

PHYS-2020: General Physics II
Course Lecture Notes
Section I

Dr. Donald G. Luttermoser
East Tennessee State University

Edition 3.2

Abstract

These class notes are designed for use of the instructor and students of the course **PHYS-2020: General Physics II** taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 8th Edition* (2009) textbook by Serway and Vuille.

I. Electric Forces & Electric Fields

A. Properties of Electric Charge.

1. Benjamin Franklin was the first to realize that there are *two types of electric charge*:
 - a) **Positive** charge: $+q$.
 - b) **Negative** charge: $-q$.
2. Like charges *repel* one another whereas unlike charges *attract* each other.
3. Electric charge is always conserved in any type of reaction or process.
4. In 1909, Robert Milliken discovered (via the **Millikan Oil-Drop Experiment**) that if an object is charged, its charge is always in multiples of the fundamental unit of charge, e .
 - a) Charge is said to be **quantized**.
$$\implies q = \pm e, \pm 2e, \pm 3e, \text{ etc.}$$
 - b) $e = 1.60219 \times 10^{-19} \text{ C}$ ($\text{C} \equiv \text{Coulomb}$, in the SI system)
 $= 4.80325 \times 10^{-10} \text{ esu}$ ($\text{esu} \equiv \text{electrostatic unit}$, in the cgs system).
 - c) Elementary particles:
 - i) Electron: $q = -e$.
 - ii) Proton: $q = +e$.
 - iii) Neutron: $q = 0$.

B. Insulators and Conductors.

1. **Conductors** are materials in which electric charges move freely (*i.e.*, they have low internal *resistance*).
 - a) Copper.
 - b) Aluminum.
 - c) Silver.
2. **Insulators** are materials in which electric charges do not freely move (*i.e.*, they have high internal *resistance*).
 - a) Glass.
 - b) Rubber.
3. **Semiconductors** are materials that lie in between these other two \implies if controlled amounts of foreign atoms are added to semiconductors, their electrical properties can be changed by orders of magnitude.
 - a) Silicon.
 - b) Germanium.
4. The Earth can be considered to be an infinite reservoir of (or for) electrons.
 - a) It can accept or supply an unlimited number of electrons.
 - b) When a conductor is connected to the Earth (*e.g.*, conducting wire or copper pipe), it is said to be **grounded** \implies lightning rods.

5. An object can be charged in one of two ways:

a) **Conduction:** Charge exchange through contact.

i) If one charged object comes in contact with a second object, charge can move from the charged object to the uncharged object.

ii) Rubbing two different materials together, called **frictional work**, can produce negative charge on one object and a positive charge on the other.

b) **Induction:** Charge exchange with no contact.

i) Charge one object.

ii) This charge produces an **electric field** to form.

iii) This electric field can then induce charges to migrate on a second object \implies the second object **polarizes** (see Figure 15.4 in the textbook).

C. Coulomb's Law.

1. **Coulomb's law** states that two electric charges experience a force between them such that:

a) It is inversely proportional to the square of the separation r between the 2 particles along the line that joins them.

b) It is proportional to the product of the magnitudes of the charges, q_1 and q_2 , on the two particles.

c) It is *attractive* if the charges are of opposite sign and *repulsive* if the charges have the same sign.

2. Mathematically, the Coulomb force F_e is

$$\boxed{\vec{F}_e = k_e \frac{|q_1| |q_2|}{r^2} \hat{r}} . \quad (\text{I-1})$$

- a) $k_e \equiv$ Coulomb's constant $= 8.9875 \times 10^9 \text{ N m}^2/\text{C}^2$ (SI units) $= 1.000 \text{ dyne cm}^2/\text{esu}^2$ (cgs units).
 - b) The sign of the electric force will depend upon the orientation of the defined coordinate system for a given problem and whether a given force is **repulsive** (*e.g.*, both q 's are positive or both q 's are negative) or **attractive** (*e.g.*, the charges are opposite in sign).
 - c) \hat{r} is the unit vector in the radial direction.
3. Coulomb's law, like Newton's law of gravity ($F = Gm_1m_2/r^2$) is:
- a) A **field force law** \implies there is no physical contact between the particles.
 - b) And an **inverse-square law** \implies the strength of the electric force falls off as the inverse of the distance squared.
4. If more than two charged particles exist in a system, then the Coulomb force exerted on one particle is the summation of all of the Coulomb forces between that particle and the rest of the particles in the ensemble:

$$\vec{F}_e = \sum_i^N F_i \hat{x} + \sum_j^N F_j \hat{y} + \sum_k^N F_k \hat{z} , \quad (\text{I-2})$$

where F_i is the component Coulomb forces of all N particles in the x direction, F_j is the component forces in the y direction, and F_k is the component forces in the z direction. Eq. (I-2) is known as the **principle of superposition**.

Example I-1. An alpha particle (charge $+2.0e$) is sent at high speed toward a gold nucleus (charge $+79e$). What is the electrical force acting on the alpha particle when it is 2.0×10^{-14} m from the gold nucleus?

Solution:

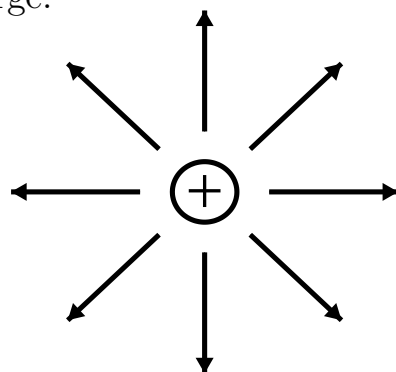
Use Coulomb's Law (*i.e.*, Eq. I-1) where $q_1 = +2e$, $q_2 = +79e$, $r = 2.0 \times 10^{-14}$ m, and $e = 1.60 \times 10^{-19}$ C:

$$\begin{aligned}
 |\vec{F}_e| &= k_e \frac{|q_1||q_2|}{r^2} = k_e \frac{(2e)(79e)}{r^2} \\
 &= \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left[\frac{(2.0 \cdot 79)(1.60 \times 10^{-19} \text{ C})^2}{(2.0 \times 10^{-14} \text{ m})^2} \right] \\
 &= \boxed{91 \text{ N (repulsion).}}
 \end{aligned}$$

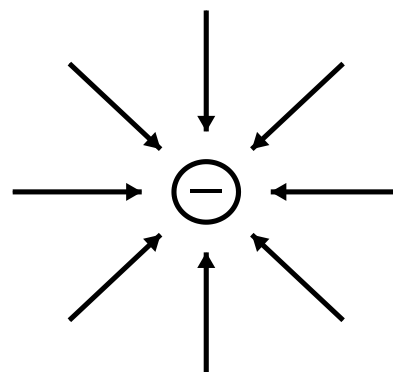
The Coulomb force is a repulsion force here since both particles have charges of the same sign.

D. The Electric Field.

1. An electric charge emits an **electric field** which always points away from a **positive** charge and points towards a **negative** charge:

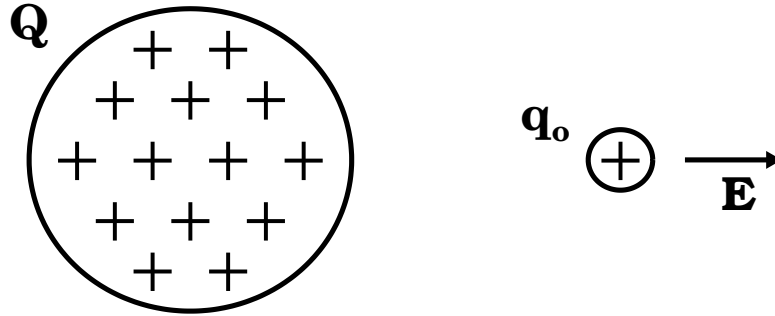


**E-field
points away**



**E-field
points to charge**

- a) Assume we take a *small* (*i.e.*, negligible) positive test charge q_o , and place it near a larger positively charged object, Q .



- b) The direction of \vec{E} (*i.e.*, the electric field) at a point is defined to be the direction of the electric force that would be exerted on a small (*i.e.*, negligible with respect to the charge on the larger object, $q_o \ll Q$) positive charge placed at that point.
- c) Note that if q_o starts to become comparable in size (in terms of charge) to Q , the electric field of Q will be altered by q_o .
2. Hence, the electric field is defined by the electric force exerted on a charged particle by another charged particle or object:

$$\boxed{\vec{E} = \frac{\vec{F}_e}{q_o}} \quad (\text{I-3})$$

- a) Note that from this equation, E always points in the same direction that the force exerted on a positively charged particle q_o by a charged object that gives rise to \vec{E} .
- b) E is measured in N/C in SI units and dyne/esu in the cgs system.

- c) The electric field \vec{E} is analogous to surface gravity (*i.e.*, acceleration due to gravity) in Newton's Theory of Gravitation since we can write Newton's 2nd law as

$$\vec{g} = \frac{\vec{F}_g}{m} ,$$

where g is measured in N/kg ($= \text{m/s}^2$) and is nothing more than the gravitational force per unit mass (*i.e.*, the strength of the gravitational field). Likewise, the electric field is the electrical force per unit charge.

3. Let the object that produces an E -field be given a charge labeled with q . Then a test charge q_o a distance r from q experiences a force from q described by Coulomb's law (*i.e.*, Eq. I-1):

$$\vec{F}_e = k_e \frac{|q| |q_o|}{r^2} \hat{r} .$$

Plugging this into Eq. (I-3) gives a second equation that describes the E -field:

$$\boxed{\vec{E} = k_e \frac{|q|}{r^2} \hat{r} .} \quad (\text{I-4})$$

- a) If q is positive (*i.e.*, $q > 0$), then E is positive and points radially away from the charge.
- b) If q is negative (*i.e.*, $q < 0$), then E is negative and points radially inward towards the charge.

Example I-2. An electron with a speed of $3.00 \times 10^6 \text{ m/s}$ moves into a uniform electric field of 1000 N/C . The field is parallel to the electron's motion. How far does the electron travel before it is brought to rest?

Solution:

When the electron, with mass $m_e = 9.11 \times 10^{-31}$ kg, enters the electric field, it experiences a retarding force given by Eq. (I-3):

$$F = -eE ,$$

negative since the E -field slows the electron. Since this force slows the electron, it produces a deceleration. Using Newton's 2nd law we can write

$$\begin{aligned} a &= \frac{F}{m_e} = \frac{-eE}{m_e} = -\frac{(1.60 \times 10^{-19} \text{ C})(1000 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} \\ &= -1.76 \times 10^{14} \text{ m/s}^2 . \end{aligned}$$

Using one of the 1-D equations of motion from General Physics I, we can solve for the distance that the electron travels before coming to a stop, Δx . We have $v = 0$ (e^- comes to rest), $v_o = 3.00 \times 10^6$ m/s, and a has been calculated above, giving

$$\begin{aligned} v^2 &= v_o^2 + 2a \Delta x \\ \Delta x &= \frac{v^2 - v_o^2}{2a} \\ &= \frac{0 - (3.00 \times 10^6 \text{ m/s})^2}{2.0(-1.76 \times 10^{14} \text{ m/s}^2)} \\ &= 0.0256 \text{ m} = \boxed{2.56 \text{ cm} .} \end{aligned}$$

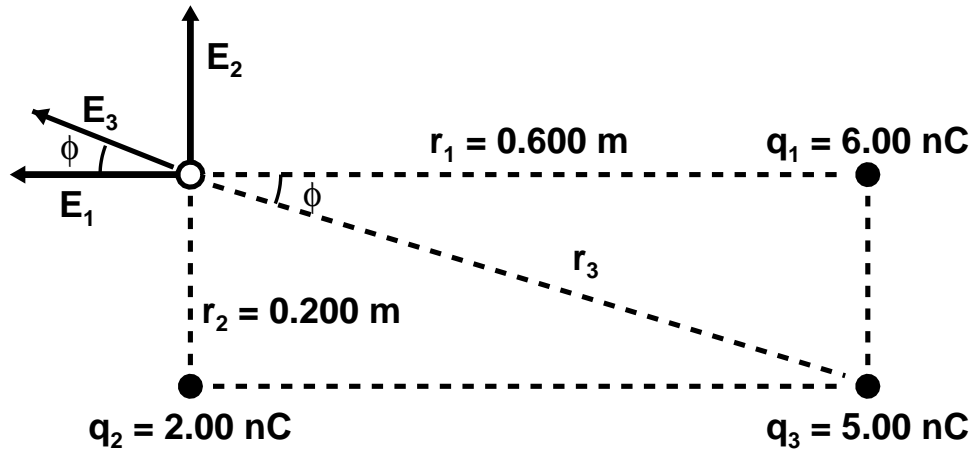
4. When summing E -fields from multiple charges, it is best to draw a vector diagram first at a point where the E -field is to be determined. Draw the E -field vector of each charge in the direction as dictated by the rules above. Then vectorially add the individual E -fields together, letting the diagram determine the sign of the

E -field (typically + to the right and + upwards). Then add using the absolute value of the charge:

$$E = \sum_1^N (\pm) \frac{k_e |q|}{r^2} , \quad (\text{I-5})$$

where either + or – is selected based upon the direction of the E -field vector (see Example I-3 below).

Example I-3. Positive charges are situated at three corners of a rectangle as shown in the figure below. Find the electric field at the fourth corner.



Solution:

From the geometry of the rectangle as shown in the figure above, we have

$$r_3^2 = r_1^2 + r_2^2 = (0.600 \text{ m})^2 + (0.200 \text{ m})^2 = 0.400 \text{ m}^2 .$$

and

$$\phi = \tan^{-1} \left(\frac{r_2}{r_1} \right) = \tan^{-1} \left(\frac{0.200 \text{ m}}{0.600 \text{ m}} \right) = 18.4^\circ .$$

The components of the individual E -field vectors are

$$\begin{aligned} \vec{E}_1 &= -E_{1x} \hat{x} = -E_1 \hat{x} \\ \vec{E}_2 &= +E_{2y} \hat{y} = +E_2 \hat{y} \\ \vec{E}_3 &= -E_{3x} \hat{x} + E_{3y} \hat{y} \\ &= -E_3 \cos \phi \hat{x} + E_3 \sin \phi \hat{y} . \end{aligned}$$

The resultant E -field at the vacant corner is

$$\vec{E}_R = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

with the resultant component vectors of

$$\begin{aligned} E_x &= \sum_{i=1}^3 E_{ix} = -E_{1x} + 0 - E_{3x} = -E_1 - E_3 \cos \phi \\ &= -\frac{k_e |q_1|}{r_1^2} - \frac{k_e |q_3|}{r_3^2} \cos \phi \\ &= -\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \left[\frac{6.00 \times 10^{-9} \text{ C}}{(0.600 \text{ m})^2} + \right. \\ &\quad \left. \frac{5.00 \times 10^{-9} \text{ C}}{0.400 \text{ m}^2} \cos 18.4^\circ \right] \\ &= -256 \text{ N/C} \end{aligned}$$

$$\begin{aligned} E_y &= \sum_{j=1}^3 E_{jy} = 0 + E_{2y} + E_{3y} = E_2 + E_3 \sin \phi \\ &= \frac{k_e |q_2|}{r_2^2} + \frac{k_e |q_3|}{r_3^2} \sin \phi \\ &= \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \left[\frac{3.00 \times 10^{-9} \text{ C}}{(0.200 \text{ m})^2} + \right. \\ &\quad \left. \frac{5.00 \times 10^{-9} \text{ C}}{0.400 \text{ m}^2} \sin 18.4^\circ \right] \\ &= 710 \text{ N/C} \end{aligned}$$

Thus,

$$E_R = \sqrt{(E_x)^2 + (E_y)^2} = 755 \text{ N/C} .$$

With $E_x < 0$ and $E_y > 0$, the resultant E -field lies in the second quadrant with an angle of

$$\theta_{(-)} = \tan^{-1} \left(\frac{E_y}{E_x} \right) = \tan^{-1} \left(\frac{710 \text{ N/C}}{-256 \text{ N/C}} \right) = -70^\circ .$$

However, this is with respect to the $-x$ axis. θ (with respect to the $+x$ axis) is then $\theta = 180^\circ + \theta_{(-)} = 180^\circ - 70^\circ = 110^\circ$. Hence,

we can express the answer in one of two ways:

$\vec{E}_R = -(256 \text{ N/C}) \hat{x} + (710 \text{ N/C}) \hat{y}$ <p style="text-align: center;">or</p> $\vec{E}_R = 755 \text{ N/C at } 110^\circ \text{ counterclockwise from the } +x \text{ axis.}$
--

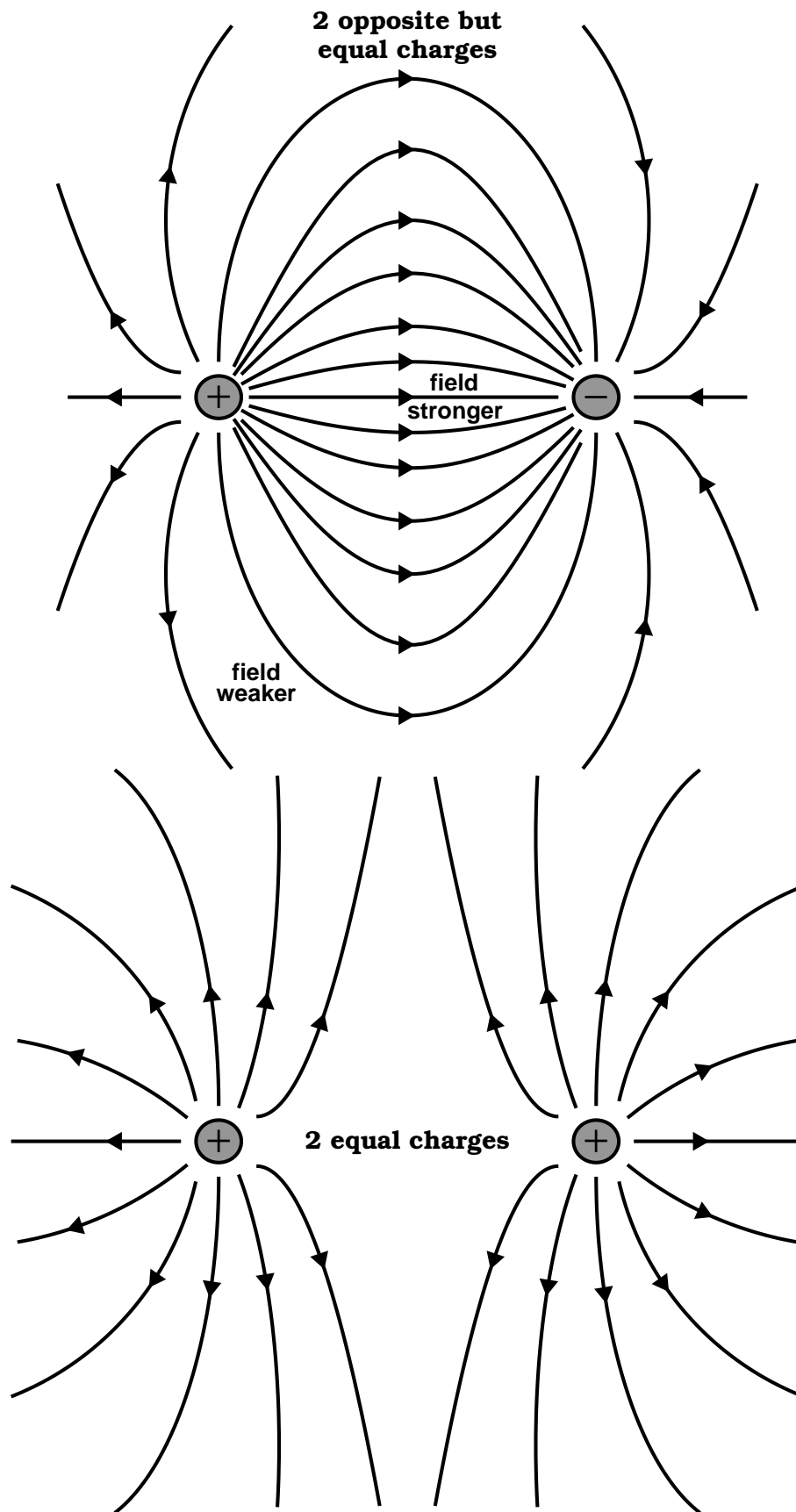
E. Electric Field Lines.

1. Drawing electric field lines:

- a) The electric field vector, \vec{E} , is tangent to the electric field lines at each point.
- b) The number of lines per unit area through a surface \perp to the lines is proportional to the strength of the electric field in a given region.
 - i) \vec{E} is large when the field lines are close together.
 - ii) \vec{E} is small when the field lines are far apart.

2. For a system of charged particles, the following rules apply:

- a) The lines must begin on positive charges (or at infinity) and must terminate on negative charges (or, in the case of excess charge, at infinity).
- b) The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge (*e.g.*, if a charge of $+q$ has n -lines per unit volume leaving the charge, a charge of $-2q$ will have $2n$ -lines per unit volume approaching the charge — see Ex. I-4).
- c) No 2 field lines can cross each other.



Example I-4. Problem 15.30 (Page 527) from the Serway & Vuille textbook: Figure P15.30 shows the electric field lines for two point charges separated by a small distance. (a) Determine the ratio q_1/q_2 . (b) What are the signs of q_1 and q_2 ?

Solution (a & b):

The magnitude of q_2 is 3 times the magnitude of q_1 since 3 times as many lines emerge from q_2 as enter q_1 . Hence, $|q_2| = 3|q_1|$. Since the field lines are emerging from q_2 , it must have a positive charge, and since they terminate on q_1 , it must have a negative charge. Thus,

$$(a) \quad \boxed{\frac{q_1}{q_2} = -\frac{1}{3} ,}$$

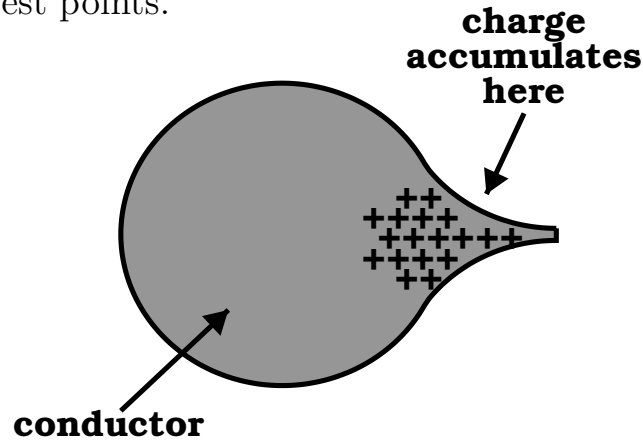
and

$$(b) \quad \boxed{q_1 < 0, \quad q_2 > 0 .}$$

F. Conductors in Electrostatic Equilibrium.

1. A good conductor contains electrons that are not bound to any one atom \implies free to move about the material.
2. **Electrostatic Equilibrium:** No net motion of charge occurs within a conductor. The following is true for such a conductor:
 - a) The electric field is zero everywhere inside the conductor.
 - b) Any excess charge on an isolated conductor resides entirely on its surface.
 - c) The electric field just outside a charged conductor is \perp to the conductor's surface.

- d) On an irregular shaped conductor, charge accumulates at its sharpest points.



G. The Electric Flux and Gauss's Law.

1. The **electric flux** is a measure of the number of E -field lines that crosses a given area.

- a) An E -field whose lines penetrate a cross-sectional (or surface) area $A \perp$ to A has an electric flux Φ_E given by

$$\Phi_E = E A . \quad (\text{I-6})$$

- b) However, if the E -field lines lie at an angle θ with respect to the normal line of area A (see Figure 15.25 in the textbook), the electric flux is given by the more general formula:

$$\Phi_E = \vec{E} \cdot \vec{A} = E A \cos \theta , \quad (\text{I-7})$$

where the ' \cdot ' is called the **dot product** operation and \vec{A} has magnitude A (the total cross-sectional or surface area) and direction given by the normal line of the area. The angle θ is the angle between that normal line and the E -field direction.

- c) When the area is constructed such that a closed surface is formed, we shall adopt the convention that the flux lines

passing into the interior of the volume are *negative* and those passing out of the interior of the volume are *positive*.

2. One of the most important law's in all of electromagnetism is Gauss's law:

- a) If we use a sphere for our enclosing volume, the sphere has a surface area of $A = 4\pi r^2$. If we place charge q at the center of this sphere, then the field lines will always be \perp to the surface of the sphere since they point radially outward. Using Eq. (I-4) in Eq. (I-6), we can write the electric flux as

$$\Phi_E = E A = k_e \frac{q}{r^2} (4\pi r^2) = 4\pi k_e q ,$$

independent of the distance from the charge!

- b) In electromagnetism, there is a constant related to the Coulomb constant called the **permittivity of free space** ϵ_o . This constant is given by

$$\epsilon_o = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} . \quad (\text{I-8})$$

- c) Using this constant in our flux equation above, we can write the electric flux as

$$\Phi_E = 4\pi k_e q = \frac{q}{\epsilon_o} .$$

- d) Using calculus, we could show that this simple result is true for any closed surface (even non-symmetrical ones) that surrounds any charge $q \implies$ such a surface is referred to as a *gaussian surface*.
- e) This bit of mathematics was first worked out by Gauss and is therefore called **Gauss's law**.

- i) In words, this law states: **The electric flux through any closed surface is equal to the net charge Q inside the surface divided by the permittivity of free space ϵ_o .**
- ii) Mathematically, it is given by

$$\boxed{\Phi_E = \frac{Q_{\text{inside}}}{\epsilon_o}} . \quad (\text{I-9})$$

Example I-5. Problem 15.41 (Page 528) from the Serway & Vuille textbook: An electric field of intensity 3.50 kN/C is applied along the x axis. Calculate the electric flux through a rectangular plane 0.350 m wide and 0.700 m long if (a) the plane is parallel to the yz plane; (b) the plane is parallel to the xy plane; and (c) the plane contains the y axis and its normal makes an angle of 40.0° with the x axis.

Solution (a):

The flux through an area is given by Eq. (I-7), where θ is the angle between the direction of the field E and the normal line of area A . The magnitude of the area of the plane is $A = (0.350 \text{ m})(0.700 \text{ m}) = 0.245 \text{ m}^2$. When the plane is parallel to the yz plane, the normal line of the area lies parallel to the x axis, so $\theta = 0$, and the flux is

$$\begin{aligned} \Phi_E = EA \cos \theta &= (3.50 \times 10^3 \text{ N/C})(0.245 \text{ m}^2) \cos 0^\circ \\ &= \boxed{858 \text{ N} \cdot \text{m}^2/\text{C}} . \end{aligned}$$

Solution (b):

When the plane is parallel to the x axis (as it must be when it is parallel to the xy plane), the normal line of the area is at a right

angle to the x axis (and hence E field), so $\theta = 90^\circ$ and

$$\Phi_E = EA \cos \theta = (3.50 \times 10^3 \text{ N/C})(0.245 \text{ m}^2) \cos 90^\circ = \boxed{0 .}$$

Solution (c):

Since the E field is along the x axis and the normal of the area make an angle of 40.0° with respect to the x axis, $\theta = 40.0^\circ$, so

$$\begin{aligned} \Phi_E = EA \cos \theta &= (3.50 \times 10^3 \text{ N/C})(0.245 \text{ m}^2) \cos 40.0^\circ \\ &= \boxed{657 \text{ N} \cdot \text{m}^2/\text{C} .} \end{aligned}$$
