Good Luck To **SOLUTION KEY** Hour **ALL** Score: **100**

Unit 1 Party: Precalculus Review Mr. Kirpes AP Calculus BC Total Points: 100

Part I: Multiple Choice **SHOW ALL WORK** Please circle the letter of the selected response. (6 points each)

1. Which of the following defines a function for which $f(-x) = -f(x)$?

A. $f(x) = (x^3+1)^2 - 1$ B. $f(x) = (x-1)^3 + 1$ C. $f(x) = \dfrac{x+1}{x}$ $\boxed{\text{D.}}$ $f(x) = -x^5 + 3x$ E. $f(x) = x^4 - 2x^2 + 6$

$f(-x) = -f(x) \Rightarrow$ function is odd

A. $f(-x) = \left((-x)^3+1\right)^2 - 1 = (-x^3+1)^2 - 1 \neq -f(x)$

B. $f(-x) = (-x-1)^3 + 1 \neq -f(x)$

C. $f(-x) = \dfrac{-x+1}{-x} = \dfrac{x-1}{x} \neq -f(x)$

$\boxed{\text{D. } f(-x) = -(-x)^5 + 3(-x) = x^5 - 3x = -f(x)}$

E. $f(-x) = (-x)^4 - 2(-x)^2 + 6 = x^4 - 2x^2 + 6 \neq -f(x)$

2. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A+B$?

A. -6 B. -3 C. 2 $\boxed{\text{D.}}$ -1 E. It cannot be determined from the given information.

$$\left.\begin{array}{l} f(2) = 16 + 4A + 2B - 5 = 3 \\ f(-2) = -16 + 4A - 2B - 5 = -37 \end{array}\right\} \Rightarrow 8A - 10 = -34,\ 8A = -24,\ \underline{A = -3}$$

$\therefore\ 16 - 12 + 2B - 5 = 3,\ 2B = 4,\ \underline{B = 2}$

$\therefore\ \boxed{A + B = -1}$

3. If $g(x_1) + g(x_2) = g(x_1 + x_2)\ \forall\ x_1, x_2 \in \mathbb{R}$, which of the following could define g ?

A. $g(x) = x + 1$ $\boxed{\text{B.}}$ $g(x) = 2x$ C. $g(x) = \dfrac{1}{x}$ D. $g(x) = x^2$ E. $g(x) = \sqrt{x}$

A. $g(x_1) + g(x_2) = x_1 + 1 + x_2 + 1 = x_1 + x_2 + 2 \neq g(x_1 + x_2)$

$\boxed{\text{B. } g(x_1) + g(x_2) = 2x_1 + 2x_2 = 2(x_1 + x_2) = g(x_1 + x_2)}$

C. $g(x_1) + g(x_2) = \dfrac{1}{x_1} + \dfrac{1}{x_2} \neq g(x_1 + x_2) = \dfrac{1}{x_1 + x_2}$

D. $g(x_1) + g(x_2) = x_1^2 + x_2^2 \neq g(x_1 + x_2) = (x_1 + x_2)^2$

E. $g(x_1) + g(x_2) = \sqrt{x_1} + \sqrt{x_2} \neq g(x_1 + x_2) = \sqrt{x_1 + x_2}$

4. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g(f(x)) =$

A. $5x^3 + 15x^2 + 20x + 25$ B. $5x^3 + 15x^2 + 25$ C. 1125 D. 225 $\boxed{\text{E.}}$ 5

$g(f(x)) = g(x^3 + 3x^2 + 4x + 5) = \boxed{5}$

5. If $f(x) = \dfrac{4}{x-1}$ and $g(x) = 2x$, then the solution set of $g(f(x)) = f(g(x))$ is

$\boxed{\text{A.}}$ $\left\{\dfrac{1}{3}\right\}$ B. $\{2\}$ C. $\{3\}$ D. $\{-1 , 2\}$ E. $\left\{\dfrac{1}{3} , 2\right\}$

$$\left.\begin{array}{l} g(f(x)) = g\left(\dfrac{4}{x-1}\right) = \dfrac{8}{x-1} \\ f(g(x)) = f(2x) = \dfrac{4}{2x-1} \end{array}\right\} \Rightarrow \dfrac{8}{x-1} = \dfrac{4}{2x-1} , \ 16x - 8 = 4x - 4 , \ 12x = 4 , \ \boxed{x = \dfrac{1}{3}}$$

6. If the domain of the function g given by $g(x) = \dfrac{1}{1-x^2}$ is $|x| > 1$, what is the range of g ?

A. $(-1 , \infty)$ B. $(0 , \infty)$ $\boxed{\text{C.}}$ $(-\infty , 0)$ D. $(-\infty , 1)$ E. $(-\infty , -1)$

Since $|x| > 1 \Rightarrow x^2 > 1 \Rightarrow 1 - x^2 < 0 \Rightarrow \dfrac{1}{1-x^2} < 0$

$\therefore$ The range of $g(x) = \dfrac{1}{1-x^2}$ is $\boxed{(-\infty , 0)}$.

7. If the fundamental period of the function $f(x) = 3\cos\left(\dfrac{kx}{2}\right)$ is $\dfrac{2\pi}{3}$, then the value of k must be

A. 2 B. 3 C. 4 $\boxed{\text{D.}}$ 6 E. 8

$$\text{Period} = \dfrac{2\pi}{\frac{k}{2}} = \dfrac{2\pi}{3} \ \therefore \ \dfrac{k}{2} = 3 , \ \boxed{k = 6}$$

8. Let f and g be odd functions. If p , r , and s are non-zero functions defined as given below, which of the following must be odd?

I. $\underline{p(x) = f(g(x))}$ II. $\underline{r(x) = f(x) + g(x)}$ III. $s(x) = f(x)g(x)$

A. I only B. II only $\boxed{\text{C.}}$ I and II only D. II and III only E. I, II, and III

I. $p(-x) = f(g(-x)) = f(-g(x)) = -f(g(x)) = -p(x)$ $\therefore$ $\underline{p \text{ is odd}}$

II. $r(-x) = f(-x) + g(-x) = -f(x) + -g(x) = -(f(x) + g(x)) = -r(x)$ $\therefore$ $\underline{r \text{ is odd}}$

III. $s(-x) = f(-x)g(-x) = (-f(x))(-g(x)) = f(x)g(x) \neq -s(x)$ $\therefore$ s is not odd (s is even)

$\therefore$ $\boxed{\text{I and II only}}$

Part II: Free Response **SHOW ALL WORK** Please circle each final response.

1. Find the center and radius of the circle represented by $4x^2+4y^2-24x+4y+9=0$. (7 points)

$4\left(x^2-6x+9\right)+4\left(y^2+y+\frac{1}{4}\right)=-9+36+1$,

$4(x-3)^2+4\left(y+\frac{1}{2}\right)^2=28$, $(x-3)^2+\left(y+\frac{1}{2}\right)^2=7$,

$\therefore$ $\boxed{\text{center: } \left(3\,,\,-\frac{1}{2}\right)\,,\ \text{radius} = \sqrt{7}}$

Center: $\left(3\,,\,-\frac{1}{2}\right)$

Radius: $\sqrt{7}$

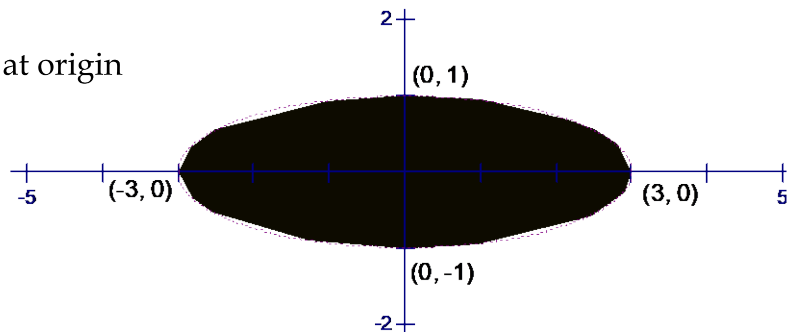
2. Sketch the graph of the conic section represented by $x^2+9y^2<9$. (7 points)
Indicate four significant points used to create the graph by including their coordinates.

$\frac{x^2}{9}+\frac{y^2}{1}<1$ Ellipse, centered at origin

Major axis length $=6$, Minor axis length $=2$

$(-3\,,\,0)$ $(0\,,\,1)$

$(3\,,\,0)$ $(0\,,\,-1)$



3. Let $f(x)=x^2$ and $g(x)=\sqrt{x-1}$. (7 points)

A. Find the domain of $g\circ f$.

Domain of f: $\mathbb{R}$

$g\circ f=g(f(x))=g(x^2)=\sqrt{x^2-1}$

Domain of $\sqrt{x^2-1}$: $|x|\ \geq\ 1$

$\therefore$ Domain of $g\circ f$: $\mathbb{R}\ \cap\ |x|\ \geq\ 1\ \Rightarrow\ \boxed{(-\infty\,,\,-1]\ \cup\ [1\,,\,\infty)}$

$(-\infty\,,\,-1]\ \cup\ [1\,,\,\infty)$

B. Find the domain of $f\circ g$.

Domain of g: $x\ \geq\ 1$

$f\circ g=f(g(x))=f\left(\sqrt{x-1}\right)=x-1$

Domain of $x-1$: $\mathbb{R}$

$\therefore$ Domain of $f\circ g$: $x\ \geq\ 1\ \cap\ \mathbb{R}\ \Rightarrow\ \boxed{[1\,,\,\infty)}$

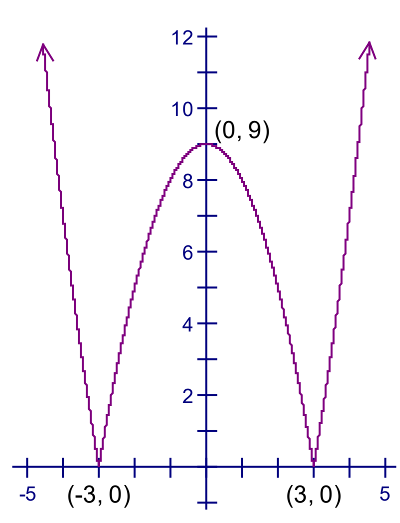
$[1\,,\,\infty)$

4. Sketch the graph of the function $K(x)=\left|9-x^2\right|$. (6 points)
Indicate three significant points used to create the graph by including their coordinates.

Note: $\left|9-x^2\right|=\left|x^2-9\right|$

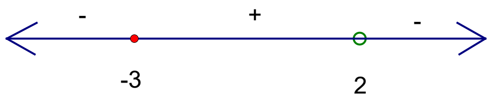
Graph x^2-9 with its negative portion reflected above the $x-$axis

$(0\,,\,9)$ $(-3\,,\,0)$ $(3\,,\,0)$



5. Find the domain of $y=\sqrt{\dfrac{x+3}{2-x}}$. (6 points)

$\dfrac{x+3}{2-x} \geq 0$, use SPA to determine correct intervals



$\Rightarrow$ Domain is $\boxed{[-3\ ,\ 2)}$ OR $\{x:\ -3\ \leq\ x\ <\ 2\}$

$[-3\ ,\ 2)$ ______________

6. Find an equation of the line containing $(4\ ,\ -2)$ that is also perpendicular to the graph of $3x-4y=33$. (6 points)

$$m_{\perp} = \frac{-A}{B} = \frac{-3}{-4} = \frac{3}{4} \ \therefore\ m_{\text{line}} = -\frac{4}{3} \Rightarrow$$

Equation of the line is $\boxed{y+2=-\dfrac{4}{3}(x-4)}$ OR $\boxed{y=-\dfrac{4}{3}x+\dfrac{10}{3}}$ OR $\boxed{4x+3y=10}$

OR $\boxed{y=-\dfrac{4}{3}x+3\dfrac{1}{3}}$

$y+2=-\dfrac{4}{3}(x-4)$ ______________

7. Give the domain of x without using the absolute value symbol: $|4-3x|\ \geq\ 11$. (6 points)

$$\begin{array}{ccc} 4-3x \geq 11 & \text{or} & 4-3x \leq -11 \\ -3x \geq 7 & \text{or} & -3x \leq -15 \\ \end{array}$$

$\boxed{x \leq -\dfrac{7}{3} \quad \text{or} \quad x \geq 5}$

$\Rightarrow$ Domain is $\boxed{\left(-\infty\ ,\ -\dfrac{7}{3}\right] \cup [5\ ,\ \infty)}$

$\left(-\infty\ ,\ -\dfrac{7}{3}\right] \cup [5\ ,\ \infty)$ ______________

8. Express the circumference, C, of a circle as a function of its area A . (7 points)

$C(r)=2\pi r\ ,\ A(r)=\pi r^2\ \therefore\ \sqrt{A}=\sqrt{\pi}r\ \ (r\ >\ 0)\ ,\ r=\dfrac{\sqrt{A}}{\sqrt{\pi}}\ \therefore\ \boxed{C=2\sqrt{A\pi}}$ OR

$\therefore\ r^2=\dfrac{A}{\pi} \Rightarrow r=\sqrt{\dfrac{A}{\pi}}\ \ (r\ >\ 0)\ \therefore\ \boxed{C=2\pi\sqrt{\dfrac{A}{\pi}}}$ OR $\boxed{C=\dfrac{2\pi\sqrt{A}}{\sqrt{\pi}}}$

$C(A)=2\pi\sqrt{\dfrac{A}{\pi}}$ ______________