PHYS-2010: General Physics I
Course Lecture Notes
Section III

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Edition 2.5

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2010: General Physics I taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 9th Edition* (2012) textbook by Serway and Vuille.

III. Motion in One Dimension

A. Displacement.

1. The **displacement** of an object is defined as the change in its position.

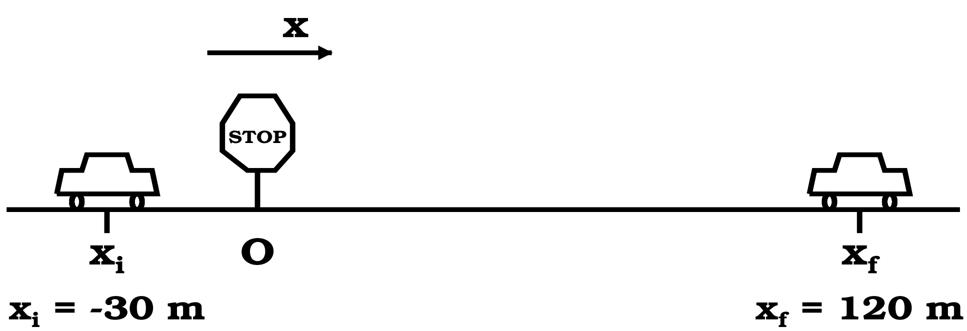
2. It is given by the difference between its final and initial coordinates:

$$\boxed{\Delta x \equiv x_{\rm f} - x_{\rm i}} = \text{displacement.} \qquad \text{(III-1)}$$

3. Displacement is a vector quantity $\Longrightarrow$ has direction and magnitude:

$$\Delta\vec{x} \equiv \vec{x}_{\rm f} - \vec{x}_{\rm i}\ . \qquad \text{(III-2)}$$

Example III–1. A car starts at a position 30 m from a stop sign and continues past the stop sign until it comes to rest 120 m past the sign. What is its displacement?



$$\Delta x = x_{\rm f} - x_{\rm i} = 120\ \text{m}\ - (-30\ \text{m}) = 120\ \text{m} + 30\ \text{m} = 150\ \text{m}$$

Note that since 150 is positive, the car is moving in the positive x direction.

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4. Unit vectors:

a) Actually in Example (III-1), displacement should have been represented as a vector $\Delta\vec{x}$ instead of a scalar Δx (note that using vector notation in one dimension is actually not needed since we always know that we will alway be along the x, or some other, axis).

b) A **unit vector** has a magnitude of unity (*i.e.*, 1) and a direction $\Longrightarrow$ it conveys the directional information of a vector quantity.

i) Cartesian x-direction: $\vec{\imath}$ or $\hat{x}$.

ii) Cartesian y-direction: $\vec{\jmath}$ or $\hat{y}$.

iii) Cartesian z-direction: $\vec{k}$ or $\hat{z}$.

iv) Polar r-direction: $\hat{r}$.

v) Polar θ-direction: $\hat{\theta}$.

c) In this class we will use the "hat" notation (*i.e.*, $\hat{x}$) for the unit vectors for Cartesian coordinates instead of the ijk notation in order to make it consistent with what we will use for polar coordinates. (Note that your textbook does not use the unit vector notation.)

d) As such, we can write a vector in component form:

$$\vec{A} = A_x\,\hat{x} + A_y\,\hat{y} + A_z\,\hat{z}\ , \tag{III-3}$$

whereas your textbook writes a vector as the sum of its Cartesian vector components (the vector arrow is <u>not</u> written in your textbook but it is implied):

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z\ .$$

Once again, we shall be using the first version of vector components in this course. In Eq. (III-3), A_x, A_y, and A_z are all scalars $\Longrightarrow$ they only contain the vector's magnitude of the given vector component.

e) In Example (III-1), we could have solved the problem using vector notation:

$$\Delta\vec{x} = \vec{x}_{\rm f} - \vec{x}_{\rm i} = (120 \text{ m})\,\hat{x} - (-30 \text{ m})\,\hat{x} = (150 \text{ m})\,\hat{x} \; .$$

B. Average Velocity.

1. The **average velocity**, $\overline{v}$, is defined as the displacement divided by the time interval during which the displacement occurred:

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_{\rm f} - x_{\rm i}}{t_{\rm f} - t_{\rm i}}$$

or in vector notation:

$$\boxed{\vec{\overline{v}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\vec{x}_{\rm f} - \vec{x}_{\rm i}}{t_{\rm f} - t_{\rm i}} \; .} \qquad \text{(III-4)}$$

2. The average velocity of an object during the time interval $t_{\rm i}$ to $t_{\rm f}$ is equal to the slope of the straight line joining the initial and final points on a graph of the position of the object plotted versus time.

Example III–2. In Example (III-1), assume it takes 5 seconds to move Δx. What is the car's average velocity?

$$\begin{aligned}\overline{v} &= \frac{\Delta x}{\Delta t} = \frac{x_{\rm f} - x_{\rm i}}{t_{\rm f} - t_{\rm i}} = \frac{120 \text{ m} - (-30 \text{ m})}{5 \text{ s} - 0 \text{ s}} = \frac{150 \text{ m}}{5 \text{ s}} \\ &= 30 \text{ m/s} \qquad \text{in the positive x direction.}\end{aligned}$$

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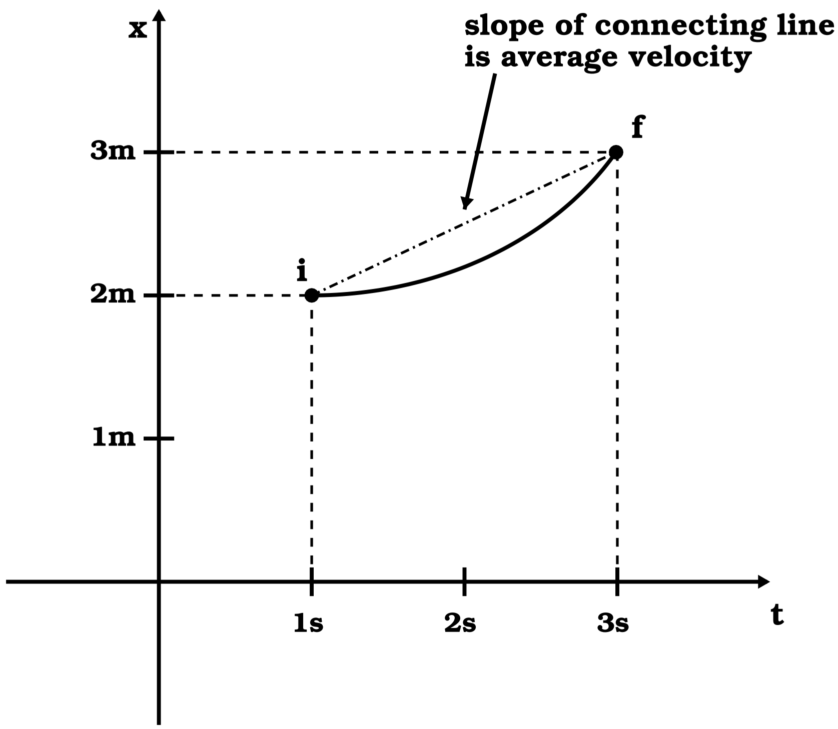
3. When plotting the value of a variable as a function of some other variable, the path on that plot that the object takes is called a **trajectory**.

Example III–3. From the graph below, calculate the average velocity of the trajectory plotted.



$$\overline{v} = \frac{x_{\rm f} - x_{\rm i}}{t_{\rm f} - t_{\rm i}} = \frac{3\ \text{m} - 2\ \text{m}}{3\ \text{s} - 1\ \text{s}} = \frac{1\ \text{m}}{2\ \text{s}}$$
$$= 0.5\ \text{m/s} \quad \text{in the positive x direction.}$$

C. Instantaneous Velocity.

1. The **instantaneous velocity**, $\vec{v}$, is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

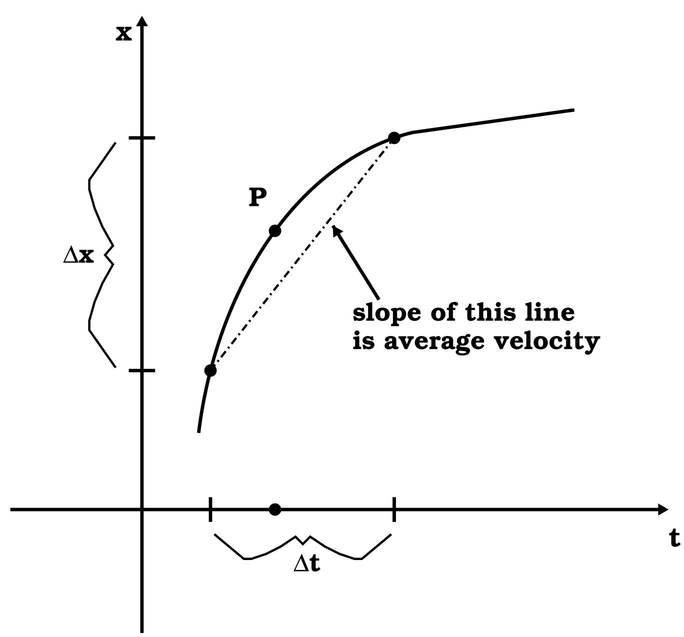
a) In equation form:

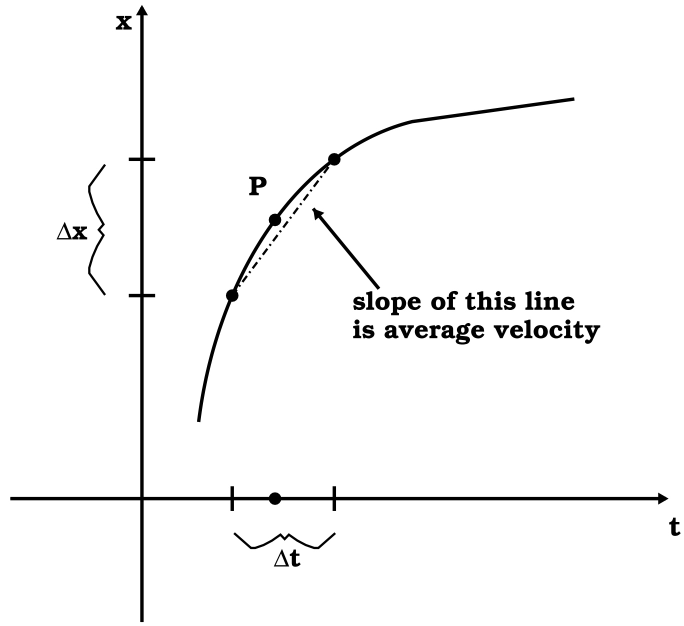
$$\boxed{\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}} \equiv \frac{d\vec{x}}{dt} \ , \qquad \text{(III-5)}$$

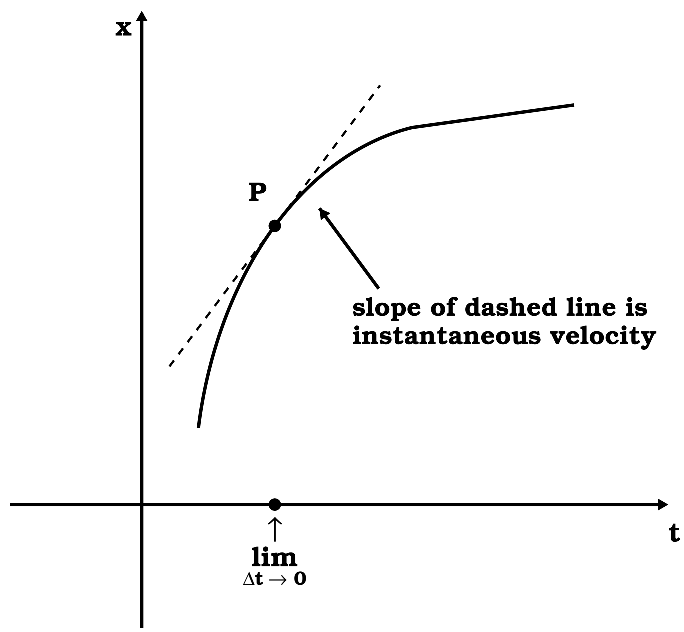
where the $d\vec{x}/dt$ notation is from calculus and is the time derivative of vector x. Note that we will not be using calculus in this course, I just introduce it here to give you a flavor of physics at the next level.

b) The notation $\lim\limits_{\Delta t \to 0}$ means that the ratio $\Delta x/\Delta t$ is to be evaluated as the time interval, Δt, get shorter and shorter approaching zero.

c) Graphically, the slope of the tangent line on the position-time curve is defined to be the instantaneous velocity at that time.







2. The **instantaneous speed** of an object, which is a scalar quantity, is defined as the absolute <u>magnitude</u> of the instantaneous velocity. Hence from this definition, speed can never be negative.

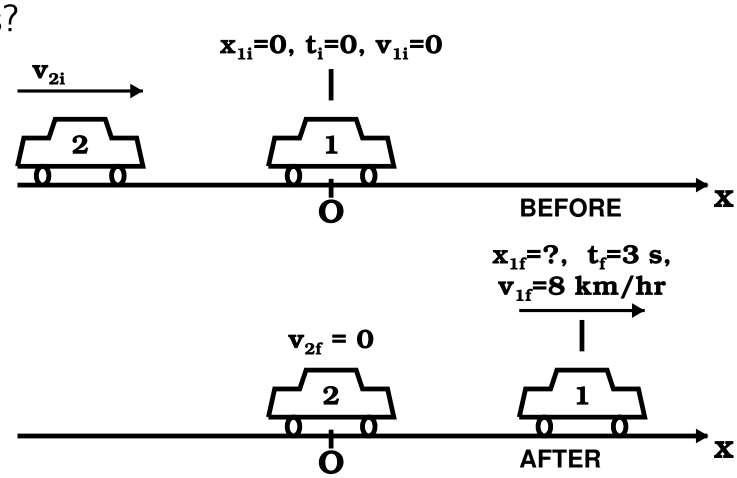
3. From this point forward, the word "**velocity**" will mean instantaneous velocity and the word "**speed**" will mean instantaneous speed.

D. Acceleration.

1. The **average acceleration**, $\overline{a}$, during a given time interval is defined as the change in velocity divided by that time interval during which the change occurs:

$$\boxed{\vec{\overline{a}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_{\rm f} - \vec{v}_{\rm i}}{t_{\rm f} - t_{\rm i}}\ .} \tag{III-6}$$

Example III–4. A parked car is hit head on by another car. Three seconds after the collision, the car that was parked is now traveling a 8 km/hr. What is the average acceleration of this car in SI units?



$$\begin{aligned}
\vec{\overline{a}} &= \frac{\vec{v}_{1\rm f} - \vec{v}_{1\rm i}}{t_{\rm f} - t_{\rm i}} = \frac{(8\ {\rm km/hr})\,\hat{x} - 0\,\hat{x}}{3\ {\rm s} - 0\ {\rm s}} = \frac{8}{3}\,\frac{\rm km}{\rm hr}\,\frac{1}{\rm s}\,\hat{x} \\
&= 2.667\ \frac{\rm km}{\rm s}\,\frac{1}{\rm hr}\cdot\frac{1\ {\rm hr}}{3600\ {\rm s}}\,\hat{x} = 7.407\times10^{-4}\ \frac{\rm km}{{\rm s}^2}\,\hat{x} \\
&= 7.407\times10^{-4}\ \frac{\rm km}{{\rm s}^2}\cdot\frac{10^3\ {\rm m}}{1\ {\rm km}}\,\hat{x} = 7.407\times10^{-1}\ \frac{\rm m}{{\rm s}^2}\,\hat{x} \\
&= (0.7407\ {\rm m/s}^2)\,\hat{x} = (0.7\ {\rm m/s}^2)\,\hat{x}\ .
\end{aligned}$$

(Remember, round the number such that it is consistent with the significant digits of the input numbers.)

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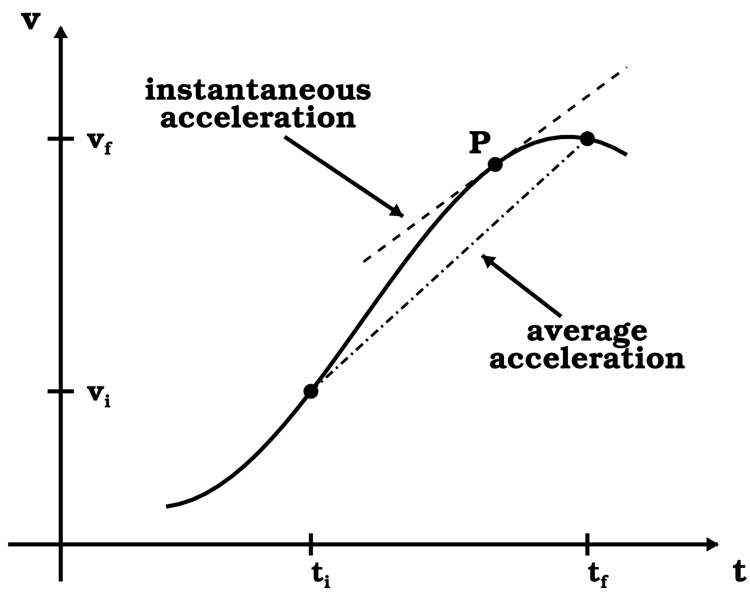
2. The **instantaneous acceleration** of an object at a certain time equals the slope of the velocity-time graph at that instant in time (*i.e.*, tangent line of $v(t)$ at that point).



$$\boxed{\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}} \equiv \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right) = \frac{d^2\vec{x}}{dt^2}\,, \qquad \text{(III-7)}$$

where the $d^2\vec{x}/dt^2$ is the double derivative of the displacement with respect to time from calculus.

E. One-Dimensional (1-D) Motion with Constant Acceleration.

1. Equations of 1-D motion:

   a) Let initial conditions be described with the subscript "$\circ$" $\Longrightarrow x_\circ, v_\circ, \&\ t_\circ = 0$ (from this point forward, with a few exceptions, we will set the initial time to zero).

   b) Let the final conditions be arbitrary $\Longrightarrow$ no subscript is written: $x, v, \&\ t$.

   c) For constant acceleration $\Longrightarrow \dfrac{\Delta v}{\Delta t}$ (*i.e.*, the slope) is constant (*i.e.*, it does not change with time).

$$\boxed{\text{average acceleration} = \text{instantaneous acceleration}}$$

i) Velocity (*i.e.*, instantaneous velocity from acceleration definition):

$$a = \frac{v_{\rm f} - v_{\rm i}}{t_{\rm f} - t_{\rm i}} = \frac{v - v_\circ}{t - 0} = \frac{v - v_\circ}{t}$$

or

$$at = v - v_\circ \ ,$$

and finally,

$$\boxed{v = v_\circ + at} \quad \text{for constant ``}a\text{''}. \qquad \text{(III-8)}$$

ii) Average velocity:

$$\boxed{\overline{v} = \frac{v_\circ + v}{2}} \quad \text{for constant ``}a\text{''}. \qquad \text{(III-9)}$$

iii) Displacement:

$$\overline{v} = \frac{x - x_\circ}{t}, \quad \text{let } x_\circ = 0 \ ,$$

$$\text{then } \overline{v} = \frac{x}{t}, \quad \text{or } x = \overline{v}t \quad \text{and}$$

$$\boxed{x = \frac{1}{2}\,(v_\circ + v)\ t} \ . \qquad \text{(III-10)}$$

Substituting Eq. (III-8) for v gives

$$\begin{aligned} x &= \frac{1}{2}\,(v_\circ + at + v_\circ)\ t \\ &= \frac{1}{2}\,(2v_\circ + at)\ t \\ &= \left(v_\circ + \frac{1}{2}at\right)\ t \end{aligned}$$

$$\boxed{x = v_\circ t + \frac{1}{2}at^2} \quad \text{for constant ``}a.\text{''} \qquad \text{(III-11)}$$

iv) Velocity (*i.e.*, instantaneous velocity, yet another way):

— Take Equation (III-8) and solve for t:

$$\begin{aligned} v &= v_\circ + at \\ &\Downarrow \\ at &= v - v_\circ \Longrightarrow t = \frac{v - v_\circ}{a} \end{aligned}$$

— Plug this value of t into Equation (III-10):

$$\begin{aligned} x &= \frac{1}{2}\,(v + v_\circ)\,t = \frac{1}{2}\,(v + v_\circ)\left(\frac{v - v_\circ}{a}\right) \\ x &= \frac{(v + v_\circ)\,(v - v_\circ)}{2a} = \frac{v^2 - vv_\circ + v_\circ v - v_\circ^2}{2a} \\ x &= \frac{v^2 - v_\circ^2}{2a} \\ 2ax &= v^2 - v_\circ^2 \end{aligned}$$

or

$$\boxed{v^2 = v_\circ^2 + 2ax} \quad \text{for constant “}a\text{.”} \qquad \text{(III-12)}$$

d) Summary:

Equation	**Information Given by Equation**
$v = v_\circ + at$	Velocity as a function of time.
$v^2 = v_\circ^2 + 2ax$	Velocity as a function of displacement.
$x = v_\circ t + \frac{1}{2}at^2$	Displacement as a function of time.
$x = \frac{1}{2}\,(v + v_\circ)\,t$	Displacement as a function of time & velocity.

Note: In the above equations, “a” is constant and the motion is in a straight line starting at the origin ($x_\circ = 0$)

at $t_\circ = 0$. If the initial position and position are not zero, the equations of motion take the following form:

Equation	**Function**
$v = v_\circ + a(t - t_\circ)$	$v(t)$
$v^2 = v_\circ^2 + 2a(x - x_\circ)$	$v(x)$
$x = x_\circ + v_\circ(t - t_\circ) + \frac{1}{2}a(t - t_\circ)^2$	$x(t)$
$x = x_\circ + \frac{1}{2}\left(v + v_\circ\right)\left(t - t_\circ\right)$	$x(v, t)$

2. Problem-Solving Strategy for Accelerated Motion in 1-D:

a) Make sure all of the units of the parameters in the problem are consistent.

b) Choose a coordinate system (and draw a picture).

c) Make a list of all of the quantities given in the problem and a separate list of those to be determined.

d) Select those equations that will allow you to determine the unknown parameters (*i.e.*, variables).

e) Make sure your answer is consistent with the diagram drawn in part (b).

Example III–5. A car starts from rest at a stop sign and accelerates (at a constant rate) to the posted speed limit of 50 km/hr. The car reaches the speed limit after traveling 107 m. What is the average velocity and acceleration of the car? How long does it take you to reach the speed limit?

a) Units consistent? NO!

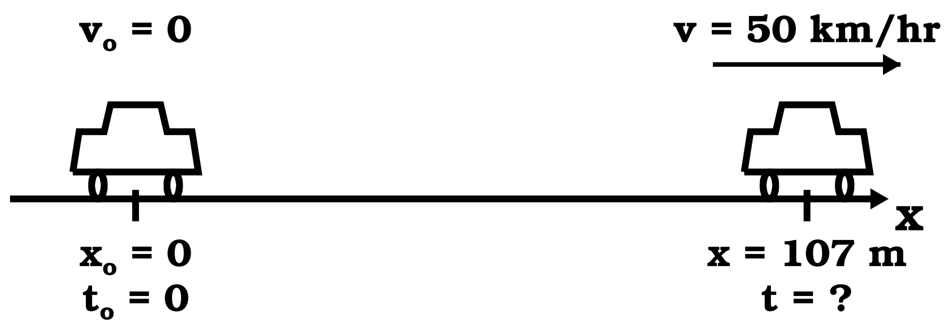
$x = 107$ m & $v = 50$ km/hr. We want SI units, so change km into m:

$$v = 50\,\frac{\text{km}}{\text{hr}} \cdot 10^3\,\frac{\text{m}}{\text{km}} = 5.0 \times 10^4 \text{ m/hr.}$$

Change hours to seconds:

$$v = 5.0 \times 10^4 \frac{\text{m}}{\text{hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} = 13.89 \text{ m/s } = 14. \text{ m/s.}$$

b) Choose a coordinate system (and draw picture):



c) Quantities given:

$$t_\circ = 0, \quad x_\circ = 0, \quad v_\circ = 0, \quad v = 14. \text{ m/s}, \quad x = 107 \text{ m}$$

Unknown quantities:

$$t, \quad \overline{v}, \quad a.$$

d) Choose equations:

$$\begin{aligned} \overline{v} &= \frac{v_\circ + v}{2} && \text{(for } \overline{v}) \\ v^2 &= v_\circ^2 + 2ax && \text{(for } a) \\ v &= v_\circ + at && \text{(for } t) \end{aligned}$$

e) Calculate results and check with figure:

$$\overline{v} = \frac{v_\circ + v}{2} = \frac{0 + 14. \text{ m/s}}{2} = \frac{14}{2} \text{ m/s}$$

$$\boxed{\overline{v} = 7.0 \text{ m/s}}$$

$$
\begin{aligned}
v^2 = v_\circ^2 + 2ax \quad \Longrightarrow \quad 2ax &= v^2 - v_\circ^2 \\
a &= \frac{v^2 - v_\circ^2}{2x} \\
a &= \frac{(14.\ \mathrm{m/s})^2 - 0^2}{2(107\ \mathrm{m})} = \frac{14^2\ \mathrm{m^2 s^{-2}}}{214\ \mathrm{m}} \\
&= 9.16 \times 10^{-1}\ \mathrm{m\ s^{-2}}
\end{aligned}
$$

$$\boxed{a = 0.92\ \mathrm{m/s^2}}$$

$$v = v_\circ + at \quad \Longrightarrow \quad at = v - v_\circ \quad \Longrightarrow \quad t = \frac{v - v_\circ}{a}$$

$$t = \frac{14\ \mathrm{m/s}\ - 0}{0.92\ \mathrm{m/s^2}} = 15.22\ \frac{\mathrm{m\ s^{-1}}}{\mathrm{m\ s^{-2}}}$$

$$\boxed{t = 15\ \mathrm{s}}$$

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3. Freely Falling Bodies.

a) **Galileo Galilei** is the father of experimental physics $\Longrightarrow$ carried out a variety of experiments in mechanics, the study of motion.

i) Near the surface of the Earth, bodies fall at the same rate of acceleration <u>independent</u> of the body's mass.

ii) Air resistance can affect the rate at which a body falls.

iii) Note that Galileo is also the first person to observe celestial objects in the sky with a telescope $\Longrightarrow$ hence is also the father of observational astronomy.

b) Bodies in free fall:

i) A freely falling body is an object moving only under the influence of gravity, regardless of its initial motion.

ii) Objects thrown upward or downward and those released from rest are all falling freely once they are released (even if they are initially going up).

iii) Once a body is in free fall, all objects have an acceleration downward $\Longrightarrow$ this free-fall acceleration is called the **surface gravity** $\vec{g}$.

iv) A gravitating body's surface gravity depends upon the total mass and size of the gravitating body.

v) The Earth's surface gravity at sea-level is

$$\boxed{g = 9.80 \text{ m/s}^2} = 980 \text{ cm/s}^2 = 32 \text{ ft/s}^2. \tag{III-13}$$

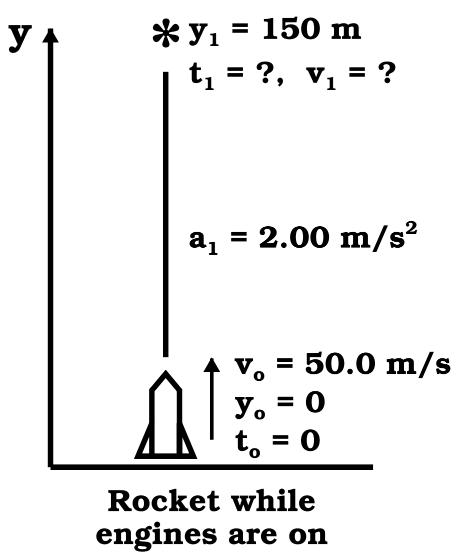
vi) Actually, bodies dropped in the Earth's atmosphere will experience a frictional force from air resistance in addition to the gravitational force. In this course, we will ignore air resistance (but it is important in reality).

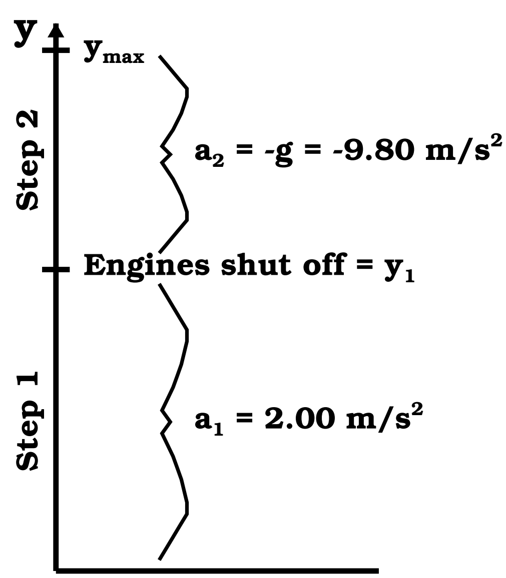
Example III–6. Problem 2.53 (Page 53) from the Serway & Vuille textbook: A model rocket is launched straight upward with an initial speed of 50.0 m/s. It accelerates with a constant upward acceleration of 2.00 m/s^2 until its engines stop at an altitude of 150 m. (a) What can you say about the motion of the rocket after its engines stop? (b) What is the maximum height reached by the rocket? (c) How long after lift-off does the rocket reach its maximum height? (d) How long is the rocket in the air?

a) What can be said about the motion of the rocket after the engines stop?

The rocket will continue upward, but start to decelerate due to the Earth's gravitational field until the upward velocity reaches zero. The rocket then begins to fall back to the ground with an acceleration equal to the Earth's surface gravity (*i.e.*, 9.80 m/s^2).

b) What is the max height?





<u>Step 1:</u>

As we will see in §IV of the notes, we use y for vertical displacement. We first need to calculate the velocity of the rocket when the engine is turned off: $v_1 \longrightarrow$ we are given $a_1 = 2.00$ m/s^2, $y_1 = 150$ m, $v_0 = 50.0$ m/s, and we can set $y_\circ = 0$ (*i.e.*, the ground) and $t_\circ = 0$. Then we can determine v_1 from the following equation:

$$v_1^2 = v_\circ^2 + 2a_1 y_1$$

$$\begin{aligned} &= (50.0 \text{ m/s})^2 + 2(2.00 \text{ m/s}^2)(150 \text{ m}) \\ &= 2.50 \times 10^3 \text{ m}^2/\text{s}^2 + 6.00 \times 10^2 \text{ m}^2/\text{s}^2 \end{aligned}$$

or

$$\begin{array}{rrll} & 25.\underline{0} & \times 10^2 & \text{m}^2/\text{s}^2 \\ & 6.0\underline{0} & \times 10^2 & \text{m}^2/\text{s}^2 \\ \hline v_1^2 = & 31.\underline{0}0 & \times 10^2 & \text{m}^2/\text{s}^2 \\ & 31.0 & \times 10^2 & \text{m}^2/\text{s}^2 \\ & 3.10 & \times 10^3 & \text{m}^2/\text{s}^2 \end{array}$$

$$v_1 = \sqrt{3.10 \times 10^3 \text{ m}^2/\text{s}^2} = 5.57 \times 10^1 \text{ m/s} = 55.7 \text{ m/s}.$$

Now we need to calculate $y_{\text{max}} \equiv y_2$.

Step 2:

We will need y_1, a_2, and v_1, also $v_2 = 0$ (rocket comes to rest). Our initial velocity is now v_1 and our final velocity is v_2, so we can write:

$$v_2^2 = v_1^2 + 2a_2 \Delta y = v_1^2 + 2a_2 \left(y_2 - y_1\right) \ ,$$

and note that $y_{\text{max}} = y_2$ and that the acceleration is now the downward acceleration due to gravity: $a_2 = -g = -9.80 \text{ m/s}^2$.

From $v_2^2 = v_1^2 + 2a_2 \left(y_2 - y_1\right)$, we solve for y_2:

$$\begin{aligned} y_2 - y_1 &= \frac{v_2^2 - v_1^2}{2a_2} = \frac{v_2^2 - v_1^2}{-2g} \\ y_2 = y_{\text{max}} &= y_1 + \frac{v_1^2 - v_2^2}{2g} \\ y_{\text{max}} &= 150 \text{ m} + \frac{(55.7 \text{ m/s})^2 - 0^2}{2(9.80 \text{ m/s}^2)} \\ &= 150 \text{ m} + \frac{3.10 \times 10^3 \text{ m}^2/\text{s}^2}{1.96 \times 10^1 \text{ m/s}^2} \\ &= 150 \text{ m} + 1.58 \times 10^{3-1} \frac{\text{m}^2\text{s}^{-2}}{\text{m s}^{-2}} \\ &= 150 \text{ m} + 1.58 \times 10^2 \text{ m} \ = 150 \text{ m} + 158 \text{ m} \end{aligned}$$

$$\boxed{y_{\rm max} = 308 \text{ m} .}$$

c) How long does it take to reach $y_{\rm max}$?

Step 1:

During this phase of our flight, we have $t_\circ = 0$, $v_\circ = 50.0$ m/s, and $v_1 = 55.7$ m/s. With these initial parameters, we can now determine t_1, which is the time when the rocket reaches y_1. Using $v_{\rm f} = v_{\rm i} + a(t_{\rm f} - t_{\rm i})$, we get $v_1 = v_\circ + a_1(t_1 - t_\circ) = v_\circ + a_1 t_1$ for Step 1 of out flight. Solving for t_1 gives:

$$\begin{aligned} t_1 &= \frac{v_1 - v_\circ}{a_1} \\ &= \frac{55.7 \text{ m/s } - 50.0 \text{ m/s}}{2.00 \text{ m/s}^2} \\ &= \frac{5.7 \text{ m s}^{-1}}{2.00 \text{ m s}^{-2}} = 2.85 \ \frac{\text{s}^2}{\text{s}} = 2.9 \text{ s} \end{aligned}$$

Step 2:

We can now solve for t_2, which is the time the rocket reaches $y_2 = y_{\rm max}$. For this Step 2 of our flight, we have $v_2 = v_1 + a_2(t_2 - t_1)$. Solving for t_2 we get

$$\begin{aligned} t_2 - t_1 &= \frac{v_2 - v_1}{a_2} \\ t_2 &= t_1 + \frac{v_2 - v_1}{a_2} \\ &= 2.\underline{9} \text{ s} + \frac{0 - 55.7 \text{ m/s}}{-9.80 \text{ m/s}^2} \\ &= 2.\underline{9} \text{ s} + \frac{-55.7 \text{ s}^2}{-9.80 \text{ s}} \\ &= 2.\underline{9} \text{ s} + 5.6\underline{8} \text{ s} = 8.\underline{5}8 \text{ s} , \end{aligned}$$

hence

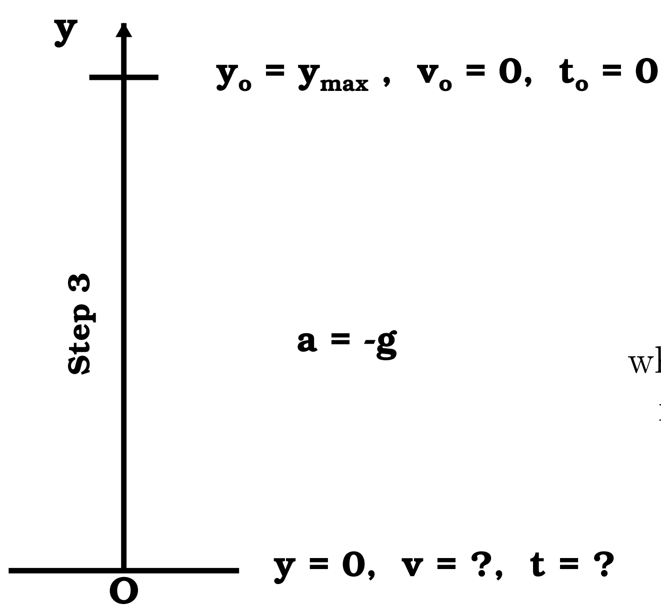
$$\boxed{t_2 = 8.6 \text{ s} .}$$

d) How long is the rocket in the air?

We just figured out how long it took to get to $y_{\rm max}$, we now have to find out how long it takes to fall, then add the two numbers together.

Step 3:

Use $y = y_\circ + v_\circ(t - t_\circ) + \frac{1}{2}a(t - t_\circ)^2$ and solve for t. Note that for this Step 3, using what we have calculated from Steps 1 and 2, we are free $t_\circ = 0$. The remaining known parameters are $v_\circ = 0$ (the velocity at the top of the trajectory $= v$ at $y_{\rm max} = y_2$ from part b), $y = 0$ (the ground), and $a = -g$ (gravitational acceleration is pointing downward with respect to y).



$\Longrightarrow$ Set up a new problem!

Want to solve for t, what we are given is shown in the figure to the left.

Carrying out the mathematics to determine the time needed to reach the ground from our maximum height (t_3), we get

$$\begin{aligned} y &= y_{\rm max} + v_\circ(t - t_\circ) + \frac{1}{2}\,a(t - t_\circ)^2 \\ 0 &= y_{\rm max} + 0 \cdot (t - 0) - \frac{1}{2}\,g(t - 0)^2 \end{aligned}$$

$$
\begin{aligned}
&= y_{\rm max} - \frac{1}{2}\,gt \\
\frac{1}{2}\,gt^2 &= y_{\rm max} \\
t^2 &= \frac{2y_{\rm max}}{g} \\
t = t_3 &= \sqrt{\frac{2y_{\rm max}}{g}} = \sqrt{\frac{2 \cdot 308 \text{ m}}{9.80 \text{ m/s}^2}} \\
t_3 &= \sqrt{62.9 \text{ s}^2} = 7.93 \text{ s}
\end{aligned}
$$

The total time in the air is then just the sum of the time found in part (b) and the time found in part (c): $t_{\rm total} = t_2 + t_3$, or

$$t_{\rm total} = 8.\underline{6} \text{ s } + 7.9\underline{3} \text{ s } = \boxed{16.5 \text{ s }.}$$
