Practice Problem #1 Solutions

Problem 1:

(a) The sample space of the experiment is
$$S = \{aaa;\ aaf;\ afa;\ faa;\ ffa;\ faf;\ aff;\ fff\}$$

(b) Event that the circuit from Z fails:
$$A = \{aaf;\ aff;\ faf;\ fff\}$$

(c) Event that the circuit from X is acceptable:
$$B = \{aaa;\ aaf;\ afa;\ aff\}$$

(d) Since $A \cap B = \{aaf,\ aff\} \neq \varnothing$, then A and B are not mutually exclusive

(e) Since $A \cup B = \{aaa;\ aaf;\ afa;\ aff;\ faf;\ fff\} \neq S$ then A and B are not collectively exhaustive.

(f) The event that more than one circuit is acceptable is
$$D = \{aaa;\ aaf;\ afa;\ faa\}$$

(g) The event that at least two circuits fail is
$$E = \{fff;\ ffa;\ faf;\ aff\}$$

(h) Since $D \cap E = \varnothing$, then D and E are mutually exclusive.

(i) Since $D \cup E = S$, then D and E are collectively exhaustive.

Problem 2:

(a) $A = \{2,3\}$.

(b) $B = \{0,1,2,3,4,5,6\}$.

(c) $C = \{x \mid x^2 - 5x + 6 = 0\} \Rightarrow C = \{2,\ 3\}$, obtained by solving the quadratic equation.

(d) $\begin{aligned} D &= \{x \mid x \text{ is the number of heads when six coins are tossed}\} \\ &= \{0,1,2,3,4,5,6\} \end{aligned}$

Solution: Comparing all four sets, it is clear that $A = C$ and $B = D$.

Problem 3:

(a) $A \cup C = \{0,2,4,6,8\} \cup \{2,3,4,5\} = \{0,2,3,4,5,6,8\}$

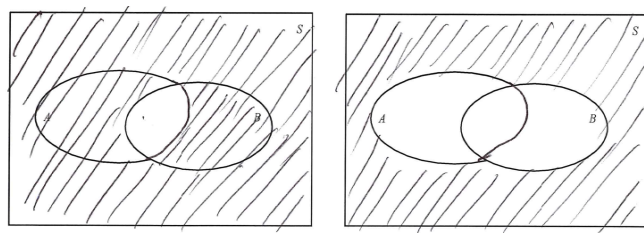(b) $A \cap B = \{0,2,4,6,8\} \cap \{1,3,5,7,9\} = \varnothing$

(c) $\bar{C} = \{0,1,6,7,8,9\}$

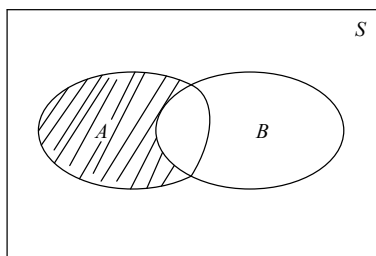(d) $\bar{C} \cap B = \{1,7,9\} \Rightarrow (\bar{C} \cap B) \cup D = \{1,6,7,9\}$

(e) $S \cap C = C = \{2,3,4,5\} \Rightarrow \overline{S \cap C} = \bar{C} = \{0,1\,6,7,8,9\}$

(f) $\bar{A} = B = \{1,3,5,7,9\} \Rightarrow \bar{A}C = \{3,5\} \Rightarrow \bar{A} \cap C \cap D = \varnothing$

Problem 4:

$C = \overline{A \cap B}$ (shaded) $D = \overline{A \cup B}$ (shaded)

$E = A \cap \overline{B}$ (shaded)

Problem 5:

Define the events:

B : customer invests in bonds M : customer invests in mutual funds

(a) $P[B \cup M] = P[B] + P[M] - P[B \cap M] = 0.6 + 0.3 - 0.15 = 0.75$

(b) $P[\text{no investment}] = P[\overline{B \cup M}] = 1 - P[B \cup M] = 1 - 0.75 = 0.25$

Problem 6:

Define the events:

A : defect in battery B : defect in electronics

C : defect in operating system D : defect in display

(a) $P[A \cup B] = P[A] + P[B] - P[A \cap B] = 0.25 + 0.18 - 0.15 = 0.28$

(b) $P[\text{no defect}] = P[\overline{A \cup B}] = 1 - P[A \cup B] = 1 - 0.28 = 0.72$

(c)

$$P[A \cap \overline{B}] = P[A \cap (S - B)] = P[A \cap S] - P[A \cap B] = P[A] - P[A \cap B]$$

$$= 0.25 - 0.15 = 0.1 \ \ (\text{assuming } P[A \cap B] = 0.15 \text{ as in part (a)})$$

Or directly from Venn diagram:

$$P[A \cap \overline{B}] = P[A] - P[A \cap B] = 0.1$$