Wed Oct 25th

Common-mode input voltage

if $V_1 = V_2$

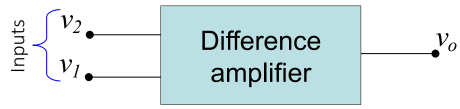
$V_{cm} = \frac{V_1 + V_2}{2}$

then would $V_0 = 0$?

$V_0 = A_v \cdot V_d$

Differential Amplifier: Basic Idea

- Most widely used building block in analog integrated circuit
- Basis of very-high-speed logic circuit family



- Ideally v_o is proportional to $(v_2 - v_1)$
- $v_o = A_v(v_2 - v_1)$
- Here, A_v = open-loop voltage gain
- $(v_2 - v_1) = v_d$
- v_d = differential input voltage

$\therefore V_0 = A_v V_d$

- If $v_1 = v_2$, the input voltage is called common-mode input voltage, denoted by v_{cm}
- If $v_1 = v_2$, $v_{cm} = v_1 = v_2$ and

$$v_{cm} = \frac{v_1 + v_2}{2}$$

$A_v = \frac{v_0}{v_d}$

output voltage

$V_2 - V_1$

open loop voltage gain

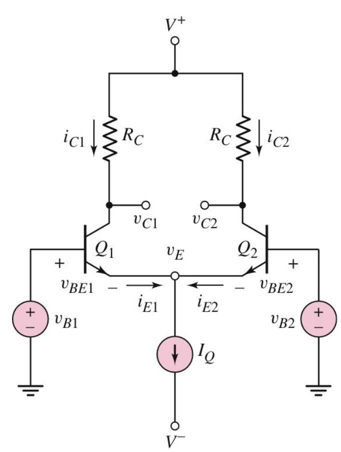
differential input voltage

1

1

A BJT Differential Pair Circuit

- The difference of the two input signals, v_{B1} and v_{B2} will be amplified at the output ✓
- For both transistors, the B-E junctions are forward biased and the C-E junctions are reversed biased, i.e., the transistors operate in active region.
- If the input signals become zero, i.e., $v_{B1} = v_{B2} = 0$, the transistors will still remain in the active region because of the current source.
- Both transistors must remain in active region for differential amplifier operation



BE is fwd

CE is rev

2

2

Differential Pair: Basic Operation (Case-A)

- Q1 and Q2 are identical
- Q1 and Q2 are in active region
- The circuit has a floating ground
- Here, both transistors are ON
- The current source I is ideal
- $i_{E1} = i_{E2} = I/2$
- $i_{C1} = i_{C2} = (\alpha I)/2$
- $V_{E1} = V_{E2} = (V_{cm} - V_{BE})$

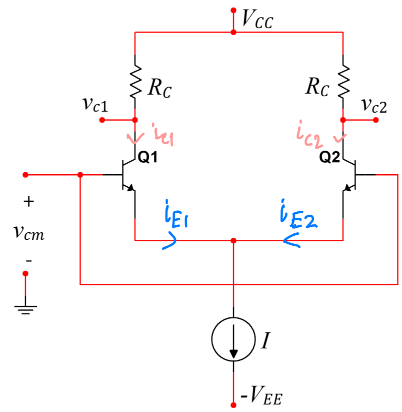
or, $V_{E1} = V_{E2} \approx (V_{cm} - 0.7)$

- $V_{C1} = V_{C2} = V_{CC} - \frac{\alpha I}{2} R_C$

i_C

- $V_{C1} - V_{C2} = 0$

$V_{C1} - V_{C2} = 0$ remains valid for any values of V_{CM} as long as Q1 and Q2 remain in active region.



Thus differential pairs REJECT the common mode input signal.

3

$I = i_{E1} + i_{E2}$

$I = 2 i_E$

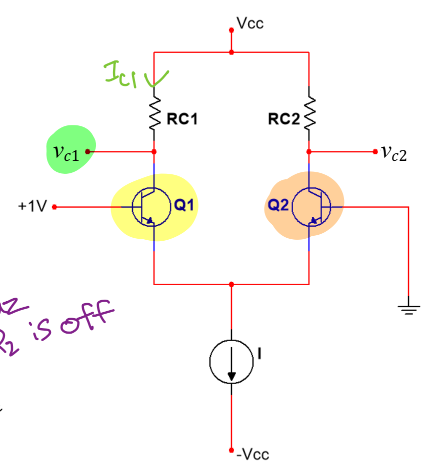
$i_E = \frac{I}{2}$

- differential pairs usually in practice don't work in common-mode

3

Differential Pair: Basic Operation (Case-B)

- Q1 and Q2 are identical
- The current source is ideal
- Transistor Q1 is ON
- Transistor Q2 is OFF



- $V_{E1} = V_{E2} \approx 1 - 0.7 = 0.3$ V
- $I_{C1} = \alpha I$ and $I_{C2} = 0$

cuz Q_2 is off

- $V_{C1} = V_{CC} - \alpha I R_C$ and $V_{C2} = V_{CC}$

Thus, $V_{C2} - V_{C1} = \alpha I R_C \approx I R_C$

cuz $\alpha \simeq 1$

$* \; V_{C2} - V_{C1} = I R_{C1}$

4

- there will be output in cases B and C

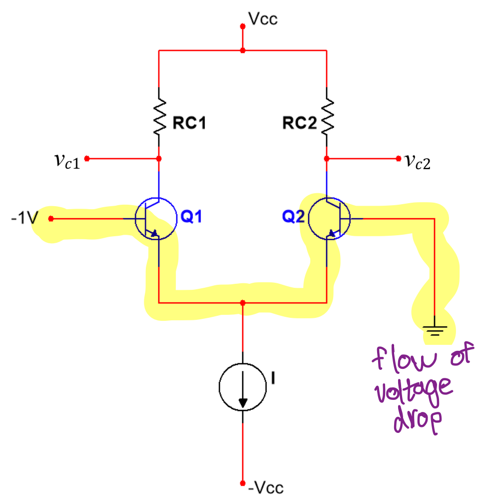
4

opposite of case B

Differential Pair: Basic Operation (Case-C)

- Q1 and Q2 are identical
- The current source is ideal
- Transistor Q1 is OFF
- Transistor Q2 is ON
- $V_{BE2} \approx 0.7$ V
- $V_{E1} = V_{E2} \approx -0.7$ V
- $V_{BE1} = -1 - (-0.7) = -0.3$ V
- $I_{C2} = \alpha I$ and $I_{C1} = 0$
- $V_{C2} = V_{CC} - \alpha I R_C$ and $V_{C1} = V_{CC}$

Thus, $V_{C2} - V_{C1} = -\alpha I R_C \approx -I R_C$



- check which is more +ve, that would be ON

flow of voltage drop

$V_{C2} - V_{C1} = -I R_{C2}$

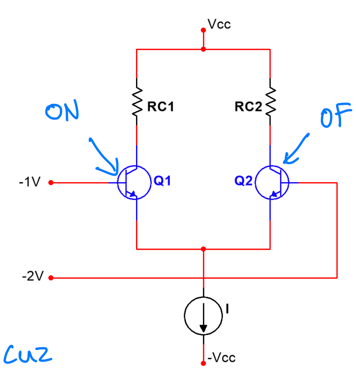
5

5

end here

Fri Oct 27

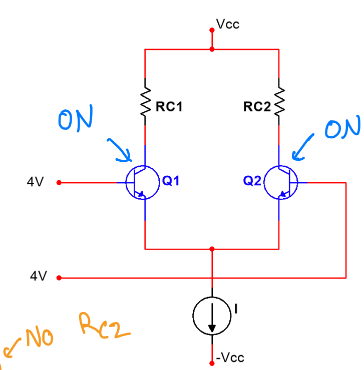
Determine the collector currents in each circuit



ON

OFF

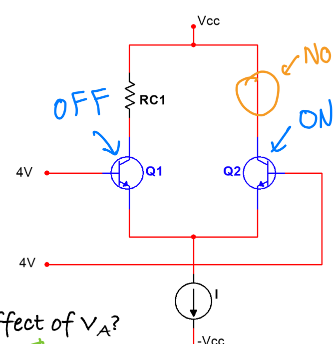
cuz $-1 > -2$



ON

ON

common mode, no output



OFF

ON

No R_{C2}

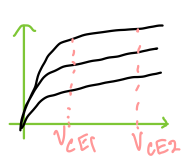
- V_{CE}'s are equal if collector resistors are equal

Effect of V_A?

6

6

there is an effect



- V_A affects r_o in small signal analysis

$r_o = \frac{V_A}{I_C}$

can perform very fast switching

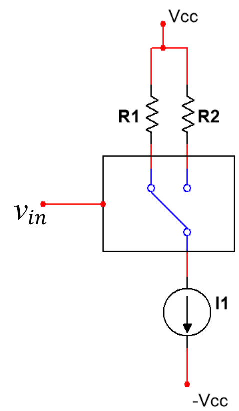
*go to textbook

Differential Pair as a Switch

Q7
- how exactly does it work?
- ex question?

~simply diagram of last slide

- A change in input signal can switch the transistors
- A single-pole double-throw (SPDT) switch
- A small signal can steer the current



7

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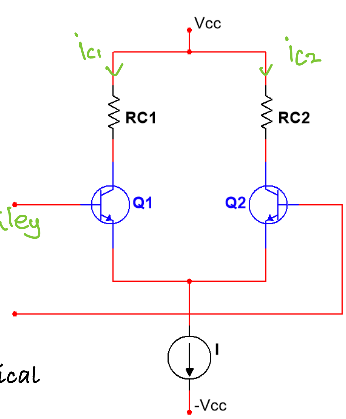
Transfer Characteristics

Consider a BJT differential pair

$$i_{C1} = I_S e^{\left(\frac{v_{B1}-v_{E1}}{v_T}\right)} \quad \dots\dots\dots (1)$$

$$i_{C2} = I_S e^{\left(\frac{v_{B2}-v_{E2}}{v_T}\right)} \quad \dots\dots\dots (2)$$

Shockley



Assumed that the two transistors are identical and operating at the same temperature.

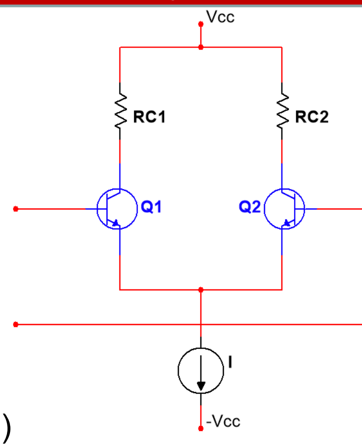
Dividing eq(1) by eq(2), we have, $V_{E1} = V_{E2}$

$$\frac{i_{C1}}{i_{C2}} = e^{\left(\frac{v_{B1}-v_{B2}}{v_T}\right)} = e^{\left(\frac{v_{id}}{v_T}\right)} \dots\dots\dots (3)$$

8

8

Transfer Characteristics (Continued ...)



$i_C \approx i_E$

One can manipulate equation (3) to yield

individual

total

$$\frac{i_{C1}}{i_{C1}+i_{C2}} = \frac{i_{C1}}{I} = \frac{1}{1+e^{-\left(\frac{v_{id}}{v_T}\right)}} \cdots\cdots\cdots (4)$$

$$\frac{i_{C2}}{i_{C1}+i_{C2}} = \frac{i_{C2}}{I} = \frac{1}{1+e^{\left(\frac{v_{id}}{v_T}\right)}} \cdots\cdots\cdots (5)$$

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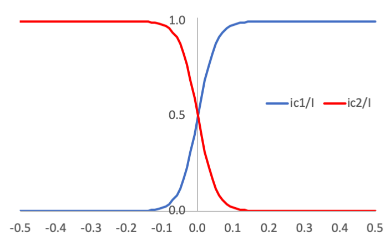
$V_{id} = V_{be1} - V_{be2}$

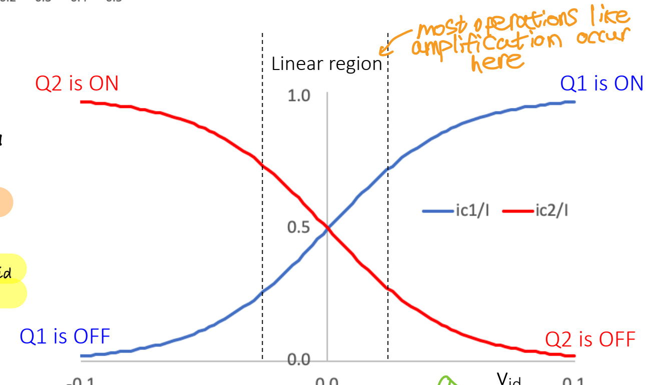
Transfer Characteristics (Continued ...)



- The amplifier responds only to V_{id}. Hence, a difference amplifier
- When V_{id} =0, I divides equally
- $V_{id} \approx$ 100 mV can switch the current (Thus, fast switching)
- In the linear region the gain is proportional to the slope

gain ∝ slope

most operations like amplification occur here



- In order to maintain a linear amplifier, the excursion of V_{id} must be kept small
- For linear operation, V_{id} is limited to $V_T/2$ ($\approx \pm$ 12.5 mV)

10

* Memorise

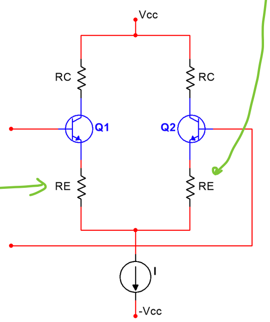
outside linear region you are at risk of producing harmonics

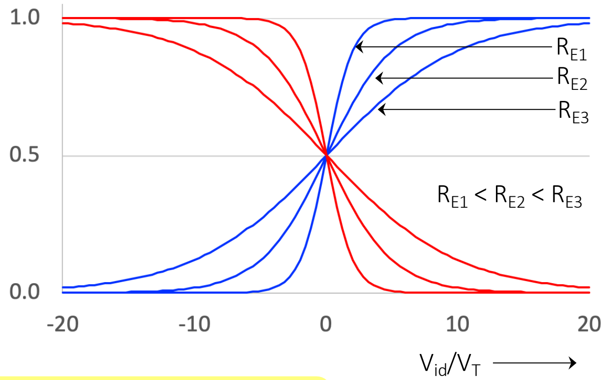
R_E must be on both sides and must be equal

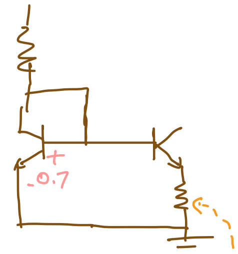
Improved Linear range

To increase linear region add R_E

∴ 12.5 is no longer valid







$V_{BE1} \neq V_{BE2}$ due to the emitter resistor

- Emitter resistance increases the linear range
- This allows larger V_{id}

Gain a Deeper Understanding (GADU)

- What changes in the collector currents may occur if the two emitter resistors are not identical

11

-what are the changes??

11

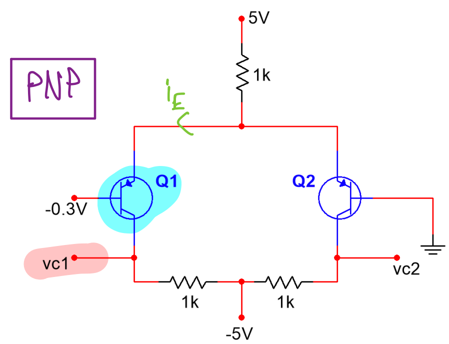
-might not be symmetrical

Example: Differential Pair

Example: $|V_{BE}|$ of a conducting transistor is 0.7V and $\alpha \approx 1$.

Find a) v_E, b) i_{C1} and i_{C2}, c) v_{C1} and v_{C2}, and d) v_{CE1} and v_{CE2}

PNP



Steps

1. Determine state of each transistor
2. Find V_E, i_E, V_C, V_{CE}

Solution:

Here, Q1 is ON and Q2 is OFF

a) $v_{BE} = v_{B1} - v_E$ $\quad V_{be} = -0.7$

or, $v_E = v_{B1} - v_{BE}$

or, $v_E = -0.3 - (-0.7) = 0.4V$

b) $i_E = (5 - 0.4)/1x10^3$

$i_E = 4.6\ mA$

Thus, $i_{C1} = 4.6\ mA$, $i_{C2} = 0$

∴ $I_C = \alpha I_E$

c) $v_{C1} = -5 + 4.6 \times 1 = -0.4$

$v_{C2} = -5V$

d) $v_{CE1} = -0.4 - 0.4 = \cancel{0.8}\ V$ $\quad -0.8V$

$v_{CE2} = -5 - 0.4 = -5.4\ V$

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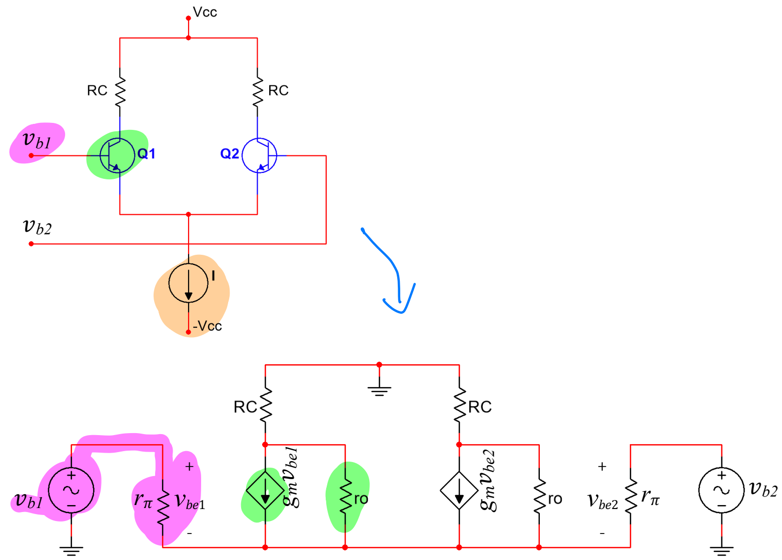
A BJT Differential Pair Circuit



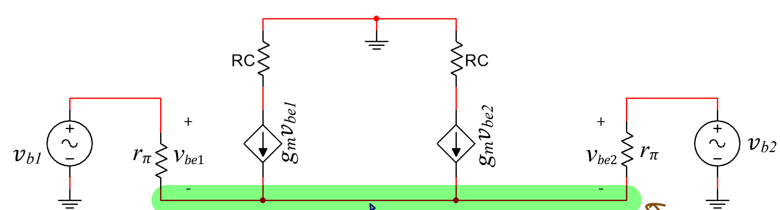
13

13

r_o is gone – since $V_A = \infty$

Simplified Small-Signal Equivalent Circuit

π - model for 2 transistors in differential circuit



This is not ground

KCL

$I = \frac{V}{R}$

$$\frac{v_{be1}}{r_\pi} + g_m v_{be1} + g_m v_{be2} + \frac{v_{be2}}{r_\pi} = 0$$

$$v_{be1}\left(\frac{1}{r_\pi} + g_m\right) = -v_{be2}\left(\frac{1}{r_\pi} + g_m\right)$$

$$v_{be1} = -v_{be2}$$

Differential input signal is split equally but have opposite sign

14

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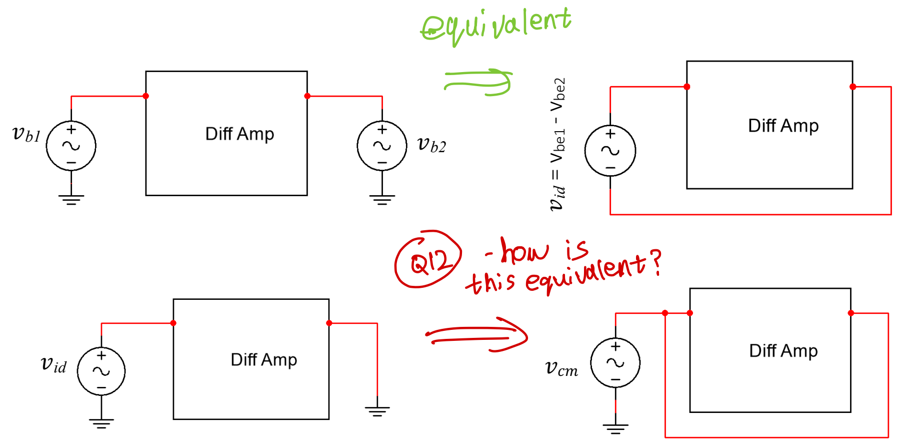
differential input signal = $V_{id} = V_{be1} - V_{be2}$

Differential Input and Common-Mode Input



$V_{cm} = \frac{V_1 + V_2}{2}$

$V_{id} = V_{be1} - V_{be2}$

$$v_{id} = v_{be1} - v_{be2} = v_{be1} - (-v_{be1}) = 2v_{be1}$$

$v_{be1} = \frac{v_{id}}{2}$ Similarly, $v_{be2} = -\frac{v_{id}}{2}$

found in previous slides

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*quiz 3 until here

15

Mon Nov 6th

Single Ended Voltage Gain

$v_{be1} = \frac{1}{2} V_{id}$

$$v_{C1} = -(g_m R_C)(v_{be1}) = -\frac{1}{2} g_m R_C v_{id}$$

$$A_{d1} \equiv \frac{v_{c1}}{v_{id}} = -\frac{1}{2} g_m R_C$$

Notice, the gain is negative, which is the same as CE amplifier

Q13 - how is gain of CE amplifier -ve?

$v_{be1} = -V_{be2}$ results in +ve gain

$$v_{C2} = -(g_m R_C)(v_{be2}) = +\frac{1}{2} g_m R_C v_{id}$$

$$A_{d2} \equiv \frac{v_{c2}}{v_{id}} = \frac{1}{2} g_m R_C$$

Notice, the gain is positive

A_{d1} and A_{d2} are single ended gain

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Differential Voltage Gain

no need to memorise

Differential gain, $A_d = A_{d1} - A_{d2}$

$$A_d = -\frac{1}{2}g_m R_C - \frac{1}{2}g_m R_C$$

$$A_d = -g_m R_C$$

If defined as, $A_d = A_{d2} - A_{d1}$

$$A_d = \frac{1}{2}g_m R_C - \left(-\frac{1}{2}g_m R_C\right)$$

$$A_d = g_m R_C$$

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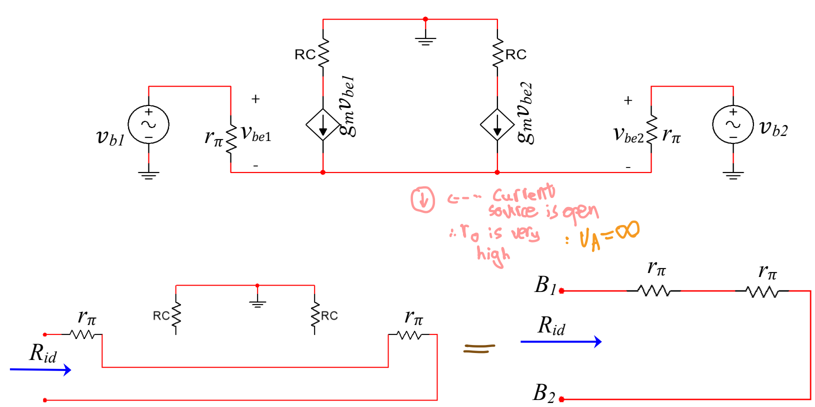
Differential Input Resistance

R_{id}



Q14
-how did we arrive here?

Differential resistance, $R_{id} = 2r_\pi$

or, $R_{id} = 2(\beta+1)r_e$

$r_\pi = (\beta+1)r_e$

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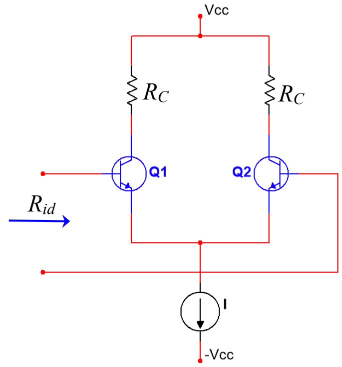
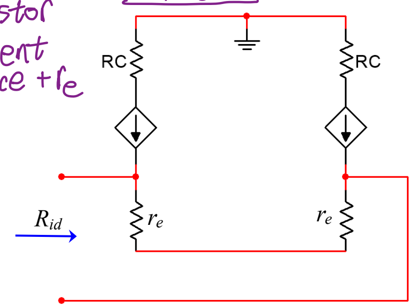
Differential Input Resistance



Transistor = current source + r_e

T-model



$R_{id} = 2 r_\pi$

Differential resistance, $R_{id} = 2(\beta + 1) r_e$

$\therefore r_\pi = (\beta+1) r_e$

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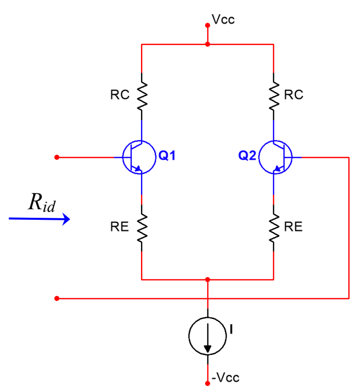
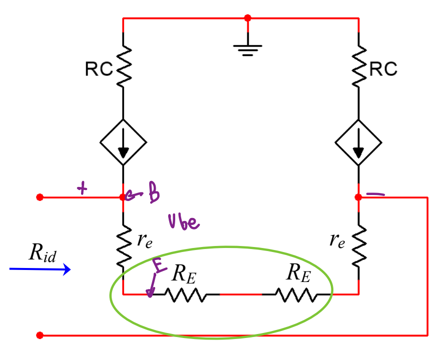
Differential Input Resistance With Emitter Resistor RE





Differential resistance, $R_{id} = 2(\beta + 1)(r_e + R_E)$

20

20

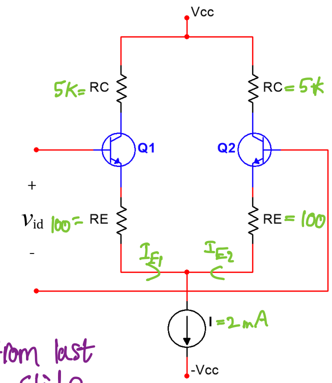
go over example again

Example: Differential Pair

Example: Given that v_{id} = 0.1V, RE=100 Ω, R_C = 5k, I = 2 mA, α=1, and β=100. $|V_{BE}|$ of a conducting transistor is 0.7V.

$V_{be1} - V_{be2}$

Find a) i_e and v_{be}, b) total emitter current, c) signal voltage at each collector, d) differential voltage gain ($v_{od} = v_{c1} - v_{c2}$), and e) differential input resistance R_{id}.



Solution:

$I = 2I_E = 2$
$\therefore I_E = 1$

a) $r_e = V_T / I_E = 25/1 = 25\ \Omega$

Here, $i_E = v_{id} / 2(r_e + R_E)$ ← $i = \frac{V}{R}$ from last slide

or, $i_E = 0.1/2(25+100) = 0.4$ mA

$v_{be1} = r_e \times i_e = 25 \times 0.4\times10^{-3}$

or, v_{be1} = 10 mV

and, $v_{be2} = -v_{be1}$

or, v_{be2} = = -10 mV

- why is Vbe1 not 0.7??

- since it's done in small signal analysis, not DC

b) i_{E1} = I/2 + 0.4 mA = 1.4 mA

and i_{E2} = I/2 - 0.4 mA = 0.6 mA

Note that $i_{E1} + i_{E2}$ = I

how??

$i_{E1} = \frac{I}{2} + i_E$

$i_{E2} = \frac{I}{2} - i_E$

- due to diagram on prev slide

- follow current flow

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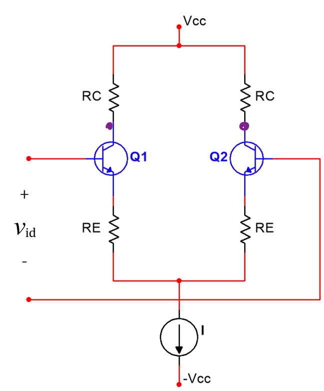
Example: Differential Pair (Continued ...)

Example: Given that v_{id} = 0.1V, 100 Ω, R_C = 5k, I = 2 mA, α=1, and β=100. $|V_{BE}|$ of a conducting transistor is 0.7V.

Find a) i_e and v_{be}, b) total emitter current, c) signal voltage at each collector, d) differential voltage gain ($v_{od} = v_{c1} - v_{c2}$), and e) differential input resistance R_{id}.



Solution:

c) $v_{c1} = -i_C R_C \approx -i_E R_C$ = -0.4mA x 5k

$v_{c1} \approx$ -2V

Similarly, $v_{c2} = +i_C R_C \approx$ 2 V

d) A_{vd} = $(v_{c1} - v_{c2})/v_{id}$

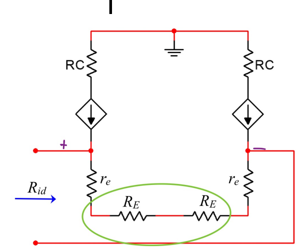
or, A_{vd} = (-2 - 2)/0.1 = - 40

diff voltage gain

e) $$R_{id} = 2(\beta+1)(r_e + R_E)$$

R_{id} = 2(25 + 100) x 101

R_{id} = 25 kΩ



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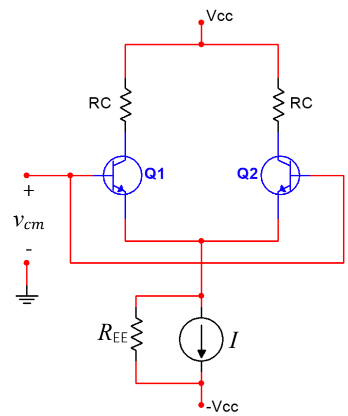
differential voltage gain = $\frac{V_{od}}{V_{id}}$

- Know how to use equations in upcoming slides, no need to know derivation

Common-Mode Gain

Ideally, the common-mode gain should be zero.

Because $V_{C1} - V_{C2} = 0$



However, it has a nonzero value in practice

$$A_{cm} \approx \frac{\Delta R_C}{2R_{EE}}$$

Here, R_{EE} = internal resistance of the current source

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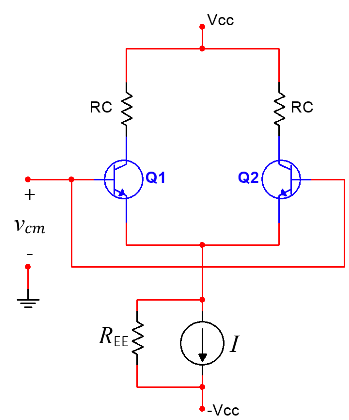
CMRR and Common-Mode Input Resistance

- defines quality of OpAmp
- found in datasheet

CMRR (Common-Mode Rejection Ratio)

$$CMMR = 20\ log\left|\frac{A_d}{A_{cm}}\right|\ dB$$



The common-mode input resistance R_{icm} is given by

$$R_{icm} = (\beta + 1)\left[(R_{EE})||\left(\frac{r_o}{2}\right)\right]$$

output resistance of individual transistor

Here, R_{EE} = internal resistance of the current source

and $r_o = r_{o1} = r_{o2}$ = output resistance

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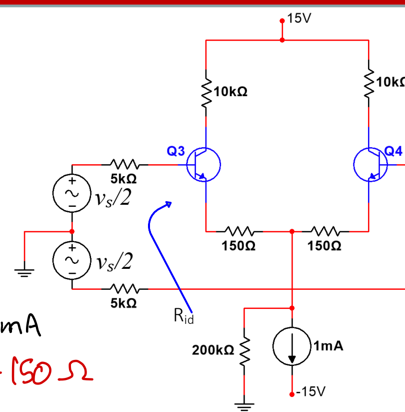
24

* solve at home *

Example: Differential Pair

Example: Given that $V_A = 100$ V and $\beta = 100$. Find a) input differential resistance, R_{id}, b) overall differential, $|v_o/v_s|$, c) the worst-case A_{cm} if R_C has ±1% tolerance, and d) the input common-mode resistance, R_{icm}.



Solution:

a) $r_{e1} = r_{e2} = r_e = V_T/I_E = 25\text{ mV}/0.5\text{ mA}$

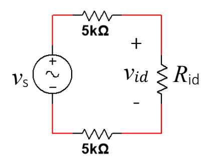
or, $r_e = 50\ \Omega$

$R_E = 150\ \Omega$

$R_{id} = 2(\beta + 1)\ (r_e + R_E) = 40\text{ k}\Omega$

- rounded

b) $v_{id} = v_s \dfrac{R_{id}}{5k\Omega + 5k\Omega + R_{id}}$



or, $v_{id} = v_s \dfrac{40\times10^3}{5k\Omega + 5k\Omega + 40\times10^3} = 0.8\ v_s$

or, $\dfrac{v_{id}}{v_s} = 0.8$

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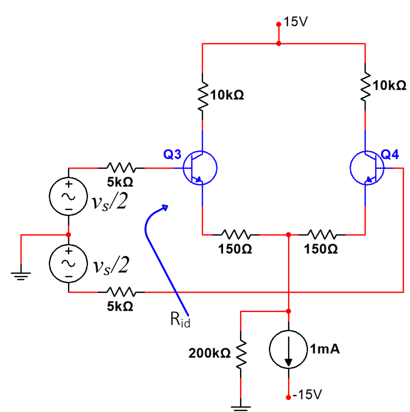
Example: Differential Pair (continued ...)

Voltage gain from the bases to the output ($v_{od} = v_{c1} - v_{c2}$) is

$$\left|\frac{v_{od}}{v_{id}}\right| = \frac{\text{total collector resistance}}{\text{total base resistance}}$$



or, $\left|\dfrac{v_{od}}{v_{id}}\right| = \dfrac{2R_C}{2(\underset{50}{r_e}+\underset{150}{R_E})} = \dfrac{20\times10^3}{2\times200} = 50$

Now,

$$\left|\frac{v_{od}}{v_s}\right| = \frac{v_o}{\cancel{v_{id}}} \times \frac{\cancel{v_{id}}}{v_s} = 50\times0.8 = 40$$

Thus, the overall voltage gain is 40

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Example: Differential Pair (continued …)

2% of $R_c = 200\,\Omega$

c) With $\pm 1\%$ tolerance,

$\Delta R_{C(max)} = 2\%$ of R_C, which is 200Ω.

Thus, the worst case A_{cm} is

$$A_{cm} = \frac{\Delta R_C}{2R_{EE}} = \frac{200}{2\times 200\times 10^3} = 5\times 10^{-4}$$

d) We know,

$$R_{icm} = (\beta+1)\left[(R_{EE})\|\left(\frac{r_o}{2}\right)\right]$$

$r_o = \frac{V_A}{I_c}$

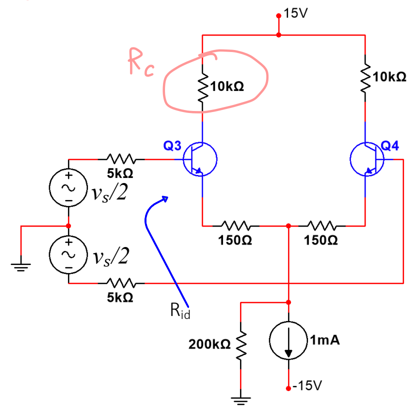
Here, $r_o = (V_A)/(I/2) = 100/(0.5\times 10^{-3})$

or, $r_o = 200\ k\Omega$ ** (r_o of Q4)

Thus, $R_{icm} = 101\,(200k\ ||\ 100k)$

or, $R_{icm} = 6.73\ M\Omega$

very high



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identical transistors → inherent parameters are equal → β, I_s

Example: Differential Pair

Example: Given that $V_A = 100$ V and β=100. Find a) input differential resistance, R_{id}, b) overall differential, $|V_o/V_s|$, c) the worst-case A_{cm} if R_C has $\pm 1\%$ tolerance, and d) the input common-mode resistance, R_{icm}.

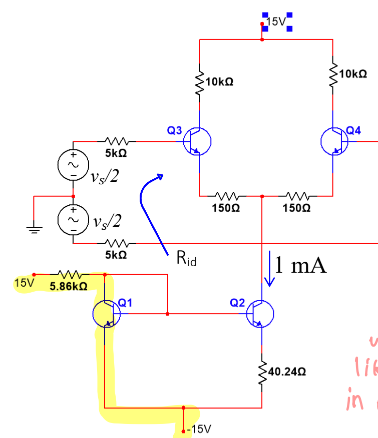
Solution:

R_{id} will remain the same ✓ $40\,k\Omega$

The overall gain will remain the same ✓ 40

The common-mode gain will have a different value

The common-mode input resistance will have a different value



very likely in exam

R_{EE} = output resistance of mirrored transistor

$r_{o2} = \frac{V_A}{I_{c2}}$

$I_{c2} = \frac{15-0.7-(-15)}{5.86\times 10^3}$

$= 5mA$

$\therefore r_{o2} = \frac{100}{5mA} = 20\,k\Omega$

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$R_{icm} = (\beta+1)\left[R_{EE} \,\|\, \frac{r_o}{2}\right]$

$= 101\,(20k \,//\, 10k)$

$= 673.3\,k\Omega$

1% of $20k = 200$

$A_{cm} = \frac{\Delta R_c}{2R_{EE}} = \frac{200}{2\times 20k} = 5\times 10^{-3}$

common mode gain