

PHYS-2020: General Physics II
Course Lecture Notes
Section XII

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Abstract

These class notes are designed for use of the instructor and students of the course **PHYS-2020: General Physics II** taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 8th Edition* (2009) textbook by Serway and Vuille.

XII. Mirrors and Lenses

A. Plane Mirrors.

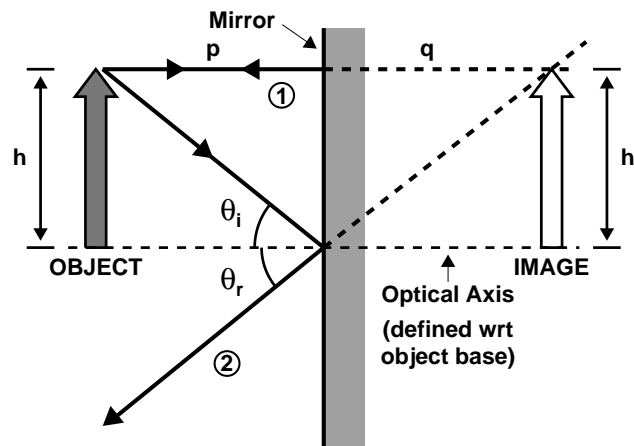
1. Images formed by plane (*i.e.*, flat) mirrors have the following properties:
 - a) The image is as far behind the mirror as the object is in front.
 - b) The image is **unmagnified**, **virtual**, and **erect**.
2. Image orientation:
 - a) **Erect**: Image is oriented the same as the object.
 - b) **Inverted**: Image is flipped 180° with respect to the object.
3. Image classification:
 - a) **Real**: Image is on the same side of mirror as the object \implies light rays actually pass through the image point.
 - b) **Virtual**: Image is on the opposite side of mirror from object \implies light rays appear to diverge from image point.
4. Image size is determined by the magnification of an object which is given by

$$M \equiv \frac{\text{image height}}{\text{object height}} = \frac{h'}{h} . \quad (\text{XII-1})$$

| | | |
|-----------|------------|---|
| $ M > 1$ | \implies | Image is bigger than object (magnified). |
| $ M = 1$ | \implies | Image is unmagnified (like a plane mirror). |
| $ M < 1$ | \implies | Image is smaller than object (demagnified). |
| $M > 0$ | \implies | Image is erect. |
| $M < 0$ | \implies | Image is inverted. |
| $M = 0$ | \implies | No image is formed. |

5. Ray Tracing Rules:

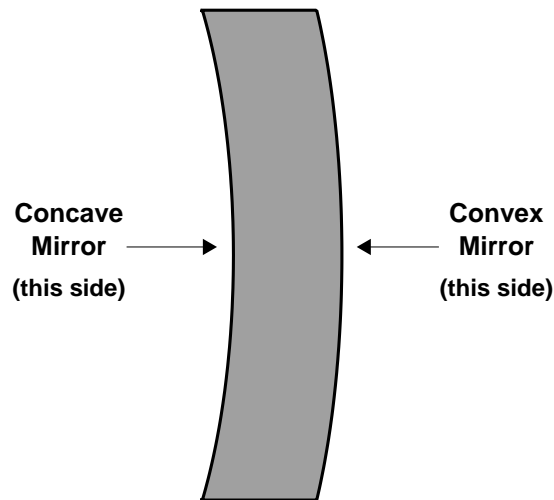
- a) Images form at the point where rays of light actually intersect (for real images) or from which they *appear* to originate (for virtual images).
- b) For plane mirrors, p (the object distance from the mirror) $= q$ (the image distance from the mirror) and $h = h'$.
- c) The following diagram shows how images are constructed for a plane mirror.
 - i) One ray runs parallel to the optical axis (\perp line to the mirror surface at the center of the mirror) from the head of the object (*e.g.*, Ray 1 in the figure).
 - ii) One ray travels from the head through the mirror at the point where the optical axis intersects the mirror (*e.g.*, Ray 2 in the figure).



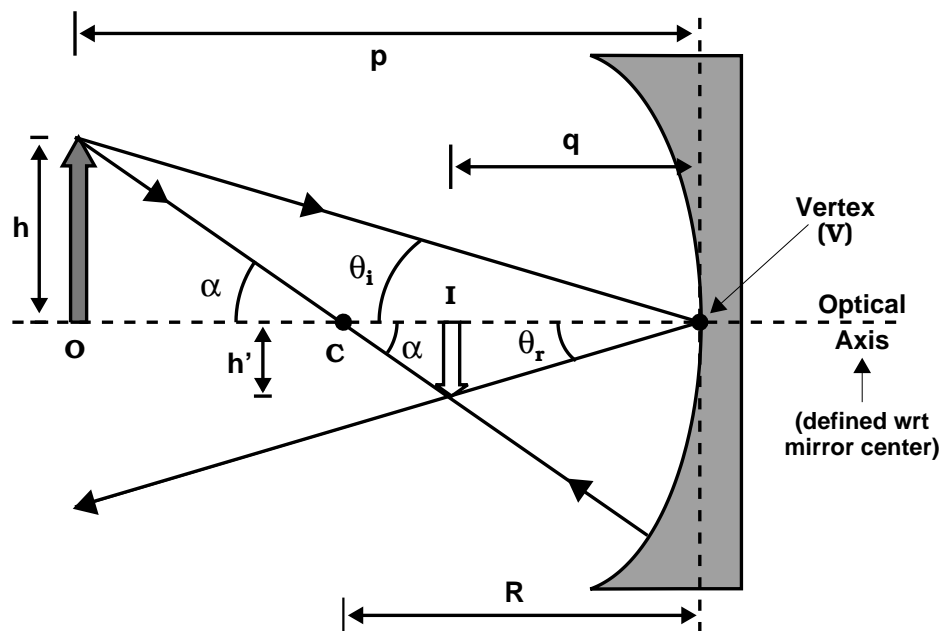
B. Spherical Mirrors

1. **Spherical mirrors** have the shape of a segment of a sphere.

- a) **Concave mirror:** Reflecting surface is on the “inside” of the curved surface.
- b) **Convex mirror:** Reflecting surface is on the “outside” of the curved surface.



2. Constructing the image. Consider the following concave mirror:



- a) The line that is normal to the mirror surface at the exact center is called the **optical axis** of the mirror.
- b) The point where the optical axis intersects the mirror surface is called the **vertex** (labeled ‘V’ in the preceding diagram).
- c) Point ‘C’ indicates the position of the **center of curvature** of the mirror \implies line CV is equal to the **radius of curvature**, R , of the mirror.
- d) Note that the lengths p , q , and R are all measured with respect to the vertex position. Also note that the object position is labeled with ‘O’ and the image position with ‘I’ in the preceding diagram.
- e) Construct the image using the law of reflection:

$$\theta_i = -\theta_r , \quad (\text{XII-2})$$

where θ is measured with respect to the normal of the mirror surface. The ‘negative’ sign is introduced here to note that the reflected angle sweeps away from the optical axis in the opposite ‘sense’ of the incident angle.

- i) All normal lines on spherical concave mirrors go through center of curvature point C ($\theta_i = \theta_r = 0$)!
 - ii) Now reflect a ray off the vertex of the mirror V.
- f) Using trigonometry, we see that

$$\frac{h'}{q} = \tan \theta_r \quad \& \quad \frac{h}{p} = \tan \theta_i .$$

- i) Since $\theta_i = -\theta_r$, we get $\tan \theta_i = -\tan \theta_r$, and hence

$$\frac{h'}{q} = -\tan \theta_i = -\frac{h}{p} \quad \text{or}$$

$$\boxed{M = \frac{h'}{h} = -\frac{q}{p}} . \quad (\text{XII-3})$$

ii) The magnification also can be determined by the ratio of the image to the object distance.

g) Using the “ α ” triangles in the preceeding diagram, we can write

$$\tan \alpha = \frac{h}{p - R} \quad \& \quad \tan \alpha = -\frac{h'}{R - q} ,$$

$$\text{or} \quad \frac{h}{p - R} = -\frac{h'}{R - q} ,$$

$$\text{or} \quad \frac{h'}{h} = -\frac{R - q}{p - R} .$$

$$\text{Finally, using Eq. (XII-3) gives} \quad \frac{q}{p} = \frac{R - q}{p - R} .$$

Solving this above expression gives

$$\begin{aligned} \frac{R - q}{q} &= \frac{p - R}{p} \\ \frac{R}{q} - 1 &= 1 - \frac{R}{p} \\ \frac{R}{q} + \frac{R}{p} &= 1 + 1 = 2 . \end{aligned}$$

Finally,

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{2}{R}} , \quad (\text{XII-4})$$

which is the **mirror equation**.

i) If $p \gg R$, then $1/p \ll 2/R$, so we say that as $p \rightarrow \infty$, $1/p \rightarrow 0$ and

$$\frac{1}{q} = \frac{2}{R} \quad \text{or} \quad q = \frac{R}{2}$$

\Rightarrow the image is formed (*i.e.*, comes to a focus) halfway out to the center of curvature.

ii) So when $p \gg R$, the focal length of the mirror is

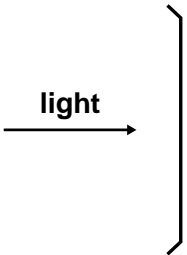
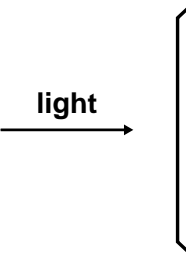
$$f = \frac{R}{2} . \quad (\text{XII-5})$$

iii) When $p \gg R$, the mirror appears “thin” to the distant object, therefore Eq. (XII-5) is called the **thin mirror approximation** and we rewrite Eq. (XII-4) as

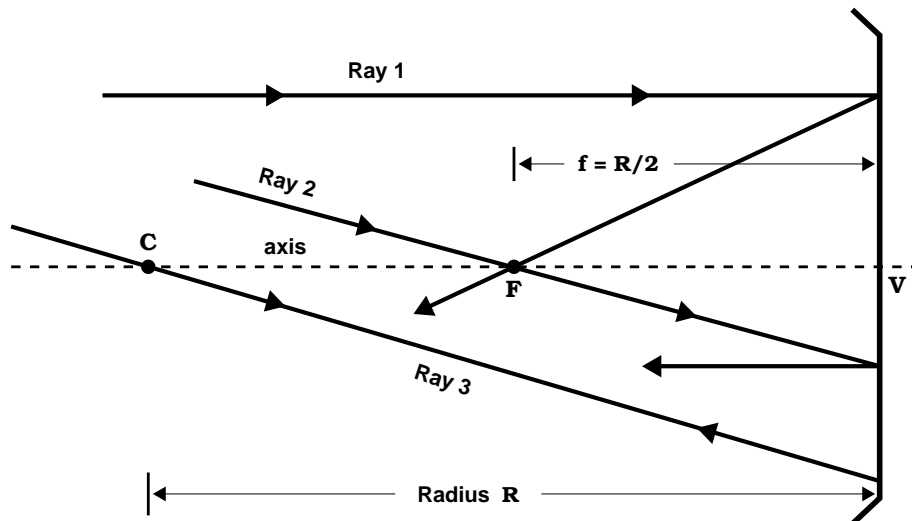
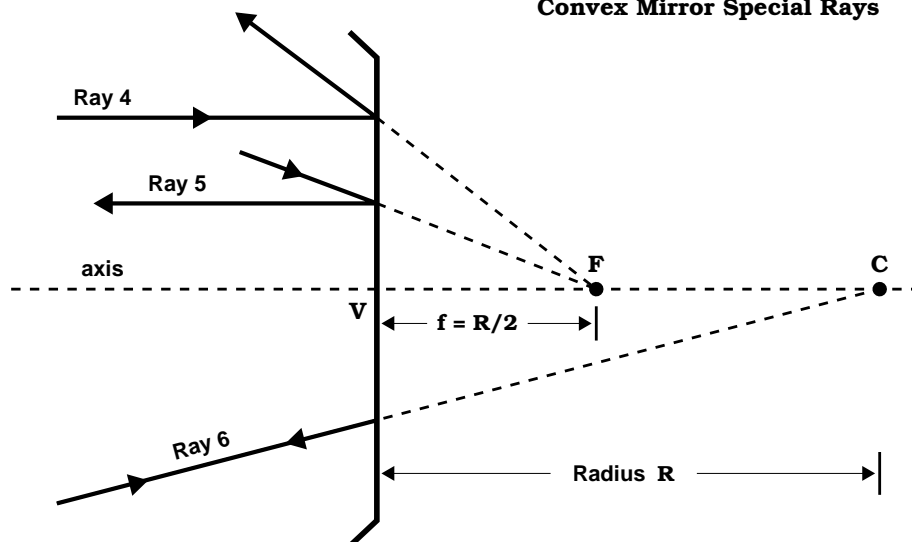
$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f}} . \quad (\text{XII-6})$$

3. Both convex and concave mirrors use Eq. (XII-6), except there is a “change” in sign for the radius and focal length of the mirror. Table XII-1 shows the sign conventions used for the geometric optics parameters for curved mirrors.
4. Image location can either be determined algebraically from Eqs. (XII-3) & (XII-6) or by drawing ray diagrams. There are three principle rays that define the image location (see figures on page XII-8):
 - a) **Concave mirror:** Ray 1 is drawn parallel to the optical axis and is reflected back through the focal point, F.
 - b) **Concave mirror:** Ray 2 is drawn through the focal point, F, and reflected parallel to the optical axis.
 - c) **Concave mirror:** Ray 3 is drawn through the center of curvature, C, and reflected back on itself.

Table XII-1: **Sign Conventions for Curved Mirrors**

| | + | SIGNS | - |
|--------|---|-------|--|
| p | object <u>left</u> of mirror (real object) | | object <u>right</u> of mirror (virtual object) |
| q | image <u>same side</u> of mirror as object (real image) | | image <u>opposite side</u> of mirror as object (virtual image) |
| h | object is <u>erect</u> | | object is <u>inverted</u> |
| h' | image is <u>erect</u> | | image is <u>inverted</u> |
| M | image is in same orientation as object | | image is inverted with respect to object |
| R | concave mirror | | convex mirror |
| f | concave mirror | | convex mirror |
| symbol |  | |  |

- d) Convex mirror: Ray 4 is drawn parallel to the optical axis and is reflected back away from the focal point, F , on the back side of the mirror.
- e) Convex mirror: Ray 5 is drawn toward the focal point, F , on the back side of the mirror and reflected back, parallel to the optical axis.
- f) Convex mirror: Ray 6 is drawn toward the center of curvature on the back side of the mirror, C , and reflected back on itself.

Concave Mirror Special Rays**Convex Mirror Special Rays**

Example XII-1. A convex spherical mirror with radius of curvature of 10.0 cm produces a virtual image one-third the size of the real object. Where is the object?

Solution:

A convex mirror ($R < 0$) will only produce an erect, virtual images of real objects, thus $M > 0$. Then Eq. (XII-3) gives

$$M = -\frac{q}{p} = +\frac{1}{3}$$

or $q = -p/3$. Using this in the mirror equation (Eq. XII-4), we only need to solve for p , the location of the object:

$$\begin{aligned}\frac{1}{p} + \frac{1}{q} &= \frac{2}{R} \\ \frac{1}{p} - \frac{3}{p} &= \frac{2}{R} \\ -\frac{2}{p} &= \frac{2}{-10.0 \text{ cm}} \\ p &= +10.0 \text{ cm}\end{aligned}$$

or the object is 10.0 cm in front of the mirror.

C. Images Formed by Refraction.

1. Using Snell's law and a little trigonometry, it can be shown that

$$\boxed{\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}} \quad (\text{XII-7})$$

for a spherical surface that transmits light.

- a) n_1 = index of refraction of medium containing the object.
- b) n_2 = index of refraction of medium containing the image.
- c) The other variables have the same meaning as they had for mirrors.

2. Furthermore, it can be shown that the magnification going through such a “transmitting” medium (*i.e.*, a lens) is

$$M = \frac{h'}{h} = -\frac{n_1 q}{n_2 p} . \quad (\text{XII-8})$$

The sign conventions are the same as mirrors except for q , where $q > 0$ when the image is on the opposite side of the lens and $q < 0$ when the image is on the same side of the lens as the object (see Table XII-2).

3. Plane refracting surfaces do not magnify images:

a) $R \rightarrow \infty$, so $(n_2 - n_1)/R \rightarrow 0 \implies n_1/p + n_2/q = 0$ or

$$q = -\frac{n_2}{n_1} p . \quad (\text{XII-9})$$

- b) As can be seen from this equation, since q is negative (assuming p is positive), the image formed by a plane refracting surface is on the same side of the surface as the object.

D. Thin Lenses

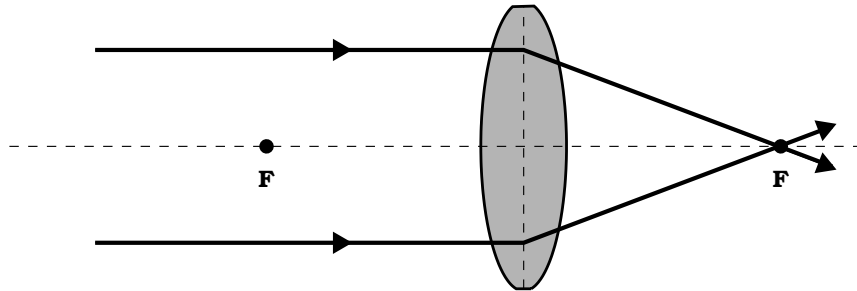
1. If an object is placed a *distance*, p , that is much farther than the focal length of a lens, f (*i.e.*, $p \gg f$), then the thickness of the lens can be considered negligible with respect to q (the image distance), p , and $f \implies$ **thin lens approximation**.
2. The image forms at the focal length when the object is “infinitely” far away:

$$p \rightarrow \infty, \quad \frac{1}{p} \rightarrow 0 \quad \text{or} \quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \implies \frac{1}{q} \rightarrow \frac{1}{f} \quad \text{as } p \rightarrow \infty.$$

3. There are 2 basic types of lenses:

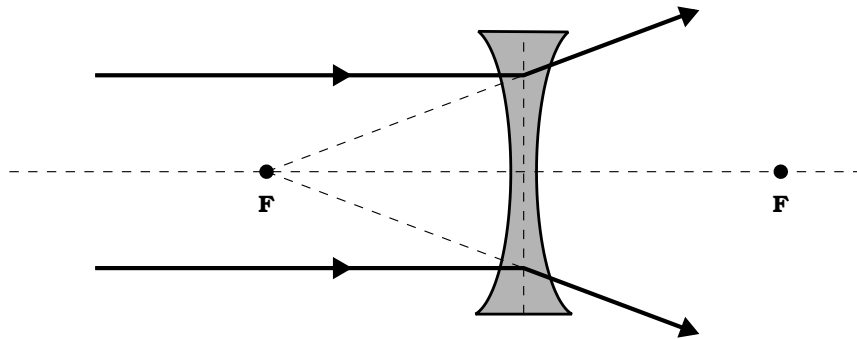
a) **Converging lens:**

- i) Lens thicker at center than edges.
- ii) Light rays are refracted towards the focal point, F , on the other side of the lens.



b) **Diverging lens:**

- i) Lens thinner at center than edges.
- ii) Light rays are refracted in a direction away from the focal point, F , on the inner side of the lens.



4. Just as we had for mirrors, the thin lens equation is

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f}} \quad (\text{XII-10})$$

with magnification being given by

$$\boxed{M \equiv \frac{\text{image height}}{\text{object height}} = \frac{h'}{h} = -\frac{q}{p}} \quad (\text{XII-11})$$



5. The focal length for a lens in air is related to the curvatures of its front and back surfaces via the **lens maker's equation**:

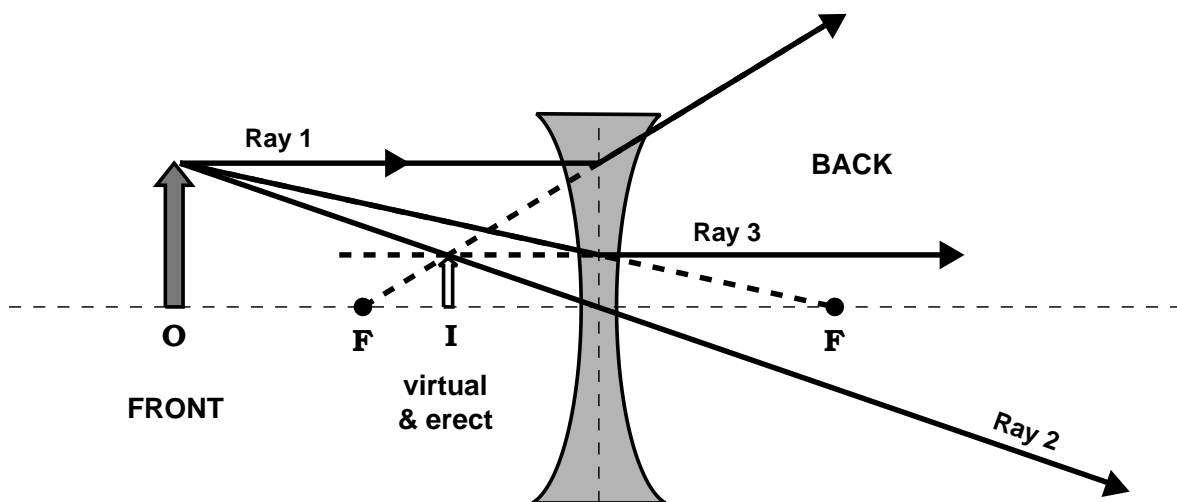
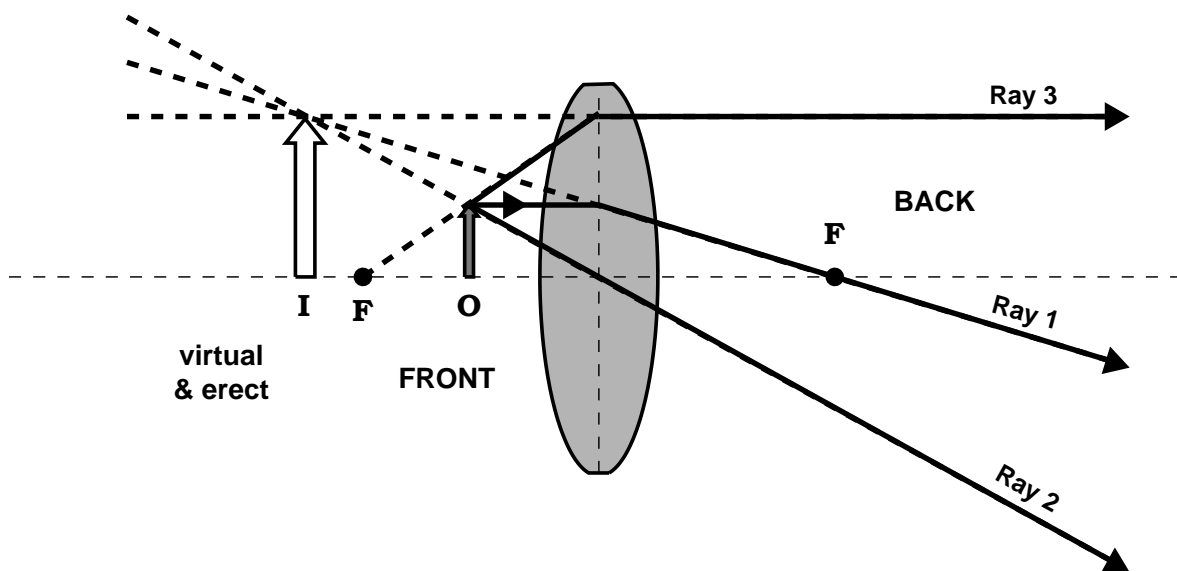
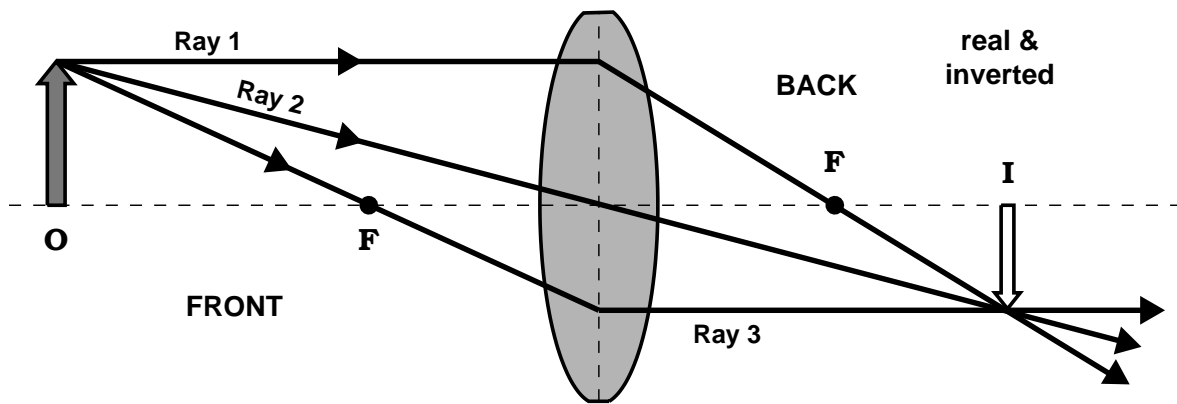
$$\boxed{\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}, \quad (\text{XII-12})$$

where $n \equiv$ index of refraction of the lens,
 $f \equiv$ focal length (*i.e.*, distance from the lens to the focal point F),
 $R_1 \equiv$ radius of curvature of front surface, and
 $R_2 \equiv$ radius of curvature of back surface.

6. The variables in Eqs. (XII-10, 11, 12) take on positive or negative values based upon their relative positions with respect to the given lens. Table XII-2 gives the sign conventions for thin lenses.
7. Ray Diagrams for Thin Lenses (see figures on page XII-14).
- a) The first ray (*i.e.*, Ray 1) is drawn parallel to the optical axis from the top of the object. After being refracted by the lens, this ray either passes through the focal point, F , on the other side of the lens (for a *converging lens*), or appears to come from the near side focal point, F , in front of the lens (for a *diverging lens*).
 - b) The second ray (*i.e.*, Ray 2) is drawn from the top of the image and through the center of the lens. This ray continues on the other side of the lens as a straight line.
 - c) The third ray (*i.e.*, Ray 3) is drawn through the focal point, F , on the near side and emerges from the lens on the opposite side, parallel to the optical axis.

Table XII-2: **Sign Conventions for Thin Lenses**

| | + | SIGNS | – |
|------------|---|-------|---|
| p | object in front of lens (real object) | | object in back of lens (virtual object) |
| q | image in back of lens (real image) | | image in front of lens (virtual image) |
| h | object is <u>erect</u> | | object is <u>inverted</u> |
| h' | image is <u>erect</u> | | image is <u>inverted</u> |
| M | image is in same orientation as object | | image is inverted with respect to object |
| R_1, R_2 | center of curvature in back of lens | | center of curvature in front of lens |
| f | converging lens | | diverging lens |
| symbol |  | |  |



8. Refracting telescopes and microscopes use 2 or more lenses in unison to magnify images.

Example XII-2. Problems 23.37 (Page 786) from the Serway & Vuille textbook: A certain LCD projector contains a single thin lens. An object 24.0 mm high is to be projected so that its image fills a screen 1.80 m high. The object-to-screen distance is 3.00 m. (a) Determine the focal length of the projection lens. (b) How far from the object should the lens of the projector be placed in order to form the image on the screen?

Solution (b):

For this problem, it is easier to do the question in Part (b) first, then use its results in Part (a). The question doesn't tell us what type of lens to use. However, since the image is real (*i.e.*, on the other side of the lens from the object) and magnified, the lens must be a **converging lens**. Also, to produce a real, magnified image, the image will always be inverted (*i.e.*, $h' < 0$) going through a converging lens. Therefore, the magnification equation gives for this set up the following

$$M = \frac{h'}{h} = \frac{-1.80 \text{ m}}{24.0 \times 10^{-3} \text{ m}} = -75.0 = -\frac{q}{p}, \text{ or } q = 75.0 p .$$

Also, we know that

$$\begin{aligned} p + q &= 3.00 \text{ m} \\ p + 75.0 p &= 3.00 \text{ m} \\ 76.0 p &= 3.00 \text{ m} \\ p &= 3.95 \times 10^{-2} \text{ m} = \boxed{39.5 \text{ mm} .} \end{aligned}$$

Solution (a):

Now we can use the solution from Part (b) to determine the answer for Part (a). The thin lens equation (Eq. XII-10) then gives

$$\begin{aligned}\frac{1}{p} + \frac{1}{q} &= \frac{1}{f} \\ \frac{1}{p} + \frac{1}{75.0 p} &= \frac{1}{f} \\ \frac{75.0}{75.0 p} + \frac{1}{75.0 p} &= \frac{1}{f} \\ \frac{76.0}{75.0 p} &= \frac{1}{f} \\ f &= \frac{75.0 p}{76.0} = \frac{75.0}{76.0} (39.5 \text{ mm}) \\ &= \boxed{39.0 \text{ mm .}}\end{aligned}$$
