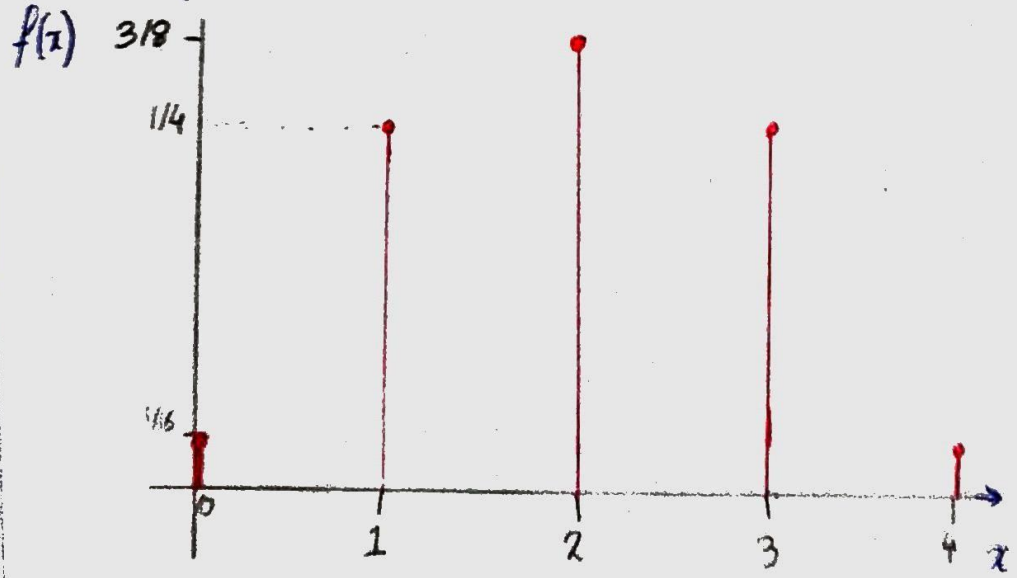
Quiz 2

Question 1

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{16} & 0 \le x < 1 \\ \frac{5}{16} & 1 \le x < 2 \\ \frac{11}{16} & 2 \le x < 3 \\ \frac{15}{16} & 3 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$

$f(x) = F(x) - F(x-1)$ in this case

a) $$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{16} & x = 0 \\ \frac{4}{16} = \frac{1}{4} & x = 1 \\ \frac{5}{8} & x = 2 \\ \frac{1}{4} & x = 3 \\ \frac{1}{16} & x = 4 \\ 0 & x > 4 \end{cases}$$



b) $P[X \geq 1 | X \leq 4] = \dfrac{P[1 \leq X \leq 4]}{P[X \leq 4]}$

$P[X \leq 4] = 1$

$P[1 \leq X \leq 4] = \frac{15}{16}$

$$\boxed{P[X \geq 1 | X \leq 4] = \frac{15}{16}}$$

Question 2

12 engines, 3 tests, 2 nonworking engines

X random variable describing nonworking engines

$$f(x) = \frac{\binom{2}{x}\binom{12-2}{3-x}}{\binom{12}{3}}$$

$$f(1) = \frac{\binom{2}{1}\binom{10}{2}}{\binom{12}{3}} = \frac{\frac{2!}{1} \cdot \frac{10!}{2!\,8!}}{\frac{12!}{3!\,9!}} = \frac{9}{22}$$

$$f(2) = \frac{\binom{2}{2}\binom{10}{1}}{\binom{12}{3}} = \frac{\frac{2!}{2!\cdot 1} \cdot \frac{10!}{9!}}{\frac{12!}{8!\,9!}} = \frac{1}{22}$$

$$f(1) + f(2) = \frac{9}{22} + \frac{1}{22} = \frac{10}{22} = \boxed{\frac{5}{11}}$$