Due Date: 4:00pm, October 4, 2019 (in Assignment box)

Assignment Problems (First 5 problems to be handed in):

Problem 1

Before the distribution of a certain software, a CD is picked, periodically, at random and tested for accuracy. The testing process consists of running four independent programs and checking the results. The failure rates for the four testing programs are, respectively, (failed test 1) 0.01, (failed test 2) 0.03, (failed test 3) 0.02 and (failed test 4) 0.01

(a) What is the probability that a CD failed at least one test?
(b) Find the probability that a CD failed program 2 or 3 or both.

Hint: Define events

- F_1, F_2, F_3, F_4 : Failed test 1, 2, 3, 4, respectively

Problem 2

A government agency employs three consulting companies (A, B, and C) with probabilities 0.4, 0.35, and 0.25, respectively. From experience it is known that the probability of cost overruns for the three companies are 0.05, 0.03, and 0.15, respectively. Suppose the agency experienced cost overrun.

(a) What is the probability that the consulting company C is involved?
(b) What is the probability that the consulting company A is involved?

Problem 3: (For consistency use A and B to represent the events)

Consider a vehicle entering the Banff National Park. The probability that it is a camper is 0.28; the probability that it has Canadian license plates is 0.12; and the probability that it is a camper and has Canadian license plates is 0.09.

(a) What is the probability that a vehicle entering the park has Canadian license plates, given that it is a camper?
(b) What is the probability that vehicle entering the Banff National Park is a camper given that it has Canadian license plates?
(c) A vehicle entering the Banff National Park neither has Canadian license plates nor is it a camper?

Problem 4

(a) If an experiment consists of drawing a letter at random from the English alphabet and then throwing a die, how many points are there in the sample space?
(b) A witness to a hit-and-run accident tells the police that the license number contained the letters ABCD followed by 3 digits, the last of which was a 7. If the witness cannot recall the first 2 digits, but is certain that all 3 digits are different, find the maximum number of automobile registrations that the police may have to check.

(c) How many ways are there to select 3 candidates from 8 equally qualified recent graduates for openings in an accounting firm?

Problem 5

(a) In a college basketball practice session, the coach needs to have 10 players standing in a row. Among these 10 players, there is one first year student, 2 second year students, 4 third year students and 3 fourth year students. How many ways can they be arranged in a row if only their class level will be distinguished?

(b) An urn contains 1 red ball, 2 green balls and 3 blue balls. Suppose three balls are taken at random from the urn. What is the probability that one is red, one is green, and one is blue?

(c) A shipment of 12 TVs contains 3 that are defective. In how many ways can a hotel purchase 5 of these sets and receive at least 2 of the defective sets? **Hint: receive 2 or 3 defective sets.**

Practice Problems (not to be handed in):

Problem 1:

The probability that an automobile needs an oil change is 0.25; the probability that it needs a new oil filter is 0.4; and the probability that both oil and the filter need changing is 0.14.

(a) If the oil needs to be changed, what is the probability that a new oil filter is needed?

(b) If a new oil filter is needed, what is the probability that the oil needs to be changed?

Problem 2:

A construction company employs two sales engineers. Engineer 1 does the work of estimating cost for 70% of jobs bid by the company. Engineer 2 does the work for 30% of jobs bid by the company. It is known from experience that the error rate for Engineer 1 is such that 0.02 is the probability of an error when he/she does the work, whereas the probability of an error in the work of engineer 2 is 0.04. Suppose a bid arrives and a serious error occurs in estimating cost. Find the relevant probabilities and conclude whether Engineer A or Engineer B most likely did the work.

Hints: Define events A: engineer 1, B: engineer 2 and E: error occurred in estimating cost, we are given:

From the statements: $P[A]=0.7$, $P[B]=0.3$; and $P[E\,|\,A]=0.02$, $P[E\,|\,B]=0.04$.

$P[E]=0.026$; $P[A\,|\,E]=0.5385$; $P[B\,|\,E]=0.4615$

Problem 3:

The number of people in a town, who have completed a college degree, are categorized as shown in the table below.

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

Suppose one of these individuals is to be selected at random for a tour to publicize the advantages of establishing new industries in the town. Let us investigate all the possible probabilities involved.

Define events:

M : a man is chosen, E : the one chosen is employed,

U : the one chosen is unemployed F : the one chosen is a female

From the problem statements, we can compute the probabilities in (a) – (e).

(a) The total number of people is 900 of which 600 are employed. Therefore,

$$P[E] = \frac{600}{900} = \frac{2}{3}$$

(b) The total number of people is 900 of which 300 are unemployed. Therefore,

$$P[U] = \frac{300}{900} = \frac{1}{3}$$

(c) The total number of people is 900 of which 500 are female. Therefore,

$$P[M] = \frac{500}{900} = \frac{5}{9}$$

(d) The total number of people is 900 of which 400 are female. Therefore,

$$P[F] = \frac{400}{900} = \frac{4}{9}$$

(e) The total number of employed people is 600 and that of employed males is 460. Therefore,

$$P[M \mid E] = \frac{460}{600} = \frac{23}{30}$$

(f) Apply conditional probability to find $P[M \cap E]$,

$$P[M \mid E] = \frac{P[M \cap E]}{P[E]} \Rightarrow P[M \cap E] = P[M \mid E]P[E] = \frac{23}{30} \times \frac{2}{3} = \frac{23}{45}$$

This result may be also found as

$$P[M \cap E] = \frac{\text{\# of employed male}}{\text{Total \# of people}} = \frac{460}{900} = \frac{23}{45}$$

(g) Apply Bayes' rule to find $P[E|M]$

$$P[E|M]=\frac{P[M|E]P[E]}{P[M]}=\frac{23}{30}\times\frac{2}{3}\times\frac{9}{5}=\frac{23}{25}$$

<u>Exercise:</u> Find $P[E|F]$, $P[U|M]$, $P[U|F]$.

<u>Problem 4:</u>

A telephone company operates three identical relay (also known as repeater) stations at various locations. During a one-year period, the number of malfunctions reported by each station and the causes are given in the table below.

Station	A	B	C
Problems with electricity supplied	2	1	1
Computer malfunction	4	3	2
Electrical equipment malfunction	5	4	2
Malfunctions caused by other human errors	7	7	5

Suppose that a malfunction was reported, and it was found to be caused by other human errors. What is the probability that it came from station C?

<u>Define events:</u>

A: station A, B: station B, C: station C, E: a malfunction by other human errors.

From the table and problem statements, we can easily compute the following probabilities:

$P[A]=\frac{18}{43}$	$P[B]=\frac{15}{43}$	$P[C]=\frac{10}{43}$
$P[E\mid A]=\frac{7}{18}$	$P[E\mid B]=\frac{7}{15}$	$P[E\mid C]=\frac{5}{10}$

We are asked to find $P[C|E]$. Using the above probabilities, <u>**Total probability**</u> of error is

$$P[E]=P[E|A]P[A]+P[E|B]P[B]+P[E|C]P[C]$$

Using <u>**Bayes' rule**</u>, the Probability that it came from station C is

$$P[C|E]=\frac{P[E|C]P[C]}{P[E]}=0.2632$$