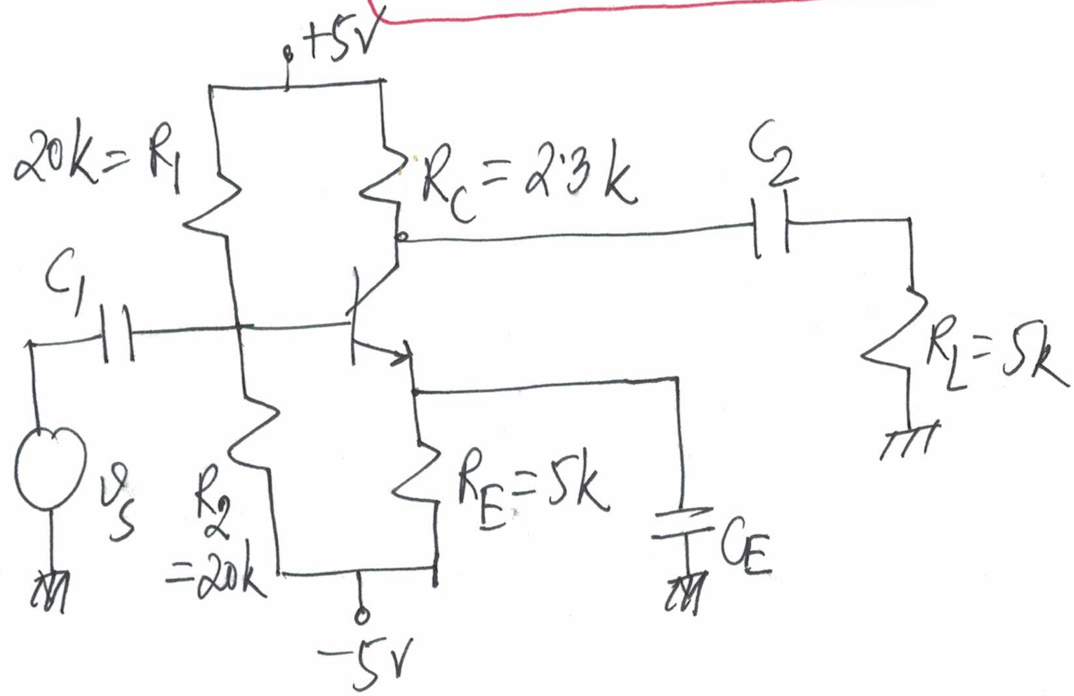
Practice Problem: 3
ENEL- 469

Problem 1:



Given:

$\beta = 125$

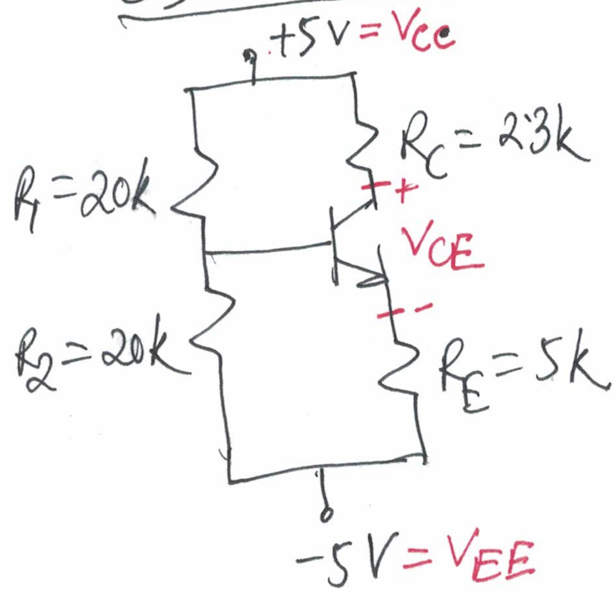
$V_{BE}(ON) = 0.7V$

$V_A = 200V$

DC Analysis:

* All the capacitors are open circuit.

Equivalent Circuit:



For DC Load Line,

Applying KVL in the collector loop:

$$V_{CC} - V_{EE} = I_C R_C + V_{CE} + I_E R_E$$

$$= I_C R_C + V_{CE} + \frac{\beta+1}{\beta} I_C R_E$$

$$\Rightarrow 5-(-5) = I_C R_C + V_{CE} + \frac{\beta+1}{\beta} I_C R_E$$

$$\Rightarrow 10 = I_C \left(R_C + \frac{\beta+1}{\beta} R_E\right) + V_{CE}$$

$$\Rightarrow V_{CE} = 10 - \left(R_C + \frac{\beta+1}{\beta} R_E\right) I_C \quad \text{--- (1)}$$

Note that DC load lines indicate the linear relationships between V_{CE} and I_C.

From 1, we get the relationship between I_C and V_{CE} of the given circuit which is ~~alternavir~~ alternatively the mathematical expression for the DC load line.

$$V_{CE} = 10 - \left(R_C + \frac{\beta+1}{\beta} R_E\right) I_C$$

$$\therefore \text{ slope} = -1 \Big/ \left(R_C + \frac{\beta+1}{\beta} R_E\right)$$

← Note this inverse is because V_{CE} in the x axis and I_C in the y-axis

$$= -\frac{1}{2.3k + 1.008 \times 5k}$$

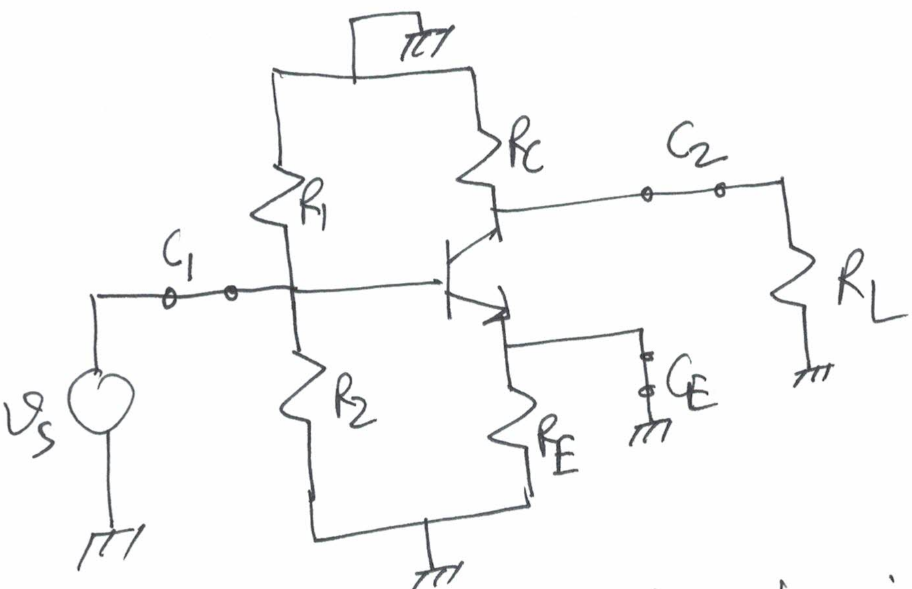
$$I_C = -\frac{V_{CE}}{R_C + \frac{\beta+1}{\beta} R_E} + \frac{10}{R_C + \frac{\beta+1}{\beta} R_E}$$

$$\boxed{\text{DC load line slope} = -\frac{1}{7.34k}}$$

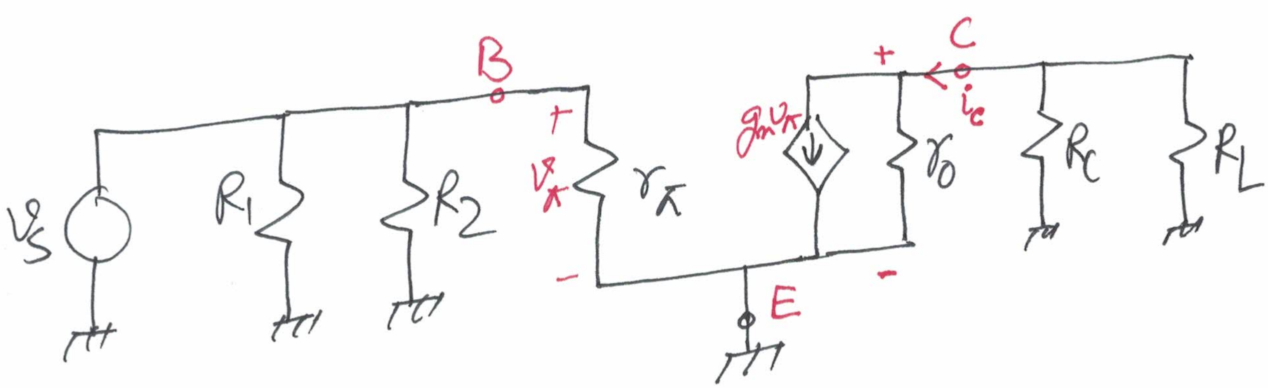
AC Analysis:

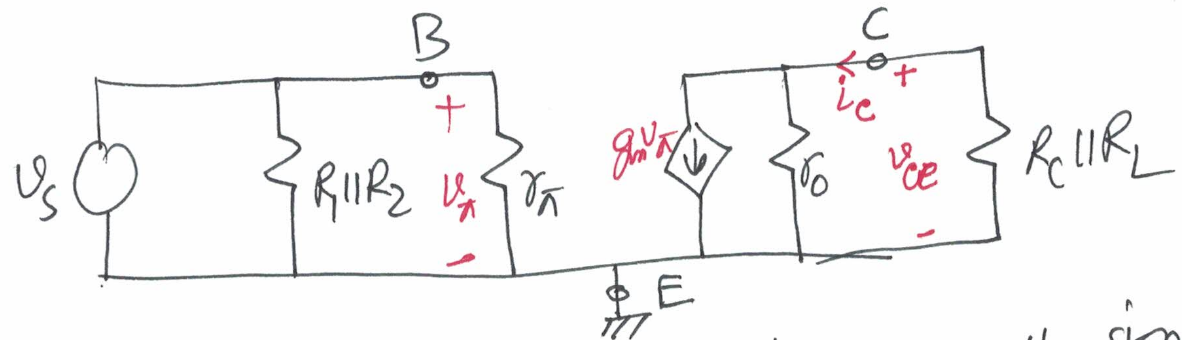
- Capacitor are short
- DC voltages are short
- ~~A~~C current sources are open

Equivalent circuit:



Small signal equivalent circuit:





We can write from above small signal equivalent circuit:

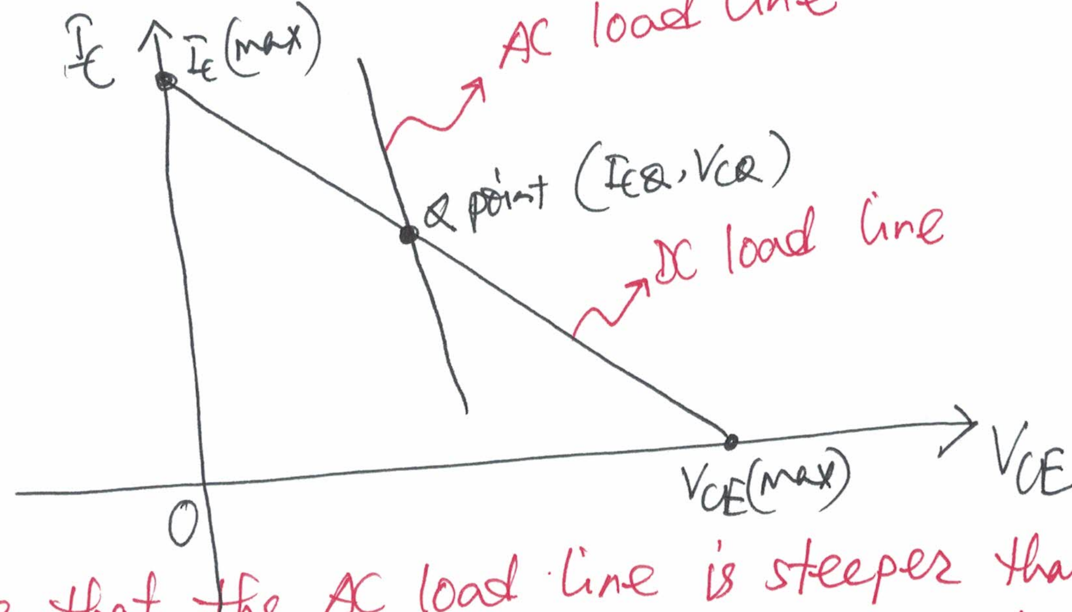
$$v_{ce} = -i_c (R_C \| R_L)$$

$$\therefore \text{AC load line slope} = -\frac{1}{R_C \| R_L}$$

$$= -\frac{1}{2.3k \| 5k}$$

$$= -\frac{1}{1.575k}$$

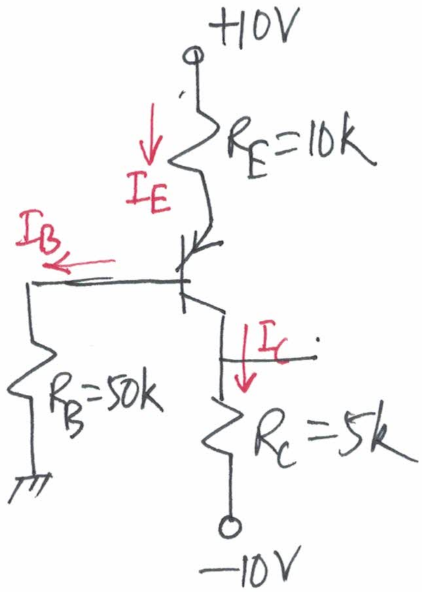
DC and AC load lines:



Note that the AC load line is steeper than the DC load line due to the slope differences.

Problem 2:

DC Analysis:



Given that:

$\beta = 150$

$V_{EB}(ON) = 0.7V$

and $V_A = \infty$

Applying KVL in the E-B loop:

$$+10V = I_E R_E + V_{EB}(ON) + I_B R_B$$

$$\Rightarrow +10V = (\beta+1) I_B R_E + V_{EB}(ON) + I_B R_B$$

$$\Rightarrow I_B = \frac{10 - 0.7}{(\beta+1) R_E + R_B}$$

$$\boxed{I_B = 5.96\,\mu A}$$

$\therefore$ $\boxed{I_C = \beta I_B = 0.894\,mA}$ and $\boxed{I_E = (\beta+1) I_B = 0.90\,mA}$

Now, Applying KVL in the E–C loop:

$$+10 - (-10) = I_E R_E + V_{EC} + I_C R_C$$

$$\Rightarrow V_{EC} = 20 - I_E R_E - I_C R_C$$

$$\Rightarrow \boxed{V_{EC} = 6.53V}$$

Slope of the DC load Line:

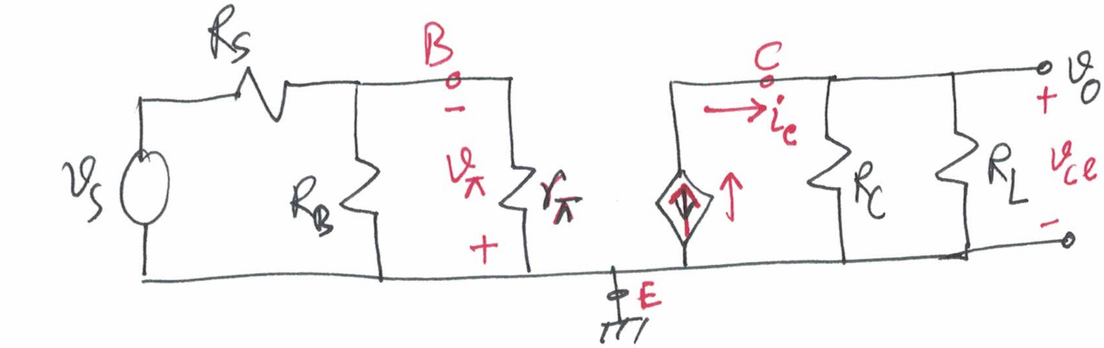
$$+10 - (-10) = I_E R_E + V_{EC} + I_C R_C$$

$$\Rightarrow 20 = \frac{\beta+1}{\beta} I_C R_E + V_{EC} + I_C R_C$$

$$\Rightarrow V_{EC} = 20 - \left(\frac{\beta+1}{\beta} R_E + R_C\right) I_C$$

$$\therefore \boxed{\text{DC load line slope} = -\frac{1}{\frac{\beta+1}{\beta} R_E + R_C} \approx -\frac{1}{15k}}$$

AC Equivalent Circuit:



Note that the voltage and current source directions are changed for PNP transistor.

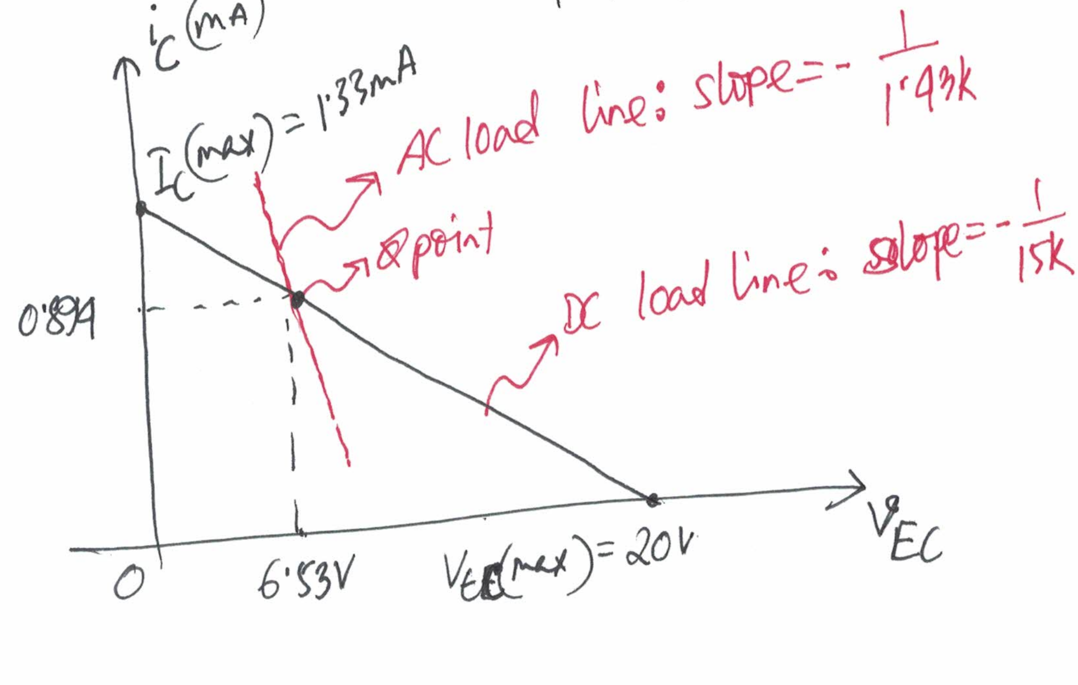
r_o is not present since, $V_A = \infty$; $r_o = \infty$

We can write: $v_{ce} = i_c (R_C \| R_L) \Rightarrow v_{ec} = -i_c (R_C \| R_L)$

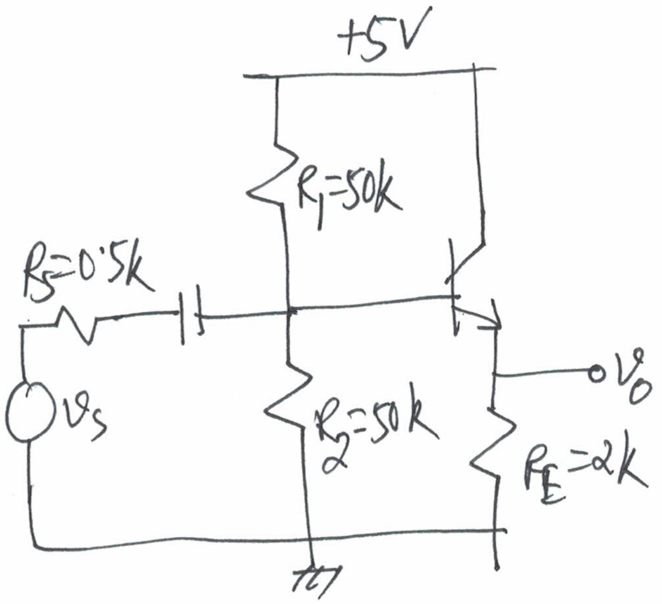
$\therefore$ AC load line slope $= -\dfrac{1}{R_C \| R_L}$

$= -\dfrac{1}{1.43k}$

Load Line:



Problem 3:



Given:

$\beta=100$

$V_{BE}(ON)=0.7V$

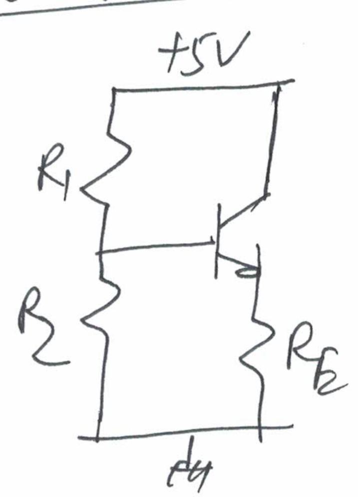
$V_A=80V$

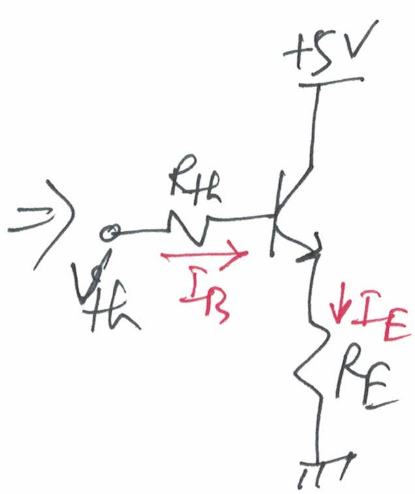
Steps:

1. find DC operating points (I_C, V_{CE})
2. find small signal parameters
3. find small-signal voltage gain from small-signal equivalent circuit.

Step 1: DC Analysis → Need to make sure the operating region of the transistor.

DC equivalent circuit:





$R_{th} = R_1 \| R_2 = 25k$

$$V_{th} = \frac{R_2}{R_1+R_2} 5 = 2.5V$$

Now, applying KVL,

$$V_{th} = I_B R_{th} + V_{BE}(ON) + I_E R_E$$

$$\Rightarrow V_{th} = \frac{I_C}{\beta} R_{th} + V_{BE}(ON) + \frac{\beta+1}{\beta} I_C R_E$$

$$\Rightarrow \boxed{I_C = 0.793\,mA}$$

Similarly; $+5 = V_{CE} + I_E R_E \Rightarrow \boxed{V_{CE} = 3.4V}$

step 2: small signal parameters

$\boxed{V_T = \frac{kT}{q} = 25mV}$

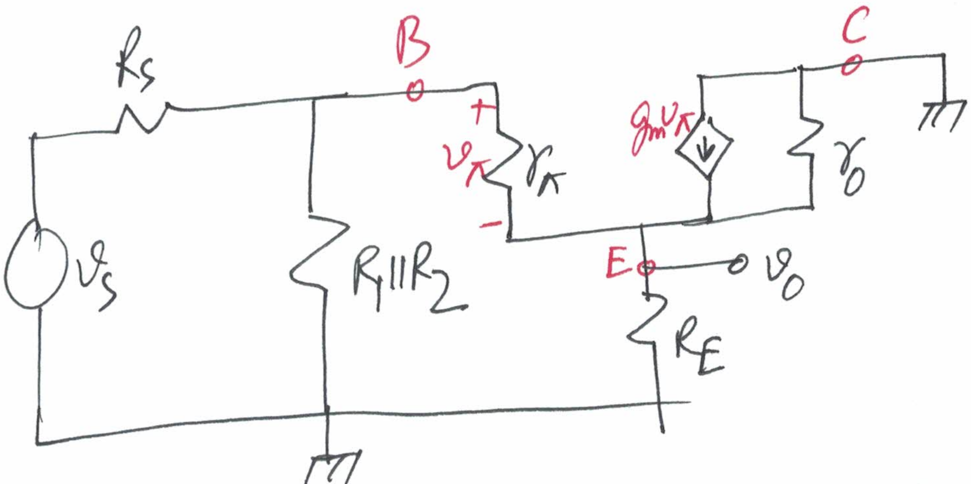
$$r_o = \frac{V_A}{I_C} = \frac{80}{0.793m} = 100\,k\Omega$$

$$r_\pi = \frac{V_T}{I_B} = \frac{V_T}{I_C/\beta} = 3.28\,k\Omega$$

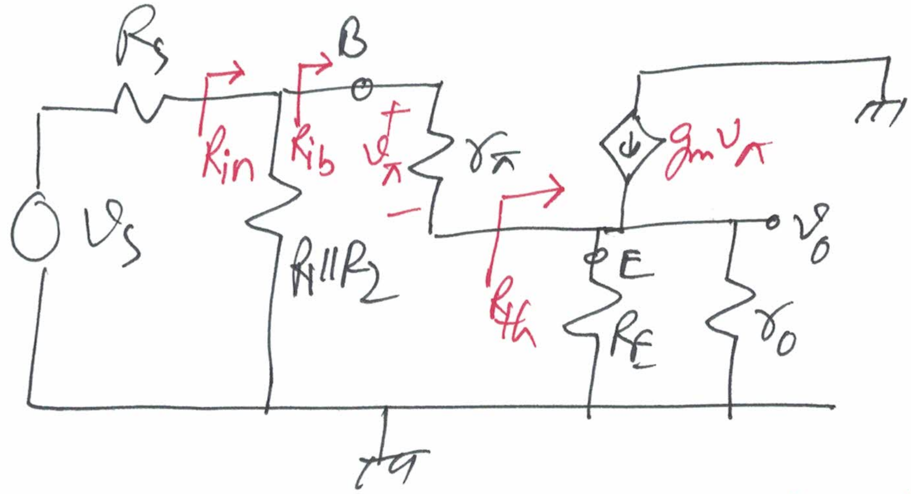
$$r_e = \frac{V_T}{I_E} = \frac{V_T}{\frac{\beta+1}{\beta} I_C} = 32.475\,\Omega$$

$$g_m = I_C/V_T = 30.5\,mA/V$$

step 3: small signal analysis



Overall small-signal gain, $A_{vo} = \dfrac{v_o}{v_s}$



$$R_{th} = (\beta+1)(r_o \| R_E)$$

$$R_{ib} = r_\pi + (\beta+1)(r_o \| R_E)$$

$$= 201\ k\Omega$$

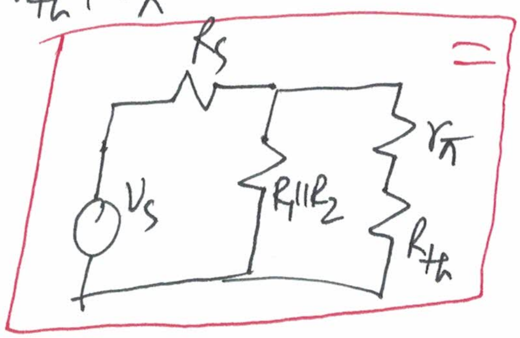
$$\therefore v_o = v_s \frac{R_{in}}{R_s + R_{in}} \cdot \frac{R_{th}}{R_{th} + r_\pi}$$

$$R_{in} = R_1 \| R_2 \| R_{ib}$$

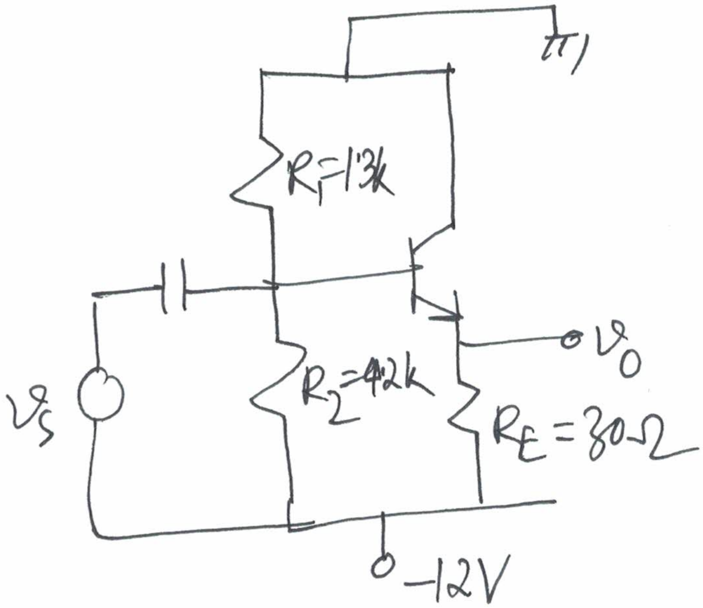
$$\Rightarrow \frac{v_o}{v_s} = \frac{R_{in}}{R_s + R_{in}} \cdot \frac{R_{th}}{R_{th} + r_\pi}$$

$$= 22.2\ k\Omega$$

$$\boxed{= 0.962}$$



Problem 4:



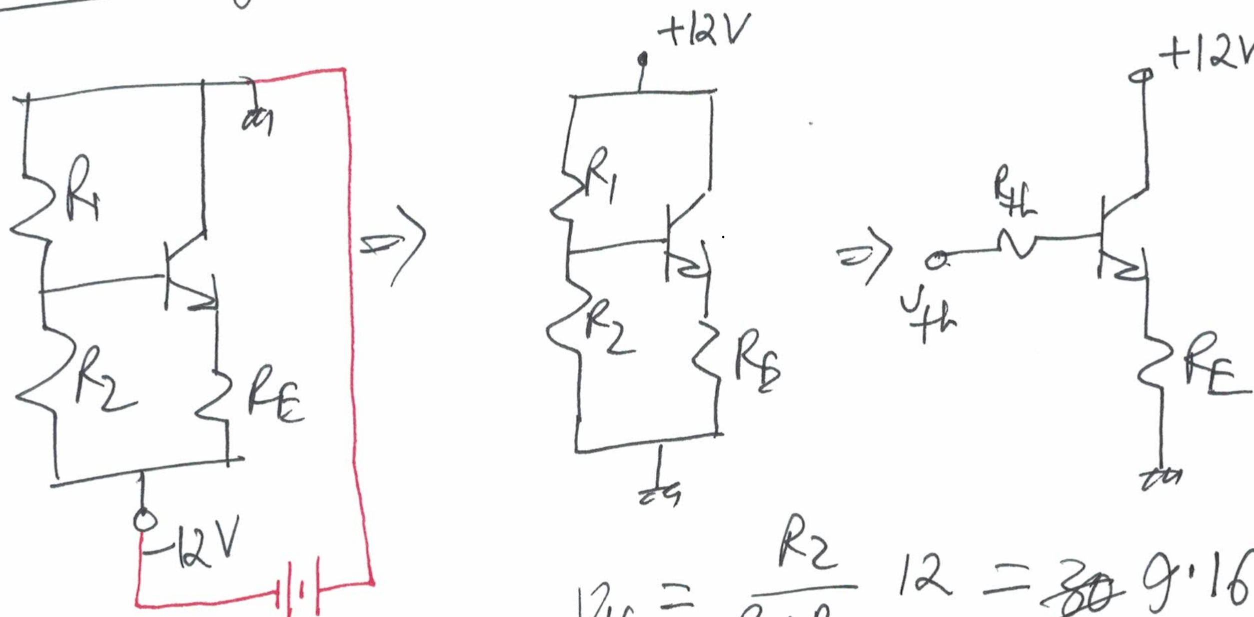
Given:

$\beta = 80$

$V_{BE}(ON) = 0.7\,V$

$V_A = 75\,V$

1. DC Analysis:



$$V_{Th} = \frac{R_2}{R_1+R_2}\,12 = \cancel{30}\ 9.16\,V$$

$$R_{Th} = R_1 \,||\, R_2 = 1k\Omega$$

$$\therefore V_{Th} = I_B R_{Th} + V_{BE} + I_E R_E \qquad \boxed{\Rightarrow I_E = 0.2\,A}$$

$$\Rightarrow V_{Th} = \frac{I_E}{\beta+1} R_{Th} + V_{BE} + I_E R_E$$

Also,

$$+12V = V_{CE} + I_E R_E$$

$$\boxed{\Rightarrow V_{CE} = 6V}$$

2. small signal parameters:

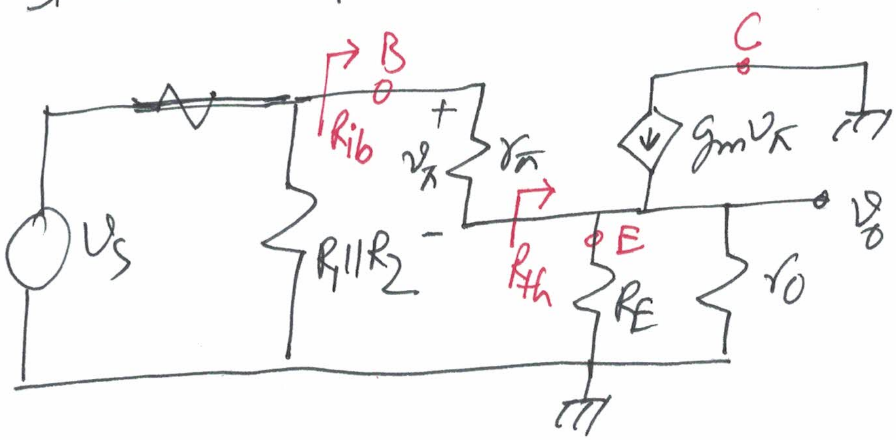
$$g_m = I_C/V_T = \frac{I_E \cdot \frac{\beta}{\beta+1}}{V_T} = 0.1975\ A/V$$

$$r_\pi = V_T/I_B = \frac{V_T}{I_E/(\beta+1)} = 10.125\ \Omega$$

$$r_o = V_A/I_C = V_A/\left(I_E \frac{\beta}{\beta+1}\right) = 379.69\ \Omega$$

3. small signal analysis:

similar to problem 3:



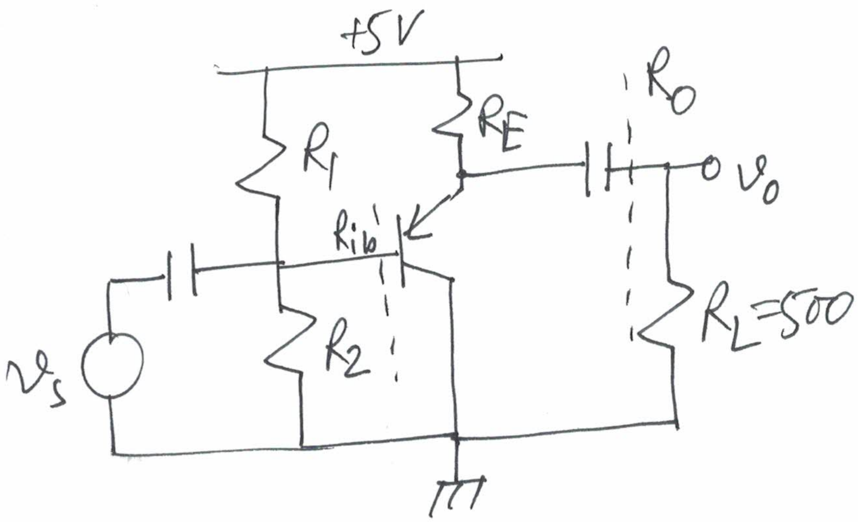
$$R_{th} = (\beta+1)(r_o \| R_E) = 2.252\ k\Omega$$

$$\boxed{R_{ib} = r_\pi + R_{th} = 2.262\ k\Omega}$$

$$\therefore v_o = \frac{R_{th}}{r_\pi + R_{th}} v_s \Rightarrow \frac{v_o}{v_s} = \frac{R_{th}}{r_\pi + R_{th}} = \frac{R_{th}}{R_{ib}}$$

$$\boxed{\Rightarrow \frac{v_o}{v_s} = \frac{2.252k}{2.262k} = 0.9956}$$

Problem 5:



Given:

$R_E = 2k$

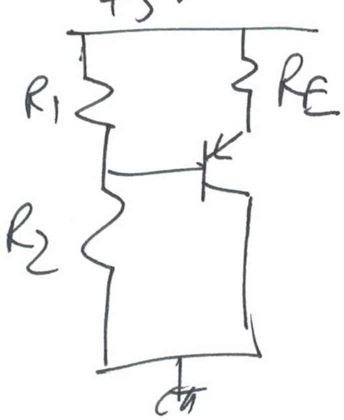
$R_1 = R_2 = 50k$

$\beta = 100$

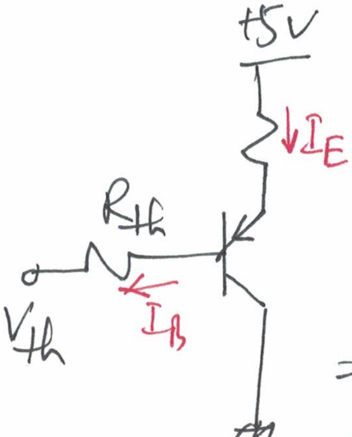
$V_{EB}(ON) = 0.7V$

$V_A = 125V$

DC Analysis:



$\Rightarrow$



$R_{th} = R_1 \parallel R_2 = 25k$

$V_{th} = 2.5V$

$$5 - V_{th} = I_E R_E + V_{EB} + I_B R_{th}$$

$$\Rightarrow 2.5 = I_E \left(R_E + \frac{R_{th}}{\beta+1}\right) + V_{EB}$$

$$\boxed{\Rightarrow I_E = 0.8mA}$$

$$\boxed{\therefore V_{CE} = 5 - I_E R_E = 3.4V}$$

small signal parameters:

$$r_\pi = \frac{V_T}{I_B} = \frac{V_T}{I_E/(\beta+1)} = 3.156\ k\Omega$$

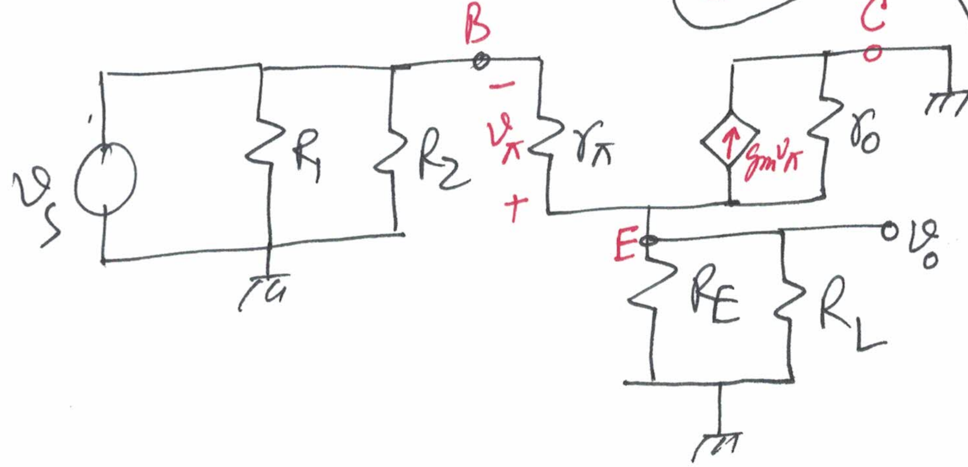
$$r_o = \frac{V_A}{I_C} = \frac{V_A}{I_E \cdot \frac{\beta}{\beta+1}} = 157.81\ k\Omega$$

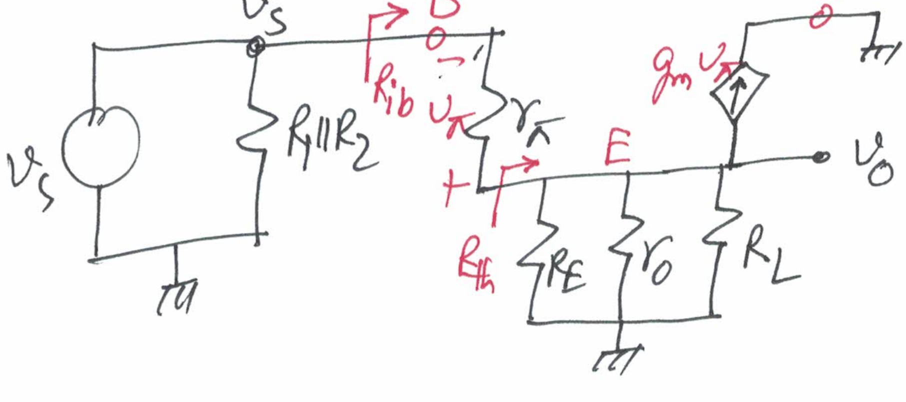
$$g_m = \frac{I_C}{V_T} = \frac{I_E \cdot \frac{\beta}{\beta+1}}{V_T} = 31.68\ mA/V$$

$$r_e = \frac{V_T}{I_E} = 31.25\ \Omega$$

small signal analysis:

Note that: for NPN transistor current source direction is from Emitter to collector and v_π is defined from E to B as well

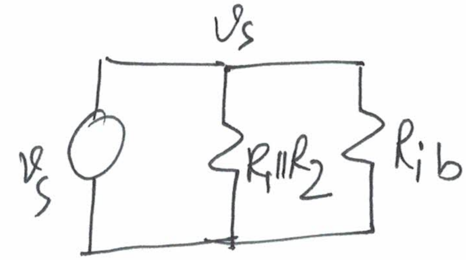


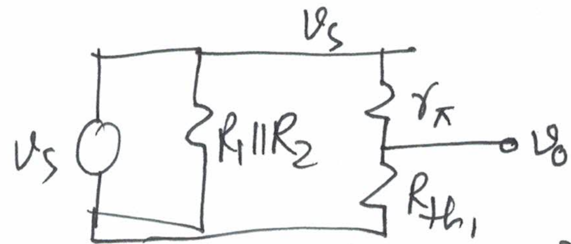


$$R_{ib} = r_\pi + (R_E \parallel r_o \parallel R_L)(\beta+1)$$

$$= 3.156\,k + 398.98.(\beta+1)$$

$$\boxed{R_{ib} = 43.455\ k\Omega}$$





$$\therefore\ v_o = v_s \frac{R_{th_1}}{r_\pi + R_{th_1}}$$

$$= v_s \frac{R_{th}}{R_{ib}}$$

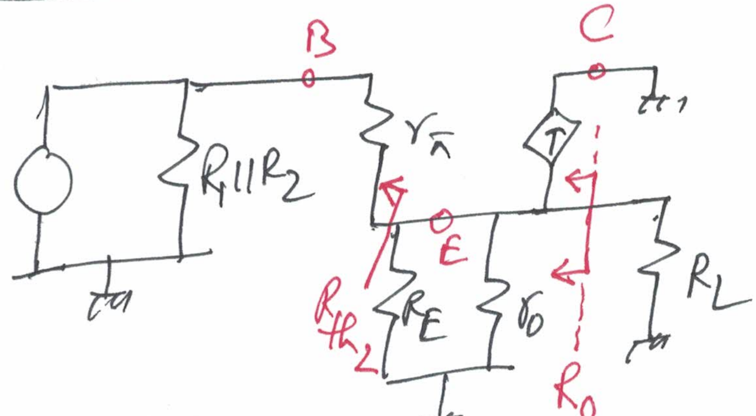
$$\Rightarrow \boxed{\frac{v_o}{v_s} = \frac{R_{th}}{R_{ib}} = 0.92}$$

$$R_{th_1} = (R_E \parallel r_o \parallel R_L)(\beta+1)$$

$$= 398.98 \times (\beta+1)$$

$$= 40.3\ k\Omega$$

Calculation of R_o :

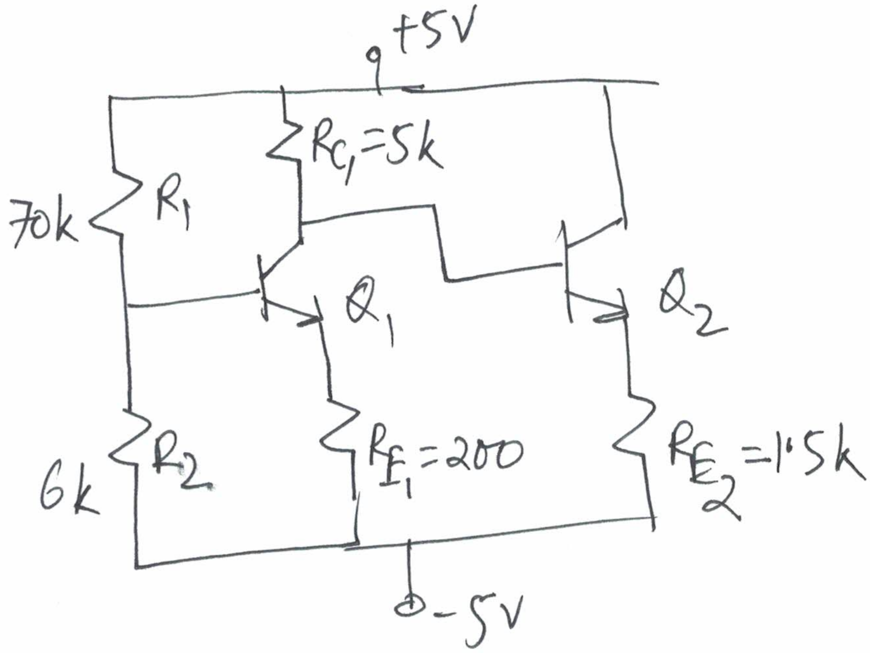


$$R_{th_2} = r_\pi = 3.156\ k\Omega$$

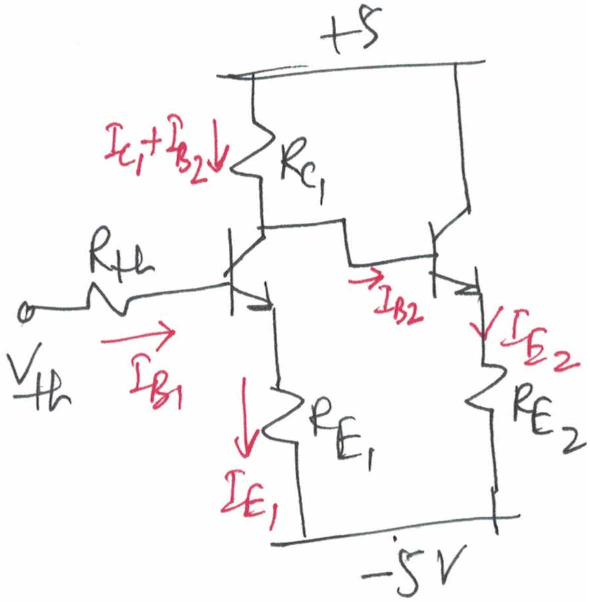
$$\therefore\ \boxed{R_o = R_{th_2} \parallel R_E \parallel r_o = 1.2146\ k\Omega}$$

Problem 6 :

Given : $\beta = 125$; $V_A = \infty$



$\Rightarrow$



$$V_{th} = \left[\frac{R_2}{R_1 + R_2}\{5 - (-5)\}\right] - 5 = \frac{R_2}{R_1 + R_2} \cdot 10 - 5$$

$$= -4.21\ V$$

$$R_{th} = R_1 \| R_2 = 5.526\ k\Omega$$

$$\therefore\ V_{th} - (-5) = V_{BE} + I_{E_1} R_{E_1} + I_{B_1} R_{th}$$

$$\Rightarrow 0.7895 - 0.7 = I_{E_1}\left(R_{E_1} + \frac{R_{th}}{\beta + 1}\right)$$

$$\Rightarrow I_{E_1} = 0.367\ mA$$

$$I_{C_1} = \alpha I_{E_1} = 0.3641 mA$$

$$\therefore \left(I_{C_1} + I_{B_2}\right) R_{C_1} + V_{BE} + I_{E_2} R_{E_2} = 10$$

$$\Rightarrow I_{C_1} R_{C_1} + I_{B_2} R_{C_1} + V_{BE} + I_{B_2} \cdot (\beta+1) R_{E_2} = 10$$

$$\Rightarrow I_{B_2} = \frac{10 - I_{C_1} R_{C_1} - V_{BE}}{(\beta+1) R_{E_2} + R_{C_1}}$$

$$= 38.55\,\mu A$$

$\therefore I_{C_2} =$ 4.82 mA 4.82 mA ; $I_{E_2} =$ 4.86 mA 4.8573 mA

Now, $10 = V_{CE_2} + I_{E_2} R_{E_2}$ $\Rightarrow V_{CE_2} = 10 - I_{E_2} R_{E_2}$

$= 2.71\,V$

$2.71\,V$

Again, $10 = \left(I_{C_1} + I_{B_2}\right) R_{C_1} + V_{CE_1} + I_{E_1} R_{E_1}$

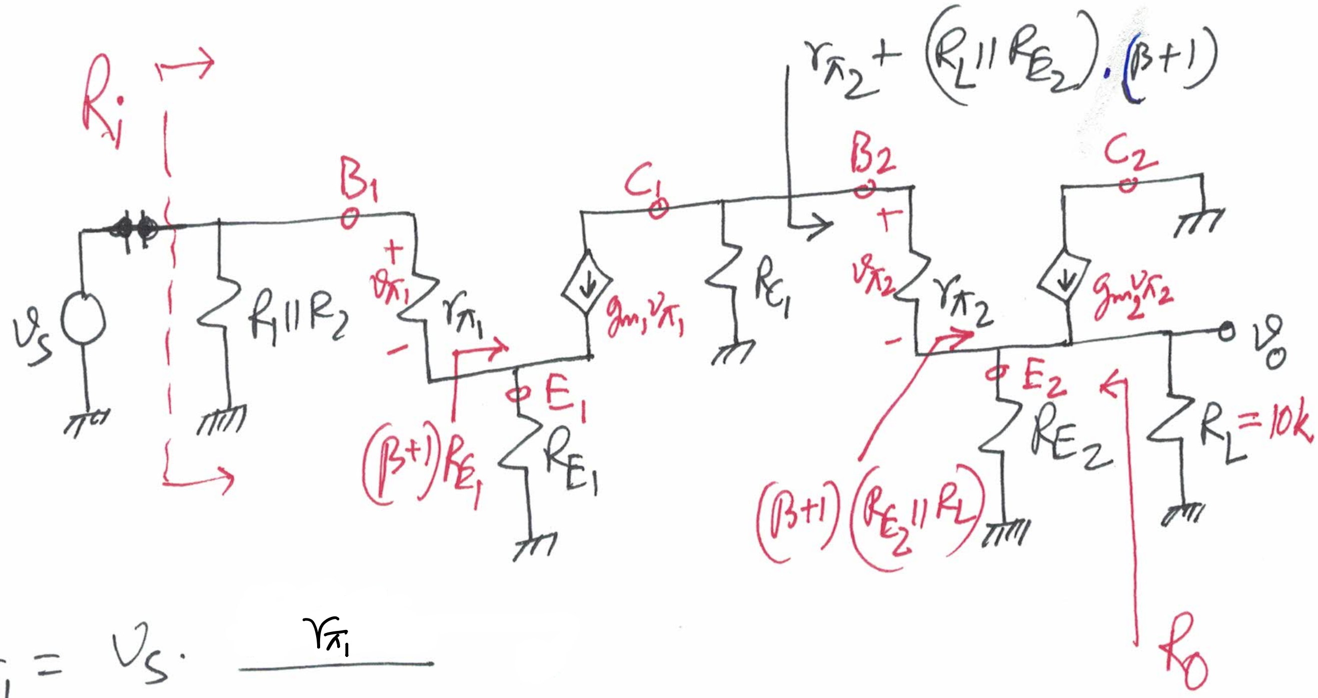
$$\Rightarrow V_{CE_1} = 10 - \left(I_{C_1} + I_{B_2}\right) R_{C_1} - I_{E_1} R_{E_1}$$

$$= 7.91\,V$$

$\therefore$ Q points: $I_{C_1} = 0.3641\,mA$, $V_{CE_1} =$ 7.91V 7.91V

$I_{C_2} =$ 4.82 mA 4.82mA $\quad V_{CE_2} =$ 2.71V 2.71V

small signal analysis :



$$v_{\pi_1} = v_s \cdot \frac{r_{\pi_1}}{(\beta+1) R_{E_1} + r_{\pi_1}}$$

$$v_{c_1} = g_{m_1} v_{\pi_1} \cdot \left\{ R_{C_1} \,\|\, \left[r_{\pi_2} + (R_L \| R_{E_2}) \cdot (\beta+1) \right] \right\}$$

$$= g_{m_1} v_{\pi_1} \cdot R_{th_1}$$

$$R_{th_1} = R_{C_1} \,\|\, \left[r_{\pi_2} + (R_L \| R_{E_2}) \cdot (\beta+1) \right]$$

$$v_o = v_{c_1} \frac{(\beta+1)(R_{E_2} \| R_L)}{r_{\pi_2} + (\beta+1)(R_{E_2} \| R_L)}$$

$$= g_{m_1} v_{\pi_1} \cdot R_{th_1} \cdot \frac{(\beta+1)(R_{E_2} \| R_L)}{r_{\pi_2} + (\beta+1)(R_{E_2} \| R_L)}$$

$$= v_s \frac{r_{\pi_1}}{(\beta+1) R_{E_1} + r_{\pi_1}} \cdot g_{m_1} R_{th_1} \cdot \frac{(\beta+1)(R_{E_2} \| R_L)}{r_{\pi_2} + (\beta+1)(R_{E_2} \| R_L)}$$

$$\Rightarrow \frac{v_o}{v_s} = g_{m_1} R_{th_1} \cdot \frac{r_{\pi_1}}{(\beta+1)R_{E_1} + r_{\pi_1}} \cdot \frac{(\beta+1)(R_{E_2} \| R_L)}{r_{\pi_2} + (\beta+1)(R_{E_2} \| R_L)}$$

Now,

$$g_{m_1} = \frac{I_{C_1}}{V_T} = 14.6 \text{ mA/V}$$

$$r_{\pi_1} = \frac{V_T}{I_{B_1}} = \frac{V_T}{I_{C_1}/\beta} = 8.583 \text{ k}\Omega$$

$$r_{\pi_2} = \frac{V_T}{I_{B_2}} = \frac{V_T}{I_{E_2}/\beta} = 0.648 \text{ k}\Omega$$

$$\therefore R_{th_1} = R_{C_1} \| \left[r_{\pi_2} + (R_L \| R_{E_2}) \cdot (\beta+1)\right]$$

$$= R_{C_1} \| 165\text{k}$$

$$= 4.853\text{k}$$

$$\therefore A_{vo} = \frac{v_o}{v_s} = g_{m_1} R_{th_1} \underbrace{\frac{r_{\pi_1}}{(\beta+1)R_{E_1} + r_{\pi_1}}}_{0.2541} \cdot \underbrace{\frac{(\beta+1)(R_{E_2} \| R_L)}{r_{\pi_2} + (\beta+1)(R_{E_2} \| R_L)}}_{0.9961}$$

$$= 17.934$$

Input Resistance:

$$R_i = R_1 \parallel R_2 \parallel \left(r_{\pi_1} + (\beta+1) R_{E_1}\right) \rightarrow 33.783k$$

$$= 5.526k \parallel 33.783k$$

$$\boxed{= 4.7492\,k\Omega}$$

Output Resistance:

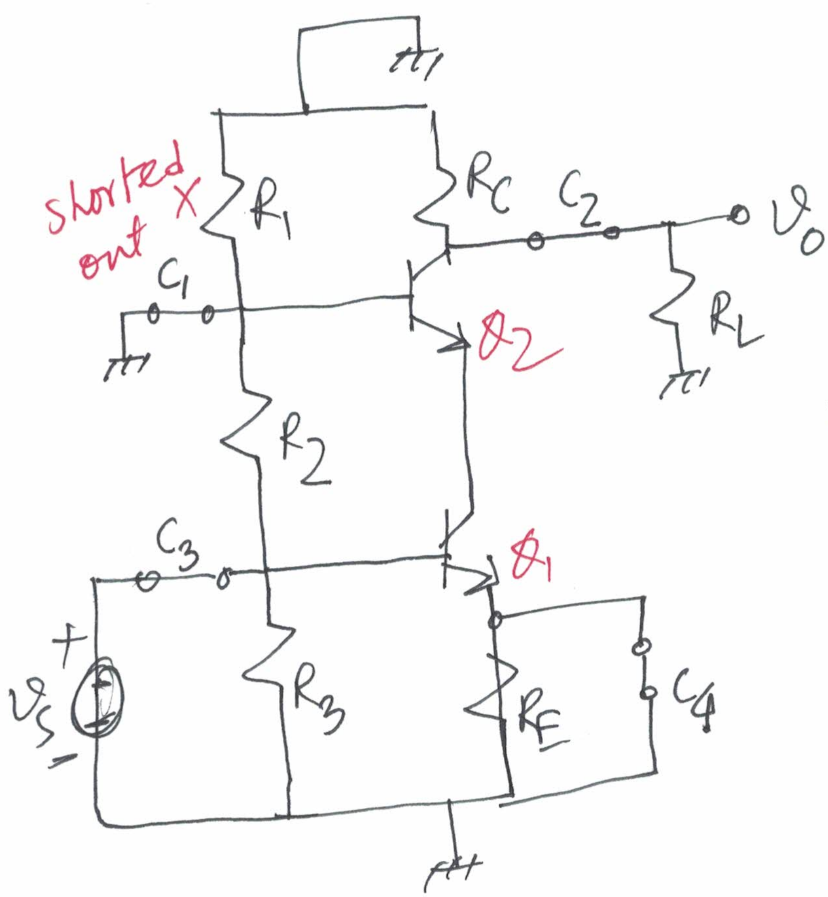
$$R_o = R_{E_2} \parallel \left(r_{\pi_2} + R_{C_1}\right) / (\beta+1)$$

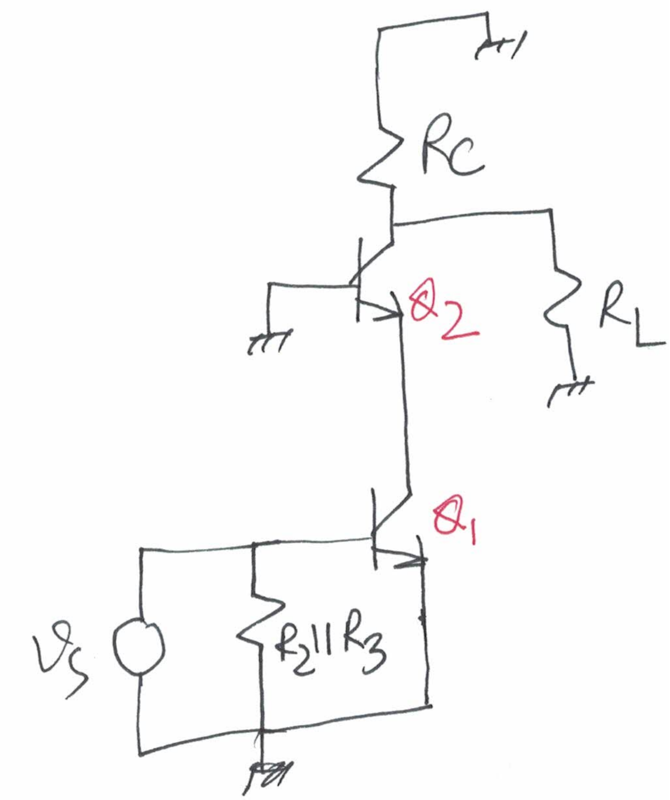
$$= R_{E_2} \parallel 44.54$$

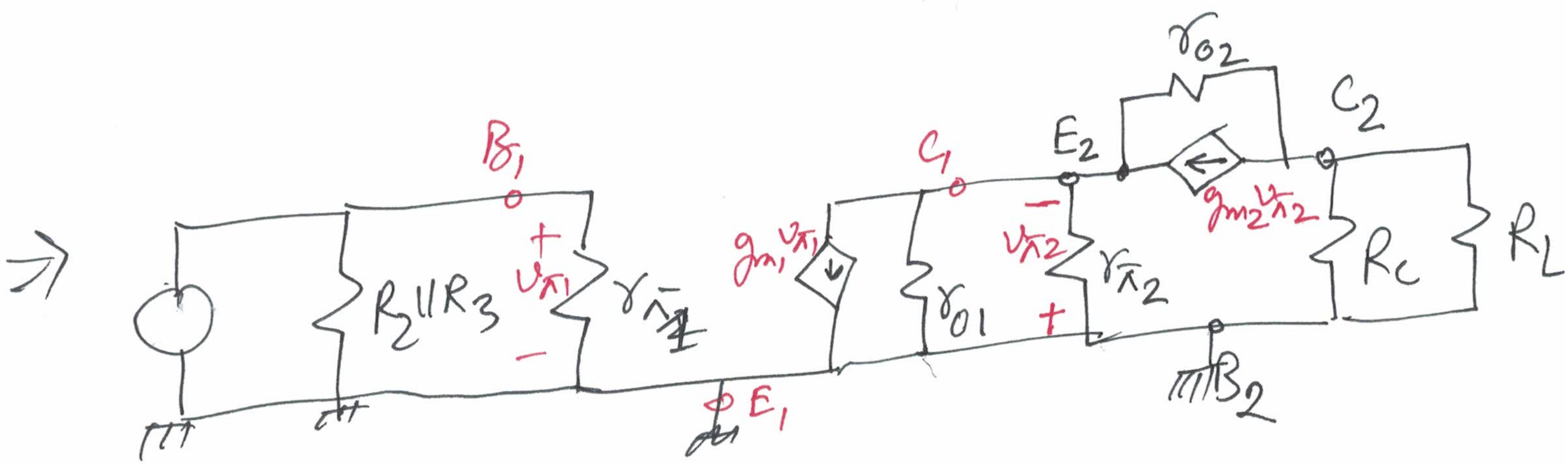
$$\boxed{= 43.256\,\Omega}$$

Problem 7 :

Equivalent AC circuit :

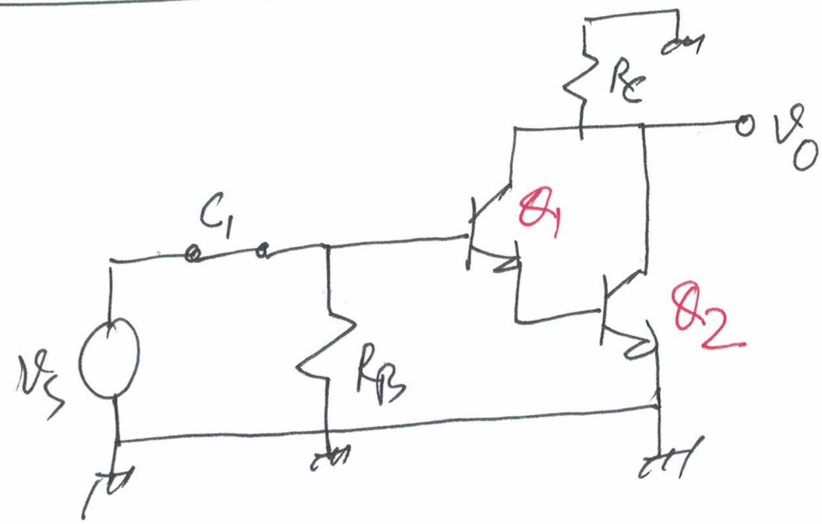


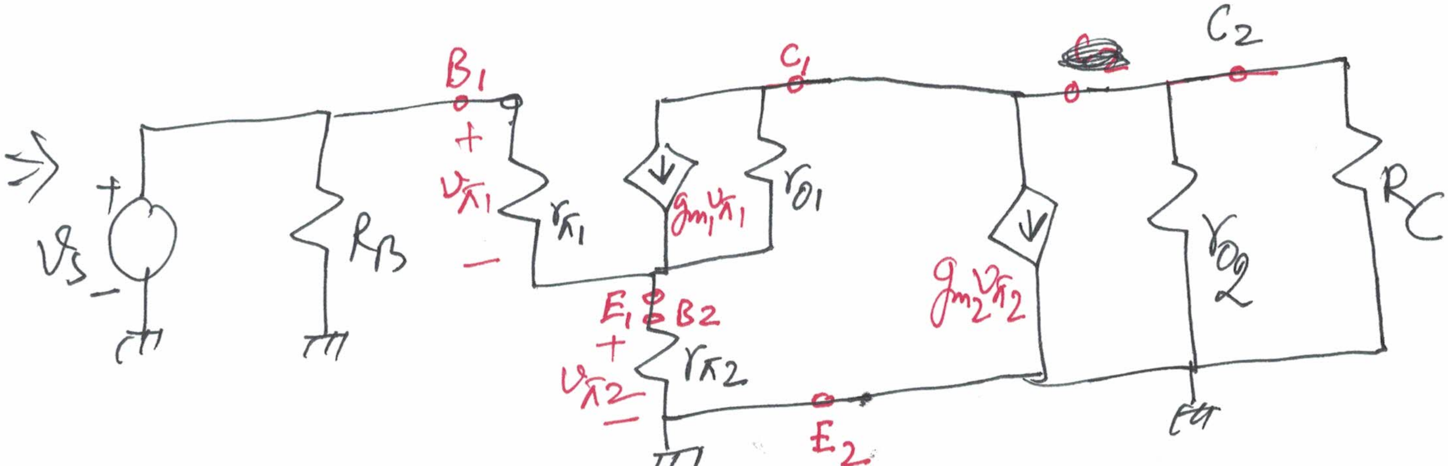




Problem 8:

AC Equivalent Circuit:





For any question, contact: sagor.dhar@ucalgary.ca