1. Using SymPy, show the statements for integrating the function

$$f(x) = \int_0^{\pi/2} x \sin(x)\, \mathrm{d}x$$

2. Consider the matrix $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ where a and b are variables entered as symbols in SymPy. Write a program that calculates the inverse of A as A^{-1} and then show that AA^{-1} is the 2×2 identity matrix. Note you can use `sympy.pprint()` to print out the matrix symbols.

3. Write a function in Sympy that, given a positive integer N determines the coefficients of $(x+1)^N$ and returns the numeric value of the coefficients of $x^{(N-1)}$. Test your function for $N = 4$

4. Create a function that calculates the N^{th} derivative of the function

$$f(x) = x^3 \sin(x^2 + 1)$$

where N is an input variable and then evaluates this at a float value of x that is also given. The function's input parameters should be N and x. Then test your function for $N = 2$ and $x = 0.1$. *Hint:* you can use `sympy.lambdify()` to get a numeric function from the symbolic function

5. Determine the power series of the function up to the term for x^5

$$f(x) = \cos(x^2 + \sqrt{x})$$

6. Find the maximum of the function

$$f(x) = x^2 \exp(-\alpha x)$$

where x is a variable, and α is a given symbolic constant that can be assumed to be a positive float.

7. Consider the following function

$$f(x, y) = \exp(-x^2 + 2x - y^2 + xy)$$

(a) Find the maximum in the $x - y$ plane using SymPy. Note that both $\frac{df}{dx} = 0$ and $\frac{df}{dy} = 0$ simultaneously at the maximum point.

(b) The maximum point of $\exp(-x^2+2x-y^2+xy)$ was found to be (x_0, y_0). Now show a numeric contour plot that shows that the maximum indeed occurs at that point.