Assignment 1 - Models and Representations

January 22, 2024

1. Read sections 1.1, 1.2 and 1.3 in the book "Introduction to Mathematical Systems Theory", by Jan Polderman and Jan Willems (see pdf called `Willems Chap1 Introductions to Mathematical Systems Theory.pdf`). Based on the text, briefly answer the following questions:

   (a) How does Willems view a model?

   (b) What is the central object specifying a mathematical model?

   (c) What are the three components of Willems' 'modelling language'?

   (d) What is the behavior of a model?

   (e) What are latent variables?

   (f) What differentiates a Dynamical System from "other" systems?

2. Unit 1 contains many concepts. It is very important to understand how all these concepts are related and interconnected. Create an infographic (i.e. a diagram, illustration, drawing, chart, etc.) with the following concepts on it:

   - Systems in the real world
   - Models of systems
   - Time domain response of a system (response due to initial conditions, step response, response to any input)
   - Representations of a system (differential equations, transfer function, state-space equations)
   - Characteristics of a system (settling time, maximum overshoot, rise time, stability)
   - Key concepts (poles, zeros)
   - Plots (time domain response, s-plane)

   The key is that your infographic should show/illustrate the relationships between all these elements. You can hand draw or use any software program that you like to make your infographic.

3. Solve the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{dt}^2} + 4\frac{\mathrm{d}x}{\mathrm{dt}} + 13x = 0$$

with initial conditions

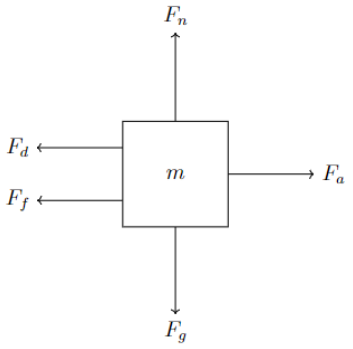
$$x(0) = 1$$
$$x'(0) = 0$$

You can find $x(t)$ by solving for the particular solution, or using Laplace transforms, it is up to you which method you prefer.

4. Calculate the inverse Laplace transform of

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}.$$

5. Consider the following free body diagram of a car. In the free body diagram the forces



are:

- $F_d = k_d v$ is the force due to drag,
- $F_g = mg$ is the force of gravity,
- $F_f = k_f v F_n$ is the force due to friction,
- $F_n = F_g$ is the normal force,
- F_a is the force applied by the wheels,

where:

- k_d is drag coefficient,
- v is speed of the car,
- m is mass of car,
- g is acceleration due to gravity,

- k_f is friction coefficient.

Based on the free body diagram, determine a transfer function that represents the car. Let F_a be the input, and v the output.

6. Find the region in the s-plane for the poles of a second order system that meets the following design requirements:

   (a) Peak overshoot $\leq 10\%$,

   (b) 2% settling time ≤ 5 seconds,

   (c) Peak time ≤ 0.5 seconds.