Start Date: 5:00pm, November 20, 2020
Due Date: 5:00pm, November 27, 2020

Problem 1:

Let random variable X denote the ratio of gallium to arsenide and Y denote the functional wafers retrieved, to develop a microchip, during a 1-hour period. Random variables X and Y have a joint PDF given by

$$f_{XY}(x,y) = \begin{cases} \dfrac{x(1+3y^2)}{4}, & 0 < x < 2,\ \ 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Show that X and Y are independent.

Problem 2:

A service facility operates with two service lines. On a randomly selected day, let X be the proportion of time that the first line is in use whereas Y is the proportion of time that the second line is in use. Suppose that their joint probability density function is

$$f_{XY}(x,y) = \begin{cases} \dfrac{3}{2}(x^2+y^2), & 0 \le x \le 1,\ \ 0 \le y \le 1, \\ 0, & \text{elsewhere} \end{cases}$$

(a) Determine whether X and Y are independent.
(b) Find $E(X + Y)$ and (ii) $E(XY)$.
(c) Find $E(XY)$
(d) Find $Var(X)$.
(e) Find $Var(Y)$.
(f) Find $Cov(X,Y)$ and ρ_{XY}.
(g) Find $Var(X+Y)$.

Problem 3

Show that $Cov(aX,bY) = abCov(X,Y)$.