PHYS-2010: General Physics I Course Lecture Notes Section XII

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Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2010: General Physics I taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics*, 9th Edition (2012) textbook by Serway and Vuille.

XII. Thermal Physics

A. Thermal Equilibrium.

- 1. If bodies A & B are separately in thermal equilibrium with a 3rd body C, then A & B will be in thermal equilibrium with each other.
 - a) This statement is often referred to as the 0th Law of Thermodynamics. It simply means that if 2 objects in thermal equilibrium with each other are at the same temperature.
 - b) Thermal equilibrium means that an object has the same temperature throughout its interior.
- 2. Temperature is nothing more than a measure of how fast particles are moving due to the heat energy stored in the system.
 - a) There are 3 different temperature scales:
 - i) Fahrenheit scale: archaic English system $\Longrightarrow 32^{\circ}F$ \equiv water freezes, $212^{\circ}F \equiv$ water boils (at atmospheric pressure).
 - ii) Celsius scale: metric system scale (once called the Centigrade scale) $\Longrightarrow 0^{\circ}C \equiv$ water freezes, $100^{\circ}C$ \equiv water boils (at atmospheric pressure).
 - iii) Kelvin scale: SI system \Longrightarrow the absolute temperature scale.
 - \Longrightarrow 0 K \equiv no atomic motions, lowest possible temperature.
 - \implies Lowest temp. recorded in lab is 10^{-6} K!

- b) Converting between the temperature scales:
 - i) ${}^{\circ}F \iff {}^{\circ}C:$

$$T_{\rm F} = \frac{9}{5}T_{\rm C} + 32$$
 (XII-1)

ii) $^{\circ}C \iff K$:

$$T_{\rm C} = T - 273.15$$
 . (XII-2)

iii) Note that T without a subscript will <u>always</u> refer to the Kelvin scale in these notes (except where noted).

Example XII–1. Problem 10.8 (Page 357) from the Serway & Vuille textbook: The temperature difference between the inside and the outside of a home is 57.0°F. Express this temperature difference on the (a) Celsius scale and (b) Kelvin scale.

Solution (a):

We need to come up with a formula that changes $\Delta T_{\rm F}$ to $\Delta T_{\rm C}$ and then to ΔT (where T is measured in Kelvin). Using Eq. (XII-1) we can write

$$(T_{\rm F})_1 = \frac{9}{5} (T_{\rm C})_1 + 32$$

and

$$(T_{\rm F})_2 = \frac{9}{5} (T_{\rm C})_2 + 32$$
.

Subtracting equation (1) from (2) yields

$$(T_{\rm F})_2 - (T_{\rm F})_1 = \frac{9}{5} [(T_{\rm C})_2 - (T_{\rm C})_1]$$

$$\Delta T_{\rm F} = \frac{9}{5} \Delta T_{\rm C} ,$$

since the two '32's cancel with each other. We are given a temperature difference of $\Delta T_{\rm F} = 57.0^{\circ} {\rm F}$, so from the equation above we get

$$\Delta T_{\rm C} = \frac{5}{9} \Delta T_{\rm F} = \frac{5}{9} (57.0) = \boxed{31.7^{\circ} \text{C}.}$$

Solution (b):

From the same argument above, Eq. (XII-2) gives

$$T_2 - T_1 = [(T_C)_2 + 273.15] - [(T_C)_1 + 273.15]$$
 $T_2 - T_1 = (T_C)_2 - (T_C)_1$

$$\Delta T = \Delta T_C = 31.7 \text{ K}.$$

B. Thermal Expansion.

- 1. Heat energy added to matter causes the particles that make up the matter to speed up.
 - a) Increased velocity of particles increases pressure.
 - b) Increased pressure causes matter to <u>expand</u> due to the increased internal force.
 - i) Liquids and gases fill a larger volume.
 - ii) Solids get longer.
 - c) A loss of heat energy causes objects to shrink in size.
- 2. For solids, the change in heat (as measured by temperature) dictates a change in linear size ΔL via

$$\Delta L = \alpha L_{\circ} \Delta T \qquad (XII-3)$$

- a) $L_{\circ} \equiv \text{initial length [m]}.$
- b) $\Delta T \equiv$ change in temp. (usually measured in °C in Eq. XII-3).
- c) $\alpha \equiv \text{coefficient of linear expansion (see Table 10.1 in textbook)}$.
- 3. Surface areas also change in size ΔA via:

$$\Delta A = \gamma A_{\circ} \Delta T \qquad (XII-4)$$

- a) $A_{\circ} \equiv \text{initial area } [\text{m}^2].$
- b) $\gamma \equiv \text{coefficient of area expansion} = 2\alpha$.
- 4. Volumes increase (or decrease), ΔV , as well when heat is added (or taken away) via:

$$\Delta V = \beta V_{\circ} \Delta T \qquad (XII-5)$$

- a) $V_{\circ} \equiv \text{initial volume [m}^3].$
- b) $\beta \equiv \text{coefficient of volume expansion} = 3\alpha$.

Example XII–2. Problem 10.11 (Page 357) from the Serway & Vuille textbook: The New River Gorge bridge in West Virginia is a 518-m long steel arch. How much will its length change between temperature extremes of -20° C and 35° C?

Solution:

Use Eq. (XII-3) here. $\Delta T = 35^{\circ}\text{C} - (-20^{\circ}\text{C}) = 55^{\circ}\text{C}$, $L_{\circ} = 518$ m, and $\alpha = 11 \times 10^{-6} \, {}^{\circ}\text{C}^{-1}$ from Table 10.1 on page 313 in the textbook. This

gives a length change of

$$\Delta L = \alpha L_{\circ} \Delta T = (11 \times 10^{-6} \, {}^{\circ}\text{C}^{-1})(518 \, \text{m})(55{}^{\circ}\text{C}) = 0.31 \, \text{m} = 31 \, \text{cm}$$

C. Heat and Internal Energy.

1. Internal energy, U, is the energy associated with the microscopic components of a system — atoms and molecules. It is given by

$$U = \overline{KE}_{trans} + \overline{KE}_{rot} + \overline{KE}_{vib} + \overline{PE}_{mol}. \qquad (XII-6)$$

- a) $\overline{\text{KE}}_{\text{trans}} \equiv \text{average translational (linear) kinetic energy of the atoms and molecules.}$
- b) $\overline{\text{KE}}_{\text{rot}} \equiv \text{average rotational kinetic energy} \Longrightarrow \text{the rotation of a molecule about an axis.}$
- c) $\overline{\text{KE}}_{\text{vib}} \equiv \text{average vibrational kinetic energy} \Longrightarrow \text{a continuous change of distance between the atoms that make up the molecule.}$
- d) $\overline{PE}_{mol} \equiv average$ intermolecular potential energy between the atoms that make up the molecules \Longrightarrow the bond energy.
- 2. Heat or thermal energy, Q, is a mechanism by which energy is transferred between a system and its environment because of a temperature difference between them.
 - a) Heat is essentially related to kinetic energy \implies energy due to the motion of particles in matter.

- b) Heat is measured in calories in the cgs unit system.
 - \implies one calorie of heat is required to raise 1 gram of water by a temperature of 1°C.
- c) Heat is measured in **kilocalories** in the SI system.
 - i) One kilocalorie (= 10³ calories) of heat is required to raise 1 kg of water by a temperature of 1°C (or 1 K).
 - ii) A kilocalorie is also called a **Calorie** (capital 'C') \Longrightarrow this is the energy unit that you see on food containers.
- d) Heat is measured in **British Thermal Units** (BTU) in the English system.
 - \implies one BTU of heat is required to raise 1 lb of water by a temperature of 1°F.
- 3. In 1843, Joule showed that heat can be used to drive a machine ⇒ mechanical energy and heat were 2 forms of the same thing.
 - a) Heat is just another form of energy.
 - **b)** 1 cal = 4.187 Joules.
 - c) 4.187 J/cal = 4187 J/kcal is known as the **mechanical** equivalent of heat.

D. Specific Heat and Calorimetry.

- 1. Calorimetry means the measurement of heat exchange.
- 2. Heat capacity is the amount of heat required to change an entire object's temperature by 1°C.

3. The specific heat c of a substance relates the amount of heat Q it takes to change the temperature T of an object of mass m by one unit degree:

$$c \equiv \frac{Q}{m\Delta T} \quad , \tag{XII-7}$$

which is the heat capacity per unit mass, so we can write the heat equation as

$$Q = m c \Delta T \qquad (XII-8)$$

- a) In the cgs system, c is measured in cal/g $^{\circ}$ C.
- b) In the SI system, c is measured in J/kg °C (note that °C is used instead of K here).
- c) Note that <u>water</u> has a specific heat defined as

$$c_{\rm w} = 1.000 \text{ cal/g} \,^{\circ}\text{C} \quad . \tag{XII-9}$$

- d) Table 11.1 in your textbook lists specific heats for various substances.
- 4. Devices used to measure heat energy are called calorimeters.
- **5.** Heat flow from one system to another obeys the conservation of energy:
 - a) Heat gained by one system (positive number) + heat loss by another system (negative number) = 0.
 - **b)** Or rephrasing: Heat loss by one system = Heat gained by another system.

$$Q_{\rm gain} = Q_{\rm loss}$$
 (XII-10)

$$m_1 c_1 (T - T_1) = m_2 c_2 (T_2 - T)$$
 . (XII-11)

i) '1' represents the system gaining heat.

- ii) '2' represents the system losing heat.
- iii) T represents the final equilibrium temperature when everything is mixed.

Example XII—3. Problem 11.9 (Page 389) from the Serway & Faughn textbook: A 5.00-gm lead bullet traveling at 300 m/s is stopped by a large tree. If half the kinetic energy of the bullet is transformed into internal energy and remains with the bullet while the other half is transmitted to the tree, what is the increase in temperature of the bullet?

Solution:

The mechanical energy transformed into internal energy of the bullet is

$$Q = \frac{1}{2} \left(\mathrm{KE_i} \right) = \frac{1}{2} \left(\frac{1}{2} m v_{\rm i}^2 \right) = \frac{1}{4} m v_{\rm i}^2 \ ,$$

where m is the mass of the bullet and $v_i = 300$ m/s is the initial speed of the bullet. From Table 11.1 on page 335 in the textbook, the specific heat of lead is 128 J/kg·°C. Using Eq. (XII-8), the change in temperature of the bullet is

$$\Delta T = \frac{Q}{mc} = \frac{\frac{1}{4}mv_{\rm i}^2}{mc} = \frac{v_{\rm i}^2}{4c} = \frac{(300 \text{ m/s})^2}{4(128 \text{ J/kg} \cdot ^{\circ}\text{C})} = \boxed{176^{\circ}\text{C}.}$$

Example XII–4. A 0.40-kg iron horseshoe that is initially at 500°C is dropped into a bucket 20 kg of water at 22°C. What is the final equilibrium temperature? Neglect any energy transfer to or from the surroundings.

Solution:

From Table 11.1 in the textbook, the specific heat of iron (Fe) is 448 J/kg·°C and water is 4186 J/kg·°C. Using Eq. (XII-11) we get an equilibrium temperature T of

$$m_{\mathrm{w}}c_{\mathrm{w}}\left(T-T_{\mathrm{w}}\right) = m_{\mathrm{Fe}}c_{\mathrm{Fe}}\left(T_{\mathrm{Fe}}-T\right)$$

$$\begin{array}{rcl} m_{\rm w} c_{\rm w} T - m_{\rm w} c_{\rm w} T_{\rm w} &=& m_{\rm Fe} c_{\rm Fe} T_{\rm Fe} - m_{\rm Fe} c_{\rm Fe} T \\ m_{\rm w} c_{\rm w} T + m_{\rm Fe} c_{\rm Fe} T &=& m_{\rm w} c_{\rm w} T_{\rm w} + m_{\rm Fe} c_{\rm Fe} T_{\rm Fe} \\ (m_{\rm w} c_{\rm w} + m_{\rm Fe} c_{\rm Fe}) \ T &=& m_{\rm w} c_{\rm w} T_{\rm w} + m_{\rm Fe} c_{\rm Fe} T_{\rm Fe} \\ T &=& \frac{m_{\rm w} c_{\rm w} T_{\rm w} + m_{\rm Fe} c_{\rm Fe} T_{\rm Fe}}{m_{\rm w} c_{\rm w} + m_{\rm Fe} c_{\rm Fe}} \end{array}$$

with

$$m_{\rm w}c_{\rm w}T_{\rm w} = (20 \text{ kg})(4186 \text{ J/kg} \cdot ^{\circ}\text{C})(22^{\circ}\text{C}) = 1.84 \times 10^{6} \text{ J},$$

 $m_{\rm Fe}c_{\rm Fe}T_{\rm Fe} = (0.40 \text{ kg})(448 \text{ J/kg} \cdot ^{\circ}\text{C})(500^{\circ}\text{C}) = 8.96 \times 10^{4} \text{ J},$

and

$$m_{\rm w}c_{\rm w} + m_{\rm Fe}c_{\rm Fe} = (20 \text{ kg})(4186 \text{ J/kg} \cdot ^{\circ}\text{C}) + (0.40 \text{ kg})(448 \text{ J/kg} \cdot ^{\circ}\text{C})$$

= $8.39 \times 10^4 \text{ J/°C}$.

Plugging this into out equilibrium temperature equation above, we get

$$T = \frac{1.84 \times 10^6 \text{ J} + 8.96 \times 10^4 \text{ J}}{8.39 \times 10^4 \text{ J/°C}}$$

= 23°C.

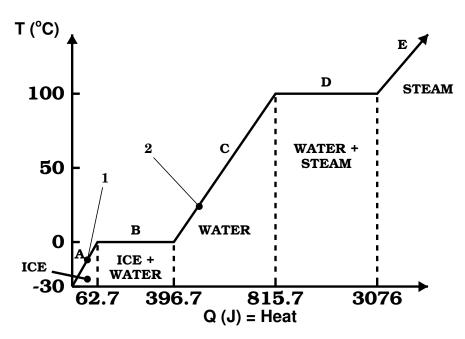
E. Latent Heat & Phase Changes.

- 1. When a substance undergoes a physical alteration from one form to another (*i.e.*, liquid to gas), it is referred to as a **phase** change.
 - a) Under such circumstances, T does <u>not</u> change when heat is added or taken away from the system \Longrightarrow the energy goes into changing the phase.
 - **b)** Mathematically:

$$Q = \pm m L \quad . \tag{XII-12}$$

- i) $Q \equiv \text{heat gained or lost to the system.}$
- ii) $m \equiv \text{mass of the system.}$
- iii) $L \equiv \text{latent heat.}$
- iv) You use the '+' sign when energy is being added during a phase change (i.e., melting or boiling).
- v) You use the '-' sign when energy is being taken away during a phase change (*i.e.*, freezing or condensation).
- c) When the phase change is from gas to liquid (or viseversa), L is called the **latent heat of vaporization**, L_v .
 - i) Changing from gas to liquid is called **condensa-**tion.
 - ii) Changing from liquid to gas is called **boiling**.
- d) When the phase change is from liquid to solid (or viseversa), L is called the **latent heat of fusion**, $L_{\rm f}$.
 - i) Changing from liquid to solid is called freezing.
 - ii) Changing from solid to liquid is called melting.
- e) Table 11.2 of the textbook shows latent heats of fusion and vaporization for some common substances.
- 2. We can deduce temperature changes in systems that undergo phase changes by using the conservation of energy for each *state* of the system.

a) Water has the following characteristics:



- **b)** In states 'A,' 'C,' and 'E,' the heat equation follows Eq. (XII-8).
- c) In states 'B' and 'D,' the phase change states, the heat equation follows Eq. (XII-12).
- d) Using the conservation of energy equation (i.e., Eq. XII-10), going from point '2' to point '1' on the above graph, we can set up the following equation

$$Q_{\text{tot}} = Q_{\text{C}}(\text{of water at '2'}) + Q_{\text{B}} + Q_{\text{A}}(\text{of ice at '1'})$$
,

$$Q_{\rm C} = m_{\rm w} c_{\rm w} (T_2 - T_{\rm B})$$

 $Q_{\rm B} = (m'_{\rm i} + m'_{\rm w}) L_{\rm f}$ (positive since energy is being added)

$$Q_{\rm A} = m_{\rm i} c_{\rm i} (T_{\rm B} - T_1) .$$

Here, $c_{\rm i}$ and $c_{\rm w}$ are the specific heats of ice and water, $L_{\rm f}$ is the latent heat of fusion of water, $m_{\rm w}$ is the initial mass of the water, $m_{\rm i}=m_{\rm w}$ is the final mass of the ice, $m_{\rm i}'+m_{\rm w}'=m_{\rm w}$ is the mass of the water freezing into ice,

 $T_2 > 0$ (the initial temperature of the water), $T_B = 0$ (the temperature of fusion in Celsius), and $T_1 < 0$ (the final temperature of the ice), so

$$Q_{\text{tot}} = m_{\text{w}} c_{\text{w}} (T_2 - T_{\text{B}}) + (m'_{\text{i}} + m'_{\text{w}}) L_{\text{f}} + m_{\text{i}} c_{\text{i}} (T_{\text{B}} - T_1)$$

= $m_{\text{w}} c_{\text{w}} T_2 + m_{\text{w}} L_{\text{f}} - m_{\text{w}} c_{\text{i}} T_1$.

Note that since $T_1 < 0$, $-m_{\rm w}c_{\rm i}T_1 > 0$ and hence the right-hand side of this equation will always be positive.

- e) For those problems where a final equilibrium temperature is being sought, One chooses a Q as either a 'gain' or a 'loss' based upon whether the fixed temperature of the Q equation is less than or greater than the equilibrium temperature:
 - i) If $T > T_{eq}$, then $Q = Q_{loss} < 0$.
 - ii) If $T < T_{eq}$, then $Q = Q_{gain} > 0$.
 - iii) As a result, note that we also could have written the conservation of heat energy (i.e., Eq. XII-10) as

$$Q_{\text{cold}} = -Q_{\text{hot}}$$
, (XII-13)

as given by the textbook.

Example XII–5. A 3.00-gm lead bullet at 30.0°C is fired at a speed of 240 m/s into a large block of ice at 0°C, in which it embeds itself. What quantity of ice melts?

Solution:

Let's assume that the large block of ice does not completely melt as a result of the bullet embedding itself, then the kinetic energy of the bullet will go towards heating the block of ice. Also, the temperature of the bullet will heat the ice block. Let m_{melt} be the amount of mass in the ice block that melts, $m_{\text{b}} = 3.00 \text{ gm} = 3.00 \times 10^{-3} \text{ kg}$ be the mass of the bullet, $T_{\text{b}} = 30.0^{\circ}\text{C}$ be the initial temperature of the bullet, $v_{\text{b}} = 240 \text{ m/s}$ be the initial speed of the bullet, and $T = 0^{\circ}\text{C}$ be the final temperature of the bullet (since it will cool to the same temperature of the block of ice). Using the tables from the textbook, the latent heat of fusion of water is $L_{\text{f}} = 3.33 \times 10^{5} \text{ J/kg}$ and the specific heat for lead is $c_{\text{b}} = 128 \text{ J/kg} \cdot ^{\circ}\text{C}$. Using the conservation of energy (and noting that $\text{J/kg} = \text{m}^2/\text{s}^2$), we get

$$\begin{split} Q_{\rm gain} &= Q_{\rm loss} \\ Q_{\rm melt} &= {\rm KE_b} + Q_{\rm b\text{-}loss} \\ m_{\rm melt} L_{\rm f} &= \frac{1}{2} m_{\rm b} v_{\rm b}^2 + m_{\rm b} c_{\rm b} \left(T_{\rm b} - T \right) \\ m_{\rm melt} &= m_{\rm b} \left[\frac{v_{\rm b}^2 / 2 + c_{\rm b} \left(T_{\rm b} - T \right)}{L_{\rm f}} \right] \\ &= \left(3.00 \ {\rm gm} \right) \left[\frac{(240 \ {\rm m/s})^2 / 2 + (128 \ {\rm J/kg \cdot ^{\circ}C})(30^{\circ}{\rm C} - 0^{\circ}{\rm C})}{3.33 \times 10^5 \ {\rm J/kg}} \right] \\ &= \boxed{0.294 \ {\rm gm} \ .} \end{split}$$

F. Transport of Heat Energy.

1. Thermal energy (i.e., heat) can only flow by one of three different mechanisms: conduction, convection, and radiation transport.

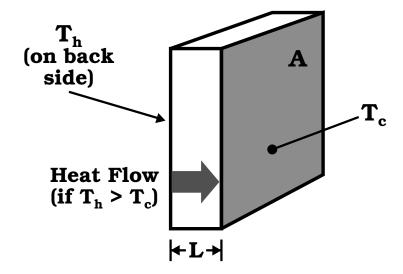
2. Heat Transfer by Conduction.

- i) Heat causes the molecules and atoms to move faster in an object.
- ii) The hotter molecules (those moving faster) collide with cooler molecules (those moving slower), which in turn, speeds up the cooler molecules making them warm.
- iii) This continues on down the line until the object reaches equilibrium.
- b) The amount of heat transferred ΔQ from one location to another over a time interval Δt is

$$\Delta Q = \mathcal{P} \, \Delta t \quad . \tag{XII-14}$$

- i) $\mathcal{P} \equiv \text{heat transfer rate.}$
- ii) \mathcal{P} is measured in watts when Q is measured in Joules and Δt in seconds.
- iii) As such, \mathcal{P} is the same thing as power since they are both measured in the same units.
- c) Heat will only flow if a temperature difference exists between 2 points in an object.
 - i) For a slab of material of thickness L and surface area A, the heat transfer rate for conduction is

$$\mathcal{P}_{\text{cond}} = \frac{\Delta Q}{\Delta t} = KA \left(\frac{T_{\text{h}} - T_{\text{c}}}{L} \right). \tag{XII-15}$$



- ii) $T_{\rm h}$ is the temperature of the hotter side and $T_{\rm c}$ is the temperature of the cooler side.
- iii) $K \equiv$ thermal conductivity of the material (see Table 11.3 in textbook) $\Longrightarrow [K] = J/(s \, m \, ^{\circ}C) = W/m/^{\circ}C$.
- iv) The larger K is, the better the material is in conducting heat.
- \mathbf{v}) The smaller K is, the better the material is as a thermal insulator.
- vi) The effectiveness of thermal insulation is rated by another quantity \Longrightarrow thermal resistance, R, where

$$R \equiv \frac{L}{K} , \qquad (XII-16)$$

where in the U.S., $[R] = \text{ft}^2 \,\text{h}\,^{\circ}\text{F/BTU}$ ('h' = hour), and elsewhere, $[R] = \text{m}^2\,^{\circ}\text{C/W}$ (see Table 11.4 in the textbook).

Example XII–6. Problem 11.41 (Page 392) from the Serway & Vuille textbook: A steam pipe is covered with 1.50-cm thick insulating material of thermal conductivity 0.200 cal/cm·°C·s. How much energy is lost every second when the steam is at 200°C and the surrounding air is at 20.0°C? The pipe has a circumference of 800 cm and a length of 50.0 m. Neglect losses through the end of the pipe.

Solution:

Listing our given parameters, we first need to convert some of these input parameters to SI units:

$$K = 0.200 \frac{\text{cal}}{\text{cm} \cdot ^{\circ}\text{C} \cdot \text{s}} \left(\frac{10^{2} \text{ cm}}{1 \text{ m}} \right) \left(\frac{4.186 \text{ J}}{1 \text{ cal}} \right) = 83.7 \frac{\text{J}}{\text{m} \cdot \text{s} \cdot ^{\circ}\text{C}}.$$

 $T_{\rm h}=200^{\circ}{\rm C}, T_{\rm c}=20^{\circ}{\rm C}, C=800~{\rm cm}=8.00~{\rm m}$ is the circumference of the pipe, $\ell=50.0~{\rm m}$ is the length of the pipe, and the thickness of the insulation is $L=1.50~{\rm cm}=0.0150~{\rm m}$. Since the problem gives us the circumference of the pipe, the total surface area of the pipe is this circumference times the length of the pipe:

$$A = C\ell = (8.00 \text{ m})(50.0 \text{ m}) = 400 \text{ m}^2$$
.

Using Eq. (XII-15) then gives the heat-loss rate via conduction as

$$\mathcal{P}_{\text{cond}} = KA \left(\frac{T_{\text{h}} - T_{\text{c}}}{L} \right)$$

$$= \left(83.7 \frac{\text{J}}{\text{m} \cdot \text{s} \cdot ^{\circ} \text{C}} \right) (400 \text{ m}^{2}) \left(\frac{200^{\circ} \text{C} - 20^{\circ} \text{C}}{0.0150 \text{ m}} \right)$$

$$= 4.02 \times 10^{8} \text{ J/s} = \boxed{402 \text{ MW}}.$$

3. Heat Transfer by Convection.

- a) When an ensemble of hot particles move in <u>bulk</u> to cooler regions of a gas or liquid, the heat is said to flow via **convection**.
- **b)** Boiling water and cumulus clouds are 2 examples of convection.
- c) Convection is complicated requiring graduate-level physics and math to describe it.
 - i) Convection will occur if an instability occurs in the gas or liquid.
 - ii) The mixing-length theory is often used to describe convection: The heat of a gas/liquid bubble will give up its heat energy to the cooler surroundings after the bubble has traveled one mixing length.

4. Heat Transfer by Radiation.

- a) Of all the heat energy transport mechanisms, only radiation does not require a medium \Longrightarrow it can travel through a vacuum.
- b) The rate at which an object emits radiant energy is given by the **Stefan-Boltzmann Law**:

$$\mathcal{P}_{em} = \sigma A e T^4 . \tag{XII-17}$$

- i) $\mathcal{P}_{em} \equiv \text{power radiated (emitted) [watts]}$.
- ii) $\sigma \equiv \text{Stefan-Boltzmann's constant} = 5.6696 \times 10^{-8}$ W/m²/K⁴.

- iii) $A \equiv \text{surface area of the object } [\text{m}^2].$
- iv) $e \equiv \text{emissivity [unitless]}$ (e = 1 for a perfect absorber or emitter).
- v) $T \equiv \text{temperature [K]}.$
- c) A body also can <u>absorb</u> radiation. If a body absorbs a power of radiation \mathcal{P}_{abs} , it will change its temperature to T_{\circ} .
 - i) The net power radiated by the system is then

$$\mathcal{P}_{\text{rad}} = \mathcal{P}_{\text{net}} = \mathcal{P}_{\text{em}} - \mathcal{P}_{\text{abs}}$$
 (XII-18)

or

$$\mathcal{P}_{\text{rad}} = \sigma A e T^4 - \sigma A e T_{\circ}^4 ,$$

$$\mathcal{P}_{\text{rad}} = \sigma A e \left(T^4 - T_{\circ}^4 \right) . \tag{XII-19}$$

- ii) In astronomy, the total power radiated by an object over its entire surface is called the **luminosity**, $L = \mathcal{P}_{rad}$, of the object. Since the amount of energy falling on the surface of the Sun (or any isolated star) from interstellar space is negligible to that of the power radiated, $L = \mathcal{P}_{em}$ for isolated stars.
- iii) If an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate ⇒ its temperature remains constant ⇒ this radiative equilibrium results in the object being in thermal equilibrium:

$$\mathcal{P}_{\rm rad} = 0 \implies T = T_{\circ} ,$$

where T_{\circ} is the temperature of the surroundings.

- d) An ideal absorber is defined as an object that absorbs all of the energy incident upon it.
 - i) In this case, emissivity (e) = 1.
 - ii) Such an object is called a blackbody:

$$\mathcal{P}_{\rm bb} = \sigma A T^4 \ . \tag{XII-20}$$

- iii) Note that a blackbody radiator can be any color (depending on its temperature \Longrightarrow red blackbodies are cooler than blue blackbodies), it does not appear "black" (unless it is very cold).
- iv) The energy flux of such a radiator is

$$F_{\rm bb} = \frac{\mathcal{P}_{\rm bb}}{A} = \sigma T^4 . \tag{XII-21}$$

- v) This radiative flux results from the condition that in order to be in thermal equilibrium, the heat gained by absorbing radiation must be (virtually immediately) radiated away by the object.
- vi) This is not the same thing as reflecting the radiation off of the surface (which does not happen in a blackbody). The incident radiation does get "absorbed" by the atoms of the object and deposited in the thermal "pool." It is just that this radiation immediately gets re-emitted just after absorption.

Example XII-7. The surface temperature of the Sun is about 5800 K. Taking the Sun's radius to be 6.96×10^8 m, calculate the total energy radiated by the Sun each second. (Assume e = 0.965.)

Solution:

We will assume the Sun's shape to be a sphere, where the surface area of a sphere is

$$A = 4\pi R^2 = 4\pi (6.96 \times 10^8 \text{ m})^2 = 6.09 \times 10^{18} \text{ m}^2$$
.

Making use of Eq. (XII-17), the total power (energy per second) that the Sun radiates to space is

$$\mathcal{P}_{em} = \sigma A e T^4 = \left(5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}\right) (6.09 \times 10^{18} \text{ m}^2) (0.965) (5800 \text{ K})^4$$

$$= 3.77 \times 10^{26} \text{ W}.$$

This is the value of the Sun's luminosity:

$$L_{\odot} = \mathcal{P}_{\rm em} = 3.77 \times 10^{26} \ {\rm W} \ .$$