Consumer Information, Heterogeneous Sellers, and Oligopolistic Competition

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Abstract

Consumers vary in price information: captive consumers may know only one price from a seller, while informed consumers know several prices. We study a homogeneous-good oligopoly market where sellers of heterogeneous costs compete on price under limited consumer information. We characterize a unique symmetric monotone Bayesian equilibrium. Our main results show that 1) the number of sellers alone does not affect prices given consumer information; 2) prices fall when captive consumers become sufficiently more informed; 3) prices of sellers with higher (lower) costs rise (fall) when informed consumers become sufficiently more informed. The effects on seller profits and consumer surplus are also examined and illustrated through a triopoly example.

JEL Codes: D11, D43, D83, L13

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1 Introduction

Consider a homogeneous-good market where consumers have limited information - they only know the prices of a subset of sellers. Some consumers are "captive" to one price from one

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seller, while others are more informed and compare prices from several sellers. On the other hand, sellers' costs are different and private. They know their own costs, but have incomplete information about other competitors' costs and consumer information. What would be an equilibrium pricing strategy for sellers? How do prices change when consumers become better informed? What are the implications for consumer welfare and seller profits?

To answer these questions, we analyze an oligopolistic model with incomplete information. There are a set of sellers, $N(n = |N|)$, in the market. Consumers only know prices from a subset of sellers, i.e., consideration set. Consideration sets vary across consumers. We assume a commonly known prior distribution, $\mu = (\mu_1, \mu_2, ..., \mu_n)$, on the size of consideration set, μ_k denotes the fraction of consumers consider k sellers. Also, sellers are anonymous and identical - every seller has same probability to be considered given the size of consideration set, e.g., the probability for every seller to be considered by a consumer with consideration set of size k is $\frac{k}{n}$. Based on the assumptions, sellers infer a posterior distribution over the number of sellers she would compete for a consumer. Furthermore, each seller pays a private marginal cost to sell a good. They don't know the costs of other sellers, which are randomly drawn from a known distribution. Under this imperfect information, each seller sets a uniform price to maximize her revenue.

First, we show that there exists a monotone pure-strategy Bayesian equilibrium such that sellers set price as an increasing function of cost. In the equilibrium, the number of sellers alone has no effect on the pricing strategy. Intuitively, sellers only consider the potential number of competitors when setting their prices. An increase/decrease in the total number of sellers has no effect on price competition if consumer information remains unchanged (μ).

Second, we examine how the consumer information affects prices. We categorize consumers into two groups, *captive consumers*, who consider only one price from one seller; *informed consumers*, who consider more than one price. We find that sellers generally reduce prices when captive consumers become sufficiently informed. In contrast, sellers with higher (lower) costs increase (decrease) prices as informed consumers become sufficiently more informed. Given the equilibrium pricing strategy, the revenue of sellers with higher costs depend more on the captive consumers and less on the informed consumers since they are more likely to lose in competition. As informed consumers become more informed, sellers with higher costs tend to "give up" on winning these informed consumers and earn more profits from captive consumers, thus raising their prices. However, the effect is reversed when costs are lower. More informed consumers intensify price competition between lower-cost sellers, and their prices fall.

Finally, we study the impact of information on seller profits and consumer surplus. First, we find that an increase in consumer information may harm the welfare of captive consumers, as the expected price may increase. Second, when consumers are more informed, the change in the value of sellers' profits exhibits a U-shape - sellers with intermediate costs lose more and sellers with higher costs or lower costs lose less[1].

2 Model

Consider an oligopoly where a set of risk neutral sellers, N ($n \equiv |N|$), sell a homogeneous good to a unit mass of consumers. Selling each good incurs a fixed marginal cost, which is privately known to each seller. The costs, $c_1, c_2, ..., c_n$, are randomly and independently drawn from a commonly-known, atomless, and differentiable upper cumulative distribution, M, on the support of $[\underline{c}, \overline{c}]$ ($0 \leq \underline{c} < \overline{c} < 1$). Each seller $j \in N$ sets a price, p_j, to maximize her profits.

Consumers demand one unit of the good and their valuations on the good are homogeneous and normalized to one. Each consumer i only considers buying a good from a subset of sellers, $K_i \subseteq N$. K_i ($k_i \equiv |K_i|$) is the consideration set of consumer i. Consumer i will buy the good at the minimum price offered by the sellers in his consideration set, $\min\{p_j : j \in K_i\}$, if it is less than or equal to one.

The collection of consideration sets takes many different forms. This creates different patterns of competition[2]. In this model, we make the simplest assumption: sellers are identical and anonymous, so all sellers are equally likely to be considered. For instance, each seller j is

[1] Sellers with lower costs may benefit from more-informed consumers.

[2] Imagine a chain store competing with some local stores. The chain store is so well known that all consumers consider buying goods from it. However, the other local stores are only considered by different subsets of consumers. The chain store competes with other local stores for different groups of consumers. Local stores hardly compete with each other.

considered by consumer i with a prior probability, $Pr(j \in K_i | k_i = k) = \frac{k}{n} = \binom{n-1}{k-1} / \binom{n}{k}$, when consumer i considers k sellers. Thus, for sellers, consumers differ only in the size of their consideration sets. We call *captive consumers* those who consider only one seller ($k_i = 1$), and *informed consumers* the others ($k_i > 1$).

Sellers do not know the sizes of consideration sets of consumers who request price quotes. Instead, we assume that there is a prior distribution on the sizes of consumers' consideration sets, $\mu \equiv (\mu_1, \mu_2, ..., \mu_n) \in \triangle^{n-1}$, where μ_k is the fraction of consumers with consideration sets of size k, or $Pr(k_i = k) = \mu_k$. Seller j infers a posterior probability on k_i when consumer i asks for a price quote,

$$\begin{aligned} Pr(k_i = k | j \in K_i) &= \frac{Pr(j \in K_i | k_i = k) Pr(k_i = k)}{\sum_{\ell=1}^{n} Pr(j \in K_i | k_i = \ell) Pr(k_i = \ell)} \\ &= \frac{\frac{k}{n}\mu_k}{\sum_{\ell=1}^{n} \frac{\ell}{n}\mu_\ell} \end{aligned} \tag{1}$$

(1) is also the probability that a seller competes with $k-1$ other sellers when she is asked for a price quote by a consumer. Each seller j sets a uniform price to compete with other sellers and maximize her profits. Let $\rho_j : [\underline{c}, \overline{c}] \to [\underline{c}, 1]$ be the pure pricing strategy of seller j. Given strategies, there exists an upper CDF of price, F, such that $F(p)^{k-1}$ is the probability that a price p is the minimum price while competing with $k-1$ other sellers. The sum, $\sum_{k=1}^{n} \frac{\frac{k}{n}\mu_k}{\sum_{\ell=1}^{n} \frac{\ell}{n}\mu_\ell} F(p)^{k-1}$, is the probability that price p is the minimum price in general. So, the quantity of demand at price p is

$$\begin{aligned} Q(p, \rho_{-j}; \mu) \equiv &\underbrace{(\sum_{\ell=1}^{n} \frac{\ell}{n}\mu_\ell)}_{\text{Expected consumers}} \underbrace{\sum_{k=1}^{n} \frac{\frac{k}{n}\mu_k}{\sum_{\ell=1}^{n} \frac{\ell}{n}\mu_\ell} F(p)^{k-1}}_{\text{Prob of being the minimum price}} \\ &= \sum_{k=1}^{n} \frac{\mu_k k}{n} F(p)^{k-1} \end{aligned} \tag{2}$$

and seller $j \in N$ sets a single price, p_j, to maximize her expected profits,

$$\Pi_j(p_j, \rho_{-j}; c_j, \mu) = (p_j - c_j) Q(p_j, \rho_{-j}; \mu) \tag{3}$$

All sellers price simultaneously (à la Bertrand). To sum up, $\langle N, M, \mu, A, (\Pi_j)\rangle$ is a Beyesian game[3]. In the following, we focus our analysis on the symmetric pure-strategy Bayesian Nash Equilibrium (BNE) - all sellers use the same pricing strategy, $\rho_j(c) = \rho(c)$ for all $j \in N$. We show that this game has a unique monotone pure-strategy BNE such that sellers with higher marginal costs set higher prices. Intuitively, sellers with lower costs are more tolerant of lower prices in exchange for greater demand. Sellers with higher costs set higher prices, which results in lower demand but ensures a reasonable amount of profit from less-informed consumers. Formally, the single-cross property of the revenue function guarantees the incentive compatibility and the existence of pure-strategy BNE (Milgrom and Weber, 1982; Athey, 2001).

Definition 1 (Athey, 2001): A function $f : \mathbb{R}^2 \to \mathbb{R}$ satisfies the *single crossing property of incremental returns* (SCP) in $(x; t)$ if, for all $x_H > x_L$ and all $t_H > t_L$, $f(x_H, t_L) - f(x_L, t_L) \geq (>)0$ implies $f(x_H, t_H) - f(x_L, t_H) \geq (>)0$.

Lemma 1: *The profit function,* $\Pi(p_j, \rho_{-j}; c_j, \mu)$*, satisfies SCP.*

Proof: Let $t = -c$ and $x = -p$. For all $p_H > p_L \geq 0$ and all $c_H > c_L \geq 0$, $(p_L - c_H)Q(p_L) \geq (>)(p_H - c_H)Q(p_H)$[4] implies that

$$
\begin{aligned}
(p_L - c_L)Q(p_L) &= (p_L - c_H)Q(p_L) + (c_H - c_L)Q(p_L) \\
&\geq (>)(p_H - c_H)Q(p_H) + (c_H - c_L)Q(p_H) = (p_H - c_L)Q(p_H)
\end{aligned}
$$

Note that $(c_H - c_L)Q(p_L) \geq (c_H - c_L)Q(p_H)$ since the quantity of demand, Q, is weakly decreasing on price; $(c_H - c_L)Q(p_L) > (c_H - c_L)Q(p_H)$ when Q is strictly decreasing on price. Q.E.D.

Lemma 1 says that, for any seller j, if another seller j' with cost $c_{j'} > c_j$ prefers some p_L to p_H ($p_L < p_H$), then seller j will prefer p_L to p_H. Logically inversely, Lemma 1 also indicates

[3] In the tuple, $A \equiv [\underline{c}, 1]$

[4] For simplicity, $Q(\bullet)$ only takes the first argument of $Q(\bullet)$ in (2).

that $(p_L - c_L)Q(p_L) < (\leq)(p_H - c_L)Q(p_H)$ implies $(p_L - c_H)Q(p_L) < (\leq)(p_H - c_H)Q(p_H)$. Thus, if another seller j' with cost $c_{j'} < c_j$ prefers p_H to p_L ($p_L < p_H$), then seller j will prefer p_H to p_L. Lemma 1 further implies Lemma 2.

Lemma 2: *For any seller with marginal cost $c \in [\underline{c}, \overline{c}]$, if a pricing strategy $\rho : [\underline{c}, \overline{c}] \to [\underline{c}, 1]$ is continuous, strictly increasing, and locally optimal, then $\rho(c)$ is a BNE.*

Proof: $\rho(c)$ is a symmetric BNE if and only if $\rho(c)$ is a globally profit-maximizing for all c. If $\rho(c)$ is continuous, strictly increasing, and locally optimal, then Q will be strictly decreasing in price, and Lemma 1 will imply that $\rho(c)$ is preferred to any $p \in [\rho(\underline{c}), \rho(\overline{c})]$.

When $0 < \mu_1 \leq 1$, $\rho(\overline{c})$ is locally profit maximizing if and only if $\rho(\overline{c}) = 1$ since $Q(\rho(\overline{c}), \rho) = Q(1, \rho)$; when $\mu_1 = 0$, $\rho(\overline{c})$ is locally profit maximizing if and only if $\rho(\overline{c}) = \overline{c}$. For any $p < \rho(\underline{c})$, $\rho(\underline{c})$ is strictly preferred to p since $Q(\rho(\underline{c}), \rho) = Q(p, \rho)$. Hence, $\rho(c)$ is preferred to any other $p \in [\underline{c}, 1]$. Q.E.D.

Using Lemma 2, we can prove the existence of the equilibrium by finding a price function that is differentiable, strictly increasing, and locally optimal.

Theorem 1: *If $0 < \mu_1 < 1$, the game possesses a symmetric pure-strategy equilibrium such that price, ρ, is an increasing and continuous function of marginal cost, c*

Proof: This is proved by construction. Suppose that $\rho(c)$ is strictly increasing, and differentiable, we obtain the necessary condition on $\rho'(c)$ for $\rho(c)$ to be locally optimal. Finally, we show that $\rho'(c)$ is positive under the condition. The first order condition for $\rho(c)$ to be locally optimal is

$$\frac{\partial \Pi(\rho(c), \rho; c, \mu)}{\partial p} = \frac{\partial Q(\rho(c))}{\partial p}(\rho(c) - c) + Q(\rho(c)) = 0 \tag{4}$$

for all $c \in [\underline{c}, \overline{c}]$. So, we have

$$\frac{\partial Q(\rho(c))}{\partial p} = \frac{\partial F(\rho(c))}{\partial p} \sum_{k=2}^{n} \frac{\mu_k k(k-1)}{n} F(\rho(c))^{k-2} = -\frac{Q(\rho(c))}{\rho(c)-c} \tag{5}$$

Since $\rho(c)$ is a strictly increasing function of marginal cost, the probability of being the lowest price is equal to the probability of being the lowest-cost seller. Hence, $F(\rho(c)) = M(c)$, and the quantity of demand can be represented as a function of marginal cost,

$$Q(\rho(c)) = \mathcal{Q}(c) \equiv \sum_{k=1}^{n} \frac{\mu_k k}{n} M(c)^{k-1} \tag{6}$$

A marginal increase in price leads to a marginal decrease in the probability of being the lowest price/being the lowest priced seller, which depends on $\rho(c)$. Specifically, $\frac{\partial F(\rho(c))}{\partial p} = \frac{\partial M(c)}{\partial p} = \frac{\partial M(c)}{\partial c}\frac{\partial c}{\partial p} = \frac{M'(c)}{\rho'(c)}$. (5) can be written as

$$\frac{\partial Q(\rho(c))}{\partial p} = \frac{M'(c)}{\rho'(c)} \sum_{k=2}^{n} \frac{\mu_k k(k-1)}{n} M(c)^{k-2} = \frac{\mathcal{Q}'(c)}{\rho'(c)} \tag{7}$$

It follows from (5) and (7) that $\rho(c)$ is given by the following differential equation and boundary condition,

$$\begin{aligned} \rho'(c) &= -\frac{\mathcal{Q}'(c)}{\mathcal{Q}(c)}(\rho(c) - c) \\ \rho(\overline{c}) &= 1 \end{aligned} \tag{8}$$

Since $\mathcal{Q}'(c)$ is negative and lower bounded, there exists a locally optimal, differentiable $\rho(c)$ satisfying $\rho(c) > c$ and $\rho'(c) > 0$ for all c. Q.E.D.

The assumption that some but not all consumers are captive ($0 < \mu_1 < 1$) can be relaxed. In the absence of captive consumers ($\mu_1 = 0$), the highest-cost seller prices at her marginal cost, $\rho(\overline{c}) = \overline{c}$, and $\rho(c)$ is still increasing in c, which is distinct from the competitive equilibrium in price dispersion models with homogeneous sellers (Varian, 1980; Burdett and Judd, 1983). When all consumers are captive ($\mu_1 = 1$), the equilibrium replicates the monopoly price equilibrium in the models with homogeneous sellers.

3 Entry, information, and price

In this section, we examine the relationship between information, price, and the number of sellers. The first finding is that the number of sellers alone does not affect prices.

Corollary 1: *The prices are invariant to the number of sellers if μ is not changed.*

From Proposition 1, we have

$$\begin{aligned}\rho'(c) &= \frac{M'(c)\sum_{k=2}^{n}\frac{\mu_k k(k-1)}{n}M(c)^{k-2}}{\sum_{k=1}^{n}\frac{\mu_k k}{n}M(c)^{k-1}}(\rho(c)-c)\\ &= \frac{M'(c)\sum_{k=2}^{n}\mu_k k(k-1)M(c)^{k-2}}{\sum_{k=1}^{n}\mu_k kM(c)^{k-1}}(\rho(c)-c)\end{aligned} \tag{9}$$

Apparently, $\rho(c)$ is independent of the number of sellers, n, given that μ is unchanged.

The entry of new sellers lifts the upper bound of sellers that can be considered; the exit of sellers necessarily results in a change of μ if $\mu_n > 0$. Nevertheless, it is not the number of sellers, but consumer information (μ) that ultimately determines prices. Some assume that the exit of a seller causes consumers who considered the seller to only consider the remaining sellers in their consideration sets (Armstrong and Vickers, 2022); the entry of a seller causes consumers include the seller into their consideration sets (Bagwell and Wolinsky, 2002; Bagwell and Lee, 2014). These models impose some dependency between consumer information (μ) and sellers in a market. On the contrary, we assume that consumer information is independent of sellers. Particularly, our assumption makes more sense in markets where a) sellers are constantly dealing with new consumers, such as the car market, the job market; b) the total number of sellers is large, but the sizes of consideration sets are generally small, such as the food and farm marketplace, the gas market.

Because the number of sellers is not critical, we focus on the information effects on prices. Hereafter, *more informed* means expanding the consideration set for some consumers. We show that the effects on prices are different between captive consumers and informed consumers when they are *sufficiently more informed*, which is defined as

Definition 2: Given μ, a consumer i becomes a) *more informed* if and only if for some $k \in \{k : k^* > k_i, \}$, $d\mu_k = -d\mu_{k^*} \to 0^+$, the other μ_ℓ are not changed, and b) *sufficiently more informed* if and only if consumer i is *more informed* and $k \in \{k^* : (k^* - 1)\sum_{\ell=1}^{n} \mu_\ell \ell \geq \sum_{\ell=1}^{n} \mu_\ell \ell(\ell - 1)\}$.

First, when captive consumers become *sufficiently more informed*, the competition among sellers increases; prices generally fall.

Theorem 2: *If captive consumers are sufficiently more informed, sellers with cost* $c \in [\underline{c}, \overline{c})$ *lower prices.*

Proof: We apply the envelope theorem (Milgrom and Segal, 2002) to the revenue function (1), that is

$$\begin{aligned} \Pi(\rho(c); c) &= \int_c^{\overline{c}} \mathcal{Q}(x)dx + \Pi(\rho(\overline{c}); \overline{c}) \\ \Rightarrow (\rho(c) - c)\mathcal{Q}(c) &= \int_c^{\overline{c}} \mathcal{Q}(x)dx + \frac{\mu_1}{n}(1 - \overline{c}) \end{aligned} \tag{10}$$

By definition, *captive consumers* become *sufficiently more informed* means that μ_1 decreases and μ_k increases ($k \in \{k^* : (k^* - 1)\sum_{\ell=1}^{n} \mu_\ell \ell \geq \sum_{\ell=1}^{n} \mu_\ell \ell(\ell - 1)\}$), while others remain unchanged. Let $\mu_1 = 1 - \sum_{\ell=2}^{n} \mu_\ell$. By taking the derivative on μ_k in (10), we obtain the effect on $\rho(c)$ of a marginal increase in μ_k and a marginal decrease in μ_1,

$$\frac{\partial \rho(c)}{\partial \mu_k}\mathcal{Q}(c) = \int_c^{\overline{c}} \frac{\partial \mathcal{Q}(x)}{\partial \mu_k}dx - \frac{1}{n}(1 - \overline{c}) - \frac{\partial \mathcal{Q}(c)}{\partial \mu_k}(\rho(c) - c) \tag{11}$$

Since $\mathcal{Q}(c) > 0$, we only care about the sign of the right-hand side,

$$\begin{aligned}\psi(c) &= \int_c^{\overline{c}} \frac{\partial \mathcal{Q}(x)}{\partial \mu_k} dx - \frac{1}{n}(1-\overline{c}) - \frac{\partial \mathcal{Q}(c)}{\partial \mu_k}(\rho(c)-c) \\ &= \int_c^{\overline{c}} \frac{k}{n} M(x)^{k-1} - \frac{1}{n} dx - \frac{1}{n}(1-\overline{c}) - [\frac{k}{n}M(c)^{k-1} - \frac{1}{n}](\rho(c)-c) \\ &= \int_c^{\overline{c}} \frac{k}{n} M(x)^{k-1} dx - \frac{1}{n}(1-\rho(c)) - \frac{k}{n}M(c)^{k-1}(\rho(c)-c)\end{aligned} \tag{12}$$

Take the derivative on c and use (8) to get

$$\begin{aligned}\frac{\partial \psi(c)}{\partial c} &= \frac{1}{n}\rho'(c) - \rho'(c)\frac{k}{n}M(c)^{k-1} - (\rho(c)-c)M'(c)\frac{k(k-1)M(c)^{k-2}}{n} \\ &= -(\rho(c)-c)M'(c)[\frac{\sum_{\ell=2}^n \mu_\ell \ell(\ell-1)M(c)^{\ell-2}}{\sum_{\ell=1}^n \mu_\ell \ell M(c)^{\ell-1}}(\frac{1}{n} - \frac{k}{n}M(c)^{k-1}) \\ &\quad + \frac{k(k-1)M(c)^{k-2}}{n}] \\ &= -(\rho(c)-c)M'(c)[\frac{1}{n}\frac{\sum_{\ell=2}^n \mu_\ell \ell(\ell-1)M(c)^{\ell-2}}{\sum_{\ell=1}^n \mu_\ell \ell M(c)^{\ell-1}} \\ &\quad + (k-1-\frac{\sum_{\ell=2}^n \mu_\ell \ell(\ell-1)M(c)^{\ell-1}}{\sum_{\ell=1}^n \mu_\ell \ell M(c)^{\ell-1}})\frac{kM(c)^{k-2}}{n}]\end{aligned} \tag{13}$$

Let $\varphi(c) \equiv \frac{\sum_{\ell=2}^n \mu_\ell \ell(\ell-1)M(c)^{\ell-1}}{\sum_{\ell=1}^n \mu_\ell \ell M(c)^{\ell-1}}$, and taking the derivative on c,

$$\begin{aligned}\frac{\partial \varphi(c)}{\partial c} &= M'(c)[\frac{(\sum_{\ell=2}^n \ell(\ell-1)^2 \mu_\ell M(c)^{\ell-2})(\mu_1 + \sum_{\ell=2}^n \mu_\ell \ell M(c)^{\ell-1})}{(\sum_{\ell=1}^n \mu_\ell \ell M(c)^{\ell-1})^2} \\ &\quad - \frac{(\sum_{\ell=2}^n \ell(\ell-1)\mu_\ell M(c)^{\ell-2})(\sum_{\ell=2}^n \ell(\ell-1)\mu_\ell M(c)^{\ell-1})}{(\sum_{\ell=1}^n \mu_\ell \ell M(c)^{\ell-1})^2}]\end{aligned} \tag{14}$$

For any $2 \le i \le j \le n$,

$$i(i-1)^2 j + j(j-1)^2 i - i(i-1)j(j-1) - j(j-1)i(i-1) = ij(j-i)^2 \ge 0 \tag{15}$$

(15) implies that (14) is negative. It in turn implies that $k - 1 > \frac{\sum_{\ell=2}^n \mu_\ell \ell(\ell-1)M(c)^{\ell-1}}{\sum_{\ell=1}^n \mu_\ell \ell M(c)^{\ell-1}}$ for all c. Thus, (13) is positive. Since $\psi(\overline{c}) = \mathcal{Q}(\overline{c})\frac{\partial \rho(\overline{c})}{\partial \mu_k} = 0$, $\frac{\partial \rho(c)}{\partial \mu_k} < 0$ for $c \in [\underline{c}, \overline{c})$. Q.E.D.

Second, when informed consumers are *sufficiently more informed*, sellers with different costs respond differently.

Theorem 3: *If informed consumers are sufficiently more informed, there is a cutoff $c^* \in (\underline{c}, \overline{c})$ such that sellers with costs $c \in (c^*, \overline{c})$ raise prices; sellers with costs $c \in [\underline{c}, c^*)$ lower prices; sellers with costs $c \in \{c^*, \overline{c}\}$ do not change prices.*

Proof: For some $2 \leq k' < k \leq n$ and $k \in \{k^* : (k^* - 1)\sum_{\ell=1}^{n} \mu_\ell \ell \geq \sum_{\ell=1}^{n} \mu_\ell \ell(\ell - 1)\}$, let $\mu_{k'} = 1 - \sum_{\ell \neq k'} \mu_\ell$, then take the derivative on μ_k,

$$\frac{\partial \rho(c)}{\partial \mu_k} \mathcal{Q}(c) = \int_c^{\overline{c}} \frac{\partial \mathcal{Q}(x)}{\partial \mu_k} dx - \frac{\partial \mathcal{Q}(c)}{\partial \mu_k} (\rho(c) - c) \tag{16}$$

Let $\psi(c) = \int_c^{\overline{c}} \frac{\partial \mathcal{Q}(x)}{\partial \mu_k} dx - \frac{\partial \mathcal{Q}(c)}{\partial \mu_k}(\rho(c) - c)$, and take the derivative on c,

$$\begin{aligned}
\frac{\partial \psi(c)}{\partial c} &= -\frac{\partial \frac{\partial \mathcal{Q}(c)}{\partial \mu_k}}{\partial c}(\rho(c) - c) - \rho'(c)\frac{\partial \mathcal{Q}(c)}{\partial \mu_k} \\
&= -M'(c)(\rho(c) - c)[(k(k-1)\frac{M(c)^{k-2}}{n} - k'(k'-1)\frac{M(c)^{k'-2}}{n}) \\
&\quad - \varphi(c)(k\frac{M(c)^{k-2}}{n} - k'\frac{M(c)^{k'-2}}{n})] \\
&= -M'(c)\frac{\rho(c) - c}{n} M(c)^{k'-2}[kM(c)^{k-k'}(k - 1 - \varphi(c)) \\
&\quad - k'(k' - 1 - \varphi(c))]
\end{aligned} \tag{17}$$

where $\varphi(c) \equiv \frac{\sum_{\ell=2}^{n} \mu_\ell \ell(\ell-1) M(c)^{\ell-1}}{\sum_{\ell=1}^{n} \mu_\ell \ell M(c)^{\ell-1}}$. From (17), $\frac{\partial \psi(c)}{\partial c} > (<)0$ if and only if

$$\frac{k}{k'} M(c)^{k-k'} - \frac{k' - 1 - \varphi(c)}{k - 1 - \varphi(c)} > (<)0 \tag{18}$$

From (15), $\varphi(c)$ is decreasing in c, then $\frac{k'-1-\varphi(c)}{k-1-\varphi(c)}$ is increasing in c. $\frac{k}{k'}M(c)^{k-k'}$ is decreasing in c. So, the left-hand side of (18) is decreasing in c. Also, we know from (18) that $\frac{\partial \psi(\underline{c})}{\partial c} > 0$ and $\frac{\partial \psi(\overline{c})}{\partial c} < 0$. There exists a $c'' \in (\underline{c}, \overline{c})$ such that $\frac{\partial \psi(c)}{\partial c} > (<)0$ if and only if $c < (>)c''$. Moreover,

$$\frac{\partial \mathcal{Q}(c)}{\partial \mu_k} = \frac{k}{n} M(c)^{k-1} - \frac{k'}{n} M(c)^{k'-1} \tag{19}$$

Let $c' = M^{-1}((\frac{k'}{k})^{\frac{1}{k-k'}})$, it is easy to see that

$$\frac{\partial Q(c)}{\partial \mu_k} = \begin{cases} 0, & c \in \{c', \overline{c}\} \\ -, & c \in (c', \overline{c}) \\ +, & c \in [\underline{c}, c') \end{cases} \tag{20}$$

(16) and (20) jointly imply that $\psi(\overline{c}) = 0$ and $\psi(c') < 0$. With previous findings about $\frac{\partial \psi(c)}{c}$, we have $c' < c'' < \overline{c}$. Since $\psi(c)$ is continuous, there must be $c^* \in (c', c'')$ such that $\psi(c) > 0$ for $c \in (c^*, \overline{c})$; $\psi(c) < 0$ for $c \in [\underline{c}, c^*)$; $\psi(c) = 0$ for $c \in \{c^*, \overline{c}\}$. Q.E.D.

As informed consumers become sufficiently more informed, the expected price may increase; prices may be more disperse.

4 Welfare

Since prices are invariant to the number of sellers, the number of sellers does not affect consumer surplus, but it does affect seller profits. From (2), we derive the profit effect directly. If consumer information (μ) remains unchanged, the entry (exit) of an additional seller decreases (increases) the profits of the existing seller. The loss (or gain) is greater for sellers with lower costs.

Consumer information affects both consumer surplus and seller profits. Consumer surplus is determined by prices and the size of consideration set. The expected surplus for consumer i with size k_i is

$$CS_i = 1 - \int_{\underline{c}}^{\overline{c}} \rho(c) d(1 - M(c)^{k_i}) \tag{21}$$

If captive consumers become sufficiently more informed, prices will fall; surpluses will increase for all consumers. Yet, the effect may be reversed if informed consumers become sufficiently more informed: the expected minimum price may increase, and consumer surplus may decrease in some cases (as shown in the next section). In contrast, the information effect on profits is more general, exhibiting a "U" shape. The following proposition summarizes our findings,

Theorem 4: *If consumers are more informed, the profit change for sellers first decreases then increases as costs are higher.*

Proof: For $1 \leq k' < k \leq n$, let $\mu_{k'} = 1 - \sum_{\ell \neq k'} \mu_\ell$.

$$\begin{aligned}\frac{\partial \Pi(\rho(c); c)}{\partial \mu_k} &= \int_c^{\overline{c}} \frac{\partial \mathcal{Q}(x)}{\partial \mu_k} dx + \frac{\partial \Pi(\rho(\overline{c}); \overline{c})}{\partial \mu_k} \\ &= \int_c^{\overline{c}} \frac{kM(x)^{k-1} - k'M(x)^{k'-1}}{n} dx - \mathbb{1}_{k'=1} \frac{1-\overline{c}}{n}\end{aligned} \tag{22}$$

Take the derivative on c,

$$\frac{\partial \frac{\partial \Pi(\rho(c),c)}{\partial \mu_k}}{\partial c} = -\frac{kM(c)^{k-1} - k'M(c)^{k'-1}}{n} \tag{23}$$

Let $c' = M^{-1}((\frac{k'}{k})^{\frac{1}{k-k'}})$, it is easy to see that

$$\frac{\partial \frac{\partial \Pi(\rho(c),c)}{\partial \mu_k}}{\partial c} = \begin{cases} +, & c \in (c', \overline{c}) \\ -, & c \in [\underline{c}, c') \end{cases} \tag{24}$$

Q.E.D.

Theorem 4 implies that medium-cost sellers suffer a greater loss of profits compared to lower-cost sellers and higher-cost sellers. Moreover, profits for some lower-cost sellers may increase under some circumstances. But profits for higher cost sellers never increase. We show these results in next section.

5 A triopoly example

Consider a triopoly (three sellers), costs distributed on the support of $[0, 0.5]$, and $M(c) = 1 - 4c^2$. From Theorem 1, we have

$$\rho(c) = \frac{576c^5(u_1 + u_2 - 1) - 80c^3(3u_1 + 2u_2 - 3) - 3u_1 + 2(u_2 - 6)}{15\,(48c^4(u_1 + u_2 - 1) - 8c^2(3u_1 + 2u_2 - 3) + 2u_1 + u_2 - 3)} \tag{25}$$

Since μ_3 always satisfies the *sufficient informed* criterion, we can derive the information effects from taking derivatives on μ_1 and μ_2 in (26). Figure 1 shows two different consumer information effects on prices from the example ($\mu_1 = 0.3, \mu_2 = 0.3, \mu_3 = 0.4$).

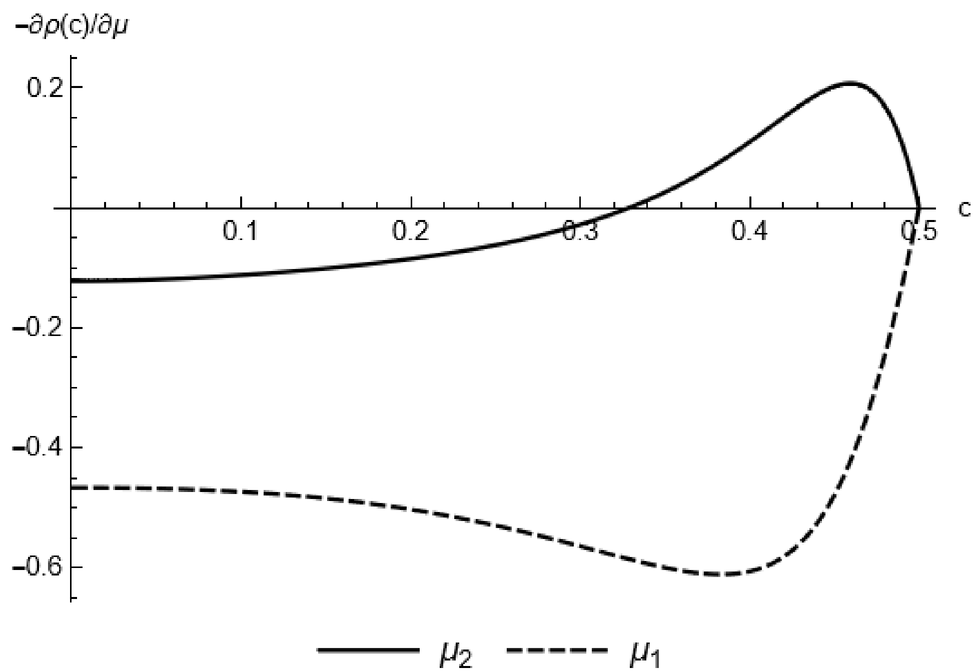

Figure 1: The information effect on prices

Numerically, we find that the expected price [5] increases from 0.6332 to 0.6413 when some informed consumers become more informed (from $\mu = (0.3, 0.6, 0.1)$ to $\mu' = (0.3, 0.3, 0.4)$). So, captive consumers become worse off. Also, the variance of price increases from 0.0201 to 0.0283. Figure 2 shows how sellers' profits change. Specifically, profits increase for sellers with

[5] Here, the expected price is equivalent to the expected minimal price for captive consumers.

lower costs as consumers are more informed (from $\mu = (0.3, 0.6, 0.1)$ to $\mu' = (0.3, 0.3, 0.4)$). The effect first decrease then increase as proposed by Proposition 5.

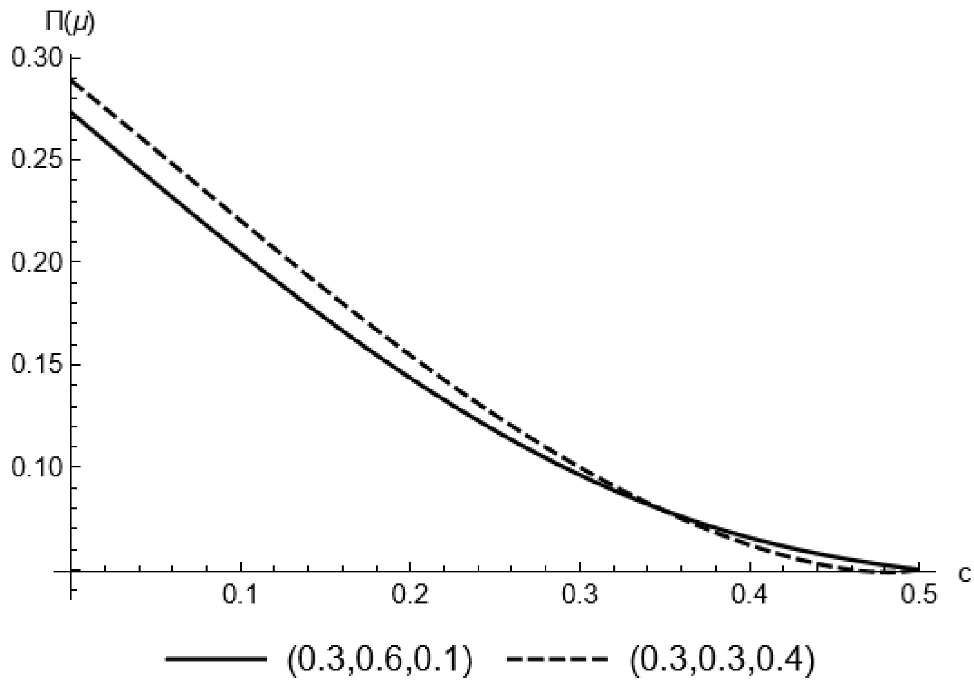

Figure 2: The information effect on profits

6 Related literature

Consumers consider only a subset of prices when purchasing a homogeneous good for many reasons. For example, sellers may be spatially dispersed, and searching for prices may be costly for some consumers. For another, consumers may pay limited attention to prices (De Clippel, Eliaz, and Rozen, 2014). A large literature empirically documents the limitations and heterogeneity of consumer attention to prices across markets. Draganska and Klapper (2011) report the size of consumer choice set on the brands of ground coffee, which is broadly distributed from one to five. Similarly, De los Santos, Hortaçsu, and Wildenbeest (2012) show that consumers consider only a few stores (mostly ≤ 4) when buying books online. Hortaçsu, Madanizadeh, and Puller (2017) find evidence of consumer inertia in finding alternative retailers for electricity. For a more comprehensive review of the relevant empirical work, see Honka, Hortaçsu, and Wildenbeest (2019). Building on the fact that consumers have limited information about prices, our paper investigates a game-theoretic model focusing on pricing strategies of heterogeneous sellers. We contribute to the literature in terms of both methodology and results.

A substantial literature has explored partial consumer information theoretically, various models have been produced. A common practice is to assume that consumers are either captive to one random seller or know prices from all sellers (Varian, 1980; Rosenthal, 1980). Another branch of studies mainly concern about consumer search frictions in explaining price dispersion (Stigler, 1961; Diamond, 1971). Burdett and Judd (1983)'s seminal paper consider a more general information framework where consumers are categorized into all countable price counts[6]. Meanwhile, the distribution over price counts was endogenized by non-sequential consumer search. The following researches explore price competition and price discrimination with respect to 1) heterogeneous search costs or 2) asymmetric information/competitive structure. Janssen and Moraga-González (2004) extend the Burdett-Judd model by introducing heterogeneous search costs - some consumers search costly while others search with zero cost. Fabra and Reguant (2020) study price discrimination in a duopoly model where consumer search costs vary within an interval. In these models, sellers have no information about the number of sellers considered by consumers. Alternatively, Armstrong and Vickers (2019) and Ronayne and Taylor (2022) examine pricing strategy in a case where sellers can discriminate against captive consumers. Bergemann, Brooks, and Morris (2021) assume that sellers might have no formation, partial information or full information about the price counts of consumers. They show the existence of a upper bound on the distribution of prices in equilibrium. Armstrong and Vickers (2022) study cases with asymmetric competitive interactions. That is, sellers are asymmetric for buyers so that the competition between sellers follows some pattern, e.g., chain store competition - one particular seller is certainly considered by consumers who consider two or more sellers.

Apart from previous works, this paper considers the heterogeneity of sellers in a Burdett-

[6] There are many types of consumer in terms of the number of sellers under consideration, one, two, three, and so on, instead of one price or all prices.

Judd framework[7]. More specifically, marginal costs of sellers are randomly drawn from a prior distribution. When setting a price, a seller faces two-sided incomplete information. On one hand, she does not know how many other sellers a consumer considers. On the other hand, she does not know her competitors marginal costs. In contrast to most existing models where sellers sell the good at the same cost which is usually normalized to zero, and sellers employ mixed strategy in equilibrium (Anderson and Renault, 2018). In our model, the symmetric Bayesian Nash equilibrium is a monotone pure strategy. This enables us to examine how sellers with different costs react (in pricing) to changes in information or competitiveness (the number of competitors).

Our findings complement the existing knowledge about price, consumer information, and competitiveness in markets of homogeneous goods. The conventional wisdom is that prices are negatively associated with the number of competitors in a market (e.g. Janssen and Moraga-González, 2004; Bagwell and Lee, 2014; Armstrong and Vickers, 2022). Some researches challenge the view and argue that prices can increase as a market is more competitive (e.g. Rosenthal, 1980; Ronayne and Taylor, 2022). We instead show that prices can be invariant to the number of competitors so long as consumer information held fixed[8]. Also, we identify the heterogeneity of information effects on prices. Particularly, we show that the expected price may increase as informed consumers are sufficiently more informed.

7 Conclusion

In this paper, we study an oligopolistic model with heterogeneous sellers and incomplete information. We examine how prices and welfare respond to the number of sellers and to consumer information. In summary, we argue that the number of sellers alone does not change prices, but the number of sellers that consumers consider does. When captive consumers consider enough sellers, prices fall. However, when informed consumers consider sufficiently more sellers, prices of higher-cost sellers increase and prices of lower-cost sellers decrease. As a result, the expected price may increase; prices may be more disperse. Con-

[7]To our best knowledge, only Bagwell and Lee (2014) studied a similar model which assumed a special case on consumer information - "one price or all prices".

[8]Stiglitz (1987) shows the uncertainty of the movement of prices in a model with sequential search.

sumers consider that more sellers may (or may not) benefit lower-cost sellers. If not, the loss in profits is smaller for lower-cost sellers and higher-cost sellers, but larger for mid-cost sellers.

There are several possible extensions beyond this paper. Following convention, one can endogenize consumer information about price by introducing search friction. Different distributions of consumer search costs imply different distributions of consumer information. Thus, one can study how changes in search costs affect prices. Relaxing the no-information assumption in our model, it would be interesting to consider partial information. Sellers receive a signal (either free or costly) about consumers' consideration sets. Sellers receive the signals to implement price discrimination. Meanwhile, static consumer information can be augmented with advertising. Sellers broadcast prices at some cost; consumers are more likely to consider sellers who advertise.

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