

Start Date: 5:00pm, October 19, 2020**Due Date: 5:00pm, October 26, 2020****Problem 1:**

Consider a random variable, X , has a PDF given by

$$f_X(x) = \begin{cases} 2x, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Now, let us construct another random variable, $Y = 0.5X + 0.25$.

- (a) Find the range of Y .
- (b) Find the CDF of X .
- (c) Find the CDF of Y .
- (d) Find the PDF of Y .
- (e) Show that the function obtained in part (d) is a valid PDF.
- (f) Find the mean values of X and Y .
- (g) Find the mean-square values of X and Y .
- (h) Find the standard deviations of X and Y .

Problem 2:

Evaluate the probability $P[\mu_X - 2\sigma_X < X < \mu_X + 2\sigma_X]$, if the PDF of X is

$$f_X(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Problem 3

Assume the length X , in minutes, of a telephone conversation is a random variable with probability density function

$$f_X(x) = \begin{cases} \frac{1}{5}e^{-\frac{x}{5}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Determine the mean length of this telephone conversation.
- (b) Find the variance and standard deviation of X .
- (c) Find $E[(X + 5)^2]$.