Quiz 4 ENEL 419

Q1] $\mu_x = 0$ $\sigma_x = 3$ $\mu_y = 1$ $\sigma_y = 4$ $C_{xy} = -3$

Find $E[W]$ and $Var[W]$

$E[W] = E[2X+2Y] = E[2X] + E[2Y] = 2E[X] + 2E[Y]$

$= 2(0) + 2(1) = \boxed{2}$

$Var[2X+2Y] = \sigma_{2x}^2 + \sigma_{2y}^2 + 2C_{2x2y}$

$\sigma_{2x}^2 = (2^2)\sigma_x^2 = 36$ $\sigma_{2y}^2 = (2^2)\sigma_y^2 = 64$

$C_{2x,2y} = 2 \times 2\, C_{xy} = (4) \cdot (-3) = -12$

$Var[2X+2Y] = 36 + 64 - 2(12) = \boxed{76}$

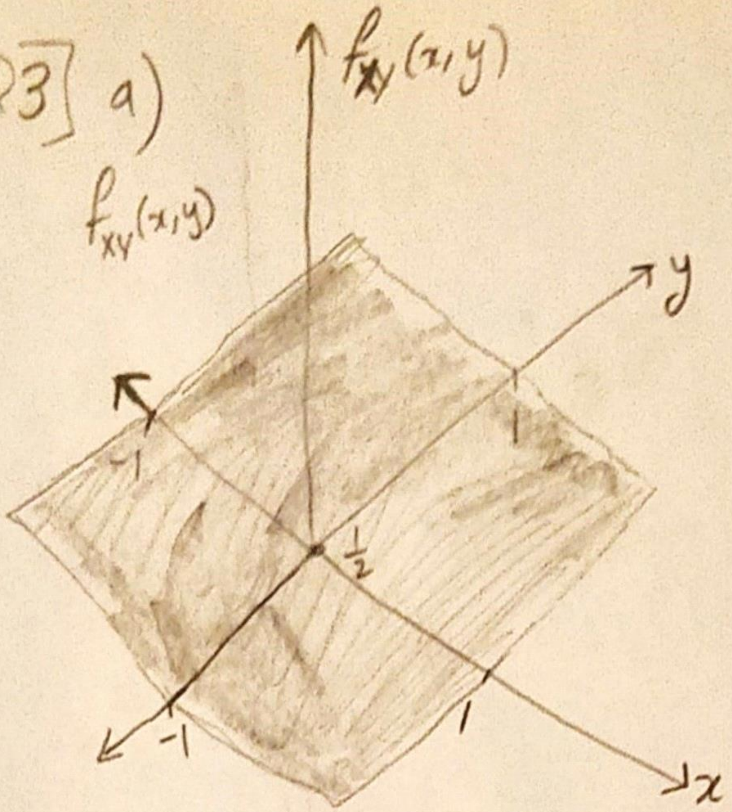
$\boxed{Q2}$

a) $\rho_{xy} = -0.9$, negative slope

b) $\rho_{xy} = 0$, no correlation

c) $\rho_{xy} = 0.9$, positive slope

Q3] a)



b) $E[XY] = 0 \quad C_{xy} \neq 0$

$$E[XY] = \int_{-1}^{1}\int_{-1}^{y} \frac{xy}{2}\,dx\,dy = \int_{-1}^{1}\left[\frac{x^2}{4}y\right]_{-1}^{y} dy = \int_{-1}^{1} \frac{y^3}{4} - \frac{y}{4}\,dy$$

$$= \left[\frac{y^4}{16} - \frac{y^2}{8}\right]_{-1}^{1} = -\frac{1}{8} - \left(-\frac{1}{8}\right) \boxed{= 0} = E[XY]$$

$$f_x(x) = \int_x^1 \frac{1}{2}\,dy = \left[\frac{y}{2}\right]_x^1 = \frac{1}{2} - \frac{x}{2}$$

$$f_y(y) = \int_{-1}^{y} \frac{1}{2}\,dx = \left[\frac{x}{2}\right]_{-1}^{y} = \frac{y}{2} + \frac{1}{2}$$

$$\mu_x = \int_{-1}^{1} x\left(\frac{1}{2} - \frac{x}{2}\right)dx = \left[\frac{x^2}{4} - \frac{x^3}{6}\right]_{-1}^{1} = \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right) - \left(\cancel{\frac{1}{4}} + \frac{1}{6}\right) = -\frac{1}{3}$$

$$\mu_y = \int_{-1}^{1} y\left(\frac{y}{2} + \frac{1}{2}\right)dy = \left[\frac{y^3}{6} + \frac{y^2}{2}\right]_{-1}^{1} = \left(\frac{1}{6} + \cancel{\frac{1}{2}}\right) - \left(-\frac{1}{6} + \cancel{\frac{1}{2}}\right) = \frac{1}{3}$$

$$C_{xy} = E[XY] - \mu_x\mu_y = 0 - \left(-\frac{1}{3}\cdot\frac{1}{3}\right) = \boxed{\frac{1}{9} \neq 0}$$

Thus X and Y are orthogonal and independent