We wish to design a controller for the elevator such that the aircraft is able to:

- maintain its elevation even in the presence of disturbances such as wind and sudden wind gusts,
- increase altitude of 1 m in 10s (the settling time to a unit step of the closed loop system should be < 10 s),

$\Longrightarrow$ I want
- $T_s < 10s$
- stability margin ≤ 1.5

$$P(s) = \frac{0.4}{s^3+0.1s^2+20s+0.5} = \frac{0.4}{(s+0.025)(s+0.0375+4.47)(s+0.0375-4.47)}$$

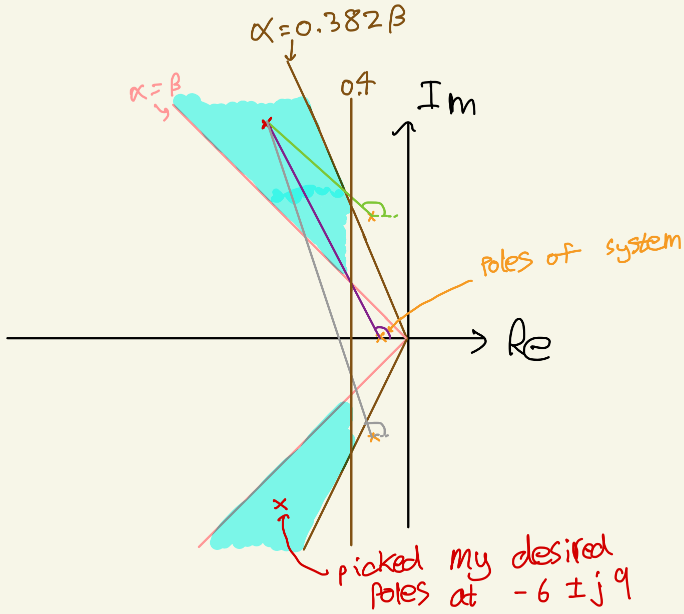
complex conjugates

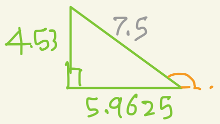
Recall:

$$\boxed{M_r = \frac{\alpha^2+\beta^2}{2\alpha\beta}}$$

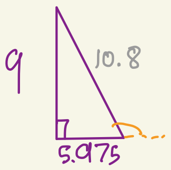
$$2\alpha\beta M_r = \alpha^2+\beta^2$$
$$\alpha^2 - 2\cdot 1.5\,\alpha\beta + \beta^2 = 0$$
$$\alpha^2 - 3\alpha\beta + \beta^2 = 0$$
$$\alpha = \frac{3\beta \pm \sqrt{(3\beta)^2 - 4\beta^2}}{2}$$
$$2\alpha = 3\beta \pm \sqrt{9\beta^2 - 4\beta^2}$$
$$2\alpha = 3\beta \pm \sqrt{5\beta^2}$$
$$\alpha = \frac{3\beta \pm \sqrt{5}\,\beta}{2}$$
$$\cancel{\alpha = 2.61\beta} \text{ ignore}$$
$$\alpha = 0.382\beta$$

$$T_s < 10$$
$$\frac{4}{\alpha} < 10$$
$$\frac{4}{10} < \alpha$$
$$0.4 < \alpha$$

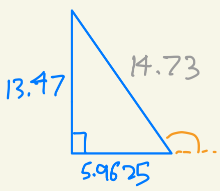




$\rightarrow 180 - \tan^{-1}\left(\frac{4.53}{5.9625}\right) = 142.8°$



$\rightarrow 180 - \tan^{-1}\left(\frac{9}{5.975}\right) = 123.6°$



$\rightarrow 180 - \tan^{-1}\left(\frac{13.47}{5.9625}\right) = 113.87°$

$$\frac{0.4}{7.5\angle 142.8°\ 10.8\angle 123.6°\ 14.73\angle 113.87°}\, K_p \frac{M_z \phi_z}{M_p \phi_p} = 1\angle -180°$$

$$-142.8 - 123.6 - 113.87 + \phi_z - \phi_p = -180$$

$$\boxed{\phi_z - \phi_p = 200.27}$$

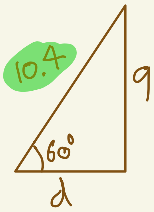
Must use compensator, but one is not enough

should aim for $\phi_z < 60°$
$\therefore$ use 4 compensators

$$\frac{200.27}{4} = 50.0675$$

$$\phi_z - \phi_p = 50^\circ$$

Pick $\phi_z = 60 \rightarrow \therefore \phi_p = 10$

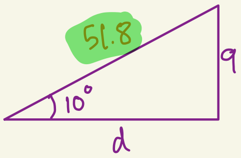


$$\tan(60) = \frac{9}{d}$$

$\therefore d = 5.2$

$\hookrightarrow$ place zero at

$6+5.2 = -11.2$

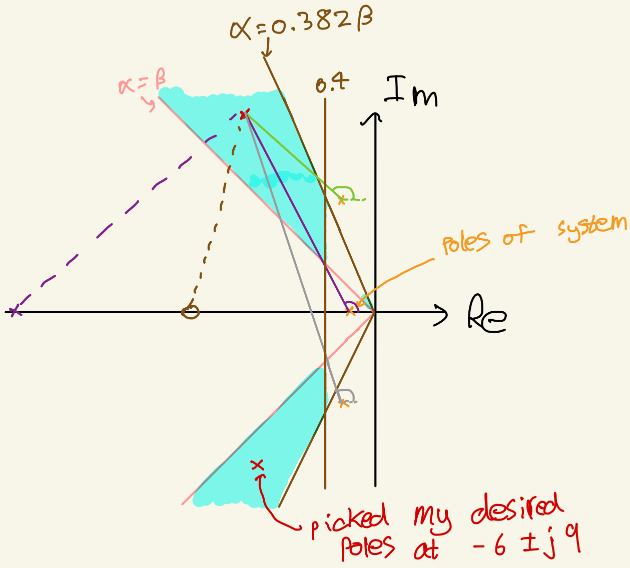


$$\tan(10) = \frac{9}{d}$$

$\therefore d = 51$

$\hookrightarrow$ place pole at

$6+51 = -57$



$$\frac{0.4}{7.5 \times 10.8 \times 14.73} \times K_{ld} \times \left(\frac{10.4}{51.8}\right)^4 = 1$$

$\hookrightarrow$ $K_{ld} = 1{,}835{,}750$

$$\boxed{\text{Final design:} \quad 1\,835\,750 \; \frac{(s+11.2)^4}{(s+57)^4}}$$