ENEL 619

Assignment 2

Problem 1

$P(F_1) = 0.01$ $\quad P(F_1') = 0.99$

$P(F_2) = 0.03$ $\quad P(F_2') = 0.97$

$P(F_3) = 0.02$ $\quad P(F_3') = 0.98$

$P(F_4) = 0.01$ $\quad P(F_4') = 0.99$

$$P(\text{pass}) = 0.99 \times 0.97 \times 0.98 \times 0.99$$
$$= 0.93168306 \approx 0.9317$$

a) $P(\text{fail}) = 1 - P(\text{pass}) \approx \boxed{0.06832}$

b) $P(\text{Failed F2 or/and F3}) = P(F_2 \text{ and } F_3) \text{ or } P(F_2) \text{ or } P(F_3)$

$= 0.03 \times 0.02 + 0.03 + 0.02$

$\boxed{= 0.0506}$

Problem 2

26 letters in alphabet
6 sides to die

_ _
↑ ↑
26 6

Total elements in sample space:
2 spaces to fill

_ _
↑ ↑
26 possibilities, 6 possibilities

26×6 possibilities $= 156$

Problem 3

a) no. of ways: $\frac{10!}{1! \times 2! \times 4! \times 3!}$

$= 12600$

_ _ _

b) 3 types, Red (R), Green (G), Blue (B)

$\frac{6!}{3! \times (6-3)!} = 20$ Total ways to arrange balls

C_3^6 or $\binom{6}{3}$

$3! = 6$ ways to get 1 Red, 1 blue, 1 green.

So probability $P(1R, 1B, 1G) = \frac{6}{20}$

$$= 0.3$$

c) number of ways to have 2 defective out of 5:

$$\binom{4}{3} \times \binom{3}{2} = \frac{4!}{3!(4-3)!} \times \frac{3!}{2!(3-2)!}$$

$$= 84 \times 3 = 252$$

number of ways to have 3 defective out of 5:

$$\binom{4}{2} \times \binom{3}{3} = \frac{4!}{2!(4-2)!} \times \frac{3!}{3!0!} = 36$$

Total ways to have atleast 2 defective

$$252 + 36 = 288$$

Problem 4

a) if order matters:

$$P_3^8 = \frac{8!}{(8-3)!} = \boxed{336}$$

b) if order doesnt matter:

$$C_3^8 = \binom{8}{3} = \frac{8!}{3!(8-3)!} = \boxed{56}$$

Problem 5

a) X random variable for no. of jazz CDs

Total combinations to select 4 CDs

$$C_4^{10} = \frac{10!}{4!(10-4)!} = 210$$

There are $C_x^5 \times C_{4-x}^5$ combinations to pick X jazz CDs into the 4 CD combination.

So,

$$P(X=x) = \frac{\binom{5}{x}\times\binom{5}{4-x}}{\binom{10}{4}} \quad \text{for } x \in \{0,1,2,3,4\}$$

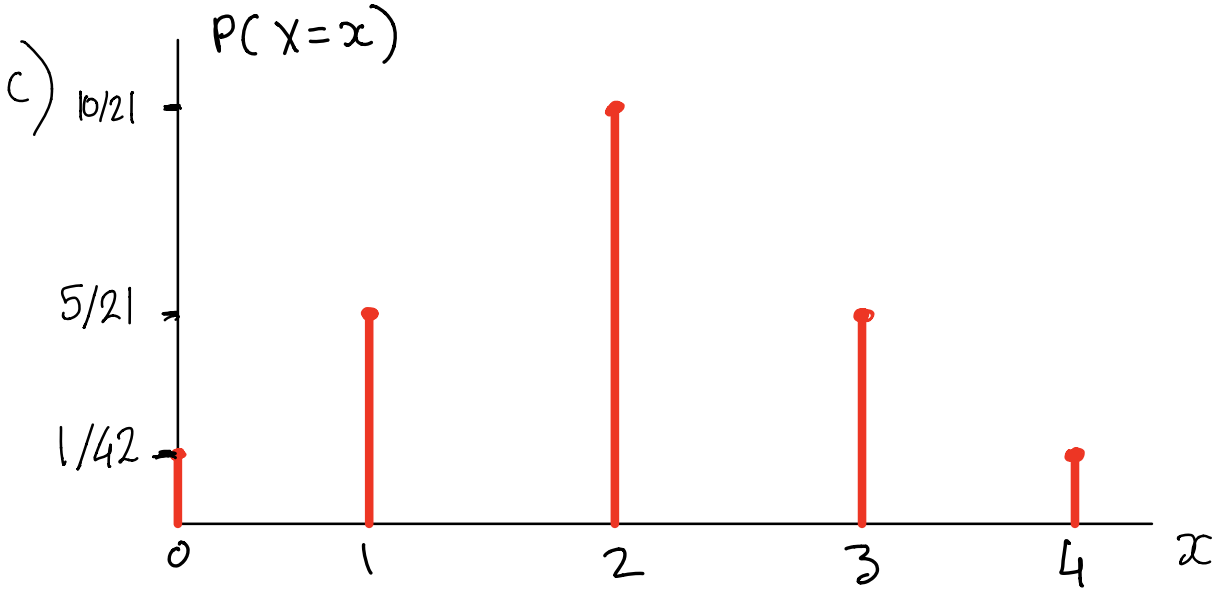
$$= \frac{\frac{5!}{x!(5-x)!} \times \frac{5!}{(4-x)!(1+x)!}}{210} = \boxed{\frac{(5!)^2}{210}\left(\frac{1}{x!(5-x)!} \times \frac{1}{(4-x)!(1+x)!}\right)}$$

b)

x	$P(X=x)$
0	1/42
1	5/21
2	10/21
3	5/21
4	1/42

$\frac{129}{210} = \frac{4}{7}$

c)



d) CDF $\rightarrow$ $F_X(X=x) = P(X \leq x) = \sum_{x_i \leq x} P(X=x_i)$

x	$F_X(X=x)$
0	1/42
1	11/42
2	31/42
3	41/42
4	1