| Quiz #1 | Start<br>Thursday, September 24, 5:00 pm | Due<br>Friday, September 25, 5:00 pm |
|---|---|---|
| ID #: [redacted] | Last Name: | First Name: [redacted] |

**Question 1:**
The number of people in a town, who have completed a college degree, are categorized as shown in the table below.

| | Employed | Unemployed | Total |
|---|---|---|---|
| Male | 460 | 40 | 500 |
| Female | 140 | 260 | 400 |
| Total | 600 | 300 | 900 |

Suppose one of these individuals is to be selected at random for a tour to publicize the advantages of establishing new industries in the town. Answer the following:

| | | Marks |
|---|---|---|
| (a) | Evaluate the probability that the individual is employed<br>$P[E] = \frac{600}{900} = \frac{2}{3} \approx \boxed{0.67}$ | /1 |
| (b) | Evaluate the probability that the individual is unemployed<br>$P[U] = \frac{300}{900} = \frac{1}{3} \approx \boxed{0.33}$ | /1 |
| (c) | Evaluate the probability that the individual is a male<br>$P[M] = \frac{500}{900} = \frac{5}{9} \approx \boxed{0.56}$ | /1 |
| (d) | Evaluate the probability that the individual is female<br>$P[F] = \frac{400}{900} = \frac{4}{9} \approx \boxed{0.44}$ | /1 |
| (e) | Evaluate the probability that the individual is a male given that he is employed<br>$P[M \mid E] = \frac{\text{number of male and employed}}{\text{number of employed}} = \frac{460}{600} = \frac{23}{30} \approx \boxed{0.77}$ | /1 |
| (f) | Evaluate the probability that the individual is a male and is employed<br>$P[M \cap E] = P[M \mid E] \cdot P[E] = \frac{23}{30} \times \frac{2}{3} = \frac{23}{45} = \boxed{0.51}$ | /1 |
| (g) | Evaluate the probability that the individual is employed given that he is a male<br>$P[E \mid M] = \frac{P[M \mid E] \cdot P[E]}{P[M]} = \frac{\frac{23}{30} \cdot \frac{2}{3}}{5/9} = \boxed{0.92}$<br>$= \frac{23}{25}$<br>$P(M \mid E)$ | /1 |

| | | |
|---|---|---|
| | $P[E\|M]$ (prev. page) $(\approx 0.92)$ | |
| (h) | Evaluate the probability that the individual is a female and is unemployed<br>$P[F \cap U] = P[F\|U] \cdot P[U] = \frac{260}{300} \cdot \frac{1}{3} = \frac{13}{45}$ 260/400=13/20<br>$\approx 0.29$ | /1 |
| (i) | Evaluate the probability<br>$P[M \cup E] = P[M] + P[E] - P[M \cap E]$<br>$= \frac{5}{9} + \frac{2}{3} - \frac{23}{45} = \frac{32}{45} = 0.71$ | /2 |
| (j) | Evaluate the probability<br>$P[\overline{M} \cup \overline{E}] = P[\overline{M \cap E}] = 1 - P[M \cap E] = 1 - \frac{23}{45} = \frac{22}{45}$<br>$= 0.49$ | /1.5 |
| (k) | Evaluate the probability<br>$P[\overline{M} \cap \overline{E}] = P[\overline{M \cup E}] = 1 - P[M \cup E] = 1 - \frac{32}{45} = \frac{13}{45}$<br>$= 0.29$ | /1.5 |
| (l) | Show whether $E$ and $M$ are independent<br>$P[E \cap M]$ must equal $P[E] \cdot P[M]$ to be independent<br>$\frac{23}{45} \neq \frac{10}{27}$<br>NOT INDEPENDENT | /1 |
| | Total | /14 |

**Question 2:**

A company employs two sales engineers $A$ and $B$. Engineer $A$ does the work of estimating cost for 70% of jobs bid by the company while Engineer $B$ does the work for 30% of jobs bid by the company. From past studies, it is known that the error rate when Engineer $A$ does the work is 2%, whereas when Engineer $B$ does the work it is 4%. Define the event $E$: Error occurs in estimating cost. Answer the following questions:

| | | Marks |
|---|---|---|
| (a) | Write down the values of the given or known probabilities | /1 |
| (b) | Evaluate the probability of an error occurring in estimating cost | /2 |
| (c) | A bid arrives and there is an error in estimating cost. What is the probability that the bid came from Engineer $A$? | /1.5 |
| (d) | A bid arrives and there is an error in estimating cost. What is the probability that the bid came from Engineer $B$? | /1.5 |

a) $P[A] = 0.7$ , $P[B] = 0.3$ ,
$P[E|A] = 0.02$ , $P[E|B] = 0.04$

b) $P[E] = P[E \cap A] + P[E \cap B]$

$P[E \cap A] = P[E|A] \cdot P[A] = 0.02 \times 0.7 = 0.014$
$P[E \cap B] = P[E|B] \cdot P[B] = 0.04 \times 0.3 = 0.012$
$P[E] = 0.014 + 0.012 = \boxed{0.026}$ or 2.6 %

c) $P[A|E] = \dfrac{P[E|A] \cdot P[A]}{P[E]}$ or $\dfrac{P[A \cap E]}{P[E]} = \dfrac{0.014}{0.026}$

$= \dfrac{7}{13} \boxed{= 0.54}$

d) $P[B|E] = \dfrac{P[E|B] \cdot P[B]}{P[B]}$ or $\dfrac{P[B \cap E]}{P[E]} = \dfrac{0.012}{0.026}$

$= \dfrac{6}{13} \boxed{= 0.46}$