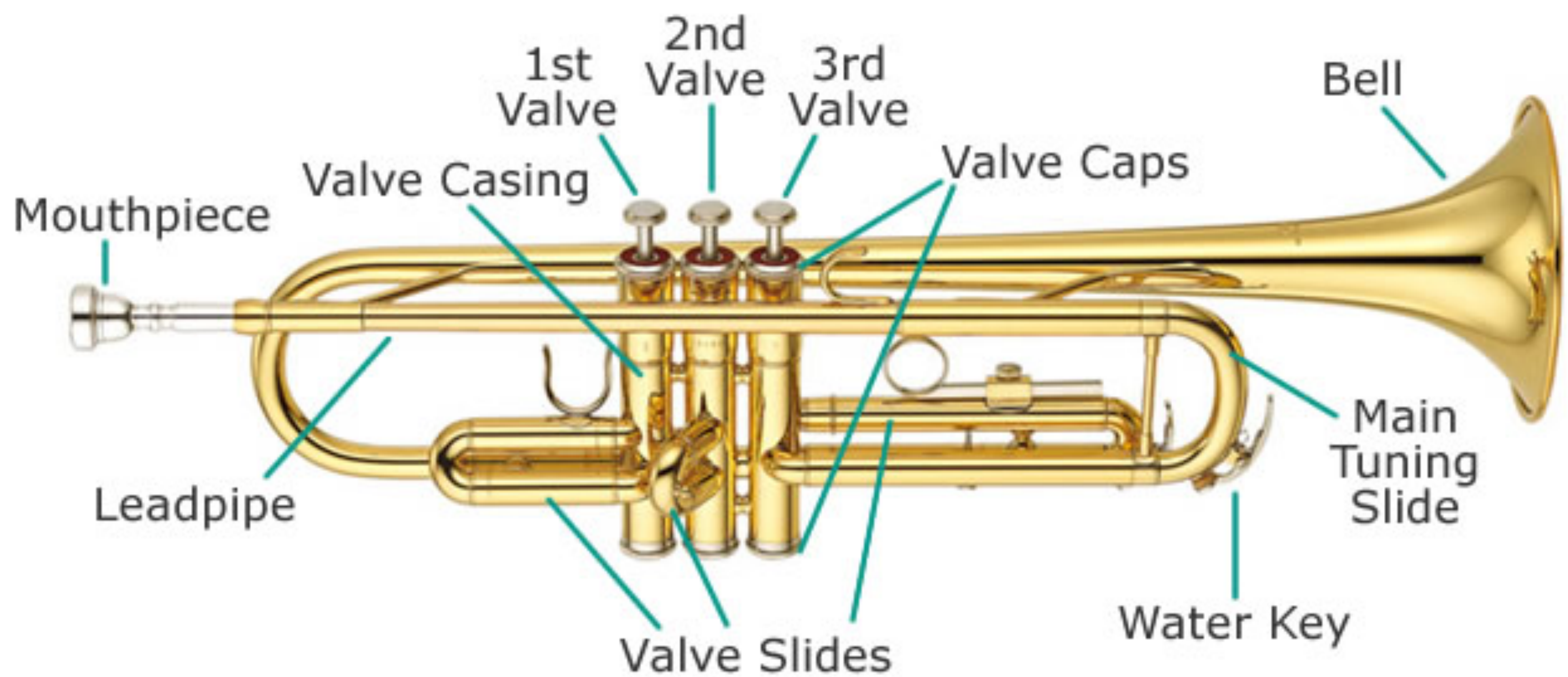
example: brass instruments

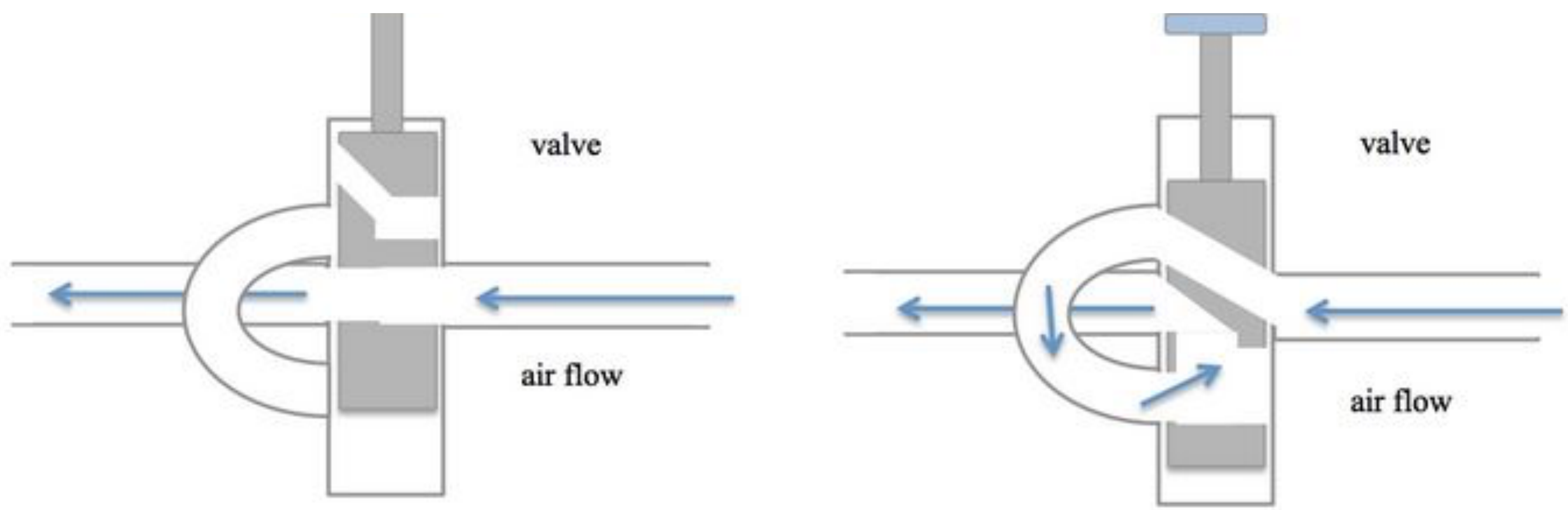


Sympathetic resonance: a system responding to external vibrations that share harmonics

demonstration: tuning forks anchored to hollow blocks of wood — https://youtu.be/sxRkOQmzLgo?t=1m31s

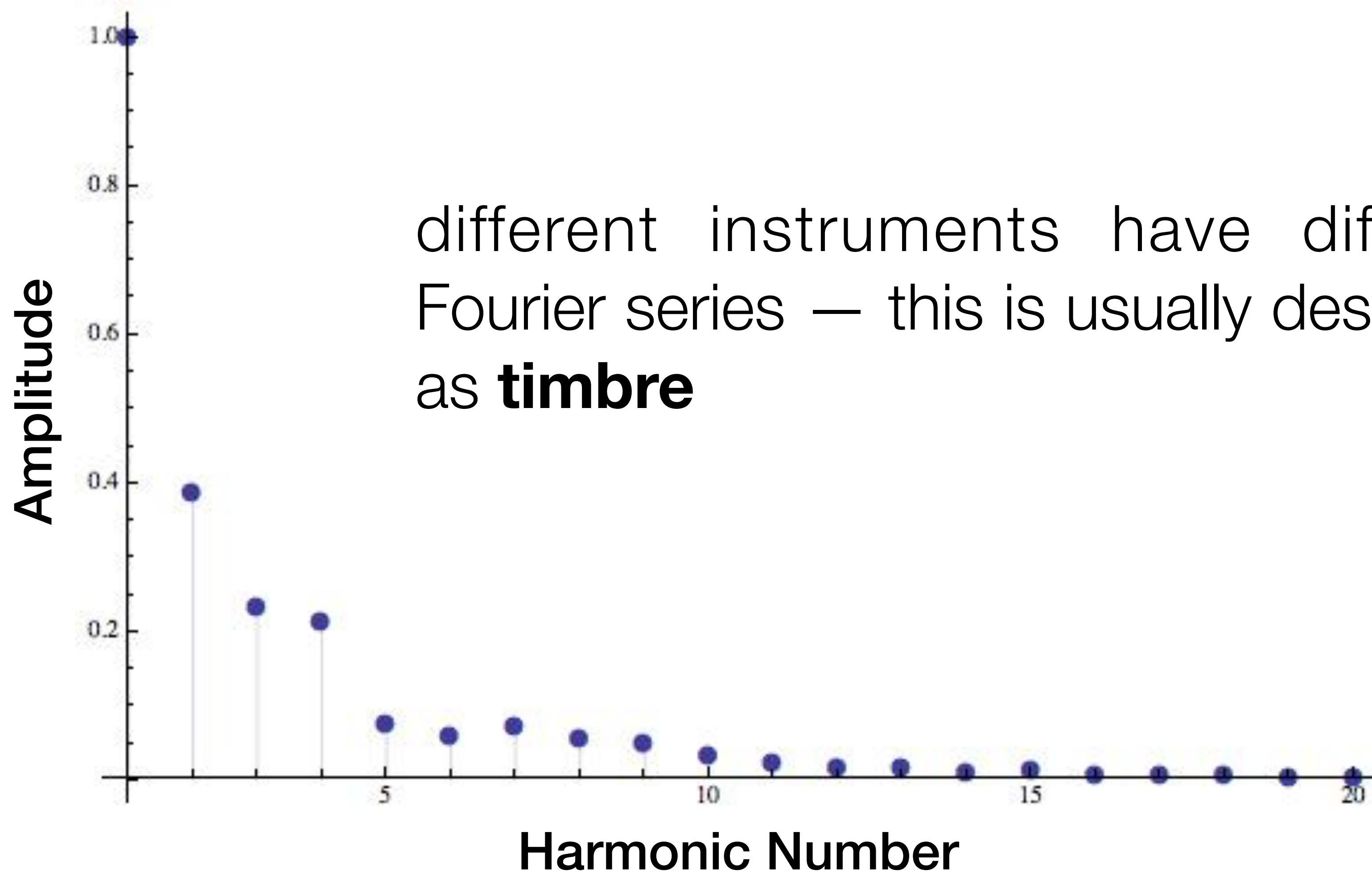
example: brass instruments





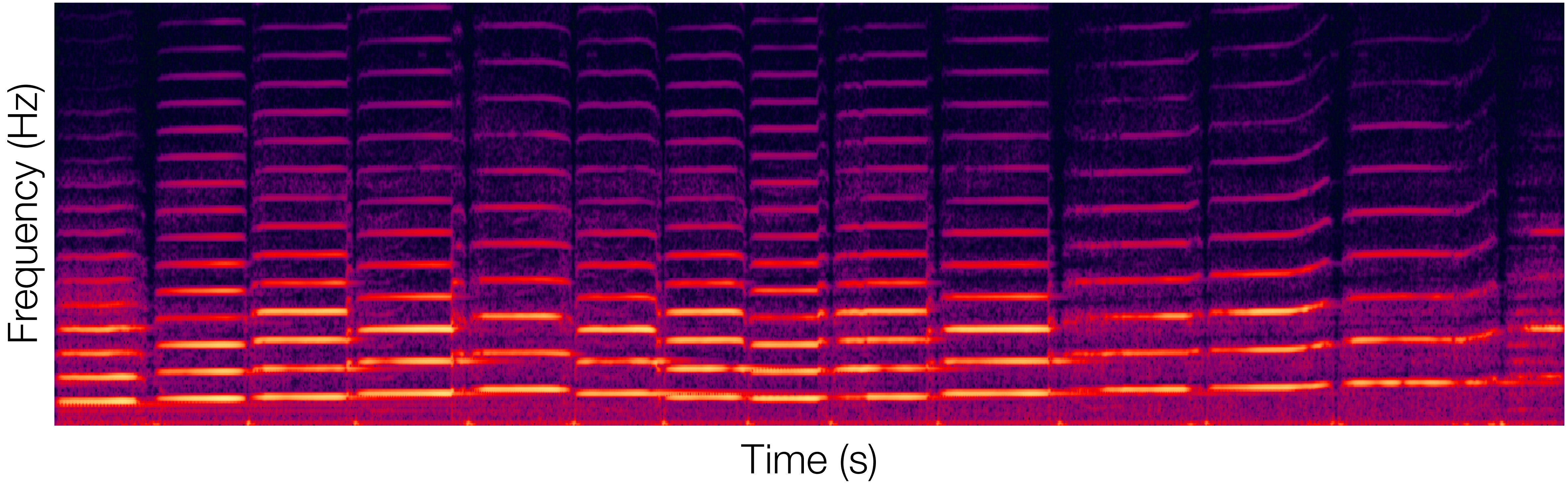
slides and valves used to change the length of tubing; embouchure, lip tension, air flow control specific harmonics

example: brass instruments



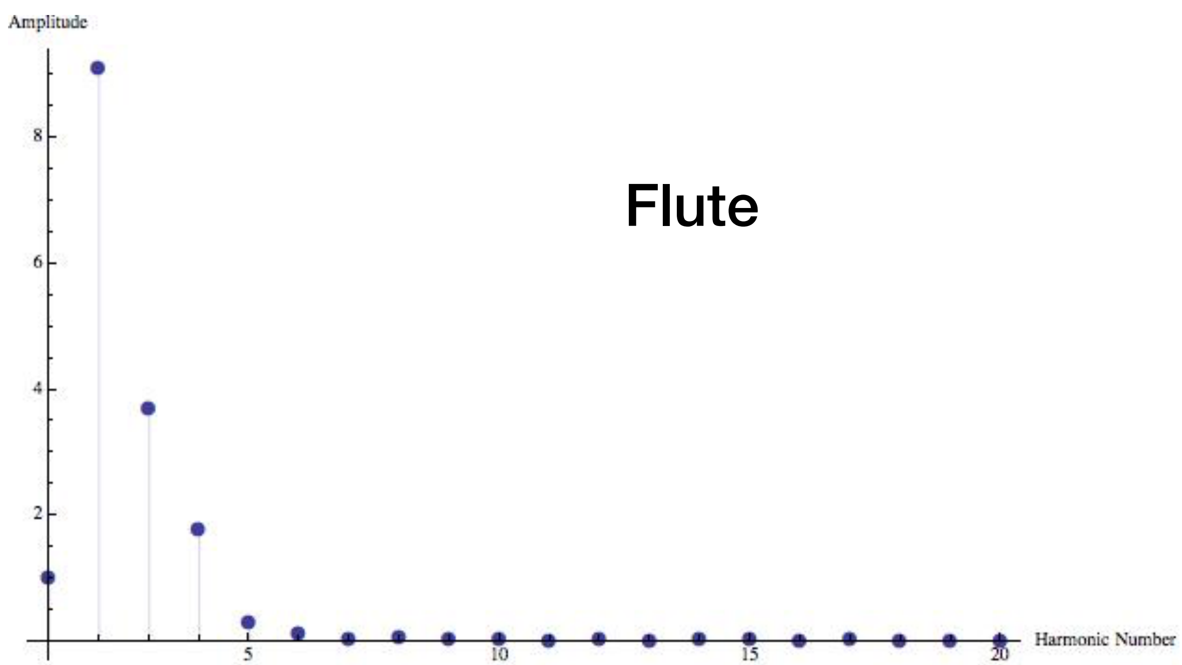
different instruments have different Fourier series — this is usually described as **timbre**

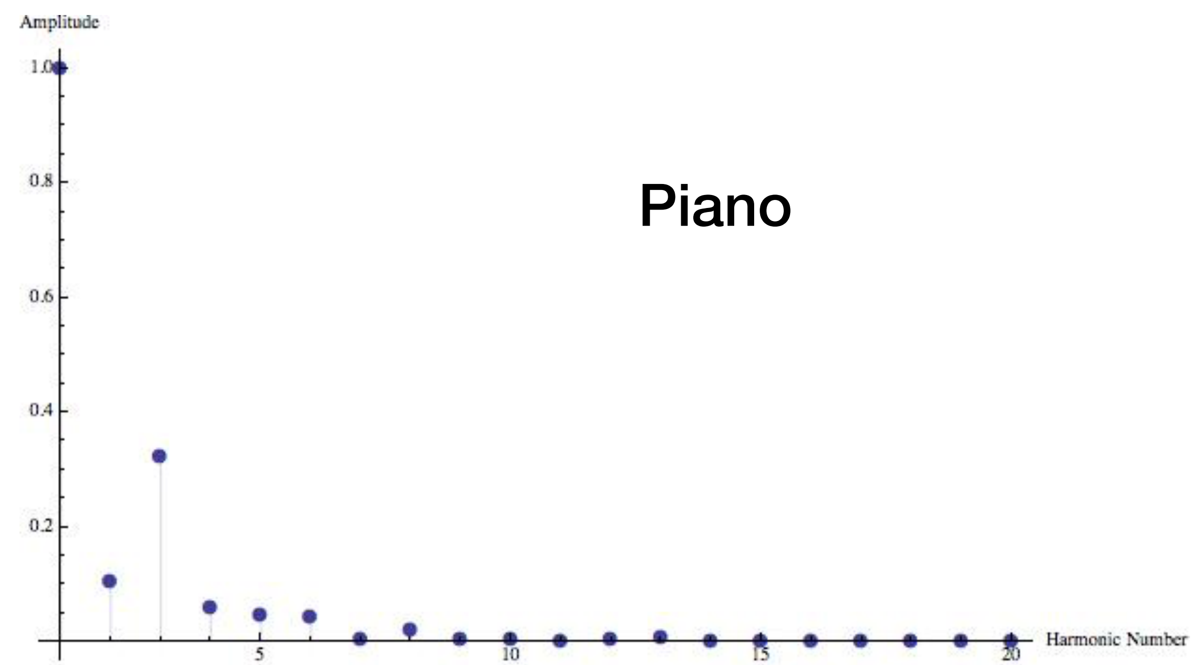
example: string instruments



why do separate instruments playing the same tone sound so different?

I’M YELLING ‘TIMBRE!’





another example
(play for sound)