CHAPTER 9

THE SCHLICK FRESNEL APPROXIMATION

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ABSTRACT

Reflection and refraction magnitudes are modeled using an approximation of the Fresnel reflectance equations called the Schlick approximation. This chapter presents the Schlick approximation, analyzes its error when compared to the full Fresnel reflectance equations, and motivates its use in ray tracing dielectric (non-conducting) materials. It further discusses the Schlick approximation in the context of metallic surfaces and shows a possible extension for the Schlick approximation to more accurately model reflectance of metals.

9.1 INTRODUCTION

When a ray hits a surface during ray tracing, we need to compute how much light should be transmitted back along it (toward the camera for a primary ray or toward the earlier path vertex for a secondary ray). Light transmitted back along a ray hitting a surface is either reflected or refracted from that surface. Some materials, like glass, will both reflect *and* refract light. For such materials, a portion of the energy of an incoming light ray is reflected—the rest is refracted.

9.2 THE FRESNEL EQUATIONS

Reflection and refraction magnitude vary with a material's refractive index[1] η and the angle of the incoming light θ_i (the incoming angle relative to the surface normal). The portion of light reflected from a ray traveling through a material with refractive index η_1 that hits a material with refractive index η_2 at

[1] A material's *refractive index* is the ratio of the speed of light in a vacuum to the speed of light in that material. For example, water has a refractive index of η = 1.333.

A. Marrs, P. Shirley, I. Wald (eds.), *Ray Tracing Gems II*, https://doi.org/10.1007/978-1-4842-7185-8_9

angle θ_i is given by the Fresnel equations:

$$R_s = \left| \frac{\eta_1 \cos\theta_i - \eta_2 \sqrt{1 - \left(\frac{\eta_1}{\eta_2} \sin\theta_i\right)^2}}{\eta_1 \cos\theta_i + \eta_2 \sqrt{1 - \left(\frac{\eta_1}{\eta_2} \sin\theta_i\right)^2}} \right|^2, \tag{9.1}$$

$$R_p = \left| \frac{\eta_1 \sqrt{1 - \left(\frac{\eta_1}{\eta_2} \sin\theta_i\right)^2} - \eta_2 \cos\theta_i}{\eta_1 \sqrt{1 - \left(\frac{\eta_1}{\eta_2} \sin\theta_i\right)^2} + \eta_2 \cos\theta_i} \right|^2. \tag{9.2}$$

The s and p subscripts denote the polarization of light: s is perpendicular to the propagation direction and p is parallel. Most ray tracers ignore light polarization by simply averaging the two equations to arrive at a final reflection magnitude $R = (R_s + R_p)/2$.

9.3 THE SCHLICK APPROXIMATION

The Fresnel equations are exact, but complicated to evaluate. In computer graphics, a simpler approximation, called Schlick's approximation, is often used instead. Schlick's approximation is given by

$$R(\theta_i) = R_0 + (1 - R_0)(1 - \cos\theta_i)^5, \tag{9.3}$$

where θ_i is again the angle of the incident ray and R_0 is the reflectivity of the material at normal incidence (you can check this by substituting $\theta_i = 0$). Here is for the Schlick approximation:

```
/** Compute reflectance, given the cosine of the angle of incidence and the
      reflectance at normal incidence. */
vec3 schlickFresnel(vec3 R0, float cos_i) {
    return lerp(R0, vec3(1.0f), pow((1.0f - cos_i), 5.0f));
}
```

The Schlick approximation is much faster to evaluate than the full Fresnel equations. In an optimized implementation, the Schlick approximation can be 32 times faster with less than 1% average error [4]. Further, the Schlick approximation only depends on the reflectance at normal incidence (R_0), which is known for many materials, whereas the Fresnel equations depend on the refractive indices η_1 and η_2.[2]

[2]Magnitude R_0 can also be computed from refractive indices like so: $R_0 = ((\eta_1 - \eta_2)/(\eta_1 + \eta_2))^2$. This makes it possible to use the Schlick approximation for reflectance at the interface of arbitrary materials. For the most common case of a material boundary with air, only reflectance at normal incidence is required.

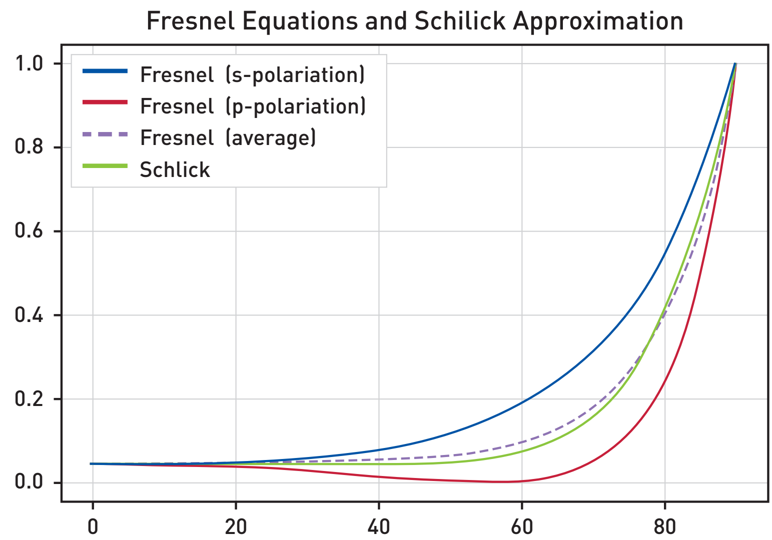

Figure 9-1. *Graph of the Fresnel equations for s-polarization (blue), p-polarization (red), the average of s- and p-polarization (dashed purple), and the Schlick approximation (green). The x-axis shows the angle of incident light (θ_i, in degrees), and the y-axis shows the portion (0–1) of incident light reflected. The Schlick approximation closely matches the averaged Fresnel equations. The Fresnel equations were computed with $\eta_1 = 1$ and $\eta_2 = 1.5$ (simulating the interface of air and glass, with incident light coming from the air). Following these, the Schlick approximation used* 0.04 *for the Fresnel reflectance at normal incidence.*

Figure 9-1 shows the magnitude of reflected radiance as a function of incident angle for a light ray traveling through air ($\eta = 1.0$) and striking glass ($\eta = 1.5$). The reflectance of *s*-polarization, *p*-polarization, their average, and the Schlick approximation are shown. Relative to the averaged Fresnel equations, the Schlick approximation gives less than 1% average error with a maximum error of approximately 3.6% at an incident angle of 85°.

The key insight is that averaging R_s and R_p already introduces an approximation to the polarized reflectance. This approximation can have high error, especially if incoming light is unevenly polarized (see Wolff and Kurlander [5] for a classic discussion of polarization in ray tracing). Given that this is already an approximation, it makes sense to fit the approximate curve with a simple polynomial instead of evaluating the full Fresnel and taking the average. The Schlick approximation achieves this.

9.4 DIELECTRICS VS. CONDUCTORS

So far when talking about reflective/refractive materials, we have been talking about *dielectrics*—materials that do not conduct electricity, such as water, wood, or stone. Materials that *do* conduct electricity, such as metals, are called *conductors*. Conductors require a more advanced form of the Fresnel equations with an additional parameter called the *extinction coefficient* (κ). The extinction coefficient represents the attenuation (amplitude reduction) of light in a volume—a lower extinction coefficient means that a material is less conductive.

The reflectance of a ray traveling through a dielectric material with refractive index η_d and striking a conductor with refractive index η_c and extinction coefficient κ at incident angle θ_i is given by

$$R_s = \frac{a^2 + b^2 - 2a\cos\theta_i + \cos^2\theta_i}{a^2 + b^2 + 2a\cos\theta_i + \cos^2\theta_i}, \tag{9.4}$$

$$R_p = R_s\frac{a^2 + b^2 - 2a\sin\theta_i\tan\theta_i + \sin^2\theta_i\tan^2\theta_i}{a^2 + b^2 + 2a\sin\theta_i\tan\theta_i + \sin^2\theta_i\tan^2\theta_i}, \tag{9.5}$$

with a^2 and b^2 given by

$$a^2 = \frac{1}{2\eta_d^2}\left\{\sqrt{\left(\eta_c^2 - \kappa^2 - \eta_d^2\sin^2\theta_i\right)^2 + 4\eta_c^2\kappa^2} + \left(\eta_c^2 - \kappa^2 - \eta_d^2\sin^2\theta_i\right)\right\}, \tag{9.6}$$

$$b^2 = \frac{1}{2\eta_d^2}\left\{\sqrt{\left(\eta_c^2 - \kappa^2 - \eta_d^2\sin^2\theta_i\right)^2 + 4\eta_c^2\kappa^2} - \left(\eta_c^2 - \kappa^2 - \eta_d^2\sin^2\theta_i\right)\right\}. \tag{9.7}$$

With extinction coefficient κ set to 0, these equations simplify to the dielectric Fresnel equations (Equations 9.1 and 9.2).

9.5 APPROXIMATIONS FOR MODELING THE REFLECTANCE OF METALS

The complex Fresnel equations are even more complicated and expensive to evaluate than the original Fresnel equations. Further, when using the complex Fresnel equations in practice, it is difficult to gain the benefit of physical correctness that should come with the higher evaluation cost (see Hoffman's discussion of these issues [2]).

For these reasons, we would prefer to continue using the Schlick approximation (or another approximation) for conductors just as we used it for

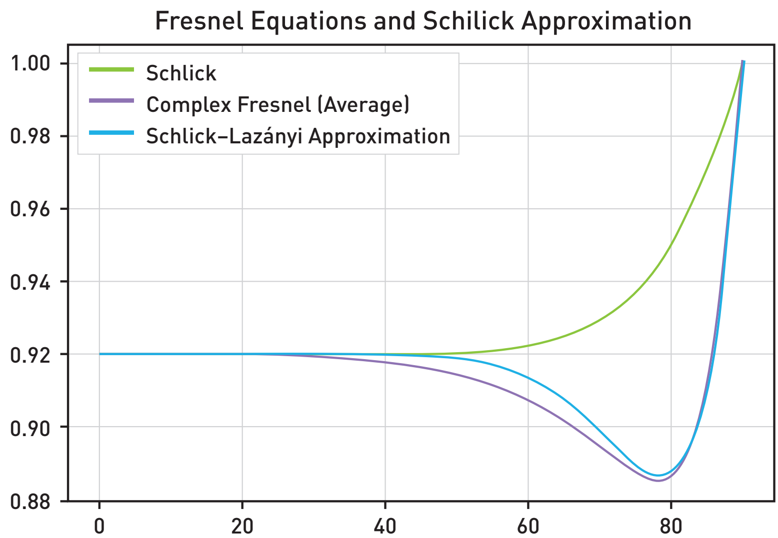

Figure 9-2. *Reflectance of aluminum at 450 nm under the complex Fresnel equations (magenta), the Schlick approximation (green), and the re-parameterized Schlick–Lazányi approximation (cyan). At a wavelength of 450 nm for incident light, aluminum has refractive index* $\eta = 0.61722$ *and extinction coefficient* $\kappa = 5.3031$*. The Schlick–Lazányi approximation was computed using* $\alpha = 6$ *and* $a = 1.136$ *(as in Hoffman [2]). Note that because the graph shows data for a specific wavelength,* a *is scalar valued.*

dielectrics. The simplest option that works well in practice is to use an RGB color value for R_0 (instead of the scalar value presented in Equation 9.3) to approximate different reflection behavior along the visible spectrum. Further, using an RGB color value allows one to use the same material shader to evaluate reflection magnitude for both metals and dielectrics.

Even when using an RGB color, the Schlick approximation can still show significant error when evaluated for conductors, especially at glancing angles (see Gulbrandsen [1] for a parameterization that addresses glancing angles specifically). This error can be decreased using the Lazányi–Schlick approximation [3], which adds an error term to the RGB Schlick approximation to account for metals:

$$R(\theta_i) = R_0 + (1 - R_0)(1 - \cos\theta_i)^5 - a\cos\theta_i(1 - \cos\theta_i)^\alpha, \tag{9.8}$$

where R_0 is the RGB reflectance at normal incidence and a and α are configurable parameters.[3] It can be used with a default parameter of $\alpha = 6$.

[3] Equation 9.8 presents the error term alongside the RGB Schlick approximation, matching Hoffman [2], and so differs slightly from the original paper by Lazányi and Szirmay-Kalos [3], which presents a Schlick approximation parameterized by η and κ.

The parameter *a* is computed per material based on the Fresnel reflectance of the material at the angle where the Lazányi error term is highest. For the exact equation for *a* and the derivation of the default value for *alpha*, see Hoffman [2]. Here is code for the Lazányi–Schlick approximation:

```
/** Return reflectance, given the cosine of the angle of incidence and the
    reflectance at normal incidence. */
vec3 schlickLazanyi(vec3 R0, float cos_i, vec3 a, float alpha) {
    return schlickFresnel(R0, cos_i) -
           a * cos_i * pow(1 - cos_i, alpha);
}
```

Figure 9-2 shows the reflectance curve for aluminum at a wavelength of 450 nm under the complex Fresnel equations, the basic Schlick approximation, and the re-parameterized Schlick–Lazányi approximation. Compared to the complex Fresnel equations, the Schlick approximation gives an average error of 1.5%, with a maximum error of 6.6%. The re-parameterized Schlick–Lazányi approximation is much closer with an average error of 0.22%, with a maximum error of 0.65%.

REFERENCES

[1] Gulbrandsen, O. Artist friendly metallic Fresnel. *Journal of Computer Graphics Techniques*, 3(4):64–72, 2014. http://jcgt.org/published/0003/04/03/.

[2] Hoffman, N. Fresnel equations considered harmful. In *Eurographics Workshop on Material Appearance Modeling*, pages 7–11, 2019. DOI: 10.2312/mam.20191305.

[3] Lazányi, I. and Szirmay-Kalos, L. Fresnel term approximations for metals. In *World Society for Computer Graphics*, pages 77–80, Jan. 2005.

[4] Schlick, C. An inexpensive BRDF model for physically-based rendering. *Computer Graphics Forum*, 13(3):233–246, 1994. DOI: 10.1111/1467-8659.1330233.

[5] Wolff, L. B. and Kurlander, D. J. Ray tracing with polarization parameters. *IEEE Computer Graphics and Applications*, 10(6):44–55, 1990. DOI: 10.1109/38.62695.