PHYS-2020: General Physics II
Course Lecture Notes
Section II

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Edition 3.2

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 8th Edition* (2009) textbook by Serway and Vuille.

II. Electrical Energy & Capacitance

A. Potential Difference & Electric Potential.

1. Like gravity, the electrostatic force is conservative.

a) From *General Physics I*, the work done from points A to B is defined by

$$W_{\rm AB} = Fd \; , \tag{II-1}$$

where F is the force supplied to a moving object and d is the distance between points A and B (*i.e.*, the distance that the particle travels).

b) If we have a uniform electric field that supplies the work on a positively charged particle, then $F = qE$ (see Eq. I-3) and the work supplied by the E-field is given by

$$W_{\rm AB} = qEd \; . \tag{II-2}$$

c) As such, as the positively charged particle moves along the E-field, it's velocity increases, which increases its kinetic energy (KE).

d) Since the electric force is conservative, as the charge particle gains KE, it loses an equal amount of potential energy (PE).

e) From the work-energy theorem, $W = \Delta{\rm KE}$, we then see that we can also write $W_{\rm AB} = -\Delta{\rm PE}$ from the statement above $\Longrightarrow$ hence, the work done on an object by the electric force is independent of path.

f) Finally, if the E-field is uniform, we can write this conservative form of the work-energy theorem as

$$\Delta{\rm PE} \; = -W_{\rm AB} = -qEd \; . \tag{II-3}$$

2. The **potential difference** V between points A and B is defined as the change in potential energy (final minus initial values) of a charge q moved from A to B divided by the charge:

$$\boxed{\Delta V \equiv V_{\rm B} - V_{\rm A} = \frac{\Delta \rm PE}{q}\ .} \tag{II-4}$$

Note that it is standard practice to express ΔV as just $V_{\rm AB}$, or even more simply as V.

a) Potential difference is <u>not</u> the same as potential energy $\Longrightarrow$ it is a *potential energy per unit charge.*

b) Electric potential (V) is a scalar quantity.

c) Electric potential is measured in volts V (do not confuse the unit V [volts] with the variable V [potential]):

$$1\ \mathrm{V}\ = 1\ \mathrm{J/C}\quad \text{(Joule/Coulomb)}. \tag{II-5}$$

d) Plugging Eq. (II-4) into Eq. (II-3) gives

$$\begin{aligned} \Delta V &= -Ed \qquad \text{(positive charge in } E\text{-field)} \\ \Delta V &= +Ed \qquad \text{(negative charge in } E\text{-field).} \end{aligned} \tag{II-6}$$

i) A positive charge *gains* electrical potential energy when it is moved in a direction <u>opposite</u> the electric field.

ii) A negative charge *loses* electrical potential energy when it is moved in a direction <u>opposite</u> the electric field.

e) As a result of Eq. (II-6), we see that the electric field can be measured with two separate sets of units:

$$[E] = 1\ \mathrm{N/C}\ = 1\ \mathrm{V/m}\ .$$

Example II–1. A 4.00-kg block carrying a charge $Q = 50.0$ μC is connected to a spring for which $k = 100$ N/m. The block lies on a frictionless horizontal track, and the system is immersed in a uniform electric field of magnitude $E = 5.00 \times 10^5$ V/m directed as in Figure P16.9 (in the textbook). (a) If the block is released at rest when the spring is unstretched (at $x = 0$), by what maximum amount does the spring expand? (b) What is the equilibrium position of the block?

Solution (a):

This problem can best be solved with the conservation of energy:

$$(\mathrm{KE} \;+\; \mathrm{PE}_s + \;\mathrm{PE}_e)_\mathrm{i} = (\mathrm{KE} \;+\; \mathrm{PE}_s + \;\mathrm{PE}_e)_\mathrm{f}\ ,$$

where $\mathrm{PE}_s = \frac{1}{2}kx^2$ is the potential energy due to the spring and $\mathrm{PE}_e = -QEx$ is the potential energy due to the E-field. Initially, the block is at rest (KE = 0) at $x_\mathrm{i} = 0$ and when it reaches its maximum extension x_max (*i.e.*, the final position), it is once again at rest (KE = 0). Substituting our equations and values into the conservation of energy and noting that V/m = N/C, we can solve for x_max:

$$\begin{aligned}
\mathrm{KE_i} + \;\mathrm{PE}_{s\mathrm{i}} + \;\mathrm{PE}_{e\mathrm{i}} &= \mathrm{KE_f} + \;\mathrm{PE}_{s\mathrm{f}} + \;\mathrm{PE}_{e\mathrm{f}} \\
0 + \frac{1}{2}kx_\mathrm{i}^2 - QEx_\mathrm{i} &= 0 + \frac{1}{2}kx_\mathrm{max}^2 - QEx_\mathrm{max} \\
0 + 0 + 0 &= 0 + \frac{1}{2}kx_\mathrm{max}^2 - QEx_\mathrm{max} \\
\frac{1}{2}kx_\mathrm{max}^2 - QEx_\mathrm{max} &= 0 \\
\frac{1}{2}kx_\mathrm{max} - QE &= 0 \\
\frac{1}{2}kx_\mathrm{max} &= QE \\
x_\mathrm{max} &= \frac{2QE}{k} \\
&= \frac{2(50.0 \times 10^{-6}\ \mathrm{C})(5.00 \times 10^5\ \mathrm{N/C})}{100\ \mathrm{N/m}}
\end{aligned}$$

$$= \boxed{0.500 \text{ m} .}$$

Solution (b):

To answer the second question, we note the keyword "equilibrium" and realized that we need to use a force equation. At equilibrium, the force imparted on the positively charged block by the E-field (pointing in the $+x$ direction), given by Eq. (I-3), is balanced by the oppositely pointing force due to the spring (as covered in *General Physics I*). Hence

$$\begin{aligned} \sum F = F_e - F_s &= QE - kx_{\text{eq}} = 0 \\ x_{\text{eq}} &= \frac{QE}{k} = \frac{1}{2}x_{\text{max}} \\ &= \boxed{0.250 \text{ m} .} \end{aligned}$$

B. Electric Potential of Point Charges.

1. The electric potential due to a point charge q at any distance r from the charge is given by

$$\boxed{V = k_e \frac{q}{r} .} \tag{II-7}$$

2. The total potential at some point P due to several point charges is the algebraic sum of the potentials due to the individual charges:

$$V_{\text{total}} = \sum_{i=1}^{N} V_i = k_e \sum_{i=1}^{N} \frac{q_i}{r_i} , \tag{II-8}$$

once again, another **superposition principle**.

3. For a system of 2 particles, the PE of the system is

$$\text{PE} = q_2 V_1 = k_e \frac{q_1 q_2}{r} . \tag{II-9}$$

a) V_1 is the electric potential due to charge q_1 at point P.

b) The work done to bring a 2nd charge q_2 from infinity to P is

$$W = -\Delta \mathrm{PE} \; = \mathrm{PE_P} - \mathrm{PE_\infty} = \mathrm{PE_P} = q_2 V_1 \; , \qquad \text{(II-10)}$$

since $\mathrm{PE}_\infty \equiv 0$, which is identical to Eq. (II-9).

c) If $q_1 > 0$ and $q_2 > 0$, PE is positive $\Longrightarrow$ positive work must be done to bring like charges together.

d) If the charges are opposite in sign, PE is negative $\Longrightarrow$ negative work must be done to bring opposite charges together $\Longrightarrow$ energy is released!

Example II–2. Two point charges are on the y-axis, one of magnitude 3.0×10^{-9} C at the origin and a second of magnitude 6.0×10^{-9} C at the point y = 30 cm. Calculate the potential at y = 60 cm.

Solution:

Let $q_1 = 3.0 \times 10^{-9}$ C with $y_1 = 0$ cm $= 0$ m, $q_2 = 6.0 \times 10^{-9}$ C with $y_2 = 30$ cm $= 0.30$ m, and $y_{\rm ref} = 60$ cm $= 0.60$ m. The distance that the reference point is from charge 1 is $r_1 = y_{\rm ref} - y_1$ = 0.60 m - 0 m = 0.60 m and from charge 2 is $r_2 = y_{\rm ref} - y_2 =$ 0.60 m - 0.30 m = 0.30 m.

Then using Eq. (II-8) we get the potential at point P as

$$\begin{aligned} V &= V_1 + V_2 = k_e \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\ &= \left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \right) \left(\frac{3.0 \times 10^{-9}\ \mathrm{C}}{0.60\ \mathrm{m}} + \frac{6.0 \times 10^{-9}\ \mathrm{C}}{0.30\ \mathrm{m}} \right) \\ &= \boxed{220\ \mathrm{V}\ .} \end{aligned}$$

Note that V = N·m/C.

C. Potentials & Charged Conductors.

1. Combining Eq. (II-3) and Eq. (II-4), we can write

$$\boxed{W = -q\,(V_\mathrm{B} - V_\mathrm{A})\ .} \tag{II-11}$$

a) No work is required to move a charge between 2 points that are at the same potential $\Longrightarrow W = 0$ when $V_\mathrm{B} = V_\mathrm{A}$.

b) The electric potential is constant everywhere on the surface of a charged conductor in equilibrium.

c) The electric potential is constant everywhere inside a conductor and is equal to its value at the surface.

2. The **electron volt** is defined as the energy that an electron (or proton) gains when moving through a potential difference of one volt.

a) 1 eV $= 1.60219 \times 10^{-19}$ C·V $= 1.60219 \times 10^{-19}$ J (SI units) $= 1.60219 \times 10^{-12}$ erg (cgs units).

b) Electronic states (electron levels) in an atomic model are sometimes listed in electron volts.

i) Ground state (lowest level) of any atom or molecule $\equiv 0$ eV.

ii) Hydrogen's 1st excited state = 10.2 eV.

iii) Hydrogen ionizes at 13.6 eV.

3. A surface on which all points are at the same potential is called an **equipotential surface**.

a) The potential difference of any 2 points on an equipotential surface is zero.

b) No work is required to move a charge at constant speed on an equipotential surface.

c) $\vec{E}$ is always $\perp$ to an equipotential surface.

D. Capacitors.

1. A capacitor is a device used in electric circuits that can store charge for a short period of time.

a) Usually consists of 2 parallel conducting plates separated by a small distance.

b) One plate is connected to positive voltage, the other to negative voltage.

c) Electrons are pulled off of one of the plates (+ plate) and are deposited onto the other plate (− plate) through (typically) a battery.

d) The charge transfer stops when the potential difference across the plates equals the potential difference of the battery.

e) A charged capacitor acts as a storehouse of charge and energy.

2. The **capacitance** C of a capacitor is the ratio of the magnitude of the charge on either conductor (*e.g.*, plate) to the magnitude of the potential difference between the conductors:

$$\boxed{C \equiv \frac{Q}{\Delta V}\ .} \qquad \text{(II-12)}$$

a) Capacitance is measured in **farads** (F) in the SI system.

$$1\ \text{F} \equiv 1\ \text{C/V}\ . \qquad \text{(II-13)}$$

b) One farad is a very large unit of capacitance. Capacitors usually range from 1 picofarad (1 pF $= 10^{-12}$ F) to 1 microfarad (1 μF $= 10^{-6}$ F).

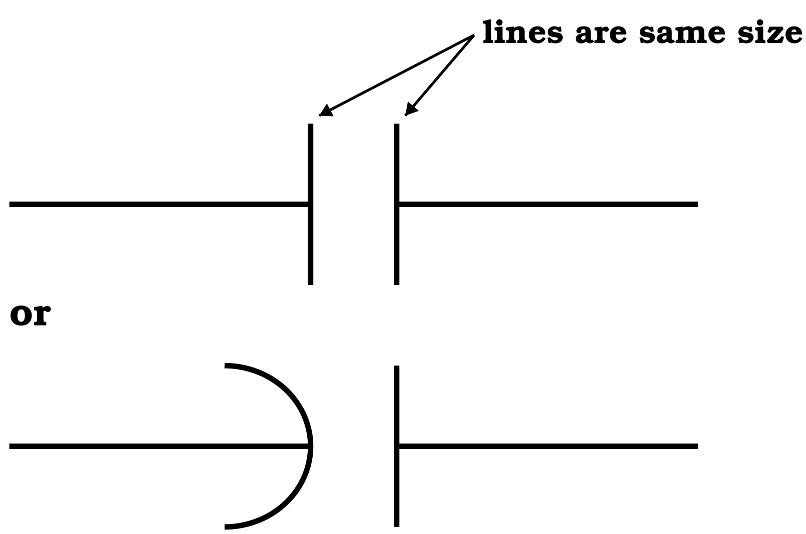
3. We also can describe capacitance based on the geometry of the capacitor.

a) For a **parallel-plate** capacitor:

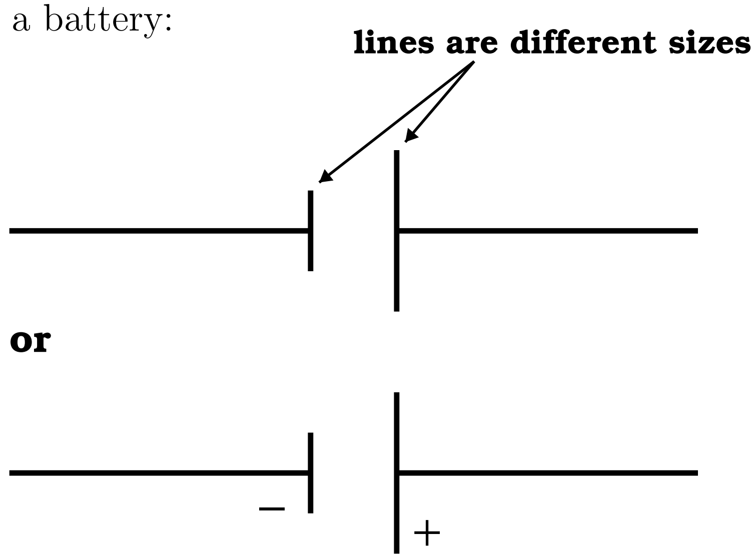
$$C = \epsilon_\circ \frac{A}{d}\ , \qquad \text{(II-14)}$$

where A is the area of one of the plates (both plates have equal areas here), d is the separation distance of the plates, and $\epsilon_\circ$ is the permittivity of free space (a constant) given in Eq. (I-8).

4. In circuit diagrams, a capacitor is labeled with



$\Longrightarrow$ Note that these symbols should not be confused with the symbol for a battery:



Example II–3. Problem 16.25 (Page 566) from the Serway & Vuille textbook: Consider Earth and a cloud layer 800 m above Earth to be the plates of a parallel-plate capacitor. (a) If the cloud layer has an area of 1.0 km^2 $= 1.0 \times 10^6$ m^2, what is the capacitance? (b) If an electric field strength greater than 3.0×10^6 N/C causes the air to break down and conduct charge (lightning), what is the maximum charge the cloud can hold?

Solution (a):

We simply need to use Eq. (II-14) here:

$$\begin{aligned} C &= \epsilon_\circ \frac{A}{d} = \left(8.85 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N{\cdot}m}^2}\right) \frac{(1.0 \times 10^6 \ \mathrm{m}^2)}{(800 \ \mathrm{m})} \\ &= 1.1 \times 10^{-8} \ \mathrm{F} \ = \boxed{11 \ \mathrm{nF}\ .} \end{aligned}$$

Solution (b):

Use Eqs. (II-12) & (II-13) in conjunction with Eq. (II-6) (with N/C = V/m), where here we use the + sign version of (II-6) since electrons are involved, thus

$$Q_{\rm max} \ = \ C(\Delta V)_{\rm max} = C(E_{\rm max} d)$$

$$= (1.11 \times 10^{-8}\ \mathrm{C/V})(3.0 \times 10^{6}\ \mathrm{V/m})(800\ \mathrm{m}) = \boxed{27\ \mathrm{C}\ .}$$

E. Combination of Capacitors.

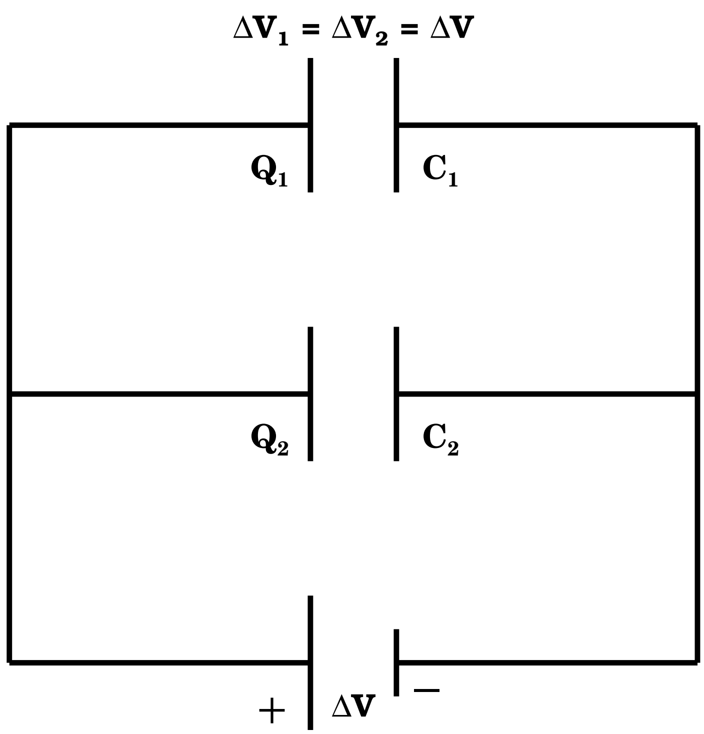
1. In circuits with multicomponents, always try to reduce the circuit to single components.

a) Combine all capacitors to one capacitor.

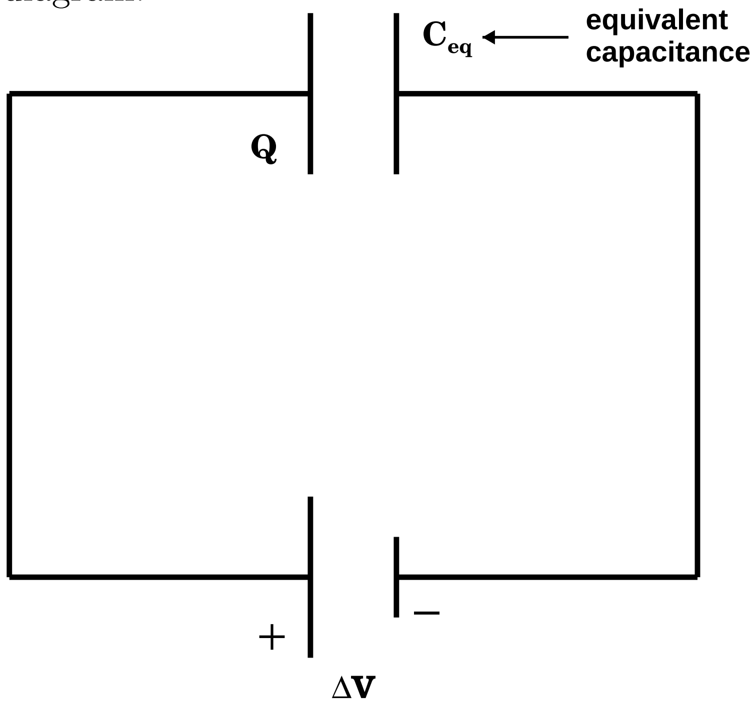
b) Combine all resistors (as shown in the next section of the notes) to one resistor.

c) Hence, we can reduce most circuits to a simple circuit of an equivalent capacitor $C_{\rm eq}$ and an equivalent resistor $R_{\rm eq}$. In this section, we will work only with capacitors.

2. Capacitors in Parallel.



a) In parallel, we can reduce the diagram above to the following diagram:



b) The potential difference across the capacitors ΔV_i in a parallel circuit are the same $\Longrightarrow$ each is equal to the battery's voltage ΔV.

$$\Delta V_1 = \Delta V_2 = \Delta V_i = \Delta V \ . \tag{II-15}$$

c) The total or equivalent charge on the capacitor in a parallel circuit is just the sum of all the charges on the individual capacitors:

$$\begin{aligned} Q_1 + Q_2 &= Q \qquad \text{(two parallel capacitors)} \\ \sum_{i=1}^{N} Q_i &= Q \qquad \text{(N parallel capacitors).} \end{aligned} \tag{II-16}$$

d) For the reduced circuit above, we can write

$$Q = C_{\rm eq}\, \Delta V \ , \tag{II-17}$$

since $Q = \sum Q_i$, we get

$$C_{\rm eq}\, \Delta V = C_1 \Delta V_1 + C_2 \Delta V_2$$

or

$$C_{\rm eq}\,\Delta V = C_1 \Delta V + C_2 \Delta V = (C_1 + C_2)\ \Delta V$$

for our circuit above, or more generally we can write

$$C_{\rm eq}\,\Delta V = \Delta V \sum_{i=1}^{N} C_i \ . \qquad \text{(II-18)}$$

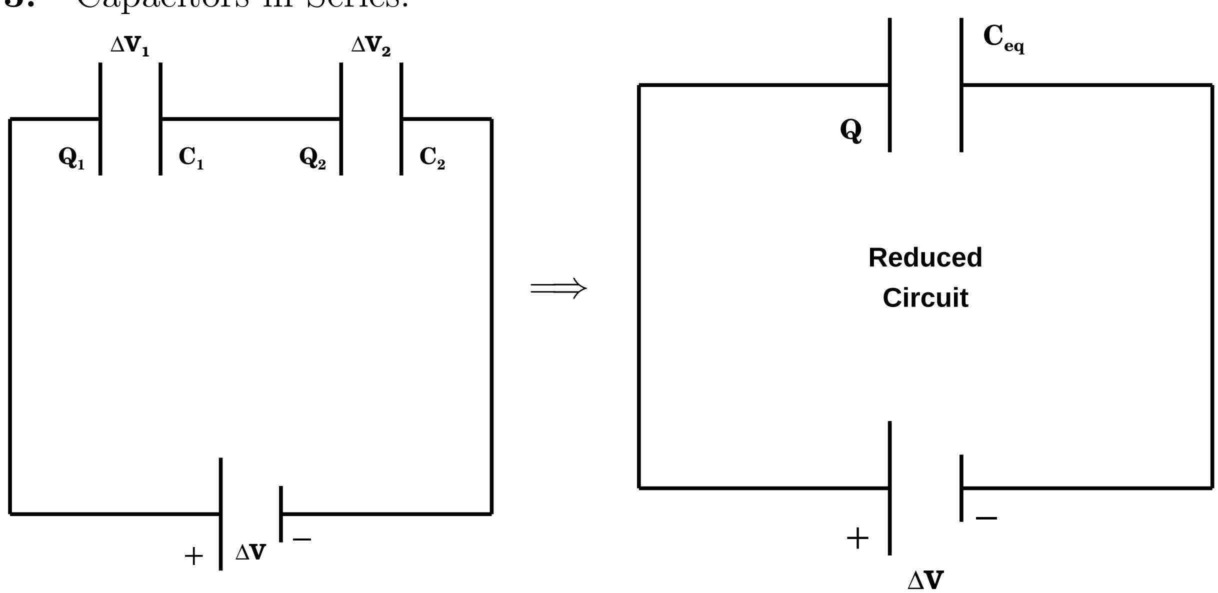
e) Finally, we can express the equivalent capacitance as

$$C_{\rm eq} = C_1 + C_2 \qquad \text{(II-19)}$$

for two parallel capacitors or more generally

$$\boxed{C_{\rm eq} = \sum_{i=1}^{N} C_i \ .} \text{ parallel circuits} \qquad \text{(II-20)}$$

3. Capacitors in Series.



a) For a series combination of capacitors, the magnitude of the charge must be the same at all plates:

$$Q_1 = Q_2 = Q_i = Q \ . \qquad \text{(II-21)}$$

b) The potential difference across any number of capacitors (or other circuit elements) in series equals the sum of the potential differences across the individual capacitors:

$$\Delta V_1 + \Delta V_2 = \Delta V \qquad \text{(two series capacitors)}$$

$$\sum_{i=1}^{N} \Delta V_i = \Delta V \qquad \text{(N series capacitors).} \tag{II-22}$$

c) For the reduced circuit above, we can write

$$Q = C_{\rm eq}\,\Delta V \quad \text{or} \quad \Delta V = Q/C_{\rm eq}\ . \tag{II-23}$$

From Eq. (II-22) for the two capacitor circuit we get

$$\begin{aligned} \Delta V &= \Delta V_1 + \Delta V_2 \\ \frac{Q}{C_{\rm eq}} &= \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \\ &= \frac{Q}{C_1} + \frac{Q}{C_2} \end{aligned}$$

or finally

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{II-24}$$

for two series capacitors or more generally

$$\boxed{\frac{1}{C_{\rm eq}} = \sum_{i=1}^{N} \frac{1}{C_i}\ .} \text{ series circuits} \tag{II-25}$$

4. Problem-Solving strategy with capacitors in circuits:

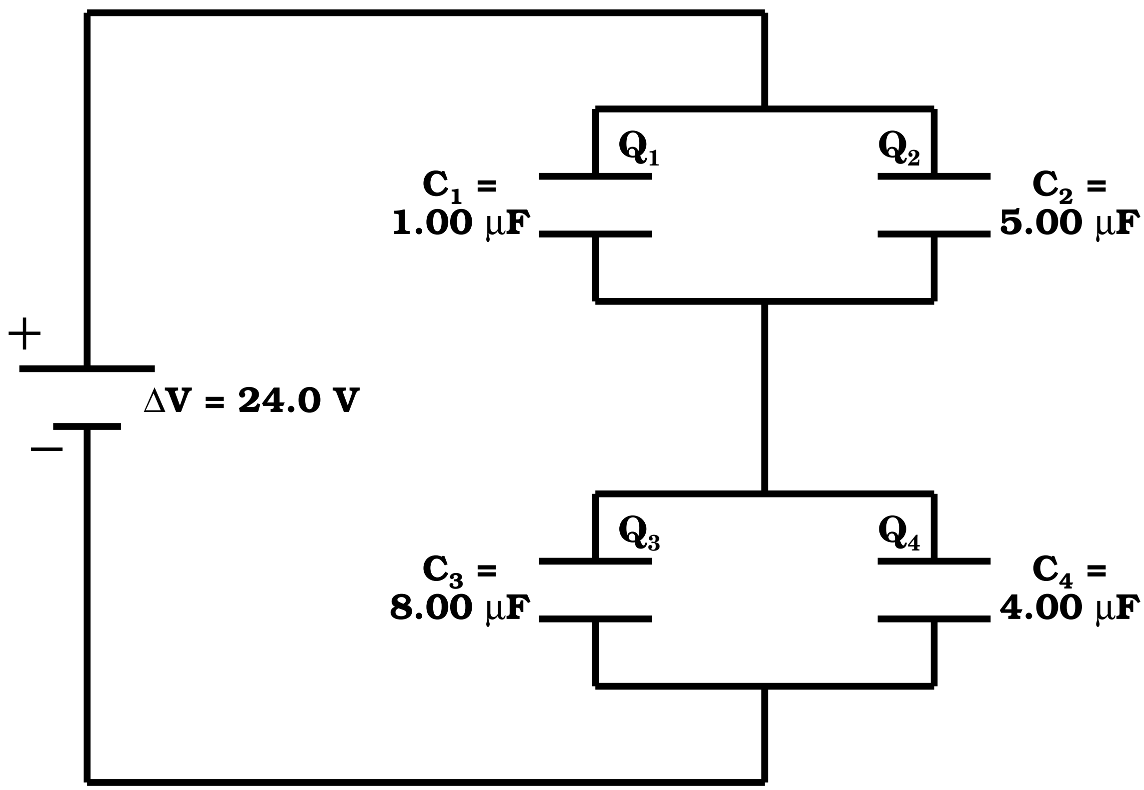
a) Make sure units are all SI — C in farads, lengths in meters, etc.

b) Make equivalent capacitors from capacitors in the circuit. Choose either sets of parallel capacitors or series capacitors first, depending on which is more obvious to chose.

c) Continue on making these equivalent capacitors until you only have one left.

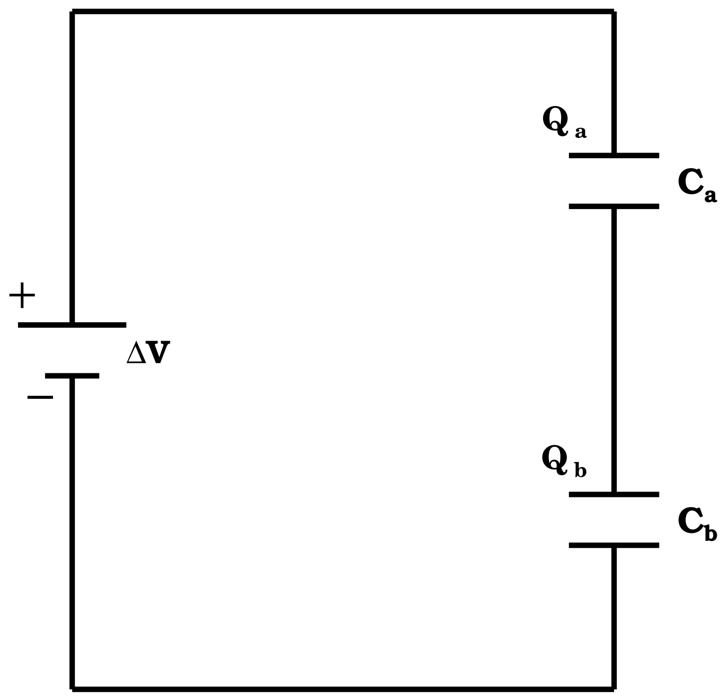
d) To find the charge on, or the potential difference across, one of the capacitors in a complicated circuit. Start with the final (*i.e.*, reduced) circuit of step (c) and gradually work your way back through the circuits using $C = Q/\Delta V$ and the rules set up for parallel and series circuits.

Example II–4. **Problem 16.39 (Page 567) from the Serway & Vuille textbook**: Find the charge on each of the capacitors in Figure P16.39 in the textbook (and shown below).



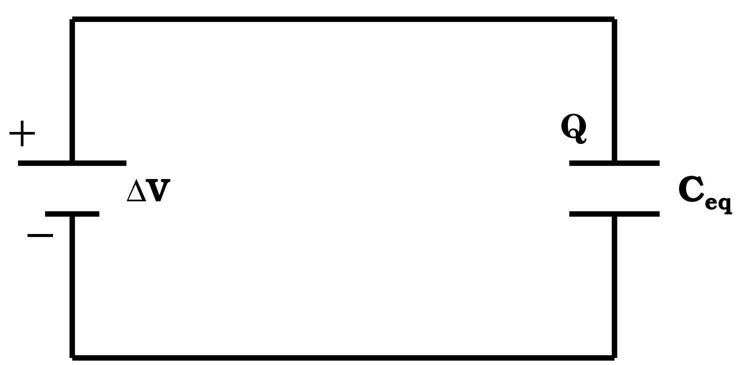
Solution:

Reduced the || capacitors first:



$$
\begin{aligned}
C_a &= C_1 + C_2 = 1.00 \ \mu\mathrm{F} \ + 5.00 \ \mu\mathrm{F} \ = 6.00 \ \mu\mathrm{F} \\
C_b &= C_3 + C_4 = 8.00 \ \mu\mathrm{F} \ + 4.00 \ \mu\mathrm{F} \ = 12.00 \ \mu\mathrm{F}
\end{aligned}
$$

Now all of the equivalent capacitors are in series, reduce these series capacitors:



$$
\begin{aligned}
\frac{1}{C_{\rm eq}} &= \frac{1}{C_a} + \frac{1}{C_b} = \frac{1}{6.00 \ \mu\mathrm{F}} + \frac{1}{12.00 \ \mu\mathrm{F}} \\
&= \frac{2}{12.00 \ \mu\mathrm{F}} + \frac{1}{12.00 \ \mu\mathrm{F}} = \frac{3}{12.00 \ \mu\mathrm{F}} \\
C_{\rm eq} &= \frac{12.00 \ \mu\mathrm{F}}{3} = 4.00 \ \mu\mathrm{F}
\end{aligned}
$$

Now find Q:

$$\begin{aligned} Q &= C_{\rm eq}\,\Delta V = (4.00 \times 10^{-6}\ \mathrm{F})(24.0\ \mathrm{V}) \\ &= 96.0 \times 10^{-6}\ \mathrm{C}\ = 96.0\ \mu\mathrm{C} \end{aligned}$$

Go back to the second reduce (*i.e.*, the series) circuit:

$$Q = Q_a = Q_b = 96\ \mu\mathrm{C}$$

and

$$\begin{aligned} \Delta V_a &= \frac{Q_a}{C_a} = \frac{96.0\ \mu\mathrm{C}}{6.00\ \mu\mathrm{F}} = \frac{96.0 \times 10^{-6}\ \mathrm{C}}{6.00 \times 10^{-6}\ \mathrm{F}} = 16.0\ \mathrm{V} \\ \Delta V_b &= \frac{Q_b}{C_b} = \frac{96.0\ \mu\mathrm{C}}{12.00\ \mu\mathrm{F}} = 8.00\ \mathrm{V} \end{aligned}$$

Finally, go back to the first reduced (*i.e.*, the parallel) circuit:

$$\Delta V_a = \Delta V_1 = \Delta V_2$$

and

$$\begin{aligned} Q_1 &= C_1 \Delta V_1 = C_1 \Delta V_a = (1.00 \times 10^{-6}\ \mathrm{F})(16.0\ \mathrm{V}) \\ &= 16.0 \times 10^{-6}\ \mathrm{C}\ = \boxed{16.0\ \mu\mathrm{C}} \\ Q_2 &= C_2 \Delta V_2 = C_2 \Delta V_a = (5.00 \times 10^{-6}\ \mathrm{F})(16.0\ \mathrm{V}) \\ &= 80.0 \times 10^{-6}\ \mathrm{C}\ = \boxed{80.0\ \mu\mathrm{C}\ .} \end{aligned}$$

$$\Delta V_b = \Delta V_3 = \Delta V_4$$

and

$$\begin{aligned} Q_3 &= C_3 \Delta V_3 = C_3 \Delta V_b = (8.00\ \mu\mathrm{F})(8.00\ \mathrm{V}) = \boxed{64.0\ \mu\mathrm{C}} \\ Q_4 &= C_4 \Delta V_4 = C_4 \Delta V_b = (4.00\ \mu\mathrm{F})(8.00\ \mathrm{V}) = \boxed{32.0\ \mu\mathrm{C}\ .} \end{aligned}$$

F. Energy Stored in Charged Capacitors.

1. An uncharged capacitor contains <u>no</u> energy $\Longrightarrow$ $\text{PE}_\text{i} = 0$. If we were to charge the capacitor, the opposite charges on either plate sets up an electric field between the plates.

a) The PE will increase as the voltage increases between the plates via Eq. (II-4):

$$\Delta\text{PE} = Q\,\Delta V \ .$$

i) At this point, let's note that the PE of a capacitor is equivalent to the *internal energy* U of the capacitor, then

$$\Delta U = Q\,\Delta V \ . \qquad \text{(II-26)}$$

ii) Using Eq. (II-12), we can write

$$\Delta V = \frac{1}{C}\,\Delta Q$$

and Eq. (II-26) becomes

$$\Delta U = \frac{Q}{C}\,\Delta Q \ . \qquad \text{(II-27)}$$

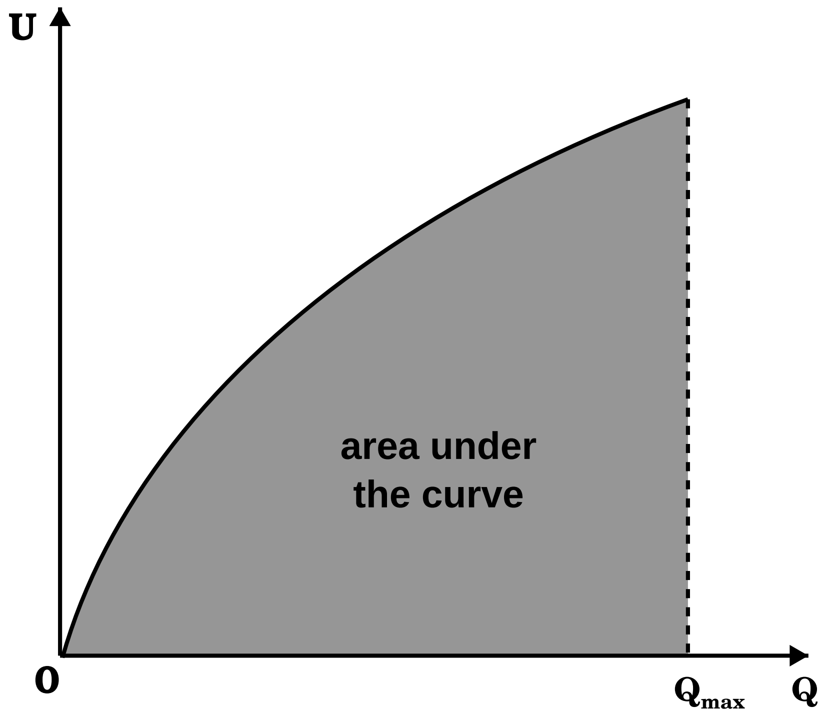
b) The next step requires a little calculus (see the text, particularly Figure 16.23, for a graphical description of this using the definition of work).

i) Let the changes in U and Q be very small so that we can change the "Δ's" with differentials "d":

$$dU = \frac{Q}{C}\,dQ \ .$$

ii) Since C is constant as U and Q changes, we can integrate this equation to find U (the internal or potential energy) as a function of Q (the charge) $\Longrightarrow$

integrating a function just means that we are finding the area under a curve described by the function.



c) The integration starts when the capacitor is discharged: $U = 0$ and $Q = 0$ and continues until the maximum charge is reached $Q \Longrightarrow Q_{\rm max}$:

$$\begin{aligned}
\int_0^{U_{\rm max}} dU &= \int_0^{Q_{\rm max}} \frac{Q}{C}\, dQ = \frac{1}{C}\int_0^{Q_{\rm max}} Q\, dQ \\
U\Big|_0^{U_{\rm max}} &= \frac{1}{C}\left(\frac{1}{2}\, Q^2\Big|_0^{Q_{\rm max}}\right) \\
(U_{\rm max} - 0) &= \frac{1}{2C}\left(Q_{\rm max}^2 - 0^2\right) \\
U_{\rm max} &= \frac{Q_{\rm max}^2}{2C} .
\end{aligned}$$

d) We can drop the "max" subscript, and remembering that $Q = C\,\Delta V$ (*i.e.*, Eq. II-12), we can write the internal energy of a capacitor in one of 3 ways:

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}Q(\Delta V)\ .} \qquad \text{(II-28)}$$

Example II–5. Consider the parallel plate capacitor formed by the Earth and a cloud layer as described in Problem 16.25 (see Example II-3). Assume this capacitor will discharge (that is, lightning occurs) when the electric field strength between the plates reaches 3.0×10^6 N/C. What is the electric energy released if the capacitor discharges completely during the lightning strike?

Solution:

This is the same electric field strength as was given in Problem 16.25, and we have already calculated the capacitance of the cloud/Earth system in Example II-3: $C = 1.1 \times 10^{-8}$ F. With an electric field strength of $E = 3.0 \times 10^6$ N/C (remember N/C = V/m) and a plate separation of $d = 800$ m, the potential difference between the plates is (remember that electrons are negatively charged)

$$\Delta V = Ed = (3.0 \times 10^6 \text{ V/m})(800 \text{ m}) = 2.4 \times 10^9 \text{ V} .$$

Thus, the energy available for release in a lightning strike is

$$\begin{aligned} U &= \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(1.1 \times 10^{-8} \text{ F})(2.4 \times 10^9 \text{ V})^2 \\ &= 3.2 \times 10^{10} \text{ J} = \boxed{32 \text{ GJ.}} \end{aligned}$$

G. Capacitors with Dielectrics.

1. A **dielectric** is any type of insulating material (*e.g.*, rubber, plastic, etc.).

a) For a capacitor with no dielectric, the voltage drop across the capacitor is $\Delta V_\circ = Q_\circ / C_\circ$. If a dielectric is inserted between the plates of a capacitor, the voltage drop is reduced by a scale factor κ (note that $\kappa > 1$):

$$\Delta V = \frac{\Delta V_\circ}{\kappa} \ .$$

b) Because the charge on the capacitor will not change when a dielectric is introduced, the capacitance in the presence of a dielectric must change to the value

$$C = \frac{Q_\circ}{\Delta V} = \frac{Q_\circ}{\Delta V_\circ/\kappa} = \frac{\kappa Q_\circ}{\Delta V_\circ}$$

or the capacitance *increases* by the amount

$$C = \kappa C_\circ \ . \tag{II-29}$$

In this equation, $C_\circ$ is the capacitance that the capacitor has when filled with air (or has a vacuum in it), and $\kappa(> 1)$ is called the **dielectric constant**. Table 16.1 in the textbook displays dielectric constants for various materials (note that a vacuum has $\kappa = 1.00000$ identically and that air has $\kappa = 1.00059$, nearly that of a vacuum).

c) When dielectrics are present in a parallel-plate capacitor, Eq. (II-14) must be rewritten as

$$C = \kappa \epsilon_\circ \frac{A}{d} \ . \tag{II-30}$$

2. For any given plate separation, there is a maximum electric field that can be produced in the dielectric before it breaks down and begins to conduct $\Longrightarrow$ this maximum electric field is called the **dielectric strength**.

a) When designing circuits, one always needs to insure that the electric field generated by the stored charge in the capacitor does not exceed the dielectric strength of the dielectric material. If this occurs, the capacitor will *short circuit* (and sometimes blow up!).

b) As can be seen by Table 16.1 in the textbook, air has a dielectric strength of 3×10^6 V/m (though this number can change somewhat depending on the water content in the air). Whenever enough charge accumulates in a cloud base with respect to the ground or a cloud top such that the E-field exceed this dielectric strength, lightning is discharged by the cloud (either to the ground or to the cloud top).