

Assignment 2

Problem 1

Before a software is distributed, a CD is picked, periodically, at random and tested for accuracy. The testing process consists of running four independent programs and checking the results. The failure rates for the four testing programs are, respectively, (failed test 1) 0.01, (failed test 2) 0.03, (failed test 3) 0.02 and (failed test 4) 0.01.

(a) Show that the probability that a CD failed at least one test is 0.07.

(b) Show that the probability that a CD failed program 2 or 3 is 0.0494.

$$\begin{aligned}P_1(\text{Pass}) &= 1 - \left[P_1 = 0.01 \right] = 0.99 \\P_2(\text{Pass}) &= 1 - \left[P_2 = 0.03 \right] = 0.97 \\P_3(\text{Pass}) &= 1 - \left[P_3 = 0.02 \right] = 0.98 \\P_4(\text{Pass}) &= 1 - \left[P_4 = 0.01 \right] = 0.99\end{aligned}$$

$$\begin{aligned}\text{(a)} \quad P(\text{F at least 1}) &= 1 - P(\text{passed all}) \\P(\text{passed all}) &= P_1 \cdot P_2 \cdot P_3 \cdot P_4 \\&= 0.99 \cdot 0.97 \cdot 0.98 \cdot 0.99 \\&= 0.932\end{aligned}$$

$$\begin{aligned}P(\text{f at least 1}) &= 1 - 0.932 \\&= 0.068 \\&\approx 0.07 \\&= 7\%\end{aligned}$$

$$\text{(b)} \quad P(2 \text{ or } 3) = P_2 + P_3 - P_2 \cdot P_3 = 0.0494$$

Problem 2:

A construction company employs two sales engineers. Engineer 1 does the work of estimating cost for 70% of jobs bid by the company. Engineer 2 does the work for 30% of jobs bid by the company. It is known that the error rate for engineer 1 is such that 0.02 is the probability of an error when he/she does the work, whereas the probability of an error in the work of engineer 2 is 0.04. Suppose a bid arrives and a serious error occurs in estimating cost. Which engineer would you consider most likely did the work and why?

$$P(E_1|A) = \frac{P(A|E_1) P(E_1)}{P(A)}$$

$$P(E_2|A) = \frac{P(A|E_2) P(E_2)}{P(A)}$$

$$\begin{aligned}P(A|E_1) &= 0.02 \\P(E_1) &= 0.7\end{aligned}$$

$$\begin{aligned}P(E_2|A) &= 0.04 \\P(E_2) &= 0.3\end{aligned}$$

$$P(A) = P(A|E_1)P(E_1) + P(A|E_2)P(E_2) = 0.026$$

$$\begin{aligned}P(E_1|A) &= 0.538 \\&= 53.8\%\end{aligned}$$

$$\begin{aligned}P(E_2|A) &= 0.462 \\&= 46.2\%\end{aligned}$$

\therefore Engineer 1 is more likely to cause the issue

Problem 3

How many ways are there to select 3 candidates from 8 equally qualified recent graduates for openings in an accounting firm?

$$C(n, u) = \frac{n!}{u!(n-u)!} \quad \leadsto \quad C(8, 3) = \frac{8!}{3!(8-3)!} = 56$$

Problem 4

An urn contains one red ball, 2 green balls and 3 blue balls. Suppose three balls are taken at random from the urn. What is the probability of one is red, one is green, and one is blue?

Total Number of ways to choose
3 balls = $C(6, 3) = 20$

$$\left. \begin{array}{l} \text{red} \rightarrow C(1, 1) = 1 \\ \text{green} \rightarrow C(2, 1) = 2 \\ \text{blue} \rightarrow C(3, 1) = 3 \end{array} \right\} \text{total} = 6$$

$$\frac{P(\text{num of ways 1 R 1 G 1 B})}{P(\text{num of ways to choose 3})} = \frac{6}{20} = \boxed{30\%}$$