Approved by Dept. Head	AmK

Student Name: [redacted]
Print Last Name, First Name

Student ID: [redacted] **Lecture Section:** 01

Instructor(s): [redacted]

TEST RULES AND INFORMATION

1. This test is: ☐ Open Course Book ☐ Open Course Notes ☐ Closed Book ☐ Closed Notes
2. This test is being made available to you on: , 2020 at MT.
3. This test is designed to be completed in no more than hour(s) and minutes.
4. **This is a timed test. Once you access the test, you have hour(s) and minutes to submit your answers/solutions.**
5. **Answers/Solutions to this test will not be accepted beyond: , 2020 at MT.**
6. This test is pages long. It has question(s) worth a total of marks.
7. You are not permitted to collaborate or consult with others when developing solutions and determining answers. The solutions/answers you submit must be your own, and developed only by you. You must abide by

   "Academic integrity is the foundation of the development and acquisition of knowledge and is based on values of honesty, trust, responsibility, and respect. We expect members of our community to act with integrity."

   "Research integrity, ethics, and principles of conduct are key to academic integrity. Members of our campus community are required to abide by our institutional code of conduct and promote academic integrity in upholding the University of Calgary's reputation of excellence."

8. You can record solutions to the test questions in the following ways (X marks all that apply):
   - ☐ Downloading the test paper as a PDF document, and writing electronically on (i.e. annotating) the PDF document using your device screen (e.g. iPAD, Surface etc.).
   - ☐ Printing out the test paper, and writing solutions by hand on the printed test paper.
   - ☐ Writing solutions by hand on loose-leaf or lined paper. If you choose this option, you do not need to include this cover page (page 1 of the examination paper) when submitting your solutions.
   - ☐
9. Write your answers neatly and legibly, show all your work and clearly state any assumptions you make. Ensure each of the following: (i) your name and/or your ID number appears at the top of each page of your solutions, (ii) each page is numbered, and (iii) you specify the question number being answered on each page.
10. You can submit solutions written on paper, or as an annotated pdf, as follows (X marks all that apply):
    - ☐ Save your annotated pdf, and upload to D2L.
    - ☐ Scan solutions written on paper, **and upload all pages (in order) as a single PDF file to D2L.** If you don't have a scanner, use the Microsoft Lens app on your phone to photograph and create a PDF of your solutions.
    - ☐ Scan solutions written on paper, **and email all pages (in order) as a single PDF file to your instructor(s).** If you don't have a scanner, use the Microsoft Lens app on your phone to photograph and create a PDF of your solutions.
11. The submitted file name format should be:
12. **By submitting solutions to the test questions, you acknowledge that the solutions you are submitting are yours, and were developed by you alone, and that you adhered to the University of Calgary Principles of Conduct.**
13. Keep your original handwritten solutions as part of your records should questions arise during marking.
14. For questions and clarifications about the **test content**, you can contact your instructor(s) by:
    ☐ Email ☐ Phone ☐ Instructor(s) will not clarify or answer questions about the test content
15. For **technical issues** that arise during submission, contact instructor(s) by: ☐ Email ☐ Phone
16. The instructor(s) will be available at the following times:
17. If during the test you become ill or receive word of domestic affliction, and feel that you are unable to continue, submit your unfinished work to your instructor with a request that it be cancelled.
18. If you submit solutions for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another test, such a request will be denied.

ENEL 419: Probability and Random Variables

Final Exam for Fall 2020

Instructor: Dr. Abu Sesay

December 19, 2020

ID NUMBER	LAST NAME (PRINTED):	OTHER NAMES

Signature:

Note: **Please read the entire instructions before you start the exam.**

INSTRUCTIONS:

- You must sign and submit the attached Academic Integrity Statement with your completed exam.
- Answer all five questions in the spaces provided after each question.
- Please print or write your answers legibly. What cannot be read cannot be marked.
- If you write anything you do not want marked, put a large "X" through it and write "rough work" beside it.
- The final will be made available for 24 hours, starting from 6:30 pm December 18 and must be completed and submitted by 6:30 pm December 19, 2020, which is the Registrar's scheduled date and time.
- You will need access to a computer and internet, as well as an ability to scan and upload handwritten work. Microsoft Office Lens is recommended when using a smartphone or tablet to scan handwritten work.
- You can use your notes and your textbook.
- You are not permitted to search the internet, communicate with classmates, or use excel or other calculation software.
- I will be available to answer questions by email ([illegible]) on December 19, between 9:00 am 4:00 pm. Please note that my response may not be instantaneous.

Marks Summary

	Q1	Q2	Q3	Q4	Q5	Total
Marks obtained						
Maximum marks	20	20	30	14	16	100

Student ID Number: [redacted]

1. Answer the following questions on the blank pages provided (or loose sheets of paper if you have difficulty printing this exam). If there are calculations involved, **you must show the steps leading to your answer**, otherwise, you will lose some points.

Marks	(a)	In an experiment C and D are independent events with probabilities $P[C\cap D]=\frac{1}{3}$, and $P[C]=\frac{1}{2}$.
/2	(i)	Find $P[D]$
/2	(ii)	Find $P[C\cap\overline{D}]$
/2	(iii)	Find $P[\overline{C\cup D}]$
/2	(iv)	Find $P[C\cup D]$
/2	(v)	Find $P[C\cup\overline{D}]$
/2	(vi)	Are C and $\overline{D}$ independent?

Note: Part (b) has no bearing to part (a)

Marks	(b)	An invigilator collects 10 cell phones during an exam and wants to check them for cheating. Among the 10 phones collected, 5 of them are known to be (security) locked. Suppose The invigilator randomly picks 5 of the 10 phones to be checked first. Answer the following questions:
/2	(i)	What is the probability that all 5 phones picked are locked?
/3	(ii)	What is the probability that at most 2 of the phones picked are locked?
/3	(iii)	What is the probability that at least 3 of the phones picked are unlocked?
/20		

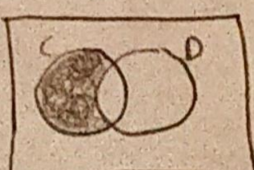
$P[C\cap D]=\frac{1}{3}$ $P[C]=\frac{1}{2}$

a) i) since C and D independent $P[C|D]=\frac{P[C\cap D]}{P[D]}=P[C]$

So: $P[C\cap D]=P[C]\cdot P[D]$

$\frac{1}{3}=\frac{1}{2}\cdot P[D] \Rightarrow \boxed{P[D]=\frac{2}{3}}$

ii) $P[C\cap\overline{D}] \Rightarrow$



$\Rightarrow P[C\cap\overline{D}]=P[C]-P[C\cap D]$

$=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

$\boxed{P[C\cap\overline{D}]=\frac{1}{6}}$

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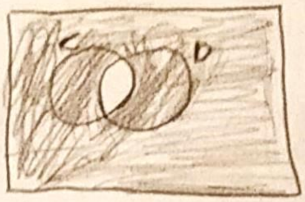
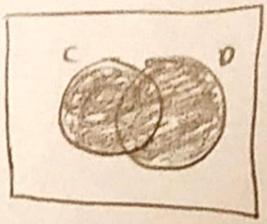
iii) $P[\bar{C} \cup \bar{D}] \Rightarrow$ 

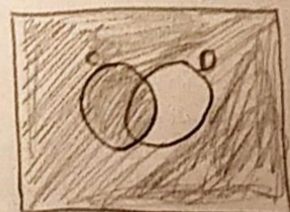
$\Rightarrow P[\bar{C} \cup \bar{D}] = 1 - P[C \cap D]$
$= 1 - \frac{1}{3}$

$\boxed{P[\bar{C} \cup \bar{D}] = \frac{2}{3}}$

iv) $P[C \cup D] \Rightarrow$ 

$\Rightarrow P[C \cup D] = P[C] + P[D] - P[C \cap D]$
$= \frac{1}{2} + \frac{2}{3} - \frac{1}{3}$

$\boxed{P[C \cup D] = \frac{5}{6}}$

v) $P[C \cup \bar{D}] \Rightarrow$ 

$\Rightarrow P[C \cup \bar{D}] = 1 - P[D] + P[C \cap D]$
$= 1 - \frac{2}{3} + \frac{1}{3}$

$\boxed{P[C \cup \bar{D}] = \frac{2}{3}}$

vi) To be independent:

$P[C \cap \bar{D}] = P[C] \cdot P[\bar{D}]$

$P[C \cap \bar{D}] = \frac{1}{6}$

$P[C] \cdot P[\bar{D}] = \frac{1}{2} \cdot (1 - \frac{2}{3}) = \frac{1}{6} \qquad \frac{1}{6} = \frac{1}{6}$

$P[C \cap \bar{D}] = P[C] \cdot P[\bar{D}]$ is true

$\boxed{C \text{ and } \bar{D} \text{ are inbependent}}$

b] i)

$\Rightarrow P[\text{All picked phones locked}] =$

$$= \frac{\text{number of combinations where all phones picked locked}}{\text{Total combinations}}$$

$$= \frac{\binom{5}{5}\cancel{\binom{5}{0}} \rightarrow 1}{\binom{10}{5}} = \frac{\frac{5!}{5!\cancel{0!}} \rightarrow 1}{\frac{10!}{5!\,5!}}$$

$$= \frac{5!\cdot 5!}{10!} \boxed{= \frac{1}{252} = 0.00397}$$

ii) $P[\text{at most 2 phones locked}] = P[\text{2 locked}] + P[\text{1 locked}] + P[\text{no locked}]$

$$\Rightarrow \frac{\binom{5}{3}\times\binom{5}{2} + \binom{5}{4}\times\binom{5}{1} + \binom{5}{5}\cancel{\binom{5}{0}}\rightarrow 1}{\binom{10}{5}} = \frac{\frac{5!}{3!\,2!}\times\frac{5!}{2!\,3!} + \frac{5!}{4!\,1!}\times\frac{5!}{1!\,4!} + \frac{5!}{5!\,0!}}{\frac{10!}{5!\,5!}}$$

$$\boxed{= 0.5 = \frac{1}{2}}$$

iii) $P[\text{at least 3 phones unlocked}] = P[\text{at most 2 phones locked}]$

$$\boxed{= 0.5 = \frac{1}{2}}$$

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2. Answer the following questions on the blank pages provided (or loose sheets of paper if you have difficulty printing this exam). If there are calculations involved, **you must show the steps** the steps leading to your answer.

Marks	(a)	A professor of Probability and Statistics drops, into a box, the same amount of money each time a student points out an error that this professor makes in a lecture. Over the professor's career of n years making errors, the total amount (in dollars) dropped in the box can be approximated by a Gaussian random variable Y_n with expected value $40n$ and variance $100n$.
/4	(i)	Evaluate the probability that the amount of dollars dropped into the box over 20 years exceeds \$1000?
/10	(ii)	Find the number of years n, that the professor must teach in order that $P[Y_n \le 1000] > 0.99$?

Note: Part (b) has no bearing to part (a)

Marks	(b)	Consider a random variable X, defined such that $E\left[(X-1)^2\right]=10$ and $E\left[(X-2)^2\right]=6$.
/6		Determine the standard deviation of X.
/20		

a) $E[Y_{20}] = 40 \times 20 = 800$ $\quad$ $Var[Y_{20}] = 2000$ $\quad$ $\sigma = \sqrt{2000} = 20\sqrt{5}$

$$P[Y_{20} > 1000] = 1 - P[Y_{20} \le 1000] = 1 - P\left[Z \le \frac{1000-800}{20\sqrt{5}}\right]$$

$$= 1 - P\left[Z \le \frac{10}{\sqrt{5}}\right] = 1 - P[Z \le 4.47] = Q(4.47)$$

$$\boxed{= 4.2935 \times 10^{-6}}$$

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ii) $P[Y_n \leq 1000] > 0.99$

$$P\left[Z \leq \frac{1000 - E[Y_{20}]}{\sigma_{20}}\right] > 0.99$$

$$\frac{1000 - E[Y_{20}]}{\sigma_{20}} > 2.33 \quad \text{from Z table}$$

$$\frac{1000 - 40n}{\sqrt{100n}} > 2.33$$

$$1000 - 40n > 10\sqrt{n} \times 2.33 = 23.3\sqrt{n}$$

$$(23.3\sqrt{n} + 40n)^2 > (1000)^2$$

$$542.89n + 1600n^2 > 10^6$$

$$1600n^2 + 542.89n - 10^6 > 0$$

$$n = \frac{-542.89 \pm \sqrt{(542.89)^2 - 4 \times 1600 \times (-10^6)}}{3200} = 22.25$$

$n < 22.25$

$\boxed{n = 22 \text{ so } P[Y_n \leq 1000] > 0.99}$

b) $E[(X-1)^2] = 10 \quad E[(X-2)^2] = 6$

$\sigma_X = \sqrt{Var(X)}$

$Var(X) = E[(X)^2] - (E[X])^2 = Var(X-1) = Var(X-2)$

$E[X^2] - (E[X])^2 = E[(X-1)^2] - (E[X-1])^2 = E[(X-2)^2] - (E[X-2])^2$

$10 - (E[X]-1)^2 = 6 - (E[X]-2)^2$

$4 - (\mu_X^2 - 2\mu_X + 1) = -(\mu_X^2 - 4\mu_X + 4)$

$4 = 2\mu_X - 3 \qquad 2\mu_X = 7 \qquad \underline{\mu_X = E[X] = 3.5 = \frac{7}{2}}$

$E[X^2] = 10 - (E[X]-1)^2 + (E[X])^2 = 16$

$Var(X) = 16 - 3.5^2 = 3.75$

$\sigma_X = \sqrt{Var(X)} = \sqrt{3.75} = 1.936$

$\boxed{\sigma_X = 1.936}$

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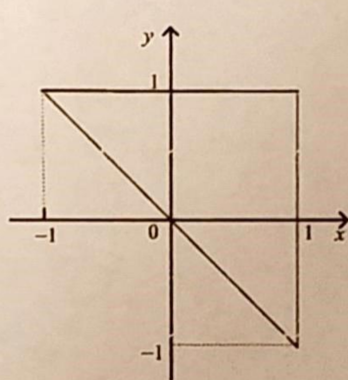
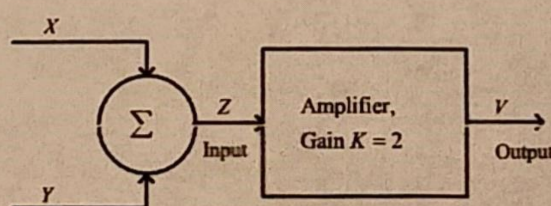
3. Answer the following questions on the blank pages provided (or loose sheets of paper if you have difficulty printing this exam). If there are calculations involved, **you must show the steps** the steps leading to your answer.

Marks	(a)	Consider two random variables X and Y with a joint probability density function $f_{XY}(x,y)=\frac{1}{2}$. The region for the values of the pair (x,y) is the inside of the triangular region shown in the graph below. 
/5	(i)	Find the marginal density functions for X and Y.
/9	(ii)	Find the correlation coefficient for X and Y.
/6	(iii)	Evaluate the probability that $P[X \geq 0, Y \leq 0]$.
/6	(iv)	Evaluate the probability $P[0.5 < Y \leq 1 \mid X = 0.5]$.
/4	(b)	We wish to investigate an amplifier with gain $K = 2$, in the diagram shown below. • The input and output voltages of the amplifier are Z and V, respectively. • The input is the sum of two random voltages X and Y.  Find the variance of the random variable V.
/30		

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a) $-1 \le -x \le y \le 1$

is the inequality describing the graph

i) $f_X = \int_{-x}^{1} f_{XY}\,dy = \int_{-x}^{1} \frac{1}{2}\,dy = \frac{y}{2}\Big|_{-x}^{1} = \frac{1}{2} - -\frac{x}{2}$

$\boxed{f_X = \frac{x+1}{2}}$

$f_Y = \int_{-y}^{1} f_{XY}\,dx = \int_{-y}^{1} \frac{1}{2}\,dx = \frac{x}{2}\Big|_{-y}^{1} = \frac{y+1}{2}$ $\boxed{f_Y = \frac{y+1}{2}}$

ii) $\rho_{XY} = \frac{C_{XY}}{\sigma_X \sigma_Y}$ $\mu_X = \int_{-1}^{1} x\left(\frac{x+1}{2}\right)dx = \int_{-1}^{1}\left(\frac{x^2}{2}+\frac{x}{2}\right)dx$

$= \frac{x^3}{6} + \frac{x^2}{4}\Big|_{-1}^{1} = \left(\frac{1}{6}+\cancel{\frac{1}{4}}\right) - \left(\frac{-1}{6}+\cancel{\frac{1}{4}}\right)$

$\mu_X = \frac{1}{3}$

$\mu_Y = \int_{-1}^{1} y\frac{(y+1)}{2}\,dy = \int_{-1}^{1} \frac{y^2}{2} + \frac{y}{2}\,dy$

$= \frac{y^3}{6} + \frac{y^2}{4}\Big|_{-1}^{1} = \left(\frac{1}{6}+\cancel{\frac{1}{4}}\right) - \left(-\frac{1}{6}+\cancel{\frac{1}{4}}\right) \Rightarrow \mu_Y = \frac{1}{3}$ $-\mu_X^2$

$\sigma_X^2 = \int_{-1}^{1} x^2\left(\frac{x+1}{2}\right)dx \underset{-\mu_X^2}{=} \int_{-1}^{1} \frac{x^3}{2}+\frac{x^2}{2}\,dx \underset{-\mu_X^2}{=} \frac{x^4}{8} + \frac{x^3}{6}\Big|_{-1}^{1} = \left(\frac{1}{8}+\frac{1}{6}\right) - \left(\frac{1}{8}-\frac{1}{6}\right) - \frac{1}{9}$

$\sigma_X^2 = \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$ $\boxed{\sigma_X = \frac{\sqrt{2}}{3}}$

$\sigma_Y = \int_{-1}^{1} \frac{y^3}{2} + \frac{y^2}{2}\,dx - \frac{1}{9} = \frac{y^4}{8} + \frac{y^3}{6}\Big|_{-1}^{1} - \frac{1}{9} = \left(\cancel{\frac{1}{8}}+\frac{1}{6}\right) - \left(\cancel{\frac{1}{8}}-\frac{1}{6}\right) - \frac{1}{9} = \frac{2}{9}$ $\boxed{\sigma_Y = \frac{\sqrt{2}}{3}}$

$E[XY] = \int_{-1}^{1}\int_{-y}^{1} xy \cdot f_{XY}\,dx\,dy = \int_{-1}^{1}\int_{-y}^{1} \frac{xy}{2}\,dx\,dy$

→ next page

$$= \int_{-1}^{1} \frac{x^2}{4} y \Big|_{-y}^{1} dy = \int_{-1}^{1} \frac{y}{4} + \frac{y^3}{4}\, dy = \frac{y^2}{8} + \frac{y^4}{16}\Big|_{-1}^{1} = \left(\frac{1}{8}+\frac{1}{16}\right) - \cancel{\left(\frac{1}{8}+\frac{1}{16}\right)} \to 0$$

$$E[XY] = 0$$

$$C_{XY} = E[XY] - \mu_X \mu_Y = -\left(\frac{1}{3}\cdot\frac{1}{3}\right) = -\frac{1}{9}$$

$$\rho_{XY} = \frac{-\frac{1}{9}}{\frac{\sqrt{2}}{3}\cdot\frac{\sqrt{2}}{3}} = \frac{-\frac{1}{9}}{\frac{2}{9}} = \frac{-1}{2} \qquad \boxed{\rho_{XY} = -\frac{1}{2}}$$

iii) $$P[X \geq 0, Y \leq 0] = \int_{-\infty}^{0}\int_{0}^{\infty} f_{xy}\, dx\, dy = \int_{-1}^{0}\int_{0}^{1} \frac{1}{2}\, dx\, dy$$

$$= \int_{-1}^{0} \frac{x}{2}\Big|_{0}^{1} dy = \int_{-1}^{0} \frac{1}{2}\, dy = \frac{y}{2}\Big|_{-1}^{0} = \frac{0}{2} - \frac{-1}{2} \boxed{= \frac{1}{2}}$$

iv) $P[0.5 < Y \leq 1 \mid X = 0.5]$

$$f_{Y|X}[y|x] = \frac{f_{XY}}{f_X} = \frac{\frac{1}{2}}{\frac{x+1}{2}} = \frac{1}{x+1}$$

$$P[0.5 < Y \leq 1 \mid X = 0.5] = \int_{0.5}^{1} f_{Y|X}(y \mid x = 0.5)\, dy = \int_{0.5}^{1} \frac{2}{3}\, dy = \frac{2}{3} y\Big|_{0.5}^{1}$$

$$= \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3}}$$

b) $Var(2X + 2Y)$

$$= \sigma_{2X}^2 + \sigma_{2Y}^2 + 2C_{2X2Y}$$

$$= 2^2\sigma_X^2 + 2^2\sigma_Y^2 + 2(2 \times 2\, C_{XY})$$

$$= 4\left(\frac{2}{9}\right) + 4\left(\frac{2}{9}\right) + 2\left(4 \times -\frac{1}{9}\right) = \boxed{\frac{8}{9} = 0.889}$$

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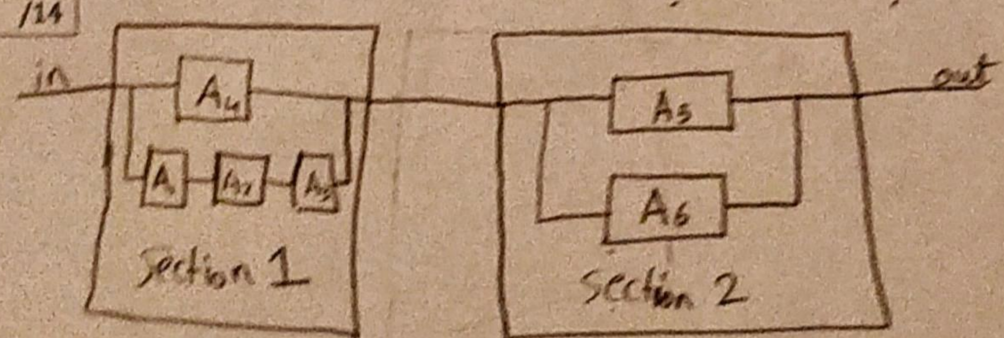
4. Answer the following questions on the blank pages provided (or loose sheets of paper if you have difficulty printing this exam). **If there are calculations involved, you must show the steps leading to your answer.**

Marks		
	(a)	You are asked to design a system for reliability. The system must use six sub-units A_1, A_2, A_3, A_4, A_5 and A_6. Each sub-unit has a failure probability equal to q, independent of other sub-units. The system is divided into 2 sections. Section 1 consists of sub-units A_1, A_2, A_3 and A_4. They are interconnected such that A_1, A_2, and A_3 all must together work, or sub-unit A_4 must work for Section 1 to function properly. Section 2 consists of sub-units A_5 and A_6. They are interconnected such that A_5 must work or A_6 must work for Section 2 to function properly. Section 2 is connected to the output of Section 1 (in series)
/2	(i)	Draw a block diagram for this operation.
/6	(ii)	Derive a formula (expressed as a function of failure probability q) for the probability $P[S]$ that the entire system operates successful.
/6	(b)	Consider the system in Part (a), above. Suppose we can replace one sub-unit (either A_1 or A_4) with a more reliable component that has a failure probability of $q_1 = 0.5q$ (assume $q = 0.2$). Which component should we replace and why?
/14		

a) i)



q is Probability of failure

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P[A] is probability of succeeding $= 1-q$

ii) $P[S] = \underbrace{P[A_4 \cup (A_1 \cap A_2 \cap A_3)]}_{P[S_1]} \cdot \underbrace{P[A_5 \cup A_6]}_{P[S_2]}$

$= \Big(1 - \overbrace{(1 - P[S_1])}^{\text{Prob of failure}}\Big) \cdot \big(1 - (1 - P[S_2])\big)$

$= \big(1 - ((q_1 + q_2 + q_3) \cdot q_4)\big) \cdot (1 - q_5 q_6)$

$\boxed{= 1 - q_5 q_6 - q_1 q_4 - q_4 q_2 - q_4 q_3 + q_4 q_1 q_5 q_6 + q_4 q_2 q_5 q_6 + q_4 q_3 q_5 q_6}$

b) assuming $q_1 = 0.1$ $q = 0.2$ (replacing A_1)

$P[S] = 1 - (0.2)^2 - (0.1)(0.2) - (0.2)^2 - (0.2)^2 + (0.2)^3(0.1) + (0.2)^4 + (0.2)^4$

$= 0.864$

assuming $q_4 = 0.1$ $q = 0.2$ (replacing A_4)

$P[S] = 1 - (0.2)^2 - (0.2)(0.1) \times 3) + ((0.2)^3(0.1) \times 3)$

$= 0.9024$

replace A_4 because it results in a higher probability of success.

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5. Answer the following questions on the blank pages provided (or loose sheets of paper if you have difficulty printing this exam). If there are calculations involved, **you must show the steps** leading to your answer.

Marks	(a)	A battery manufacturing company makes two types of laptop batteries, A and B. The manufacturer claims that the average lifetime of battery type A exceeds the average lifetime of battery type B by exactly 12 months. Battery type A has a standard deviation of 6.28 months while type B has a standard deviation of 5.61 months The University of Calgary purchases a large batch of both types of batteries for their computer labs and decides to test the manufacturer's claim. A quality control engineer of the university randomly picks 30 batteries of each type and tests them under similar conditions. This test shows that battery type A has an average life of 86.7 months, and battery type B has an average life of 77.8 months. Test the manufacturer's claim using a 98% level of significance.
/8		

Part (b) has no bearing to Part (a)

	(b)	A circuits lab wants to study the average lifetime of a batch of the same electronic units. A quality control engineer conducts an experiment where he/she picks 15 units, operates them under similar stress conditions until they fail. The times the units run before they fail are recorded (in months) below. 22.0, 26.0, 25.6, 23.8, 22.7, 24.8, 24.9, 22.1, 26.1, 24.5, 23.5, 21.0, 21.4, 23.5, 20.0 At what level of significance (0.1, 0.01, 0.02 or 0.05) does the quality control engineer fail to reject the hypothesis that the mean lifetime is equal to 22.2 months.
/8		
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a) $\mu_{OA} = \mu_{OB} + 12 \qquad \underline{\mu_{OA} - \mu_{OB} = 12}$

$$\bar{X}_A - \frac{\sigma_A}{\sqrt{n}} \cdot y_c \leq \mu_{OA} \leq \bar{X}_A + \frac{\sigma_A}{\sqrt{n}} \cdot y_c$$

$\alpha = 0.02 \qquad y_c = 2.462$

$$86.7 - \frac{6.28}{\sqrt{30}}(2.462) \leq \mu_{OA} \leq 86.7 + \frac{6.28}{\sqrt{30}}(2.462)$$

$$\underline{83.877 \leq \mu_{OA} \leq 89.523}$$

$$\bar{X}_B - \frac{\sigma_B}{\sqrt{n}} \cdot y_c \leq \mu_{OB} \leq \bar{X}_B + \frac{\sigma_B}{\sqrt{n}} \cdot y_c$$

$$77.8 - \frac{5.61}{\sqrt{30}}(2.462) \leq \mu_{OB} \leq 77.8 + \frac{5.61}{\sqrt{30}}(2.462)$$

$$\underline{75.278 \leq \mu_{OB} \leq 80.322}$$

$\mu_{OA} - \mu_{OB} = 12$

$$83.877 - 80.322 \leq \mu_{OA} - \mu_{OB} \leq 89.523 - 75.278$$

$$\boxed{3.555 \leq \mu_{OA} - \mu_{OB} \leq 14.245}$$

$\mu_{OA} - \mu_{OB} = 12$
$\therefore$ falls within the range

$\boxed{\text{manufacturer's claim is true}}$

Student ID Number:

b) $n=15$

$$\bar{X} = \frac{1}{15}\sum_{i=1}^{15} X_i = \frac{22+26+25.6+23.8+22.7+24.8+\dots+20}{15}$$

$$= \frac{351.9}{15} = 23.46 = \bar{X}$$

$$S_X^2 = \frac{1}{n-1}\sum_{i=1}^{15}(X_i-\bar{X})^2 = \frac{(22-23.46)^2+\dots\dots+(20-23.46)^2}{15-1}$$

$$S_X^2 = \frac{49.896}{14} = 3.564 \qquad \sigma_X = \sqrt{S_X} = 1.888$$

α	y_c
0.1	1.761
0.01	2.977
0.02	2.624
0.05	2.145

$$\bar{X} - \frac{\sigma_X}{\sqrt{n}}y_c \le \mu_{0X} \le \bar{X} + \frac{\sigma_X}{\sqrt{n}}y_c$$

Try $0.1=\alpha$, $y_c = 1.761$

$$23.46 - \frac{1.888}{\sqrt{15}}(1.761) \le \mu_{0X} \le 23.46 + \frac{1.888}{\sqrt{15}}(1.761)$$

$$22.602 \le \mu_{0X} < 24.318$$

Try $\alpha = 0.01$, $y_c = 2.977$

$$23.46 - \frac{1.888}{\sqrt{15}}(2.977) \le \mu_{0X} \le 23.46 + \frac{1.888}{\sqrt{15}}(2.977)$$

$$\boxed{22.00877 \le \mu_{0X} \le 24.911} \rightarrow \mu_{0X} = 22.2 \text{ fall within this range}$$

when $\alpha = 0.01$ (ie 99% level of significance), we accept the hypothesis