PHYS-2020: General Physics II
Course Lecture Notes
Section IX

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Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 8th Edition* (2009) textbook by Serway and Vuille.

IX. Electromagnetic Radiation (Photons)

A. Properties of Electromagnetic (E/M) Radiation.

1. Maxwell showed in 1865 that electric phenomena were related to magnetic phenomena $\implies$ **Maxwell's equations** (or **laws**):

   a) Electric fields originate on positive charges and terminate on negative charges. The electric field due to a point charge can be determined by applying Coulomb's law.

   $$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_\circ} \; .$$

   i) This is similar to Gauss's law given in Eq. (I-7).

   ii) $\rho = q/V$ is the electric charge density (charge per unit volume).

   iii) The "del" symbol is from *vector calculus* and is defined by

   $$\vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \; ,$$

   in Cartesian coordinates, where '∂' is the *partial* derivative symbol from calculus.

   iv) The '$\vec{\nabla}\cdot$' operation is called the *divergence* in higher mathematics — it is the 'dot product' of the 'del' operator and the vector in question, here the electric-field vector.

   b) Magnetic-field lines always form closed loops $\implies$ they do not begin or end anywhere $\implies$ there are no **magnetic monopoles**. Mathematically, this is given by Maxwell's

equation for the divergence of the B-field:

$$\vec{\nabla} \cdot \vec{B} = 0 \; ,$$

the 'zero' simply means that there are no magnetic monopoles.

c) A varying B-field induces an emf and hence electric (E) field $\Longrightarrow$ Faraday's law re-expressed in Maxwell's form:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \; .$$

The '$\vec{\nabla} \times$' operation is called the **curl** in higher mathematics. Whereas the divergence produces a scalar, the curl produces a vector.

d) Magnetic fields are generated by moving charges (or currents) $\Longrightarrow$ Ampere's law re-expressed in Maxwell's form:

$$\vec{\nabla} \times \vec{B} = \mu_\circ \vec{J} \; ,$$

where $J(\propto \partial q/\partial t)$ is the current density.

2. Maxwell's last 2 laws allow electromagnetic waves (*i.e.*, radiation) to **self-propagate** at a velocity of

$$\boxed{c = \frac{1}{\sqrt{\mu_\circ \epsilon_\circ}} = 2.99792 \times 10^8 \text{ m/s} \; .} \qquad \text{(IX-1)}$$

a) "c" is called the **speed of light**, since visible light is a form of electromagnetic radiation.

b) An oscillating electric charge produces an E-field that varies in time, which produces a B-field that varies in time, which produces a new E-field that varies in time,

and so on.
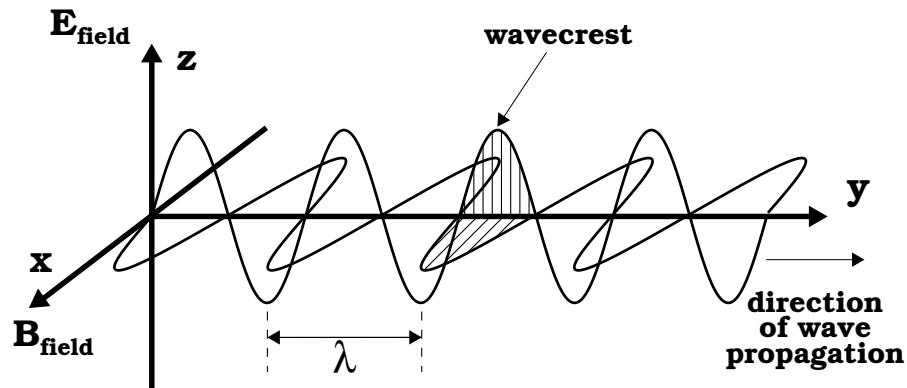
c) The ratio of the maximum magnitude E-field to the maximum magnitude of the B-field of an E/M wave is

$$\boxed{\frac{E_{\text{max}}}{B_{\text{max}}} = c \ .} \qquad \text{(IX-2)}$$

d) We will see in the Optics portion of this course that the above value of c is the value of the speed of light in a *vacuum*. Light slows when it enters a medium (*i.e.*, air [though not by much], glass, etc.).

e) Even though the solution to Maxwell's equations clearly showed that light (or any E/M radiation) can self-propagate, it took another 50 years before Einstein demonstrated this with the *Special Theory of Relativity* in 1905. Prior to this, light was assumed to propagate on a medium in space called the *Ether*.

f) In the late-1800s, Michelson and Morley tried to measure the Earth's motion through the Ether and was unable to detect it — either the Ether moved with the Earth, or the Ether didn't exist. Einstein chose the latter and used this fact as one of the underlying principles to relativity.

g) In the *Special Theory of Relativity*, Einstein also showed that this vacuum speed of light represents the fastest speed in the Universe!

i) Energy (*e.g.*, E/M radiation, gravitational) always travels at this speed in a vacuum, no slower, no faster.

ii) Matter must always travel slower than c, it can never go this speed nor any faster!

iii) This results from the following reduced mass equation derived in the relativistic momentum equation from relativity:

$$m = \frac{m_\circ}{\sqrt{1 - v^2/c^2}},$$

where $m_\circ$ is the mass an object has when at rest, and v is the velocity of the mass.

iv) As can be seen in this equation, if $v \ll c$, then $m \approx m_\circ$ — the mass remains relatively constant at the rest mass value.

v) However, as $v \to c$, $v^2/c^2 \to 1$, $1 - v^2/c^2 \to 0$, and finally $m_\circ/\sqrt{1 - v^2/c^2} \to \infty$! The mass gets infinity big! Since an infinite amount of energy would be required to push even the smallest mass to the speed of light in a vacuum, it is impossible for even the smallest mass to be accelerated to the speed of light. Matter can be accelerated to speeds very close to c, but not right at c!

vi) This is not just a technical barrier, such as the sound barrier, **it is impossible for anything with mass, even an electron, to be accelerated to** $v = c$.

vii) Also note that if $v > c$, then we get a negative number in the square-root $\Longrightarrow$ an *imaginary* number, which is a meaningless quantity for velocity in our physical Universe.

3. E/M waves also behave like **particles**!

a) Sometimes joking called a *wavicle*!

b) As such, Planck called a packet of electromagnetic radiation a **photon** in the early 1900s and this term has been used ever since to describe E/M radiation.

c) The **photoelectric** effect demonstrates this "particle" picture nicely: If a photons hits a certain type of metal, it can "knock" an electron off of an atom in the metal and produce an electric current. (Einstein won his Nobel Prize for figuring this out. He never won a Nobel Prize for either the Special or General Theories of Relativity since at the time of their development, they could not be fully tested due to the lack of technology.)

d) At this point we need to distinguish between two types of radiation. The word *radiation* simply means something flowing from one point to another. This "something," however, can either be particles of energy or particles of matter:

i) The flow of photons (energy particles) is called *radiation*.

ii) Also the motion of atomic and subatomic (matter) particles given off during radioactivity events is called *radiation*. This was discovered by the Curies in the early 1900s. The Curies discovered three types of particle radiation:

- **Alpha particles** have positive charge and were later determined to be helium (He) nuclei. As such, He nuclei are often referred to as alpha particles.

- **Beta particles** have negative charge and were later found to be the simple electron.

- **Gamma particles** have no charge and were later realized to be high-energy photons. These particles were renamed **gamma rays** and represent the highest energy form of E/M radiation.

4. E/M waves are transverse waves, since the electric and magnetic fields are ⊥ to the direction of propagation of the wave and to each other.

5. E/M waves carry both energy and momentum (despite have no mass).

   a) The energy is proportional to frequency and inversely proportional to wavelength.

$$\boxed{E = h\nu \ .} \qquad\qquad \text{(IX-3)}$$

   i) E is the energy (SI unit of J, cgs unit of erg) of the E/M wave.

ii) ν (Greek 'nu') is the frequency (unit of Hz = 1/s) of the E/M wave. Note that the textbook uses 'f' for a photon's frequency just as we did in the case of sound. However, it is more common to use 'ν' for the frequency of an E/M wave. We will just use the variable f as the frequency of waves requiring a "medium" (*i.e.*, sound, water waves, etc.)

iii) h = Planck's constant = 6.62620×10^{-34} J·s = 6.62620×10^{-27} erg·s.

iv) There is another form of Planck's constant, the angular frequency ($\omega = 2\pi\nu$) version: $\hbar$ (called 'h-bar') = $h/2\pi$ = 1.05459×10^{-34} J·s = 1.05459×10^{-27} erg·s, where a photon's energy is given by $E = \hbar\omega$ in terms of angular frequency.

v) Also note that the amplitude of the wave is related to the intensity of the E/M radiation, which is related to the total number of photons. The amplitude (hence intensity) has nothing to do with the energy of the wave.

vi) The frequency of the wave is related to the wavelength (*i.e.*, the distance between wavecrests) with

$$\boxed{\nu = \frac{c}{\lambda}\ .}\qquad\text{(IX-4)}$$

As such, we also can write the energy of a photon as

$$\boxed{E = \frac{hc}{\lambda}\ .}\qquad\text{(IX-5)}$$

b) The momentum of a photon (E/M wave) is

$$p = \frac{h}{\lambda} \qquad\qquad (IX\text{-}6)$$

or

$$p = \frac{h\nu}{c} \ . \qquad\qquad (IX\text{-}7)$$

c) Note that if we use Eq. (IX-3) in Eq. (IX-6), we get

$$p = \frac{E}{c} \quad \text{or} \quad E = pc \ . \qquad\qquad (IX\text{-}8)$$

Example IX–1. An electromagnetic wave in vacuum has an electric field amplitude of 220 V/m. Calculate the amplitude of the corresponding magnetic field.

Solution:

We just need to use Eq. (IX-2) and solve for B_{max}:

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{220 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 7.33 \times 10^{-7} \text{ T} = \boxed{733 \text{ nT} \ .}$$

Example IX–2. Compare the wavelength, energy, and momentum of a radio wave at 10^8 Hz (*e.g.*, a TV signal) to that of visible light at 500 nm (the yellow part of the spectrum).

Solution:

First use Eq. (IX-4) to get the wavelength of the radio wave:

$$\nu_{\text{radio}} = \nu_R = 10^8 \text{ Hz}$$
$$\lambda_R = \frac{c}{\nu_R} = \frac{3.00 \times 10^8 \text{ m/s}}{10^8 \text{ s}^{-1}} = 3.00 \text{ m}$$
$$\lambda_V = 500 \text{ nm} = 500 \times 10^{-9} \text{ m} = 5.00 \times 10^{-7} \text{ m}$$
$$\frac{\lambda_R}{\lambda_V} = \frac{3.00 \text{ m}}{5.00 \times 10^{-7} \text{ m}} = \boxed{6.00 \times 10^6}$$

$\Longrightarrow$ the radio wave is 6 million times longer than the visible light wave.

Now use Eqs. (IX-3) and (IX-5) to compare the energies:

$$E_R = h\nu_R = (6.63 \times 10^{-34} \text{ J·s})(10^8 \text{ s}^{-1}) = 6.63 \times 10^{-26} \text{ J}$$

$$E_V = \frac{hc}{\lambda_V} = \frac{(6.63 \times 10^{-34} \text{ J·s})(3.00 \times 10^8 \text{ m/s})}{5.00 \times 10^{-7} \text{ m}} = 3.98 \times 10^{-19} \text{ J}$$

$$\frac{E_R}{E_V} = \frac{6.63 \times 10^{-26} \text{ J}}{3.98 \times 10^{-19} \text{ J}} = \boxed{1.67 \times 10^{-7}} \, ,$$

or $E_V = 6.00 \times 10^6 E_R \Longrightarrow$ the visible light photon is 6 million times more energetic than the radio photon.

Finally, use Eq. (IX-6) to compare the momenta:

$$\frac{p_R}{p_V} = \frac{h/\lambda_R}{h/\lambda_V} = \frac{\lambda_V}{\lambda_R} = \frac{5.00 \times 10^{-7} \text{ m}}{3.00 \text{ m}} = \boxed{1.67 \times 10^{-7}}$$

or $p_V = 6.00 \times 10^6 p_R$!

B. The Electromagnetic Spectrum.

1. The **E/M spectrum** (and spectra in general) is defined to be the intensity, or flux, of an E/M wave as a function of wavelength, frequency, or energy.

    a) In spectroscopy, very small units of length are often used:

        i)   1 micrometer (μm or μ) = 10^{-6} m.

        ii) 1 nanometer (nm) = 10^{-9} m.

        iii) 1 Ångstrom (Å) = 10^{-10} m.

    b) Visible light ranges from 4.0×10^{-7} m to 7.0×10^{-7} m, or 0.4 μm – 0.7 μm, or 400 nm – 700 nm, or 4000 Å – 7000 Å.

2. The continuous E/M spectrum (called the **continuum**) is split into different regimes. Below, we list the E/M spectrum from shortest to longest wavelengths with distinct wavelength dividing lines. Often, however, these dividing lines are *fuzzy* such that one scientist might call a long IR photon a microwave whereas another would refer to it as a microwave.

a) **Gamma rays:** Highest energy, highest frequency, and shortest wavelengths: $\lambda < 0.01$ nm. These types of photons are given off by nuclei during high-energy nuclear reactions and by extremely hot gas ($T > 3.0 \times 10^8$ K — over 300 million degrees Kelvin!).

b) **X-rays:** 0.01 nm $< \lambda < 10$ nm. These photons are given off by low-energy nuclear reactions and very hot gas (3.0×10^5 K $< T < 3.0 \times 10^8$ K).

c) **Ultraviolet (UV):** 10 nm $< \lambda < 400$ nm. These photons are given off by electron transitions in atoms, typically involving the ground state and by hot gas (7000 K $< T < 3.0 \times 10^5$ K).

d) **Visible (visual):** 400 nm $< \lambda < 700$ nm. These photons are given off by electron transitions in atoms and warm gas (4000 K $< T < 7000$ K).

e) **Infrared (IR):** 700 nm $< \lambda < 1$ mm. These photons are given off by electron transitions in atoms (between excited levels) and molecules. Cool gas (3 K $< T < 4000$ K) can also thermally emit these photons.

f) **Microwaves:** 1 mm $< \lambda < 10$ cm. These photons are given off by electronic devices or from cold gas (0.03 K $< T < 3$ K).

g) **Radio waves:** $\lambda > 10$ cm. These photons have the lowest energy, lowest frequency, and longest wavelengths and can be created by electrical circuits and from extremely cold gas $(T < 0.03$ K$)$. The longest radio waves $(\lambda > 1$ km$)$ are sometimes just referred to as *long waves.*

3. Some spectral regions are subdivided into smaller bands. In the case of visible light, we refer to those bands as colors since our eyes perceive them that way. The wavelength delineation (which are approximate) for each color from shortest to longest wavelengths are:

a) **Violet** $(400$ nm $< \lambda < 450$ nm$)$.

b) **Blue** $(450$ nm $< \lambda < 490$ nm$)$.

c) **Green** $(490$ nm $< \lambda < 520$ nm$)$.

d) **Yellow** $(520$ nm $< \lambda < 590$ nm$)$.

e) **Orange** $(590$ nm $< \lambda < 630$ nm$)$.

f) **Red** $(630$ nm $< \lambda < 700$ nm$)$.

4. The visible region is the spectrum is the narrowest. It corresponds to the wavelengths to which the human eye is sensitive.

a) The Earth's atmosphere is transparent (assuming clouds are not blocking your view) at visible wavelengths.

b) The human eye is most sensitive to the green-yellow part of the visible spectrum between 500–570 nm which is also the wavelengths where the Sun emits the peak of its intensity in the E/M spectrum (*natural selection at work!*).

Example IX–3. The eye is most sensitive to light of wavelength 5.50×10^{-7} m, which is in the gree-yellow region of the visible electromagnetic spectrum. What is the frequency of this light?

Solution:

We just need to use Eq. (IX-4):

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{5.50 \times 10^{-7} \text{ m}} = \boxed{5.45 \times 10^{14} \text{ Hz} .}$$


C. The Formation of Spectra and the Doppler Effect of Light.

1. In the 1860s, Kirchhoff realized that there are 3 types of spectra that objects emit which depend upon the *state* and *orientation* the object is in $\Longrightarrow$ **Kirchhoff's Laws**:

    a) **Law 1:** A luminous opaque object (solid, liquid, or gas) emits light at all wavelengths (E/M spectrum), thus producing a **continuous spectrum** — a complete rainbow of colors without any spectral lines.
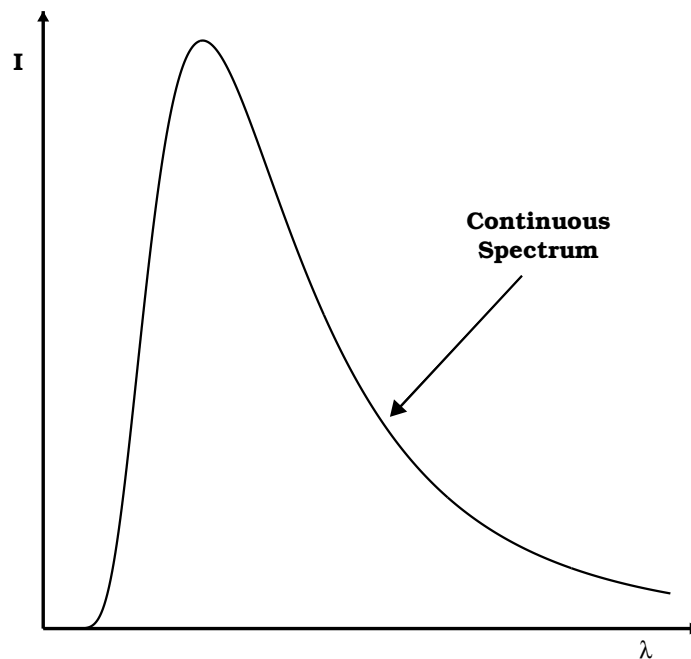
b) **Law 2:** A rarefied luminous gas emits light whose spectrum shows an **emission-line spectrum** — a series of bright spectral lines against a dark background.
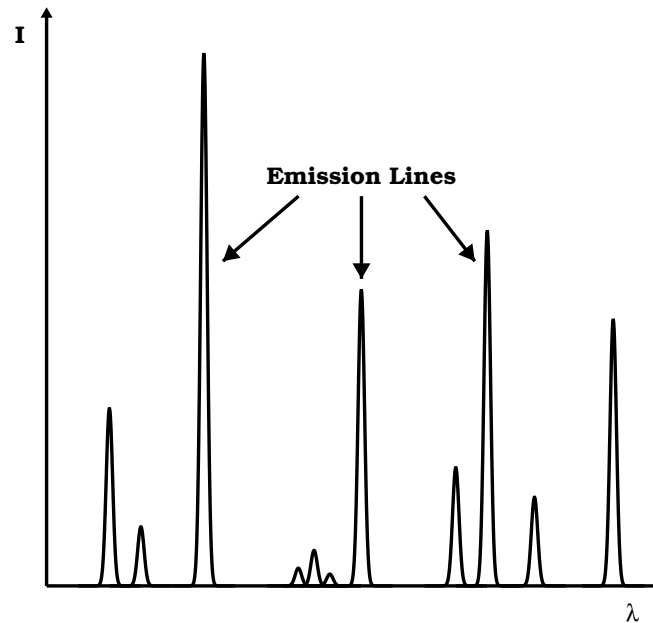
c) **Law 3:** If white light (*i.e.*, a continuum) from a luminous source passes through a lower-density gas, an **absorption-line spectrum** appears — a series a dark spectral lines among the colors of the continuous spectrum.
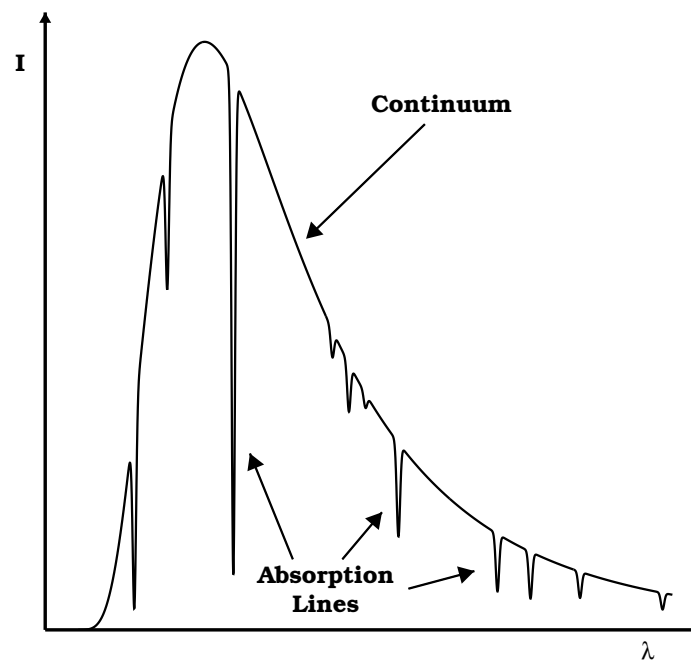
d) Kirchhoff's laws can be summarized with the following cartoon:
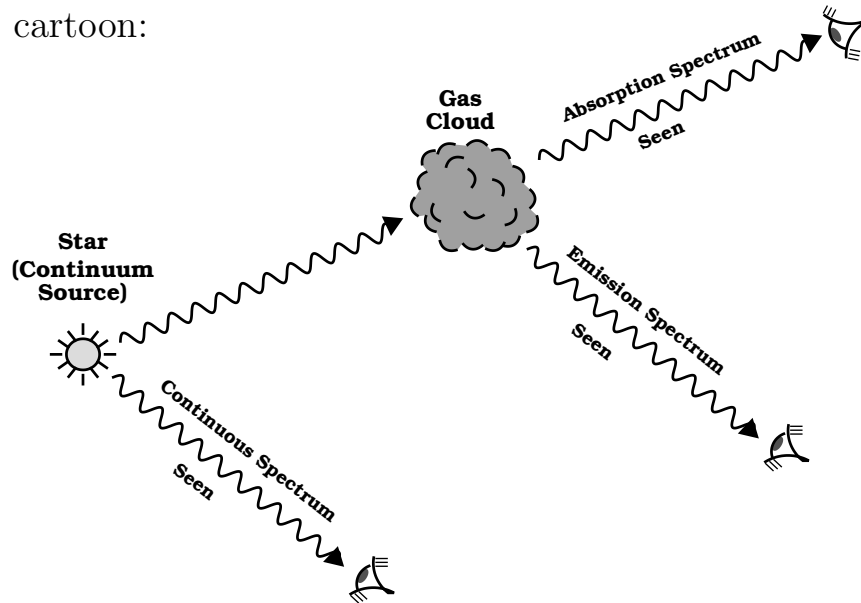
Kirchhoff's Radiation Laws

2. The Doppler effect for E/M Radiation:

a) The spectrum of an object will be **blueshifted** if it is <u>approaching</u> the observer.

b) The spectrum of an object will be **redshifted** if it is <u>receding</u> from the observer.

c) The wavelength shift in a spectral line is given by:

$$\frac{\Delta\lambda}{\lambda_\circ} = \frac{v_r}{c}, \qquad \text{(IX-9)}$$

where $\Delta\lambda = \lambda - \lambda_\circ$ (negative shift = blueshift), $\lambda_\circ$ = rest (lab) wavelength, v_r = radial (*i.e.*, line-of-sight) velocity of the object, and c = speed of light.

Example IX–4. We observe a hydrogen spectral line of Polaris (the North Star) with a wavelength of 6562.48 Å, which in the laboratory is measured to be at 6562.85 Å. What is the radial velocity of Polaris?

Solution:

We are given $\lambda = 6562.48$ Å and $\lambda_\circ = 6562.85$ Å, so $\Delta\lambda = 6562.48$ Å -6562.85 Å $= -0.37$ Å.

$$
\begin{aligned}
v &= \frac{\Delta\lambda}{\lambda_\circ} c = \frac{-0.37 \text{ Å}}{6562.85 \text{ Å}} 3.00 \times 10^8 \text{ m/s} \\
&= (-5.638 \times 10^{-5}) \, (3.00 \times 10^8 \text{ m/s}) \\
&= -1.69 \times 10^4 \text{ m/s} = \boxed{-16.9 \text{ km/s} \; .}
\end{aligned}
$$

Polaris is moving towards us (negative sign and the line was blueshifted) at 16.9 km/s.

D. Blackbody Radiation.

1. Late in the 1800s and in the early part of the 20th century, Boltzmann, Planck, and others investigated E/M radiation that was given off by hot objects.

    a) In general, matter can **absorb** some radiation (*i.e.*, photons converted to thermal energy), **reflect** some, and **transmit** some of the energy.

    b) The color of cool objects, objects that don't emit their own <u>visible</u> light, is dictated by the wavelengths of light they either reflect, absorb, or transmit.

        i)   A blue sweater is "blue" because the material <u>reflects</u> blue light (from either room lights or the Sun) more effectively than the other colors of the rainbow.

        ii) Coal is black because it <u>absorbs</u> visible light and reflects very little.

iii) Glass is transparent because visible light is almost completely <u>transmitted</u> through the glass with little absorption and reflection.

2. To make the physics a little less complicated, these scientists invented the concept of an **ideal** or **perfect radiator**.

a) A hypothetical body that completely absorbs every kind of E/M radiation that falls on it.

b) This absorption continues until an equilibrium temperature is reached.

c) At that point, all incoming radiation is immediately re-radiated away as soon as it is absorbed (note that this is <u>not</u> the same as being reflected).

d) Such a perfect radiator or absorber is called a **blackbody**.

3. A blackbody radiator has an **energy flux** F (defined as the energy emitted per unit area each second and related to the intensity) that is radiated away which is proportional to the 4th power of temperature via

$$\boxed{F = \sigma T^4 \ .}$$ (IX-10)

a) $\sigma = 5.67 \times 10^{-8}$ W/m^2/K^4 is the Stefan-Boltzmann constant.

b) The units of energy flux is W/m^2 in the SI system and erg/cm^2/s in the cgs system.

c) The temperature in Eq. (IX-10) is in units of Kelvin (K) and is the equilibrium or **effective temperature**.

d) Eq. (IX-10) is the **Stefan-Boltzmann law**.

4. The total brightness, or **luminosity** (L), of a blackbody is just the flux integrated over all of the surface of the object. For a spherical object, the surface area is $4\pi\,R^2$, where R is the radius of the spherical blackbody, so the luminosity is

$$L = 4\pi\,R^2\,F = 4\pi\,\sigma\,R^2\,T^4. \qquad \text{(IX-11)}$$

Note that if we treat stars as blackbodies, we can eliminate the constants in the above equation by dividing both sides by *solar* values:

$$\frac{L}{L_\odot} = \frac{4\pi\,\sigma\,R^2\,T^4}{4\pi\,\sigma\,R_\odot^2\,T_\odot^4}$$

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4. \qquad \text{(IX-12)}$$

5. In 1893, Wien discovered a simple relationship between T of a blackbody and the wavelength where the maximum amount of light is emitted $\Longrightarrow$ **Wien's displacement law** (usually called Wien's law for short):

$$\boxed{\lambda_{\text{max}} = \frac{2.897 \times 10^{-3} \text{ m} \cdot \text{K}}{T}}\,, \qquad \text{(IX-13)}$$

or in the other wavelength units we have discussed:

$$\begin{aligned}
\lambda_{\text{max}} &= \frac{0.2897 \text{ cm} \cdot \text{K}}{T} \\
&= \frac{2.897 \times 10^6 \text{ nm} \cdot \text{K}}{T} \\
&= \frac{2.897 \times 10^7 \text{ \AA} \cdot \text{K}}{T}\,.
\end{aligned}$$

6. Blackbody radiators, being in thermal equilibrium, emit continuous spectra that are called **Planck curves**:
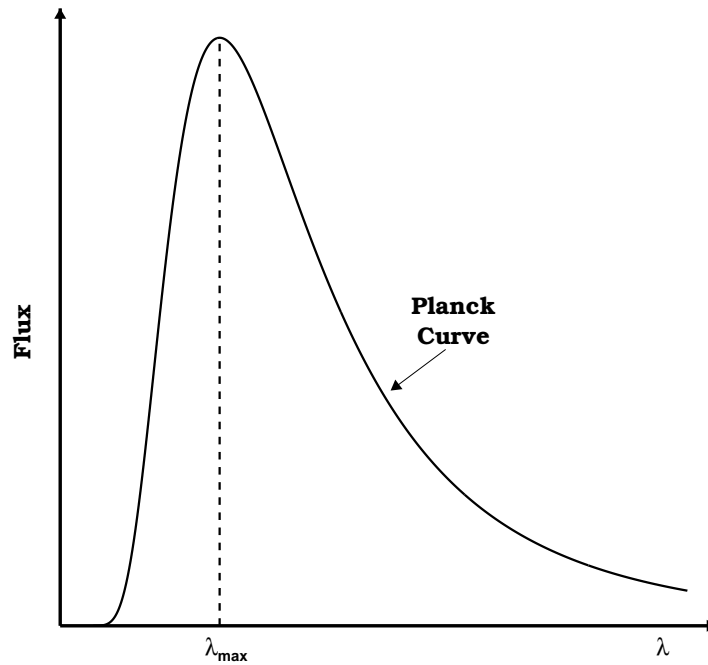
Example IX–5. A star has a temperature of 10,000 K and a radius of 20 $R_\odot$, what is its energy flux and wavelength of maximum flux? What is its luminosity with respect to the Sun ($\odot$)? (Note that $R_\odot = 6.96 \times 10^8$ m and $T_\odot = 5800$ K.)

Solution:

For this problem, we will assume that stars are blackbody radiators. We then use the Stefan-Boltzmann law (Eq. IX-10) to determine the energy flux of this star:

$$
\begin{aligned}
F &= (5.67 \times 10^{-8} \text{ W m}^{-2}\text{K}^{-4})\,(10{,}000 \text{ K})^4 \\
&= (5.67 \times 10^{-8} \text{ W m}^{-2}\text{K}^{-4})\,(10^4 \text{ K})^4 \\
&= (5.67 \times 10^{-8} \text{ W m}^{-2}\text{K}^{-4})\,(10^{16} \text{ K}^4) \\
&= \boxed{5.67 \times 10^8 \text{ W m}^{-2}}\,.
\end{aligned}
$$

The wavelength of maximum brightness is given by Wien's law

(Eq. IX-13, using nm for wavelength instead of m):

$$\lambda_{\text{max}} = \frac{2.897 \times 10^6 \text{ nm K}}{10,000 \text{ K}} = \boxed{290 \text{ nm}} \implies \text{UV light!}$$

Finally, the luminosity is given by Eq. (IX-12):

$$\begin{aligned}
\frac{L}{L_\odot} &= \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4 \\
&= \left(\frac{20\,R_\odot}{R_\odot}\right)^2 \left(\frac{10,000 \text{ K}}{5800 \text{ K}}\right)^4 = (400)\,(1.72)^4 \\
&= (400)\,(8.84) = 3500 \\
L &= \boxed{3500\ L_\odot\ .}
\end{aligned}$$

E. Spectroscopy: The Study of Spectra.

1. In 1814, Joseph von Fraunhofer discovered about 600 dark lines in the solar spectrum $\implies$ **spectral lines**. The darkest he labeled from "A" (in the red) to "H" (in the blue) [note that a "K" line was added later].

2. In 1859, Gustav Kirchhoff and Robert Bunsen discovered that each element contained a unique set of lines in their spectra $\implies$ spectral lines are the *fingerprints* of chemical elements and compounds. This is the basis behind **spectral analysis**.

3. A device that breaks white light into its component colors, hence displaying spectral lines if they are present, is called a **spectrograph** or **spectroscope**.

4. Bohr, early in the 20th century, demonstrated that absorption lines result from photons being absorbed by an atom, causing an electron to jump from a lower to a higher energy state.

a)   Electrons can only orbit nuclei in **allowed** orbits or states. This is the basis of the branch of modern physics called **quantum mechanics**.

b)   Bohr's model atom corresponds to the hydrogen atom which will be discussed in §X of these notes.

5.   Likewise, an emission line results when an electron jumps from an upper to a lower level in an atom or molecule.

6.   In both cases, the energy separation between the two levels must be equal to the energy of the photon that is either absorbed or emitted following

$$\Delta E = E_u - E_\ell = h\nu_{u\ell} \, , \qquad\qquad \text{(IX-14)}$$

where E_u is the energy of the upper level, E_ℓ the energy of the lower level, $\nu_{u\ell}$ is the frequency of the photon that is absorbed or emitted, and $h\nu_{u\ell}$ is the energy of this photon.

7.   The interaction of E/M radiation and matter will be discussed in more detail in the next section of these notes.