ADDITIONAL PRACTICE PROBLEMS SET #2:

Problem 1:

The population of adults (divided into 2 groups A and B) in a small town, who have completed a college degree, are categorized in as shown in the table below.

	Employed (E)	Unemployed (U)	Total
A	460	40	500
B	140	260	400
Total	600	300	900

One of these individuals is to be selected at random for a tour to publicize the advantages of establishing new industries in the town. Determine all the various probabilities involved:

$P[A], P[E], P[A|E], P[A \cap E]$; $P[B], P[E], P[B|E], P[B \cap E]$

$P[U], P[A|U], P[A \cap U]$; $P[B|U], P[B \cap U]$

$$P[E|A], P[E|B], P[U|A], P[U|B]$$

Hint: Define events

A: {the one chosen is from A}, E: {the one chosen is employed},
U: {the one chosen is unemployed} B: {the one chosen is from B}

Example answers:

$$P[A] = \frac{500}{900} = \frac{5}{9}, P[E] = \frac{600}{900} = \frac{2}{3}, P[B] = \frac{400}{900}, P[U] = \frac{300}{900} = \frac{1}{3}$$

$P[A|E] = \frac{460}{600} = \frac{23}{30}$, $P[B|E] = \frac{140}{600} = \frac{7}{30}$, $P[A \cap E] = \frac{460}{900}$

Problem 2:

A telephone company operates three identical relay (also known as repeater) stations. During a one-year period, the number of malfunctions reported by each station and the causes are given in the table below.

Station	A	B	C
Problems with electricity supplied	2	1	1
Computer malfunction	4	3	2
Electrical equipment malfunction	5	4	2
Problems caused by other human errors	7	7	5

Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from station C?

Hint: Define events

A: station A, B: station B, C: station C, E: a malfunction by other human errors.

Probability that it came from station C is

$$P[A] = \frac{18}{43}, P[B] = \frac{15}{43}, P[C] = \frac{10}{43}$$

$$P[E|A] = \frac{7}{18}, P[E|B] = \frac{7}{15}, P[E|C] = \frac{5}{10}$$

$$P[E] = P[E|A]P[A] + P[E|B]P[B] + P[E|C]P[C]$$

$$P[C \mid E] = \frac{P[E \mid C]P[C]}{P[E]} = 0.2632$$

Problem 3:

A manufacturing company produces IC chips. Batches of chips are tested by three different departments having rejection rates of 0.10, 0.08 and 0.12, respectively. The tests by the three departments are sequential and independent.

(a) What is the probability that a batch of IC chips survives the first departmental test but is rejected by the second departmental test?

$$0.9 \times 0.08 = 0.072$$

(b) What is the probability that a batch of IC chips is rejected by the third department?

$$0.9 \times 0.92 \times 0.12 = 0.099$$

Problem 4:

A town has two fire engines operating independently. The probability that a specific engine is available when needed is 0.96.

(a) What is the probability that neither is available when needed? (Answer: 0.0016)

$$P[\bar{A} \cap \bar{B}] = P[\bar{A}]P[\bar{B}] = 0.04 \times 0.04 = 0.0016$$

(b) What is the probability that a fire engine is available when needed? (answer: 0.9984). Note that 0.96+0.96>1 so it does not work; try something else.

$$P[A \cup B] = 1 - P[\bar{A} \cap \bar{B}] = 1 - 0.0016 = 0.9984$$

Problem 5:

Police plan to enforce speed limits by using radar traps at four different locations within the city limits. The radar traps at each of the locations L_1, L_2, L_3, and L_4 will be operated 40%, 30%, 20%, and 30% of the time. If a person who is speeding on her way to work has probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, what is the probability that she will receive a speeding ticket?

Hint: Define events

(1) S_1, S_2, S_3, S_4 that the person is speeding as he/she passes through the respective locations $L_1, L_2, L_3, L_4 \Rightarrow P[S_1]=0.2, P[S_2]=0.1, P[S_3]=0.5, P[S_4]=0.2$ and (2) R: data trap is working. If the person is speeding, the radar must be working to get caught, therefore, the probability that a radar trap is working given a person is speeding through location L_i is $P[R \mid S_i] \Rightarrow P[R \mid S_1]=0.40, P[R \mid S_2]=0.30, P[R \mid S_3]=0.20, P[R \mid S_4]=0.30.$ You now need to compute the total probability $P[R]=0.27$.

Let $S_1, S_2, S_3,$ and S_4 represent the events that a person is speeding as he passes through the respective locations and let R represent the event that the radar traps is operating resulting in a speeding ticket. Then the probability that he receives a speeding ticket: $P(R) = \sum_{i=1}^{4} P(R \mid S_i)P(S_i) = (0.4)(0.2) + (0.3)(0.1) + (0.2)(0.5) + (0.3)(0.2) = 0.27.$

Problem 6:

If the person in ***Problem 5*** received a speeding ticket on her way to work, what is the probability that he/she passed through the radar trap located at L_2? (**Hint:** we need $P[S_2|R]$

$$P[S_2|R] = \frac{P[R|S_2]P[S_2]}{P[R]}$$

Problem 7:

A telephone company operates three identical relay stations at various locations. During a one-year period, the number of malfunctions ions reported by each station and the causes are shown below.

Station	A	B	C
Problems with electricity supplied	2	1	1
Computer malfunction	4	3	2
Malfunctioning electrical equipment	5	4	2
Malfunction Caused by other human errors	7	7	5

Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from station C?

Hint: Follow Procedure in Problem 2.

Problem 8: A construction company employs two sales engineers. Engineer 1 does the work of estimating cost for 70% of jobs bid by the company. Engineer 2 does the work of estimating cost for 30% of jobs bid by the company. It is known that when engineer 1 does the work the probability of an error is 0.02, whereas the probability of an error in the work of engineer 2 is 0.04. Suppose a bid arrives and serious error occurs in estimating the cost. Which engineer would you guess did the work?

Hint: A similar examples was done in class and is also found in the notes.

Problems from Recommended ENEL 419 textbook: Yates and Goodman:
Solutions to these problems can be found at the following website:

> Students' solution manual (odd numbered problems) at Author's website:
> http://www.winlab.rutgers.edu/~ryates/student3e/studentsolns3.pdf

1.3.9● A student's score on a 10-point quiz is equally likely to be any integer between 0 and 10. What is the probability of an A, which requires the student to get a score of 9 or more? What is the probability the student gets an F by getting less than 4?

1.5.1● Given the model of handoffs and call lengths in Problem 1.4.1,

(a) What is the probability $P[H_0]$ that a phone makes no handoffs?

(b) What is the probability a call is brief?

(c) What is the probability a call is long or there are at least two handoffs?

1.4.1● Mobile telephones perform *handoffs* as they move from cell to cell. During a call, a telephone either performs zero handoffs (H_0), one handoff (H_1), or more than one handoff (H_2). In addition, each call is either long (L), if it lasts more than three minutes, or brief (B). The following table describes the probabilities of the possible types of calls.

	H_0	H_1	H_2
L	0.1	0.1	0.2
B	0.4	0.1	0.1

(a) What is the probability that a brief call will have no handoffs?

(b) What is the probability that a call with one handoff will be long?

(c) What is the probability that a long call will have one or more handoffs?

1.5.3◆ Suppose a cellular telephone is equally likely to make zero handoffs (H_0), one handoff (H_1), or more than one handoff (H_2). Also, a caller is either on foot (F) with probability 5/12 or in a vehicle (V).

(a) Given the preceding information, find three ways to fill in the following probability table:

	H_0	H_1	H_2
F			
V			

(b) Suppose we also learn that 1/4 of all callers are on foot making calls with no handoffs and that 1/6 of all callers are vehicle users making calls with a single handoff. Given these additional facts, find all possible ways to fill in the table of probabilities.

1.6.1● Is it possible for A and B to be independent events yet satisfy $A = B$?

1.6.3● At a Phonesmart store, each phone sold is twice as likely to be an Apricot as a Banana. Also each phone sale is independent of any other phone sale. If you monitor the sale of two phones, what is the probability that the two phones sold are the same?

1.6.5■ In an experiment, A and B are mutually exclusive events with probabilities $P[A] = 1/4$ and $P[B] = 1/8$.

(a) Find $P[A \cap B]$, $P[A \cup B]$, $P[A \cap B^c]$, and $P[A \cup B^c]$.

(b) Are A and B independent?

1.6.7■ In an experiment, A and B are mutually exclusive events with probabilities $P[A \cup B] = 5/8$ and $P[A] = 3/8$.

(a) Find $P[B]$, $P[A \cap B^c]$, and $P[A \cup B^c]$.

(b) Are A and B independent?