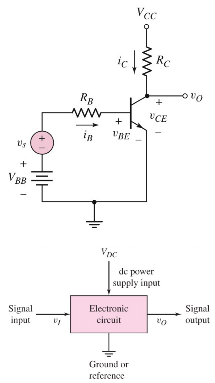
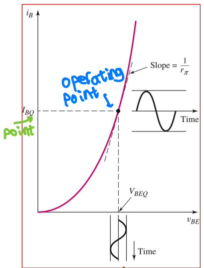
Application of an Input Signal

- DC biasing condition (not changing with time)

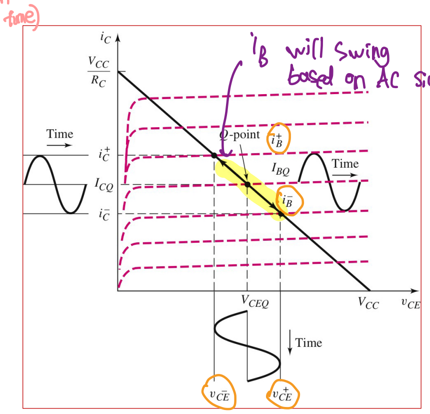




operating point

Q-point

* The current and voltage changes based on signal applied



i_B will swing based on AC signal

* i_b will change V_c

$V = IR$

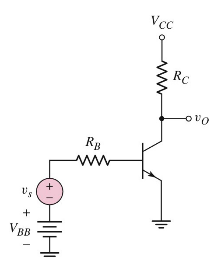
$R = \frac{V}{I}$

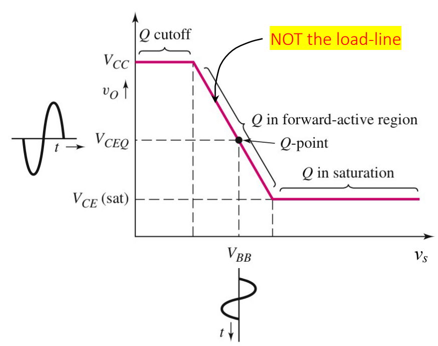
- At $v_s = 0$ (no input signal), $V_{CE} = V_{CEQ}$ and $I_C = I_{CQ}$ (Q-point)
- The operating point moves along the load-line with the changes of input signal

1

1

Transfer Characteristics





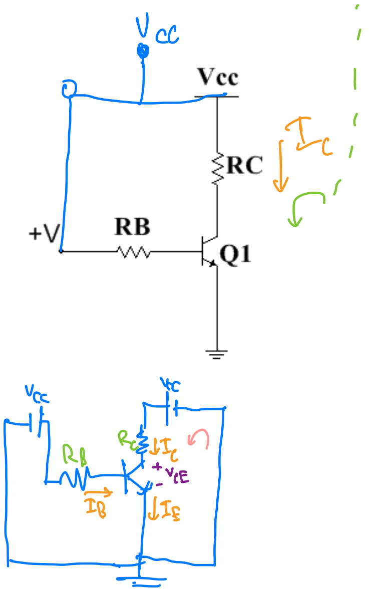
- When an alternating input signal (v_s) is applied V_{CE} and I_C start changing with time in accordance with the input signal
- Large v_s can take the transistor into saturation mode
- Large negative v_s can take the transistor into cut-off mode
- A small input signal can keep the transistor in active region

2

2

Small-Signal Operation: Basic Equations

$V_{CC} - I_C R_C - V_{CE} = 0$



Recall that

$$I_C = I_S e^{\left(\frac{V_{BE}}{V_T}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Shockley's

$$I_E = \frac{I_C}{\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2)$$

$$I_B = \frac{I_C}{\beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3)$$

$$V_C = V_{CE} = V_{CC} - I_C RC \ \ldots \ (4)$$

$V_{CC} - I_C R_C - V_{CE} = 0$

$V_{CE} = V_{CC} - I_C R_C$

3

3

Small-Signal Operation: Conventions

I or V	Upper case means DC current or voltage	DC only
$I_B, V_C, V_E, or I_E$	Upper case suffix means respective DC current or voltage	
i or v	Lower case means signal current (AC) or voltage	AC only
$i_b, v_c, v_e,$ or i_e	Lower case suffix means respective signal current or voltage	
$i_B, v_C, v_E,$ or i_E	This means DC + Signal = total current or total voltage	mix

4

4

Small Signal Analysis of BJT

Neso Academy #92

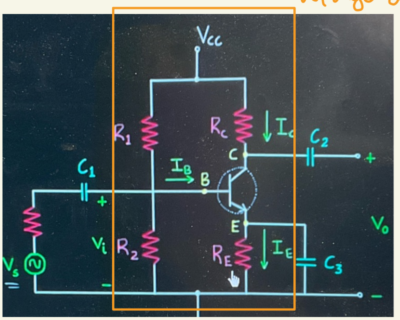
- A small signal is one that will keep transistor in active region (transistor acts as amplifier)

Active → CB is rev biased
Active → EB is fwd biased

Total response = DC response + AC response

BJT amplifier circuit:

voltage divider bias



- In DC we only consider circuit inside rectangle, cuz C_1 and C_2 will act as open circuits
- In AC, must consider rest of circuit cuz capacitors will act as short circuits

C_1 and C_2 are coupling capacitors

C_3 is bypass capacitor

} they all have high capacitance values, ∴ short circuit for AC

- C_1 is coupling previous stage with orange rect
- C_2 is coupling next stage with orange rect

∴ needed so that V_s and V_o don't interfere with DC biasing voltage (V_{cc}), or else operating point will change

reactance of capacitor

$$X_C = \frac{1}{2\pi f C}$$

freq, capacitance

for DC signal:
$f = 0$
$X_C = \frac{1}{0} \approx \infty$
∴ all 3 capacitors will act as open circuits

for AC signal:
$f \neq 0$
capacitance is high
$X_C = \frac{1}{\text{large num}} \approx 0$
∴ all capacitors act as short circuits

- C_3 bypasses AC signal

- AC signal can either go thru R_E or C_3, but it picks C_3 cuz no resistance. so R_E is short circuited.

allowing gain to increase

Ask prof
- what model is this?
π, hybrid π, re, T

10/2/20

Small-Signal Parameters: Transconductance

Equations hold in active region

The total base-emitter voltage is

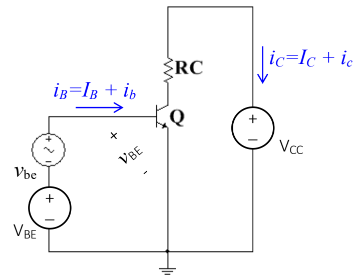
$$v_{BE} = \underbrace{V_{BE}}_{DC} + \underbrace{v_{be}}_{AC}$$

Thus, the total collector current is

$$i_C = I_S e^{\left(\frac{v_{BE}}{V_T}\right)} = I_S e^{\left(\frac{\overbrace{V_{BE}}^{DC}+\overbrace{v_{be}}^{AC}}{V_T}\right)}$$

$$i_C = I_S e^{\left(\frac{v_{BE}}{V_T}\right)} = \underbrace{\overbrace{I_S e^{\left(\frac{V_{BE}}{V_T}\right)}}^{DC}}_{=I_C} \times \overbrace{e^{\left(\frac{v_{be}}{V_T}\right)}}^{signal/AC}$$

$$i_C = I_C \times e^{\left(\frac{v_{be}}{V_T}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$



DC collector current

Here, g_m = Transconductance

$$\boxed{g_m = \frac{I_C}{V_T}} \quad \ldots\ldots\ldots\ldots \quad (7)$$

what is transconductance? what effect does it have on circuit?

indicates biasing methods

it has to do with expansion of exponential series, no need to know math behind it

If $v_{be} \ll V_T$, eq (5) can be written as

$$i_C \approx I_C \left(1 + \frac{v_{be}}{V_T}\right) = I_C + \frac{I_C}{V_T} v_{be} \quad \ldots\ldots (6)$$

$$\boxed{i_C = I_C + g_m v_{be}}$$

Note that the input signal must be very small ($v_{be} \ll V_T$)

Small-Signal Parameters: Current Source

signal current doesn't depend on β

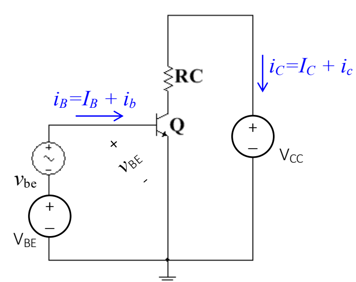
We have just obtained that

$$i_C = I_C + g_m v_{be}$$

$$i_C = I_C + i_c$$

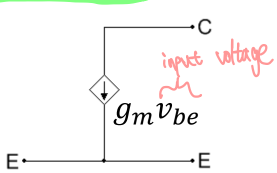
Total current and AC signal, are the same symbol?

Total current = DC + signal

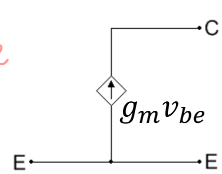


Thus, transistor is a voltage controlled current source as shown below

between collector and emitter



For npn transistor
downwards



For pnp transistor
upwards

$$i_c = \frac{I_C}{V_T} v_{be} = g_m v_{be}$$

$\ldots\ldots\ldots\ldots$ (8)

* g_m is constant factor

Small-Signal Parameters: The Input Resistance at the Base

The signal component of base current is

signal component of collector current

$$i_b = \frac{i_c}{\beta} = \frac{1}{\beta}\times\frac{I_C}{V_T}\times v_{be} \quad \text{From eq (8)}$$

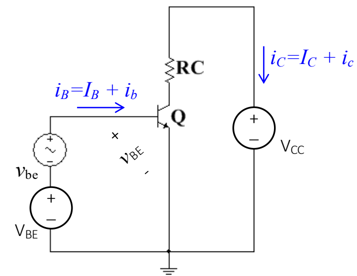
$g_m V_{be}$

$I_C = \beta I_B$ — I_C

$$\text{or,}\quad i_b = \frac{1}{\beta}\times(\beta I_B)\times\frac{1}{V_T}\times v_{be}$$

$$\text{or,}\quad \frac{v_{be}}{i_b} = \frac{V_T}{I_B}$$

$$\text{or,}\quad \boxed{r_\pi = \frac{V_T}{I_B}} \quad \ldots\ldots\ldots \ (9)$$



- $r_\pi = (v_{be})\,/\,(ib)$ = small-signal resistance at the base
- r_π is also the small-signal input resistance for CE and CC configurations
- This is the resistance experienced by the signal at the base, i.e., between base and emitter
- r_π depends on biasing condition

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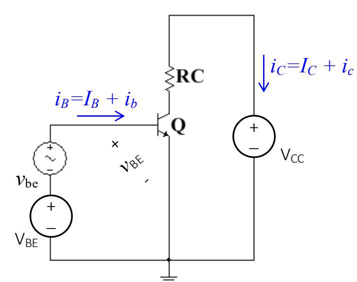
* when you bias you fix these parameters. AC doesn't change the parameters

7

Small-Signal Parameters: The Input Resistance at the Base (Continued)

The signal component of base current is

$$i_b = \frac{i_c}{\beta} = \frac{1}{\beta}\times\frac{I_C}{V_T}\times v_{be} \quad \text{From eq (8)}$$



$$\text{or,}\quad i_b = \frac{1}{\beta}\times(g_m)\times v_{be}$$

$$\text{or,}\quad \frac{v_{be}}{i_b} = \frac{\beta}{g_m}$$

$$\text{or,}\quad \boxed{r_\pi = \frac{\beta}{g_m}} \quad \ldots\ldots\ldots \ (10)$$

$= \dfrac{V_T}{I_B}$

So g_m and r_π depends on our biasing, as I_B is in the term

Transconductance

- g_m depends on biasing condition
- β is an inherent property of a transistor

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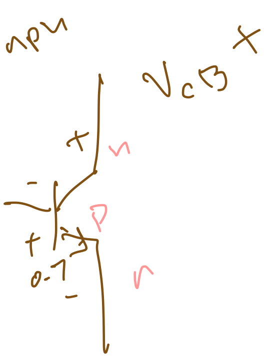
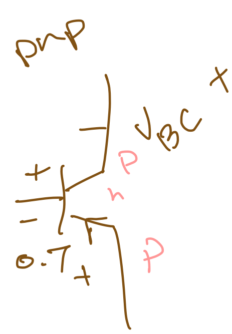
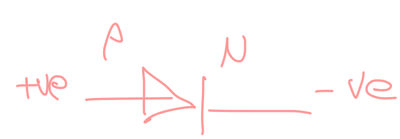




- if p is +ve wrt n then it's fwd



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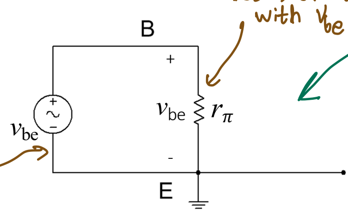
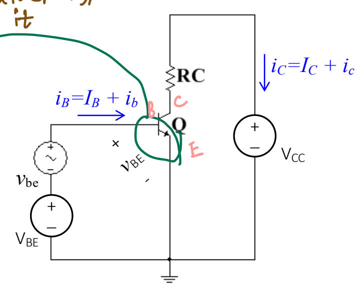
Small-Signal Parameters: The Input Resistance at the Base (Continued)

$$\boxed{r_\pi = \frac{V_T}{I_B}}$$

resistor with value r_π with v_{be} across it





DC voltage source removed

The small-signal equivalent between base and emitter

9

9

Small-Signal Parameters: Emitter Resistance

The signal component of emitter current is

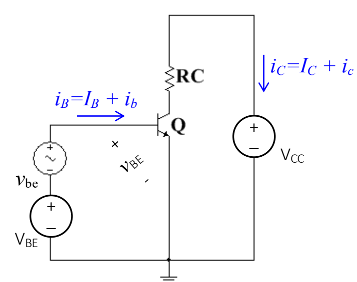
$$i_e = \frac{i_c}{\alpha} = \frac{1}{\alpha} \times \frac{I_C}{V_T} \times v_{be}$$ From eq (8)

or, $i_e = \frac{I_E}{V_T} \times v_{be}$

(Ohm's law) Resistor equation

or, $\frac{v_{be}}{i_e} = \frac{V_T}{I_E}$

or, $\boxed{r_e = \frac{V_T}{I_E}}$ ……… (11)



- $r_e = (v_{be})$ / (ie) = small-signal resistance at the emitter
- r_e depends on biasing condition

- what will the small signal equivalent look like??
- does it replace r_π??

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Small-Signal Parameters: Emitter Resistance (Continued)

We have just derived

$$r_e = \frac{V_T}{I_E}$$

$\alpha = \frac{I_C}{I_E}$

$\downarrow I_E = \frac{I_C}{\alpha}$

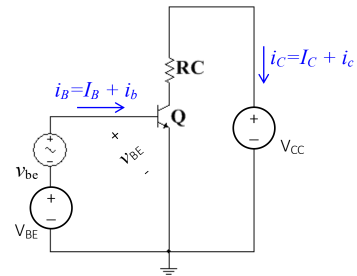
or, $r_e = \dfrac{V_T\alpha}{I_C}$ [using eq (2)]

or, $\boxed{r_e = \dfrac{\alpha}{g_m}}$ ……… (12)

$= \frac{I_C}{V_T}$

$\alpha \approx 1$ in most cases

or, $\boxed{r_e \approx \dfrac{1}{g_m}}$ ……… (13)



Notice that

$$v_{be} = i_b r_\pi = i_e r_e$$

or, $r_\pi = \dfrac{i_e}{i_b} r_e = \dfrac{(\beta + 1) i_b}{i_b} r_e$

$\boxed{r_\pi = (\beta + 1) r_e}$ ……… (14)

11

11

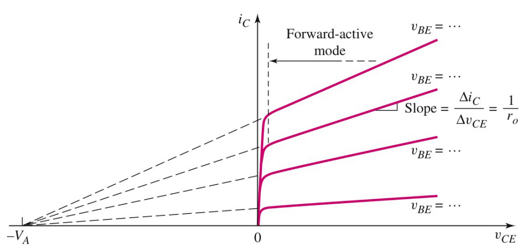
Huh??

Transistor Output Resistance

Recall Early voltage. The linear dependence of i_C versus V_{CE} is expressed by

$$i_C = I_S e^{\left(\frac{v_{BE}}{V_T}\right)}\left(1 + \frac{v_{CE}}{V_A}\right) \quad \text{……… (15)}$$

where did this come from?



The output (collector) resistance is obtained from the slope as

$$\frac{1}{r_o} = \left.\frac{\delta i_C}{\delta v_{CE}}\right]_{v_{BE} = constant}$$

how did he go from here to here?

Using eq (15) we have,

$$\boxed{r_o \cong \frac{V_A}{I_C}}$$

- If $V_A \to \infty$, $r_o = \infty$ and the curves get horizontal (zero slope)
- Transistor is a current source
- r_o is the internal resistance of the current source
- Recall that the internal resistance of an ideal current source is infinite

how??

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Do I need to memorise derivatives?

Small-Signal Parameters: Summary

IMPORTANT

Transconductance	$g_m = \dfrac{I_C}{V_T}$
Input resistance at base	$r_\pi = \dfrac{V_T}{I_B}$ or $\dfrac{\beta}{g_m}$ or $(\beta+1)r_e$
Input resistance at emitter	$r_e = \dfrac{V_T}{I_E} \approx \dfrac{1}{g_m}$
Output resistance	$r_o \cong \dfrac{V_A}{I_C}$

Additional relationships

$r_\pi = \dfrac{\beta}{g_m}$ $\quad$ $r_e = \dfrac{\alpha}{g_m}$ $\quad$ $r_e \approx \dfrac{1}{g_m}$ $\quad$ $r_\pi = (\beta+1)r_e$

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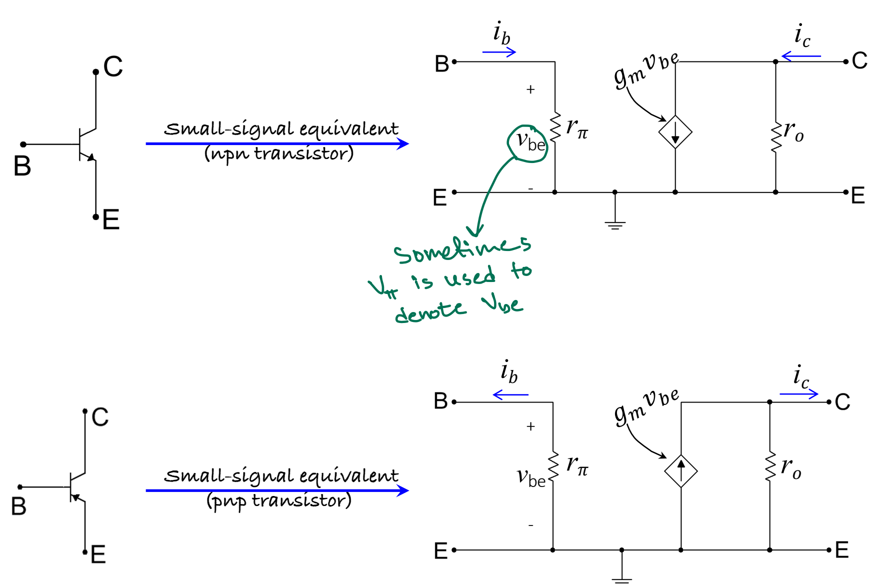
Small-Signal Model: π-Model



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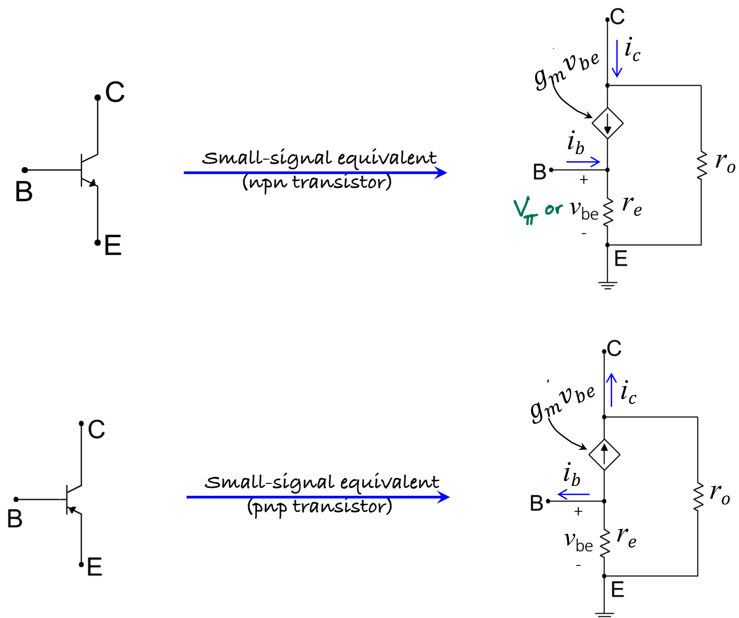
Small-Signal Model: T-Model



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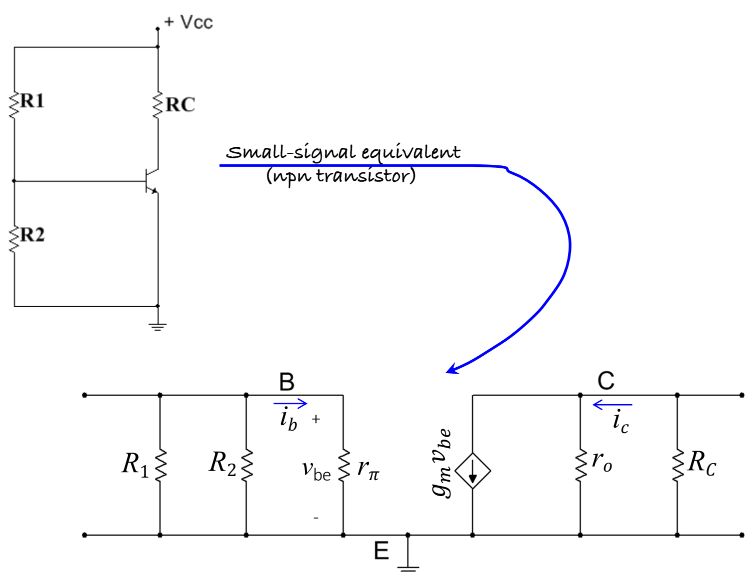
Small-Signal π-Model of a CE Amplifier



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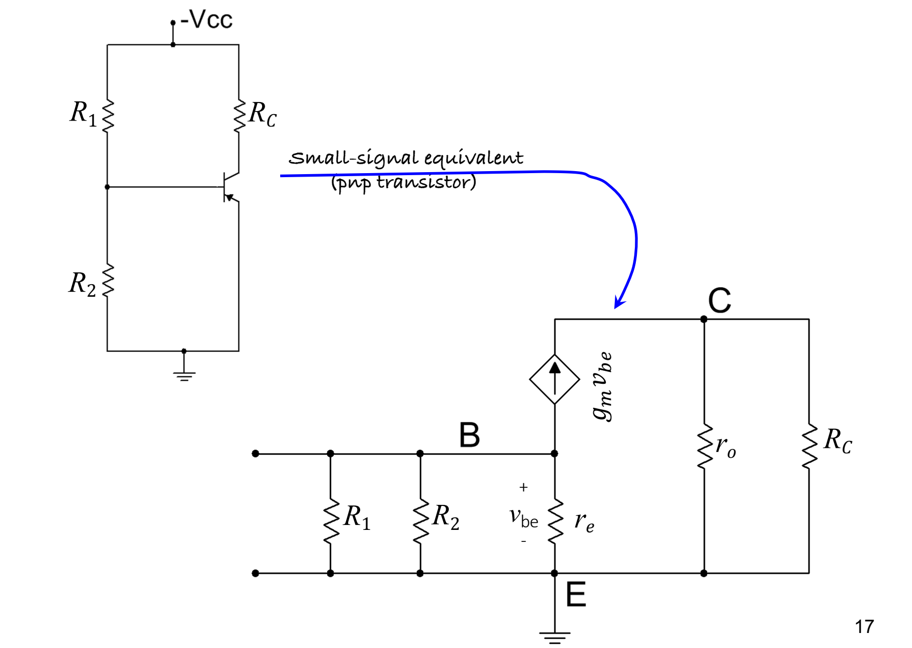
Small-Signal T-Model of a CE Amplifier



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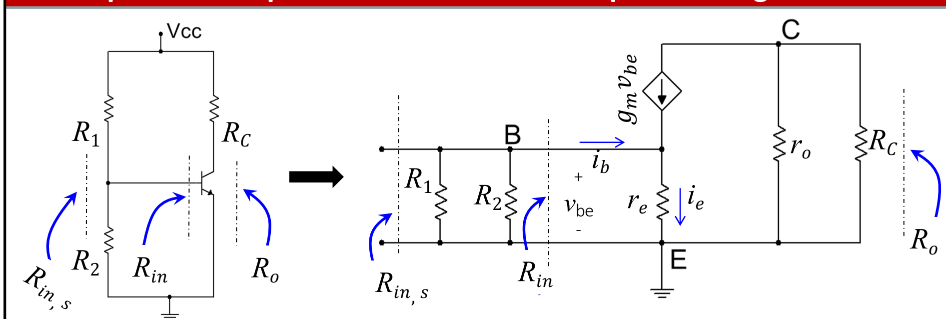
Input and Output Resistance of a CE Amplifier Using T-Model



Input resistance at the base, R_{in}, is

$$R_{in} = \frac{v_{be}}{i_b} = \frac{i_e r_e}{i_b} = \frac{i_b(\beta+1)r_e}{i_b}$$

$\boxed{R_{in} = (\beta+1)r_e = r_\pi}$ [using eq (14)]

Notice that $R_{in} \neq r_e$, but it is multiplied by a factor $(\beta+1)$. Very IMPORTANT

Now $\boxed{R_{in,s} = R_1 || R_2 || R_{in}}$

Output resistance, R_o is given by

$\boxed{R_o = R_C || r_o}$

The current source $g_m v_{be}$ is considered open for determining resistance because it has become an ideal source after separating the internal resistance r_o.

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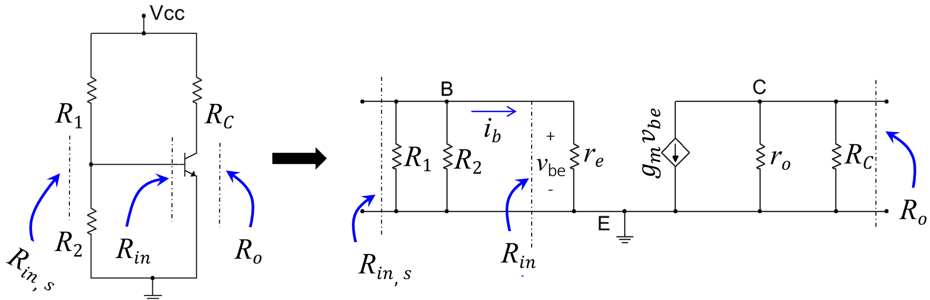
Input and Output Resistance of a CE Amplifier Using π-Model



Input resistance at the base, R_{in}, is

$$R_{in} = \frac{v_{be}}{i_b} = \frac{i_b r_e}{i_b} \qquad \text{or,} \quad \boxed{R_{in} = r_\pi}$$

Now $\boxed{R_{in,s} = R_1 || R_2 || R_{in}}$

Output resistance, R_o is given by

$$\boxed{R_o = R_C || r_o}$$

The current source $g_m v_{be}$ is considered open for determining resistance because it has become an ideal source after separating the internal resistance r_o.

Note that the π-model and T-model produce the same result

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