

### Exercises on projections onto subspaces

**Problem 15.1:** (4.2 #13. *Introduction to Linear Algebra*: Strang) Suppose  $A$  is the four by four identity matrix with its last column removed;  $A$  is four by three. Project  $\mathbf{b} = (1, 2, 3, 4)$  onto the column space of  $A$ . What shape is the projection matrix  $P$  and what is  $P$ ?

**Solution:**  $P$  will be four by four since we are projecting a 4-dimensional vector to another 4-dimensional vector. We will have:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This can be seen by observing that the column space of  $A$  is the  $wxy$ -space, so we just need to subtract the  $z$  coordinate from the 4-dimensional vector  $(w, x, y, z)$  we're projecting. The projection of  $\mathbf{b}$  is therefore:

$$\mathbf{p} = P\mathbf{b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}.$$

**Problem 15.2:** (4.2 #17.) If  $P^2 = P$ , show that  $(I - P)^2 = I - P$ . For the matrices  $A$  and  $P$  from the previous question,  $P$  projects onto the column space of  $A$  and  $I - P$  projects onto the \_\_\_\_\_.

**Solution:**

$$(I - P)^2 = I^2 - IP - PI + P^2 = I - 2P + P^2 = I - 2P + P = I - P.$$

Using the matrices  $A$  and  $P$  from the previous question,

$$I - P = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

projects onto the **left nullspace** of  $A$ .

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