

**Due Date: 4:00pm, November 22, 2019 (in Assignment box)**

**Problem 1:** (6 marks)

Random variables  $X$  and  $Y$  have joint PDF given by

$$f_{XY}(x, y) = \begin{cases} 4xy, & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{otherwise} \end{cases}.$$

- (a) Find  $\mu_X$  and  $\sigma_X^2$ .
- (b) Find  $\mu_Y$  and  $\sigma_Y^2$ .
- (c) Find  $\text{Cov}[X, Y]$ .
- (d) Find the correlation coefficient.
- (e) Find  $E[X + Y]$ .
- (f)  $\text{Var}[X + Y]$ .

**Problem 2:** (5 marks)

The random variables  $X$  and  $Y$  have a joint PDF given by

$$f_{XY}(x, y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Show whether  $X$  and  $Y$  are independent or dependent.
- (b) Show whether  $X$  and  $Y$  are uncorrelated or correlated.

**Problem 3:** (5 marks)

Two refills,  $X$  and  $Y$ , for a ballpoint pen are selected at random from a certain box, and the following is the joint probability distribution:

$f_{XY}(x, y)$	$x = 0$	$x = 1$	$x = 2$
$y = 0$	3 / 28	9 / 28	3 / 28
$y = 1$	3 / 14	3 / 14	
$y = 2$	1 / 28	0	0

- (a) Find  $\mu_X$ ,  $\mu_Y$ ,  $E[X^2]$  and  $E[Y^2]$ .
- (b) Find  $\sigma_X^2$  and  $\sigma_Y^2$ .
- (c) Find  $E[XY]$ ,  $C_{XY}$  and  $\rho_{XY}$ .
- (d) Find  $P[X \leq 0]$ .

**Problem 4:** (4 marks)

The random variables have a joint PDF given by

$$f_{XY}(x, y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the correlation coefficient between  $X$  and  $Y$ .