

**Due Date: 4:00pm, Friday September 20, 2019 (in Assignment box, ICT second floor)**

**Note: Problems 1, 2, 4 and 6 are to be handed in as Assignment #1. Problems 3 and 5 are for practice only.**

**Problem 1:**

An integrated circuit manufacturer has three machines  $X$ ,  $Y$  and  $Z$ . A circuit selected from each machine can be either defective (not acceptable) or non-defective (acceptable). Suppose we run an experiment by randomly pick three circuits in sequence from  $X$ ,  $Y$  and  $Z$ , respectively. We test each one and record the observation as “ $a$ ” if acceptable or “ $f$ ” if it fails. For example if the circuits from  $X$  and  $Y$  are acceptable (pass the test) but the circuit from  $Z$  is defective (fails the test), then the observation is  $aaf$ . Answer the following questions:

- What are the elements of the sample space of the experiment?
- What are the elements of the set  $A = \{\text{circuit from } Z \text{ fails}\}$
- What are the elements of the set  $B = \{\text{circuit from } X \text{ acceptable}\}$
- Are  $A$  and  $B$  mutually exclusive? Why?
- Are  $A$  and  $B$  collectively exhaustive? Why?
- What are the elements of the set  $D = \{\text{more than one circuit is acceptable}\}$ ?
- What are the elements of the set  $E = \{\text{at least two circuits fail}\}$ ?
- Are  $D$  and  $E$  mutually exclusive? Why?
- Are  $D$  and  $E$  collectively exhaustive? Why?

**Problem 2:**

Comparing all the events listed below, which events are equal?

- $A = \{2, 3\}$ .
- $B = \{0, 1, 2, 3, 4, 5, 6\}$ .
- $C = \{x \mid x^2 - 5x + 6 = 0\}$
- $D = \{x \mid x \text{ is the number of heads when six coins are tossed}\}$

**Problem 3: (For practice only)**

Consider the following sets:

$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$	$A = \{0, 2, 4, 6, 8\}$	$B = \{1, 3, 5, 7, 9\}$
$C = \{2, 3, 4, 5\}$	$D = \{1, 6, 7\}$	

List the elements of the sets corresponding to the following events:

(a) $A \cup C$	(b) $A \cap B$	(c) $\overline{C}$
(d) $(\overline{C} \cap B) \cup D$	(e) $(\overline{S \cap C})$	(f) $\overline{A} \cap C \cap D$

**Problem 4:**

Consider two non-disjoint events  $A$  and  $B$  and sample space  $S$ . Sketch Venn diagrams and shade the regions for each of the events resulting from the following operations:

(a) $C = \overline{A \cap B}$	(b) $D = \overline{A \cup B}$	(c) $A \cap \overline{B}$
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**Problem 5: (For practice only)**

From past experience, a stockbroker believes that under present economic conditions a customer will invest in bonds (event  $B$ ) with a probability of 0.6, will invest in mutual funds (event  $M$ ) with a probability of 0.3, and invest in both bonds ( $B$ ) and mutual funds ( $M$ ) with a probability of 0.15. Note that events are not disjoint or mutually exclusive. Find the probability that a customer will make

- (a) An investment in either bonds or mutual funds or both; (answer: 0.75)
- (b) No investment at all; (answer: 0.25).

**Problem 6:**

A cell phone manufacturer is concerned about a possible recall of its best-selling phone. If there were a recall, there is a probability 0.25 of a defect in the battery, 0.18 of a defect in the electronics, 0.17 of a defect in the operating system, and 0.40 of a defect in the display. Define events:  $A$  = defect in the battery and  $B$  = defect in the electronics,  $C$  = defect in the operating system and  $D$  = defect in display.

- (a) What is the probability that the defect is in the battery or the electronics, or both, if the probability of defects in both battery and electronics, simultaneously, is  $P[A \cap B] = 0.15$ ?
- (b) What is the probability that no defects will be involved in any of the two (battery or electronics)?
- (c) What is the probability that battery will be defective, and the electronics will not be defective, if the probability of defects in both battery and electronics, simultaneously, is  $P[A \cap B] = 0.15$ ?