Appendices

I. Appendix A, Numbers, Inequalities, and Absolute Values, p. A2

A. Example 1, p. A4

1. $1+x < 7x+5 \Rightarrow -6x < 4 \,,\ \dfrac{-6x}{-6} > \dfrac{4}{-6} \,,\ \boxed{x > -\dfrac{2}{3}}$ <u>OR</u>

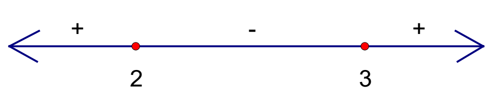
2. $1+x < 7x+5 \Rightarrow -4 < 6x \,,\ \dfrac{-4}{6} < \dfrac{6x}{6} \,,\ -\dfrac{2}{3} < x \,,\ \boxed{x > -\dfrac{2}{3}}$

B. Example 2, p. A5

1. $4 \le 3x-2 < 13 \Rightarrow 6 \le 3x < 15 \,,\ \boxed{2 \le x < 5}$ <u>OR</u> $\boxed{[2\,,\,5)}$: half-closed interval

C. Example 3, p. A5

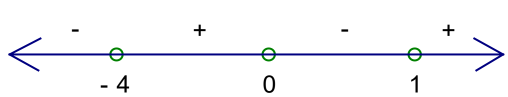
1. $x^2-5x+6 \le 0$; check boundary values: $\Rightarrow (x-2)(x-3) = 0 \Rightarrow \underline{x=2\,,\ 3}$

2. 

$\boxed{2 \le x \le 3}$ <u>OR</u> $\boxed{[2\,,\,3]}$: closed interval

D. Example 4, p. A6

1. $x^3+3x^2 > 4x$; check boundary values: $\Rightarrow x\left(x^2+3x-4\right)=0 \,,\ x(x+4)(x-1)=0 \,,\ \Rightarrow \underline{x=0\,,\ -4\,,\ 1}$

2. 

$\boxed{-4 < x < 0 \ \text{ or } \ x > 1}$ <u>OR</u> $(-4\,,\,0) \cup (1\,,\,\infty)$

3. Each of these two is an open interval.

E. Extra example #1

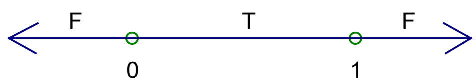
1. $2x+1 < 4x-3 \leq x+7 \Rightarrow 4 < 2x$ and $3x \leq 10 \Rightarrow x > 2$ and $x \leq \frac{10}{3}$

2. $\boxed{2 < x \leq \frac{10}{3}}$ <u>OR</u> $\left(2 \,,\, \frac{10}{3}\right]$: half-open interval

F. Extra example #2

1. $\frac{1+x}{1-x} > 1$; check boundary values: $\Rightarrow 1+x=1-x \,,\; 2x=0 \,,\; \underline{x=0}$

2. Observe: $x \neq 1$



$\boxed{0 < x < 1}$ <u>OR</u> $(0 \,,\, 1)$

G. Extra example #3

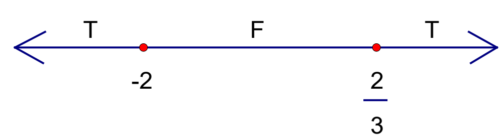
1. $|2x+1| = |5x+3| \Rightarrow 2x+1 = 5x+3$ or $2x+1 = -5x-3$,

2. $3x = -2$ or $7x = -4$, $x = -\frac{2}{3}$ or $x = -\frac{4}{7}$, $\boxed{x = -\frac{2}{3} \,,\, -\frac{4}{7}}$

H. Example 8, p. A8

1. $|3x+2| \geq 4$; check boundary values: $\Rightarrow 3x+2 = 4$ or $3x+2 = -4$,

2. $3x = 2$ or $3x = -6$, $x = \frac{2}{3}$ or $x = -2$, $\underline{x = \frac{2}{3} \,,\, -2}$



3. $\boxed{x \leq -2 \text{ or } x \geq \frac{2}{3}}$ <u>OR</u> $\boxed{\left(-\infty \,,\, -2\right] \cup \left[\frac{2}{3} \,,\, \infty\right)}$

I. Properties of absolute value

1. If $a \geq 0$, $|a| = a$.

2. If $a < 0$, $|a| = -a$.

3. $\sqrt{a^2} = |a|$.

4. <u>Triangle Inequality Theorem</u>: $|a+b| \leq |a| + |b|$.

II. Appendix B, Coordinate Geometry and Lines, p. A10

A. Abscissa: $x-$coordinate, 1st coordinate, Ordinate: $y-$coordinate, 2nd coordinate

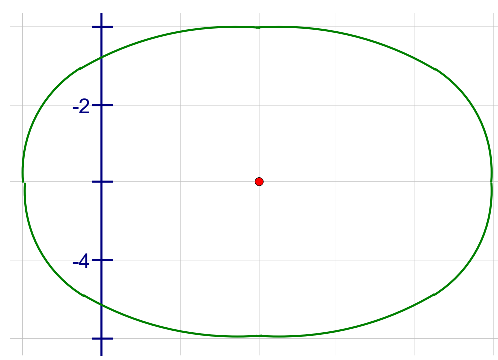
III. Appendix C, Graphs of 2nd Degree Equations, p. A16

A. #32, p. A23

1. Identify and sketch: $4x^2 + 9y^2 - 16x + 54y + 61 = 0 \Rightarrow 4\left(x^2 - 4x + \quad\right) + 9\left(y^2 + 6y + \quad\right) = -61$,

2. $4\left(x^2 - 4x + 4\right) + 9\left(y^2 + 6y + 9\right) = -61 + 16 + 81$, $4(x-2)^2 + 9(y+3)^2 = 36$, $\dfrac{(x-2)^2}{9} + \dfrac{(y+3)^2}{4} = 1$,

3. $(2, -3)$, $a = 3$, $b = 2 \Rightarrow \boxed{\text{ellipse}}$



IV. Appendix D, Trigonometry, p. A24

A. Know trigonometric identities, pp. A28-A29

Optional: $\tan(x+y)$ and $\tan(x-y)$

V. Appendix Extras

A. Binomial Theorem

1. $(a+b)^n =$

$$\binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + \binom{n}{n-3}a^3 b^{n-3} + \binom{n}{n-2}a^2 b^{n-2} + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n =$$

$$a^n b^0 + na^{n-1}b^1 + \frac{n(n-1)}{2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{6}a^{n-3}b^3 + \cdots + \frac{n(n-1)(n-2)}{6}a^3 b^{n-3} + \frac{n(n-1)}{2}a^2 b^{n-2} + na^1 b^{n-1} + a^0 b^n$$

$$= a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{6}a^{n-3}b^3 + \cdots + \frac{n(n-1)(n-2)}{6}a^3 b^{n-3} + \frac{n(n-1)}{2}a^2 b^{n-2} + nab^{n-1} + b^n$$

B. Factoring the difference of Perfect n th Powers

1. $x^n - a^n = (x-a)\bullet\left(x^{n-1}a^0 + x^{n-2}a^1 + x^{n-3}a^2 + x^{n-4}a^3 + \cdots + x^3 a^{n-4} + x^2 a^{n-3} + x^1 a^{n-2} + x^0 a^{n-1}\right)$

$$= (x-a)\bullet\left(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + x^{n-4}a^3 + \cdots + x^3 a^{n-4} + x^2 a^{n-3} + xa^{n-2} + a^{n-1}\right)$$

Chapter 1: Functions and Models

I. 1.1, Four Ways to Represent a Function, p. 11

A. Functional representation

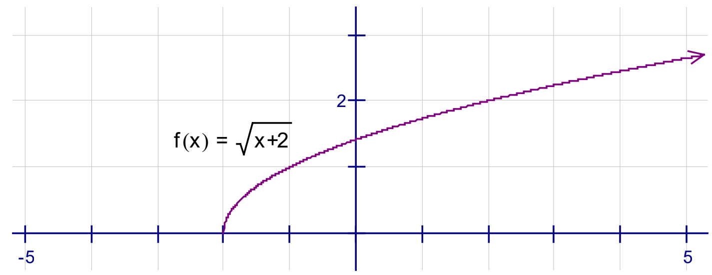
1. Equation
2. Table
3. Graph
4. Words

B. Topics are shown geometrically: graphically or visually, numerically, algebraically, and verbally: descriptive

C. Example 6, p. 17

1. a. Find the domain and sketch $f(x)=\sqrt{x+2}$;

Domain: $x+2 \ge 0$, $\boxed{x \ge -2}$ OR $[-2, \infty)$

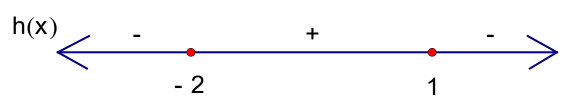


b. Find the domain of $g(x)=\dfrac{1}{x^2-x}$; $\Rightarrow \dfrac{1}{x^2-x} = \dfrac{1}{x(x-1)} \Rightarrow x \neq 0, 1$; Domain: $\{x \mid x \neq 0, x \neq 1\}$

D. Extra example

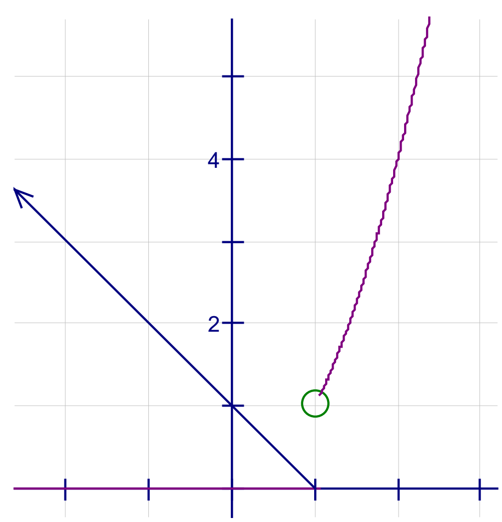
1. Find the domain of $h(x)=\sqrt{2-x-x^2}$; $\Rightarrow 2-x-x^2 \ge 0$; check boundary values: $\Rightarrow 2-x-x^2 = 0$,

$\Rightarrow (2+x)(1-x) = 0$, $\Rightarrow \underline{x = -2, 1}$



$\boxed{-2 \le x \le 1}$ <u>OR</u> $\boxed{[-2, 1]}$

E. Sketch the piecewise function $f(x)=\begin{cases} 1-x, & x \le 1 \\ x^2, & x > 1 \end{cases}$; $\Rightarrow$

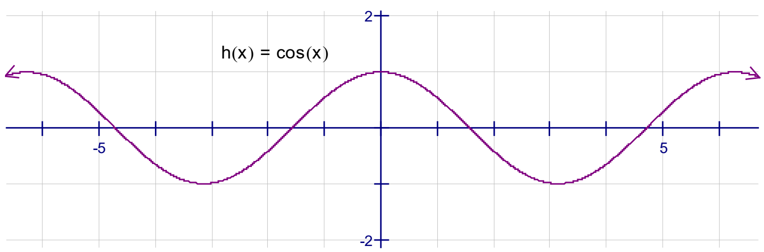


F. Function symmetry

1. Even function: $f(-x) = f(x)$

   a. E.g. $\cos x$, x^2 , $7x^6 + 4x^2 - 3$

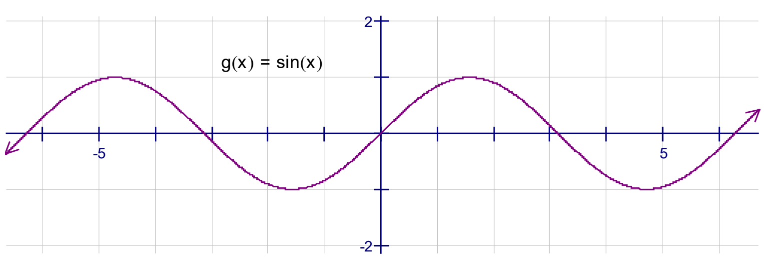
   b. Includes all polynomials with even exponents



2. Odd function: $f(-x) = -f(x)$

   a. E.g. $\sin x$, x^3 , $8x^7 - 5x^3 + 2x$

   b. Includes all polynomials with odd exponents



G. Increasing and decreasing functions

1. A function f is <u>increasing</u> on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I .

2. A function f is <u>decreasing</u> on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I .

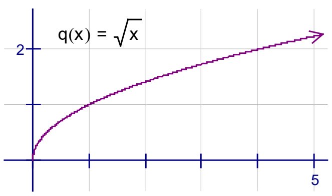
II. 1.3, Transforming Functions: New Functions from Old Functions, p. 38

A. Transformations

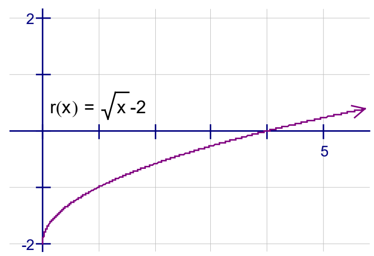
1. Analyze Figure 1, Figure 2, and Figure 3, p. 39

B. Example 1, p. 40

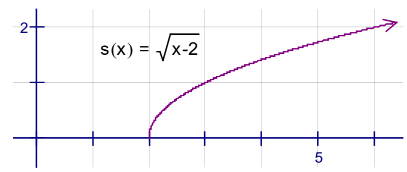
1. Use $y = \sqrt{x}$ to graph the following:



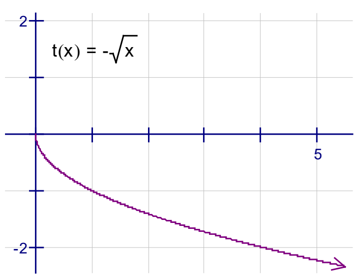
a.



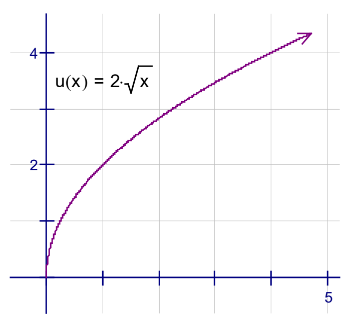
b.



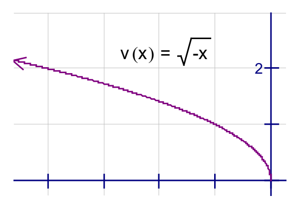
c.



d.

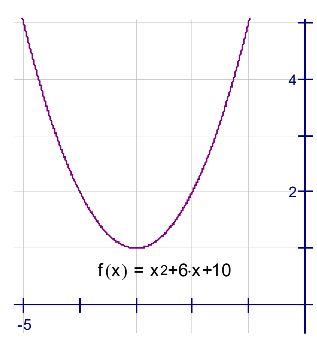


e.



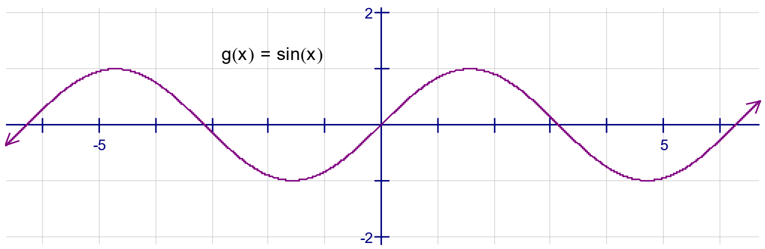
C. Example 2, p. 40

1. Sketch $x^2 + 6x + 10 \Rightarrow x^2 + 6x + 9 + 1 = (x+3)^2 + 1$; vertex: $(\,-3\ ,\ 1)$

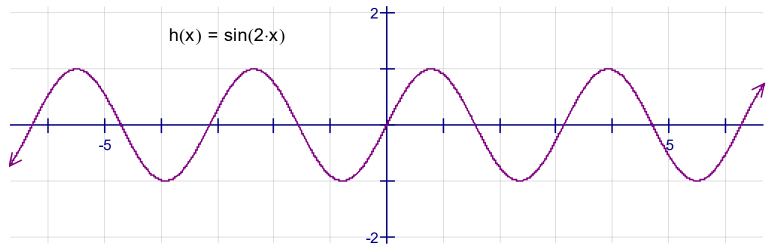


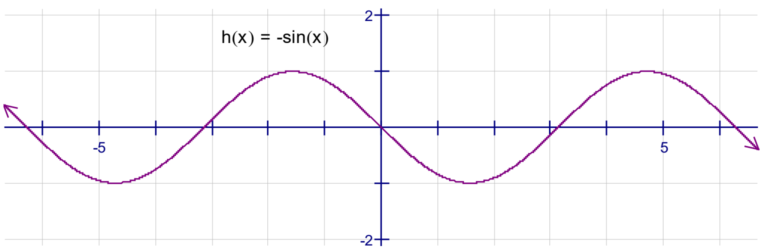
D. Example 3, p. 40

1. Sketch using the sine function

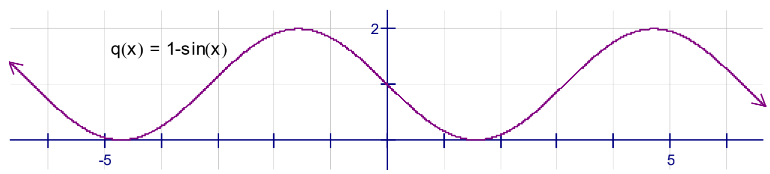


a. $y = \sin 2x$



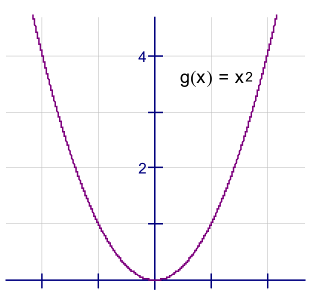


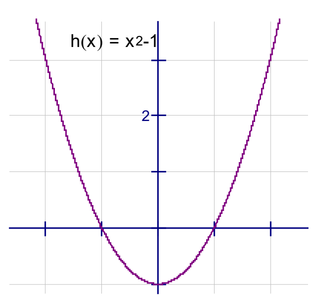
b. $y = 1 - \sin x$

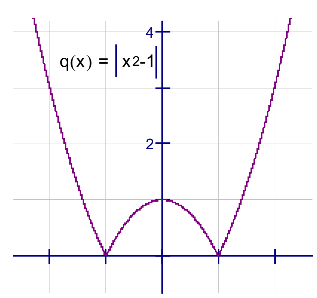


E. Example 5, p. 42

1. Sketch $y = |x^2 - 1|$







F. Composition of functions

1. Example 8, p. 44

2. $f(x)=\sqrt{x}$, $g(x)=\sqrt{2-x}$, find each function and domain

a. $f \circ g$

i. $f(g(x))=\sqrt{\sqrt{2-x}}=\boxed{\sqrt[4]{2-x}} \Rightarrow 2-x \ge 0 , 2 \ge x , \boxed{x \le 2}$

b. $g \circ f$

i. $g(f(x))=\boxed{\sqrt{2-\sqrt{x}}} \Rightarrow 2-\sqrt{x} \ge 0 , 2 \ge \sqrt{x} , \sqrt{x} \le 2 , \underline{x \le 4} \therefore x \le 4 \cap x \ge 0 , \boxed{[0 , 4]}$

c. $f \circ f$

i. $f(f(x))=\sqrt{\sqrt{x}}=\boxed{\sqrt[4]{x}} \Rightarrow \boxed{x \ge 0}$

d. $g \circ g$

i. $g(g(x))=\boxed{\sqrt{2-\sqrt{2-x}}} \Rightarrow 2-\sqrt{2-x} \ge 0 , 2 \ge \sqrt{2-x} , \sqrt{2-x} \le 2 , 2-x \le 4 , -x \le 2 ,$

$x \ge -2 \therefore x \ge -2 \cap x \le 2 , \boxed{[-2 , 2]}$

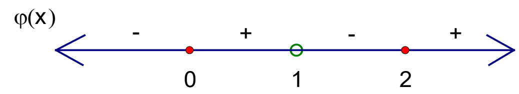
G. Example 9, p. 45

1. Find $f \circ g \circ h$ if $f(x)=\dfrac{x}{x+1}$, $g(x)=x^{10}$, $h(x)=x+3 \Rightarrow$

$$f \circ g \circ h=f(g(h(x)))=f(g(x+3))=f((x+3)^{10})=\frac{(x+3)^{10}}{(x+3)^{10}+1}$$

H. Extra example #1

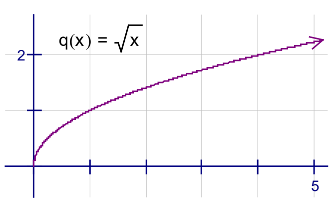
1. $\phi(x)=\sqrt{\dfrac{x^2-2x}{x-1}}$, find domain $\Rightarrow$ $\dfrac{x^2-2x}{x-1} = \dfrac{x(x-2)}{x-1} \Rightarrow$

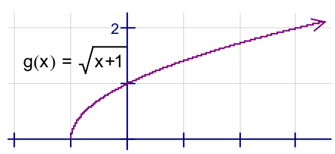


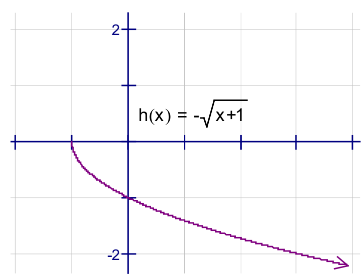
2. Domain: $\boxed{[0\ ,\ 1)\ \cup\ [2\ ,\ \infty)}$

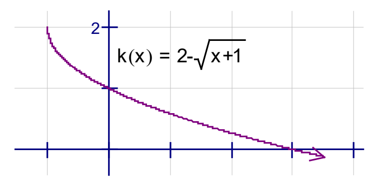
I. Extra example #2

1. Graph $y=2-\sqrt{x+1}\ \Rightarrow$









J. Extra example #3

1. Graph $y=\big|\ |x|-1\ \big|\ \Rightarrow$