

Quiz 4 ENEL 419

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Q1] $\mu_x = 0$ $\sigma_x = 3$ $\mu_y = 1$ $\sigma_y = 4$ $C_{xy} = -3$

Find $E[W]$ and $\text{Var}[W]$

$$\begin{aligned} E[W] &= E[2X+2Y] = E[2X] + E[2Y] = 2E[X] + 2E[Y] \\ &= 2(0) + 2(1) = \boxed{2} \end{aligned}$$

$$\text{Var}[2X+2Y] = \sigma_{2X}^2 + \sigma_{2Y}^2 + 2C_{2X,2Y}$$

$$\sigma_{2X}^2 = (2^2) \sigma_x^2 = 36$$

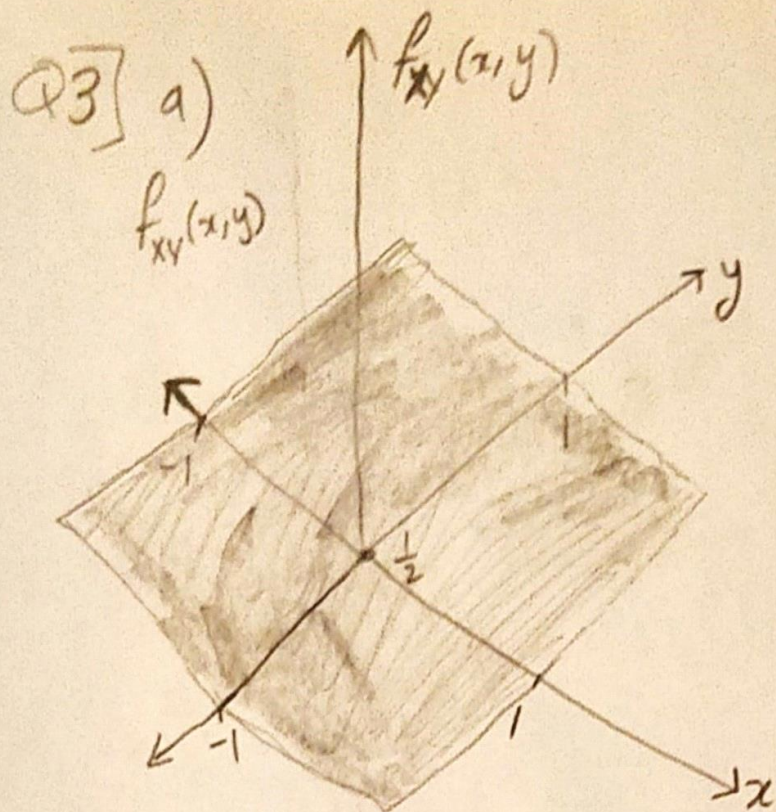
$$\sigma_{2Y}^2 = (2^2) \sigma_y^2 = 64$$

$$C_{2X,2Y} = 2 \times 2 C_{XY} = (4) \cdot (-3) = -12$$

$$\text{Var}[2X+2Y] = 36 + 64 - 2(12) = \boxed{76}$$

Q2

- a) $\rho_{xy} = -0.9$, negative slope
- b) $\rho_{xy} = 0$, no correlation
- c) $\rho_{xy} = 0.9$, positive slope



b) $E[XY] = 0 \quad (xy \neq 0)$

$$E[XY] = \int_{-1}^1 \int_{-1}^1 \frac{xy}{2} dx dy = \int_{-1}^1 \left[\frac{x^2 y}{4} \right]_{-1}^1 dy = \int_{-1}^1 \frac{y^3}{4} - \frac{y}{4} dy$$

$$= \left[\frac{y^4}{16} - \frac{y^2}{8} \right]_{-1}^1 = -\frac{1}{8} - \left(-\frac{1}{8}\right) = \boxed{0} = E[XY]$$

$$f_x(x) = \int_{-1}^1 \frac{1}{2} dy = \left[\frac{y}{2} \right]_{-1}^1 = \frac{1}{2} - \frac{-1}{2}$$

$$f_y(y) = \int_{-1}^1 \frac{1}{2} dx = \left[\frac{x}{2} \right]_{-1}^1 = \frac{1}{2} + \frac{1}{2}$$

$$\mu_x = \int_{-1}^1 x \left(\frac{1}{2} - \frac{x}{2} \right) dx = \left[\frac{x^2}{4} - \frac{x^3}{6} \right]_{-1}^1 = \left(\frac{1}{4} - \frac{1}{6} \right) - \left(\frac{1}{4} + \frac{1}{6} \right) = -\frac{1}{3}$$

$$\mu_y = \int_{-1}^1 y \left(\frac{y}{2} + \frac{1}{2} \right) dy = \left[\frac{y^3}{6} + \frac{y^2}{2} \right]_{-1}^1 = \left(\frac{1}{6} + \frac{1}{2} \right) - \left(-\frac{1}{6} + \frac{1}{2} \right) = \frac{1}{3}$$

$$C_{xy} = E[XY] - \mu_x \mu_y = 0 - \left(-\frac{1}{3} \cdot \frac{1}{3} \right) = \boxed{\frac{1}{9} \neq 0}$$

Thus X and Y are orthogonal and independent