1. Consider the single-variable equation of

$$f(x) = \exp(0.45x) - x^2 + 5$$

The objective is to determine the two extremum points of this function.

1. Use the `sympy.diff()` to determine the $\frac{df(x)}{dx}$. Plot this out over the range $-10 < x < 12$. This function has two stationary points (zero slope points) in this region.
2. Use `scipy.optmize.root_scalar()` to determine the solutions of $\frac{df(x)}{dx} = 0$, which are the extremum points.

2. Consider the single variable equation of

$$f(x) = 1.2\exp(0.53x) - 2.3x + 1.01$$

Generate a plot of this function over $-10 < x < 8$. Use `scipy.optimize.root` to find the two zeros of the function and indicate the location of these zeros with a suitable marker on the plot. Complete your plot with a legend.

3. Consider the curve parameterized by the equations

$$x(t) = \sin(2t)$$
$$y(t) = \cos(t)$$
$$z(t) = t$$

and assume a range in the parameter t of $0 \leq t \leq 10$. Create a three-dimensional plot of this curve using `plot3()`. Then determine the arc length of the curve over the parameter range of $0 \leq t \leq 10$. *Hint:* Recall from calculus that the arc length is given as:

$$L = \int_t \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,\mathrm{d}t$$

Then for this problem, the arc line length is given as

$$L = \int_0^{10} \sqrt{4\cos^2(2t) + \sin^2(t) + 1}\,\mathrm{d}t$$

4. In this problem, you are to calculate the integration of the function $f(x, y, z)$ over a volume defined by its surface boundaries. A simple problem is selected that has an algebraic solution. Here the integrand function is $f(x, y, z) = xyz^2$, and the volume is a prism in Figure 1.

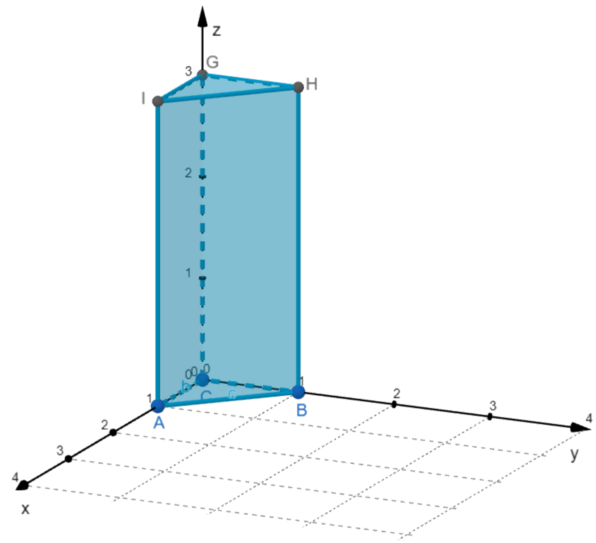

Figure 1: Prism for Q4

The analytical solution of the volume integration is given as

$$\int f(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \int_0^3 z^2 \left\{ \int_0^1 y \left[\int_0^{1-y} x\,\mathrm{d}x \right] \mathrm{d}y \right\} \mathrm{d}z$$

$$= \frac{1}{2}\int_0^3 z^2\,\mathrm{d}z \int_0^1 (1-y)^2 y\,\mathrm{d}y$$

$$= \frac{1}{2} \times 9 \times \frac{1}{12} = \frac{3}{8}$$

Note the treatment of the prism boundaries. For this problem, use `np.tplquad()` to do the triple integration over the prism volume.

5. Consider a series circuit of a battery connected to a series circuit consisting of a diode with a current-voltage relation of

$$i = 0.001(\exp(7.5v) - 1.1)$$

(a) Plot the current through the diode i as a function of the voltage across the diode v.

(b) Find the voltage across the diode (v) and the current in the series circuit (i). Generate a plot of v and i as the supply battery voltage, denoted as V_b (supplied to the series resistor-diode circuit), which is varied between $-10\,\mathrm{V}$ to $10\,\mathrm{V}$. Note that the overall equation is

$$V_b = v + iR$$

where $R = 120\,\Omega$ and i is the current through diode.

Hint: solve $V_b = 120(0.001\exp(7.5v) - 1.1) + v$ to get v then solve for i

6. Consider a series circuit consisting of a voltage source, $v(t)$, that switches from 0 V to 1 V at $t = 0$, which is connected in series with an inductor L and a resistor R. Assume that the current is to be determined.

The DEQ for this circuit is given as

$$v(t) = L\frac{di}{dt} + Ri$$

Assume that $L = 0.5\,\text{H}$ and $R = 0.7\,\Omega$. Solve for the current $i(t)$ for the range $0 < t < 10$ with the initial condition of $i(t) = 0$. Use `scipy.integrate.odeint()` to integrate the DEQ.

Hint: The derivative of the current state variable is given as

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v(t)$$

7. Consider a mass of $M = 1.3\,\text{kg}$ that is suspended on a spring with a stiffness constant of $k = 150\,\text{N}\,\text{m}^{-1}$. The weight is initially held such that the tension through the spring is zero. Then at time $t = 0$, the weight is released. Find the weight displacement over the $0 < t < 5$ interval. Note that the DEQ for this example is given as

$$\begin{aligned} M\frac{d^2x}{dt^2} + kx &= gM \\ \frac{d^2x}{dt^2} + \frac{k}{M}x &= 9.8 \\ \frac{d^2x}{dt^2} + 100x &= 9.8 \end{aligned}$$

To solve a second-order DEQ, you must reduce it to a first-order format. Hence you set up a state space formulation as

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

Then input this first-order state equation into `odeint()`.

First, we have to set this up as a first-order DEQ with state variables of x and dx/dt such that

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{M} & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 9.8 \end{bmatrix}$$

The crux of the problem is to determine the derivative function of the state variables...