ENEL 419
Assignment 5

Problem 1

a) $\overline{X} = \frac{1}{10}\sum_{n=1}^{10} X_n = \frac{1}{10}(112+77+113+83+95+105+102+120+73+95)$

$= \frac{975}{10} = \boxed{97.5 = \overline{X}}$

b) $S_x = \sqrt{S_x^2} = \sqrt{\frac{1}{10-1}\sum_{n=1}^{10}(X_n - \overline{X})^2} = \sqrt{\frac{1}{9}\left(\sum_{n=1}^{10} X_n^2 - \sum_{n=1}^{10} 2X_n\overline{X} + \sum_{n=1}^{10}\overline{X}^2\right)}$

$= \sqrt{\frac{1}{9}\left(\left(\sum_{n=1}^{10} X_n^2\right) - \frac{2}{10}\left(\sum_{n=1}^{10} X_n\right)^2 + \frac{1}{10}\left(\sum_{n=1}^{10} X_n\right)^2\right)}$

$= \sqrt{\frac{1}{9\times 10}\left(10\left(\sum_{n=1}^{10} X_n^2\right) - \left(\sum_{n=1}^{10} X_n\right)^2\right)}$

$= \sqrt{\frac{1}{90}(973390 - 950625)} = \boxed{15.904 = S_x}$

2] a) $\bar{X} = \frac{1}{10}\sum_{n=1}^{10} X_n = 2.34$

$\bar{Y} = \frac{1}{10}\sum_{n=1}^{10} Y_n = 17.63$

b) $S_x^2 = \frac{1}{9\times10}\left(10\sum_{n=1}^{10} X_n^2 - \left(\sum_{n=1}^{10} X_n\right)^2\right) = 2.325$

$S_y^2 = \frac{1}{9\times10}\left(10\sum_{n=1}^{10} Y_n^2 - \left(\sum_{n=1}^{10} Y_n\right)^2\right) = 24.551$

c) $C_{xy} = \frac{1}{9}\sum_{n=1}^{10}(X_n-\bar{X})(Y_n-\bar{Y}) = \frac{10\sum_{n=1}^{10} X_nY_n - \left(\sum_{n=1}^{10} X_n\right)\left(\sum_{n=1}^{10} Y_n\right)}{10\times9}$

$= \frac{10(673.5918) - (23.4\times176.3)}{90} = \boxed{6.783}$

d) $\rho_{xy} = \frac{C_{xy}}{S_x S_y} = \frac{6.783}{\sqrt{2.325\times24.551}} = \boxed{0.898}$

3) $\bar{X} = \frac{1}{9}\sum_{n=1}^{9} X_n = \frac{1}{9}(9.05) = \underline{1.0056}$

$S_x = \sqrt{\frac{1}{8\times 9}\left(9\sum_{n=1}^{9} X_n^2 - \left(\sum_{n=1}^{9} X_n\right)^2\right)} = \underline{0.02455} = \sigma_x$

$\alpha = 1 - 0.99 = 0.01$

$$-z_c\left(\frac{\sigma_x}{\sqrt{n}}\right) - \bar{X} \leq -\mu_x \leq z_c\left(\frac{\sigma_x}{\sqrt{n}}\right) - \bar{X}$$

$$-z_c\left(\frac{\sigma_x}{\sqrt{n}}\right) + \bar{X} \leq \mu_x \leq z_c\left(\frac{\sigma_x}{\sqrt{n}}\right) + \bar{X}$$

$z_c = Q^{-1}\left(\frac{\alpha}{2}\right)$ $\quad Q(z_c) = 0.005$

$z_c \approx 3.35539$

$$1.0056 - 3.35539\left(\frac{0.02455}{\sqrt{9}}\right) \leq \mu_x \leq 1.0056 + 3.35539\left(\frac{0.02455}{\sqrt{9}}\right)$$

$$\boxed{0.9781 \leq \mu_x \leq 1.0331}$$

4) $\overline{X} = 23000$ $\sigma_x = 3900$

a)

$z_c \simeq 2.5755$ $n = 100$

$$\overline{X} - z_c\left(\frac{\sigma_x}{\sqrt{n}}\right) \leq \mu_x \leq \overline{X} + z_c\left(\frac{\sigma_x}{\sqrt{n}}\right)$$

$$23000 - 2.5755\left(\frac{3900}{10}\right) \leq \mu_x \leq 23000 + 2.5755(390)$$

$$21995.555 \leq \mu_x \leq 24004.645$$

b) $\overline{X} = 23500$ $\sigma_x = 3900$

$z_c = 2.5755$ $n = 100$

$$23500 - 2.5755(390) \leq \mu_x \leq 23500 + 2.5755(390)$$

$$22995.555 \leq \mu_x \leq 24504.445$$