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Fall 2023 Due: October 2, 2023

1. Give the Python code to calculate x as well as the value of x

$$x = \sum_{k=-3}^{4} k^2 \sin\left(0.1k^2\right)$$

2. Give the Python statements to calculate

$$x = \sum_{j=1}^{3} \sum_{k=-3}^{4} \sqrt{jk^2} \sin(0.1(k-j)^2)$$

- **3.** Define the variables $x = \sqrt{3}$ and $y = 0.3x^2 + \sqrt{x}$, $z = \sqrt{e} + x \ln(x) \log_{10}(x)$ and then compute $v = \sqrt{\tanh(xyz)}$
- 4. Generate a plot for $y = \tanh x$ for x in the range of 0 to 400 with 500 points. Use linspace() to generate x and annotate the plot with labels and a grid.
- 5. Let x = -4 + j1 and y = j3 (that is y is a complex number with $j = \sqrt{-1}$). Determine the magnitude squared of the vector of $z = \begin{bmatrix} x^y & xy^2 & \exp(\sqrt{x}) \end{bmatrix}$.
 - (a) Determine the elements of z, in polar coordinates of magnitude in an array m and phase (in radians) in an array p.
- **6.** A matrix is given as

$$x = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 8 & 8 \\ 2 & 2 & 4 \end{bmatrix}$$

Determine the matrix $y = x + x^{T}x + x^{3}$. The superscript \top implies a matrix transpose. Assume algebraic matrix multiplication operations and not element-wise multiplication.

7. A matrix is given as

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 8 & 8 \\ 2 & 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 5 & -3 \\ 4 & 8 & 8 \\ 2 & 2 & 4 \end{bmatrix}$$

Determine the solution to the system of linear equations given as

$$\begin{bmatrix} A & B \\ 0 & 0 & 0 \\ 0 & 0 & 0 & A \\ 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Hint: use numpy.linalg.solve

- **8.** Assume a row vector of samples $x = [-50, -49, \dots, 30]$, a vector y calculated with elements of $y = 3x^2 + 2$ and a matrix $Q = \begin{bmatrix} x \\ y \end{bmatrix}$. Give the Python code for calculating $z = QQ^{\top}$. The superscript \top implies a matrix transpose. Print out z.
- 9. Three vectors are given as

$$\vec{u} = -3\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\vec{v} = 2\hat{i} - 5\hat{j} - 4\hat{k}$$

$$\vec{w} = \hat{i} - \hat{j} - \hat{k}$$

Use np.dot() and np.cross() to compute the vector $Q = (u \cdot v)^2 + |(u \times v) \times w|$

10. Two matrices are given as

$$X = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 7 & 7 \\ 1 & 2 & 1 \end{bmatrix} \quad Y = \begin{bmatrix} 2 & 2 & 3 \\ 7 & 6 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Determine $Q = X^{-1}(Y + X^2)$ where algebraic matrix multiplications are assumed.

11. Solve the linear set of equations using linalg.solve() from the numpy library

$$4x + y + z = 3$$
$$2x + 2 + 13z = 4 - y$$
$$3x - z + 3y = 11 + 3y$$

Output should be $Q = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

12. Solve the recursion equation for the first 101 points

$$x_n = \sin(x_{n-1}) - 0.3x_{n-2} + 1$$

with $x_1 = x_0 = 0$. Then plot x_n as a function of n for n = 1 to 100