RECOMMENDED PRACTICE PROBLEMS:

Problems (From ENEL 419 text book: Yates and Goodman):

3.4.1, 3.4.2, 3.4.3, 3.5.3, 3.5.5, 3.5.7, 3.5.8, 3.5.13, 3.6.1, 3.6.3, 3.6.5, 3.6.7, 3.8.1, 3.8.3.
Students' solution manual (odd numbered problems) at Author's website:
http://www.winlab.rutgers.edu/~ryates/student3e/studentsolns3.pdf

ADDITIONAL PROBLEMS:

Problem 1:

A section of an electrical circuit has two relays, numbered 1 and 2 operating in parallel. Let the random variable X, denote the number of relays that close properly, with a probability distribution (PMF) given by

$$P[X=0]=0.04$$
$$P[X=1]=0.32$$
$$P[X=2]=0.64$$

Find and sketch the CDF for X.

Solution:

The intervals to be considered are intervals: $x<0,\ 0\le x<1,\ 1\le x<2,$ and $x\ge 2$.

$$F_X(x)=\begin{cases} 0, & x<0 \\ 0.04, & 0\le x<1 \\ 0.36, & 1\le x<2 \\ 1, & x>2 \end{cases}$$

Problem 2:

A large university uses some of the student fees to offer free use of its health centre to all students. Let X, represent the number of times that a randomly selected student visits the health centre during a semester. Based on historical data, the CDF of X is given by

$$F_X(x)=\begin{cases} 0, & x<0 \\ 0.6, & 0\le x<1 \\ 0.8, & 1\le x<2 \\ 0.95, & 2\le x<3 \\ 1, & x\ge 3 \end{cases}$$

(a) Graph $F_X(x)$, (b) Verify that $F_X(x)$ is a CDF function, and (c) Find the PMF associated with $F_X(x)$.

(b) To verify that $F_X(x)$ is a CDF, we must confirm it satisfies four conditions:

- $\lim_{x\to-\infty} F_X(x) = 0$
- $\lim_{x\to+\infty} F_X(x) = 1$,
- $F_X(x)$ is non-decreasing,
- $F_X(x)$ is discontinuous at four points: $0,\ 1,\ 2,$ and 3. At each point $F_X(x)$ is right-hand continuous.

$F_X(x)$ satisfies all four conditions, therefore, $F_X(x)$ is a CDF function.

(c) The points of positive probability occur at the points of discontinuity: $0,\ 1,\ 2,$ and 3. The probability is the height of the "jump" at each of these points. This gives the PMF as follows:

x	$p_X(x)$
0	$0.6-0=0.6$
1	$0.8-0.6=0.2$
2	$0.95-0.8=0.15$
3	$1-0.95=0.05$

Problem 3:

Determine the value of c so that each of the following functions can serve as a PMF of random variable X:

(a) $p_X(x) = c\left(x^2+4\right),\ \ x=0,1,2,3.$

(b) $p_X(x) = c\binom{2}{x}\binom{3}{3-x},\ \ x=0,1,2.$

Problem 4:

A shipment of 7 TV sets contains 2 defective sets. A hotel makes a random purchase of 3 sets. Suppose random variable X is the number of defective sets purchased by the hotel.

(a) Find the expression for the PMF of X.

(b) Express the results in tabular form and graphically as a probability histogram.

x	0	1	2
$p_X(x)$	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$

Problem 5:

The probability distribution of random variable X, of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given in the table below.

x	0	1	2	3	4
$p_X(x)$	0.41	0.37	0.16	0.05	0.01

Construct the cumulative distribution function.

Problem 6:

Find the CDF of random variable X representing the number of defective TVs in Problem 4. Then using the CDF $F_X(x)$, find

(a) $P[X=1]$ (Answer: 4/7)

(b) $P[0<X\le 2]$ (Answer: 5/7)

Problem 9: From a box containing 4 dimes and 2 nickels, 3 coins are selected at random without replacement. Find the PMF for the total value, T, of the 3 coins selected. Express the PMF in table form and graphically as a probability histogram.

Solution: T: total of 3 coins, D: dime, N: nickel. Sample space: $\{T=t=20,\ 25,\ 30\}$

There are three ways we can pick. We can pick 2 nickels + 1dime (20 cents), 1 nickel + 2 dimes (25 cents) or 3 dimes (30 cents).

$$P[T=20]=\frac{\binom{2}{2}\binom{4}{1}}{\binom{6}{3}};\quad P[T=25]=\frac{\binom{2}{1}\binom{4}{2}}{\binom{6}{3}};\quad P[T=30]=\frac{\binom{2}{0}\binom{4}{3}}{\binom{6}{3}}$$