PHYS-2020: General Physics II
Course Lecture Notes
Section XIII

Dr. Donald G. Luttermoser

East Tennessee State University

Edition 3.2

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 8th Edition* (2009) textbook by Serway and Vuille.

XIII.   Wave Optics

## A. Wave Interference and Young's Double Slit Experiment.

1. We have already discussed wave interference (see VII.E.2 on Page VII-15 and VIII.E on Page VIII-12). In this section we will expand upon the physics of interference.

2. In order to carry out experiments concerning constructive or destructive interference of waves of two or more sources, the waves from the sources must be **coherent**.

   a) This means that the waves they emit must maintain a constant **phase** with respect to each other.

   b) In addition to this, the waves must have identical wavelengths to be coherent.

   i) Both of these conditions require the sources to emit a single emission-line spectrum.

   ii) A **laser** (**l**ight **a**mplification of **s**timulated **e**mission of **r**adiation) is a good source of coherent radiation.

   c) Most ordinary light sources produce a continuous spectrum, hence produce **incoherent** light. Incoherent light can be converted to coherent light through the use of two "pinholes" or "slits" at a specific separation as described below.

3. In 1801, Thomas Young first demonstrated interference in light waves from two sources produced from a single source of light passing through two pinholes in a screen as shown in Figure 24.1

in your textbook. (From this point forward, we will describe this experiment with slits instead of pinholes.)

a) This experiment showed that the resulting wavefronts from the two slits produced as series of bright and dark parallel bands called **fringes**.

b) In order for the wavefronts (*i.e.*, wavecrests) of the light from the two different slits to arrive on a projection screen at the same time at the same place, the following equation must be true

$$\delta = r_2 - r_1 = d \sin \theta , \qquad \text{(XIII-1)}$$

where r_1 is the distance that the wavefront traveled from slit 1, r_2 the distance traveled from slit 2, δ is the path difference between r_2 and r_1, d the distance between the slits, and θ is the angle shown in the figure below.
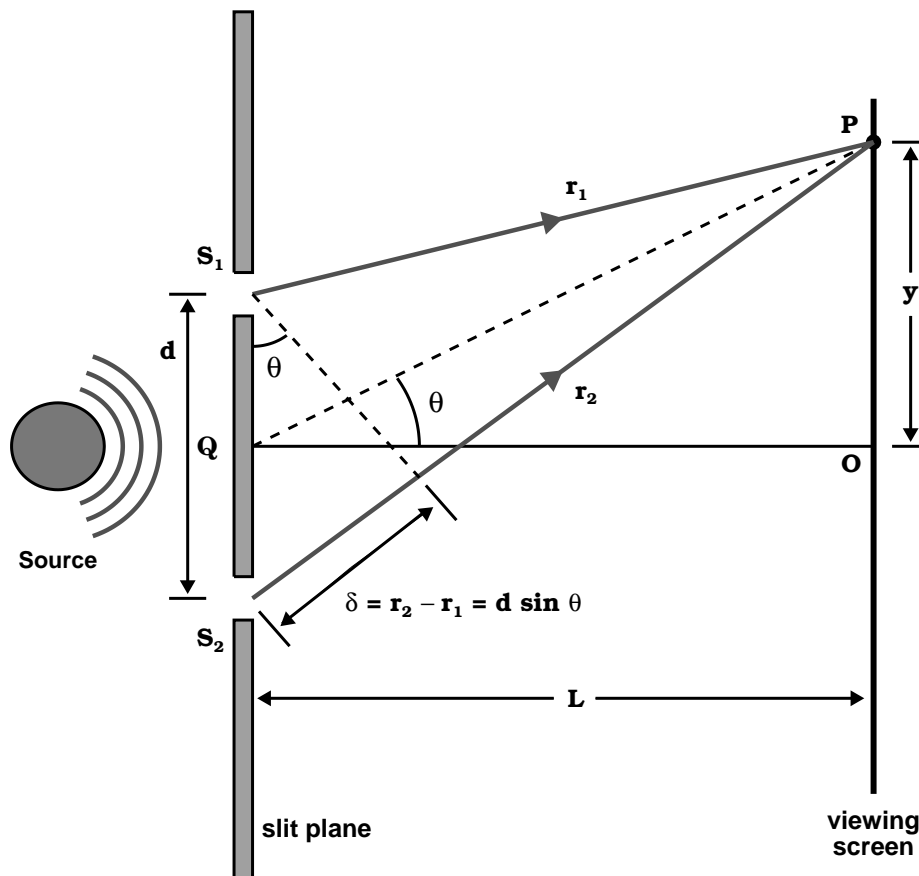
c) The "bright fringes" correspond to regions of **construc-tive interference**. This will occur at a position on the viewing screen when $\delta = \pm m\lambda$:

$$\boxed{\delta = d\,\sin\theta_{\text{bright}} = m\lambda\;, \quad m = 0, \pm 1, \pm 2, \ldots} \quad \text{(XIII-2)}$$

i) The number m is called the **order number**.

ii) The central bright fringe at $\theta_{\text{bright}} = 0$ is called the *zeroth-order maximum.*

iii) The first maximum on either side, where $m = \pm 1$, is called the *first-order maximum,* and so forth.

d) The "dark fringes" correspond to regions of **destructive interference**. This will occur at a position on the viewing screen when the waves are $180°$ out of phase (*i.e.,* δ is an odd multiple of $\lambda/2$):

$$\boxed{\delta = d\,\sin\theta_{\text{dark}} = \left(m + \frac{1}{2}\right)\lambda\;, \quad m = 0, \pm 1, \pm 2, \ldots}$$
$$\text{(XIII-3)}$$

i) Here, the <u>first</u> dark fringe occurs at $m = 0$ giving a path difference of $\lambda/2$.

ii) The second dark fringe occurs at $m = 1$ giving a path difference of $3\lambda/2$, and so on.

e) Let L be the distance between the "slit plane" and the viewing screen and y be the distance that a fringe is away from the zeroth-order maximum on the viewing screen. Then, if $L \gg d$ and $d \gg \lambda$, which is typically the case in these types of experiments, we note from trigonometry that

$$y = L\tan\theta \approx L\sin\theta\;. \qquad \text{(XIII-4)}$$

i) Using this approximation in Eq. (XIII-2), we find that the positions of the *bright fringes* measured from the zeroth-order bright fringe is

$$y_{\text{bright}} = \frac{\lambda L}{d}\, m, \quad m = 0, \pm 1, \pm 2, \ldots \qquad \text{(XIII-5)}$$

ii) Now, using this approximation in Eq. (XIII-3) gives

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right), \quad m = 0, \pm 1, \pm 2, \ldots$$

$$\text{(XIII-6)}$$

for the location of the *dark fringes* as measured from the zeroth-order bright fringe.

4. Young's double-slit experiment provides a method for measuring the wavelength of light. In addition, his experiment gave the wave model of light a great deal of credibility.

Example XIII–1. Problems 24.3 (Page 817) from the Serway & Vuille textbook: A pair of narrow, parallel slits separated by 0.250 mm is illuminated by the green component from a mercury vapor lamp (λ = 546.1 nm). The interference pattern is observed on a screen 1.20 m from the plane of the parallel slits. Calculate the distance (a) from the central maximum to the first bright region on either side of the central maximum and (b) between the first and second dark bands in the interference pattern.

Solution (a):

Here, we have $\lambda = 546.1$ nm $= 546.1 \times 10^{-9}$ m, $L = 1.20$ m, and $d = 0.250$ mm $= 0.250 \times 10^{-3}$ m. The distance between the central maximum and the first-order bright fringe can be determined with the use of Eq. (XIII-5):

$$\Delta y = y_{\text{bright}}\big|_{m=1} - y_{\text{bright}}\big|_{m=0} = \frac{\lambda L}{d} \cdot 1 - 0$$

$$= \frac{\lambda L}{d} = \frac{\left(546.1 \times 10^{-9} \text{ m }\right)(1.20 \text{ m})}{0.250 \times 10^{-3} \text{ m}}$$

$$= 2.62 \times 10^{-3} \text{ m } = \boxed{2.62 \text{ mm }}.$$

Solution (b):

The distance between the first and second dark bands can be determined with Eq. (XIII-6):

$$\begin{aligned}
\Delta y &= y_{\text{dark}}\big|_{m=1} - y_{\text{dark}}\big|_{m=0} \\
&= \left[\frac{\lambda L}{d}\left(1 + \frac{1}{2}\right)\right] - \left[\frac{\lambda L}{d}\left(0 + \frac{1}{2}\right)\right] \\
&= \frac{3\lambda L}{2d} - \frac{\lambda L}{2d} = \frac{\lambda L}{d} \\
&= 2.62 \times 10^{-3} \text{ m } = \boxed{2.62 \text{ mm }}.
\end{aligned}$$

B. Interference in Thin Films.

1.   Consider light reflected off of a surface. Let n_i be the index of refraction of the incident side of the surface and n_o be the index of refraction on the opposite side (*i.e.*, the "other" side). With this set up, we can write the **Rules of Phase Change of Reflected Light:**

   a)   If $n_o > n_i$, any light reflected off of this surface will undergo a 180° phase change upon reflection.

   b)   If $n_o < n_i$, any light reflected off of this surface will experience <u>no</u> phase change upon reflection.

2.   Consider a film of uniform thickness ℓ (note that your textbook uses t for this variable) with an index of refraction n (as shown

in Figure 24.7 in the textbook). We will label the top side of the film surface A and the bottom side surface B. On either side of this film, we will assume that air is the medium and hence set $n_{\text{outside}} = n_{\text{air}} = 1$.

a) An incident beam passes through air and interacts with surface A. Part of the beam gets reflected (which we will call ray '1'), where the reflected ray has an $180°$ phase change since $n_o > n_i$, and part of the ray is refracted into the medium (*i.e.*, film).

b) In the medium, the wavelength of the light changes to

$$\lambda_n = \frac{\lambda}{n} \, , \qquad\qquad \text{(XIII-7)}$$

where λ is the wavelength of the light in a vacuum (and in the air since we have assumed that $n_{\text{air}} = 1$).

c) The refracted beam in the medium then interacts with surface B. Once again, part of the beam gets reflected, but this time the reflected ray has no phase change since $n_o < n_i$, and part of the ray is transmitted (and once again refracted) into the air on the other side of the thin film following a path that is parallel to the original incident beam.

d) This surface B reflection then once again interacts with surface A and gets refracted such that it is following a path parallel to the $180°$ out of phase surface A reflected beam (*i.e.*, ray 1). We'll label this second ray '2'.

3. Since these two rays are completely out of phase with each other, we will get *constructive interference* when

$$2\ell = \left(m + \frac{1}{2}\right) \lambda_n \, , \quad m = 0, 1, 2, \ldots \qquad \text{(XIII-8)}$$

a) This condition takes into account two factors:

    i) The difference in path length for the two rays (*i.e.,* the $m\lambda_n$ term).

    ii) The 180° phase change upon reflection (*i.e.,* the $\lambda_n/2$ term).

b) Using Eq. (XIII-7) in Eq. (XIII-8) gives

$$2n\ell = \left(m + \frac{1}{2}\right)\lambda\,, \quad m = 0, 1, 2, \ldots \qquad \text{(XIII-9)}$$

4. Likewise, if the extra distance 2ℓ that ray 2 takes is a multiple of λ_n, the two waves combine out of phase resulting in *destructive interference*:

$$
\begin{aligned}
2\ell &= m\lambda_n & m &= 0, 1, 2, \ldots \\
&\text{or} & & \\
2n\ell &= m\lambda & m &= 0, 1, 2, \ldots
\end{aligned}
\qquad \text{(XIII-10)}
$$

5. **Eqs. (XIII-8, 9, 10) for constructive and destructive interference are valid when there is only one phase reversal.**

6. **If the film is placed between two different media, one of lower refractive index than the film and one of higher refractive index, Eqs. (XIII-8, 9) and Eqs. (XIII-10) are reversed:**

    a) Equations (XIII-8, 9) are used for destructive interference.

    b) Equations (XIII-10) are used for constructive interference.

Example XIII–2.   Problems 24.20 (Page 818) from the Serway & Vuille textbook: A transparent oil with index of refraction 1.29 spills on the surface of water (index of refraction 1.33), producing a maximum of reflection with normally incident orange light (wavelength 600 nm in air). Assuming the maximum occurs in the first order, determine the thickness of the oil slick.

Solution:

Since $n_{\text{air}} < n_{\text{oil}} < n_{\text{water}}$, light reflected from both top and bottom surfaces of the oil film experiences phase reversal, resulting in zero net phase difference due to reflections. Therefore, the condition for constructive interference (which will give us maximum light) for reflective light will be given by Eq. (XIII-10b):

$$2n_{\text{oil}}\ell = m\lambda \ , \ \text{or} \ \ell = m\left(\frac{\lambda}{2n_{\text{oil}}}\right) \ \text{where} \ m = 0, 1, 2, \ldots$$

Since we are told that the first order will produce the maximum intensity, $m = 1$ and the thickness of the oil slick is

$$\ell = (1)\left(\frac{600 \ \text{nm}}{2(1.29)}\right) = \boxed{233 \ \text{nm} \ .}$$

C. Diffraction.

1. If we allow monochromatic (*i.e.*, one wavelength) light to fall on a single slit whose width 'a' is of the order of magnitude of the wavelength λ of the light, the light will flare out as it passes through the slit as a result of Huygen's Principle of wave optics.

   a) This spreading out of light is from its initial line of travel is called **diffraction**.

   b) When viewed on an image screen, a **diffraction pattern** appears consisting of a broad intense central band

flanked by a series of narrower, less intense, secondary bands (called **secondary maxima**) and a series of dark bands, or **minima**.

c) The physics of diffraction is similar to the physics of interference. Here though, instead of two slits, **each portion of the single slit acts as a source of waves — light from one portion of the slit can interfere with light from another portion of the slit.**

2. Diffraction was discovered by Francesco Maria Grimaldi in the mid-1600s.

3. We can describe the physics of single-slit diffraction analogous to Young's double-slit interference experiment by describing the single slit of width 'a' as two slits each with a width of $a/2$.

a) Waves from one side of the slit will *destructively* interfere with waves from the other side of the slit when

$$\frac{a}{2} \sin \theta = \frac{\lambda}{2} \ ,$$

where both the wavelength λ and the slit width 'a' must be measured in the same units, or

$$\sin \theta = \frac{\lambda}{a} \ ,$$

where θ is the angle subtended from the normal line of the slit to the radial direction of the first dark band.

b) We can continue this line of logic by breaking the slit into four parts instead of two, then

$$\sin \theta = \frac{2\lambda}{a} \ ,$$

θ now giving the angle to the second dark band, or six

parts instead of four,

$$\sin \theta = \frac{3\lambda}{a} \ ,$$

with θ giving the angle to the third dark band, etc.

c)   Therefore, the general condition for **destructive inter-ference** for a single slit of width 'a' is

$$\sin \theta_{\mathrm{dark}} = m\frac{\lambda}{a} \ , \quad m = \pm 1, \pm 2, \pm 3, \ldots \qquad \text{(XIII-11)}$$

4.   Diffraction will occur either for light passing through narrow slits, or for light traveling *around* thin obstacles. It is this second type of diffraction that is used to construct **diffraction gratings**.

a)   Diffraction gratings have parallel lines *etched* ("scratched") on the surface of a glass (or some other transparent material) plate.

b)   These etched lines act like a series of parallel slits. As such, the 'grating equation' just takes on the form of Eq. (XIII-2), except here, d is the separation between the etched lines.

c)   We will describe diffraction gratings in more detail in §XIV of these notes.

Example XIII–3.   Problems 24.32 (Page 819) from the Serway & Vuille textbook: Light of wavelength 600 nm falls on a 0.40-mm-wide slit and forms a diffraction pattern on a screen 1.5 m away. (a) Find the position of the first dark band on each side of the central maximum. (b) Find the width of the central maximum.

Solution (a):

Following Eq. (XIII-11), dark bands will occur where $\sin\theta = m\,(\lambda/a)$. For the first dark band, $m = 1$. Recalling Eqs. (XIII-4) and (XIII-5), the distance from the center of the central maximum to the first dark band is

$$
\begin{aligned}
y_1 &= L\tan\theta \approx L\sin\theta = L\left(\frac{\lambda}{a}\right) \\
&= (1.5 \text{ m})\left(\frac{600 \times 10^{-9} \text{ m}}{0.40 \times 10^{-3} \text{ m}}\right) = 2.25 \times 10^{-3} \text{ m} \\
&= \boxed{2.3 \text{ mm}.}
\end{aligned}
$$

Solution (b):

The width of the central maximum is just

$$
2y_1 = 2(2.25 \text{ mm}) = \boxed{4.5 \text{ mm}.}
$$

D. Polarization.

1. As discussed in §IX of these notes, E/M radiation is composed of an electric field and a magnetic field that oscillate at right angles to each other as it propagate as a transverse wave.

   a) An ordinary beam of light consists of a large number of such E/M waves emitted by atoms and/or molecules contained in the source of the light.

   b) The vibrating charges associated with the atoms and molecules act as tiny antennas. As such, each atom and molecule in a source produces a wave with its own orientation of $\vec{E}$ (and of course a $\perp\vec{B}$) corresponding to the direction of the vibration.

c) However, since there are a large number of atoms and molecules in the source, the light emitted by the source has a superposition of all the possible orientations of the $\vec{E}$ and $\vec{B}$ fields $\Longrightarrow$ the emitted light is **unpolarized** as a result.

2. A wave is said to be **linearly polarized** if the resultant $\vec{E}$-field vibrates in the same direction *at all times* at a particular point. Such a wave is often referred to being **plane polarized** or just simply **polarized**.

3. Unpolarized light can be changed into polarized light by passing the light through, or reflecting off of, a special material called a **polarizer**.

a) In 1932, E.H. Land discovered such a transmitting material, which he called **Polaroid**, that polarized light through **selective absorption** by molecules oriented in a specific manner in the material as unpolarized light passed through it.

b) When light gets reflected from a surface (especially water and oil), it becomes either partially or completely polarized due to the interaction of the light's electric field and the flat surface.

4. Finally, the process of **scattering** can also polarize light (see your textbook for details). The N_2 and O_2 molecules in the Earth's atmosphere are very effective at polarizing the Sun's light through a process known as **Rayleigh scattering**.

a) This is why most sunglasses are made with polarizing film coating the optical surface — it cuts down on the glare of the sky as well as glare from reflected sunlight off of

liquids lying on the ground.

b) The amount of Rayleight scattering that an atom or a molecule will perform is proportional to $1/\lambda^4$. This is why the sky is blue — blue light from the Sun is scattered much more efficiently than red light.