Schulich School of Engineering Academic Integrity Statement

Academic integrity is the foundation of the development and acquisition of knowledge and is based on values of honesty, trust, responsibility, and respect. We expect members of our community to act with integrity.

Research integrity, ethics, and principles of conduct are key to academic integrity. Members of our campus community are required to abide by our institutional code of conduct and promote academic integrity in upholding the University of Calgary's reputation of excellence.

The University of Calgary Principles of Conduct can be found in Section K of the University Calendar.

You are expected to write this exam on your own, without consultation with your peers. The answers on this exam should be reflective of your work and your understanding of the course content.

"Integrity is doing the right thing, even when no one is watching"
-C.S. Lewis

I, Yousif Al-Khoury, ID# 30080225 (First Name Last Name, U of C Student ID Number) do solemnly swear that I have not and will not communicate about this final examination with anyone, especially other students in the course, until after the deadline for submission of exam solutions. The answers on this exam are my own. I did not consult with any other person about the content on this exam prior to submitting my answers. I have conducted myself in an ethical manner that upholds the integrity and dignity of the engineering profession.

I fully understand that disciplinary action may be taken against me if I am discovered to have communicated with anyone about the content or solution of this final examination.

I am / am not (*circle one*) living with students currently enrolled in an engineering program. Their names are provided below:

____________________ (name/program/university)

____________________ (name/program/university)

____________________ (name/program/university)

By submitting this final examination, I agree that I have abided by the university's principles of conduct as outlined above.

Signature

Yousif Al-Khoury

ENEL 419: Probability and Random Variables

Midterm Exam

Instructor: Dr. Abu Sesay

November 5, 2020

Last Name (printed):	First Name:	ID #:
AL-khoury	Yousif	30080225

Signature: Yousif Alkhoury

Instructions:

- Sign, attach and submit the Academic Integrity Statement with your completed exam.
- Answer all three questions in the spaces provided after each question.
- Please print or write your answers legibly. What cannot be read cannot be marked.
- If you write anything you do not want marked, put a large "X" through it and write "rough work" beside it.
- The midterm will be made available for 24 hours, starting from 5 pm November 5 and must be completed and submitted by 5 pm November 6, 2020.
- You will need access to a computer and internet, as well as an ability to scan and upload handwritten work. Microsoft Office Lens is recommended when using a smartphone or tablet to scan handwritten work.
- You are allowed to use your notes and your textbook.
- You are not permitted to search the internet, communicate with classmates, or use excel or other calculation software.

Marks Summary

	Q1	Q2	Q3	Total
Marks				
Out of	35	35	30	100%

Question 1:

Marks		A computer system uses passwords that contain exactly 8 characters. Each character is one of the 26 lowercase letters $(a-z)$ or 26 uppercase letters $(A-Z)$ or 10 integers $(0-9)$. Let Ω denote the set of all possible passwords, and let A denote the events that consist of passwords with only letters and B denote the events that consist of passwords with only integers. Letters and numbers are repeatable. Suppose that all passwords in Ω are equally likely. Answer the following questions:
/3	(a)	Determine the cardinality of (1) the sample space, (ii) the event A and (iii) the event B
/3	(a)	Determine $P[A]$ and $P[B]$
/5	(a)	Determine $P[A\|\bar{B}]$. Round your answers to three decimal places.
/12	(b)	Determine $P[\bar{A}\cap\bar{B}]$. Round your answers to three decimal places.
/12	(c)	Determine the probability that password contains exactly 2 integers given that it contains at least 1 integer. **Note:** Let C denote the event that consists of passwords with exactly 2 integers, and let D denote the event that consists of passwords with at least one integer.
/35		

a) (1) sample space $= \Omega = (26+26+10)^8 = \boxed{2183401105584896}$

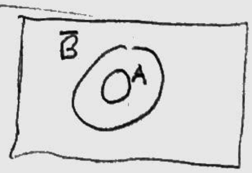
(2) event A $= (26+26)^8 = 53459728531456$

(3) event B $= (10)^8 = 100000000$

b) $P[A] = \dfrac{53459728531456}{218340105584846} = \dfrac{(26+26)^8}{(26+26+10)^8} \simeq 0.24485$

$P[B] = \dfrac{10^8}{(26+26+10)^8} = 4.58 \times 10^{-7}$

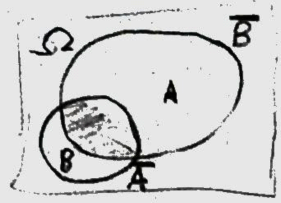
c) $P[A|\bar{B}] = \dfrac{P[A\cap\bar{B}]}{P[\bar{B}]} = \dfrac{P[A]}{P[\bar{B}]} = \dfrac{0.24485}{1-4.58\times10^{-7}}$



$\boxed{\approx 0.245}$

d) $P[\bar{A} \cap \bar{B}] = P[\bar{A}] + P[\bar{B}] - P[\bar{A} \cup \bar{B}]$

$= \dfrac{\Omega - A - B}{S} = 1 - P[A] - P[B]$



Note: $P[\bar{A} \cup \bar{B}] = 1$

$= 1 - 0.24485 - (4.58 \times 10^{-7}) = \boxed{0.755}$

e] $P[C|D] = \dfrac{P[C \cap D]}{P[D]} =$

$P[C \cap D] = P[C] = \dfrac{10^2 \cdot 52^6}{62^{10}} \cdot \binom{8}{2} = \dfrac{2800 \cdot 52^6}{62^{10}}$

$P[D] = 1 - P[A] = 1 - \dfrac{52}{62^8} = 0.75515$

$P[C|D] = \dfrac{2800 \cdot 52^6}{62^8 - 52^8} = \boxed{0.335745}$

$\boxed{\approx 0.336}$

Question 2:

Marks		A circuit consists of two devices, A_1 and A_2, connected in series. The probabilities that devices function correctly are $R_1 = 0.80$ and $R_2 = 0.90$. Assume that devices fail independently and let the random variable X denote the number of failed devices.
/12	(a)	Determine, and express in table form, the probability mass function of X
/10	(b)	Find the expected value of X
/13	(c)	Find the variance of X
/35		

a) PMF

$$f(x) = \begin{cases} 0 & 0 > x \\ 0.72 = 0.9 \times 0.8 & x = 0 \\ 0.26 = (0.2 \times 0.9) + (0.8 \times 0.1) & x = 1 \\ 0.02 = 0.1 \times 0.2 & x = 2 \\ 0 & x > 2 \end{cases}$$

X	0	1	2	otherwise
f(x)	0.72	0.26	0.02	0

$$\sum_x f(x) = 0.72 + 0.26 + 0.02 = 1$$

b) $E(X) = \mu = \sum_x x f(x) = 0(0.72) + 1(0.26) + 2(0.02)$ $\boxed{= 0.3}$

c) $V(x) = \sigma^2 = E(X-\mu)^2 = \sum_x (x-\mu)^2 f(x) = \sum_x (x^2 f(x)) - 2\mu \overbrace{\sum_x x f(x)}^{\mu} + \mu^2 \overbrace{\sum_x f(x)}^{1}$

$= \sum_x (x^2 f(x)) - \mu^2 = 0^2(0.72) + 1^2(0.26) + 2^2(0.02) - 0.3^2 = \boxed{0.25}$

Question 3:

Marks		
		The talk time (in hours) on a cell phone in a month is approximated by the probability density function $$f(x)=\begin{cases}\frac{(x-10)}{5h}, & 10<x<15\\ \frac{1}{h}, & 15\le x\le 20\\ -\frac{x-25}{5h}, & 20\le x\le 25\end{cases}$$ Find
/10	(a)	The value of h,
/6	(i)	The expected value of X,
/6	(ii)	The second moment or mean-square value X,
/4	(iii)	The variance of X,
/4	(iv)	The probability $P[15\le X\le 20]$.
/30		

a) $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{10}^{15} \frac{x}{5h} - \frac{2}{h}\,dx + \int_{15}^{20} \frac{1}{h}\,dx + \int_{20}^{25} \frac{-x}{5h} + \frac{5}{h}\,dx = 1$

$= \left[\frac{1}{10h}x^2 - \frac{2x}{h}\right]_{10}^{15} + \left[\frac{x}{h}\right]_{15}^{20} + \left[\frac{-x^2}{10h} + \frac{5x}{h}\right]_{20}^{25} = 1$

$= \frac{25}{2h} - \frac{10}{h} + \frac{5}{h} - \frac{45}{2h} + \frac{25}{h} = \frac{10}{h} = 1$ $\boxed{h=10}$

i) $E[X] = \mu = \int_{-\infty}^{\infty} x\,f(x)\,dx = \int_{10}^{15} \frac{x^2}{50} - \frac{x}{5}\,dx + \int_{15}^{20} \frac{x}{10}\,dx + \int_{20}^{25} \frac{-x^2}{50} + \frac{x}{2}\,dx$

$\mu = \left[\frac{x^3}{150} - \frac{x^2}{10}\right]_{10}^{15} + \left[\frac{x^2}{20}\right]_{15}^{20} + \left[\frac{-x^3}{150} + \frac{x^2}{4}\right]_{20}^{25}$

$\mu = \frac{10}{3} + \frac{35}{4} + \frac{65}{12} = \frac{35}{2} = \boxed{17.5}$

ii) $E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx =$

$$= \int_{10}^{15} \frac{x^3}{50} - \frac{x^2}{5}\, dx + \int_{15}^{20} \frac{x^2}{10}\, dx + \int_{20}^{25} -\frac{x^3}{50} + \frac{x^2}{2}\, dx$$

$$= \left[\frac{x^4}{200} - \frac{x^3}{15}\right]_{10}^{15} + \left[\frac{x^3}{30}\right]_{15}^{20} + \left[-\frac{x^4}{200} + \frac{x^3}{6}\right]_{20}^{25}$$

$$= \frac{1075}{24} + \frac{925}{6} + \frac{2825}{24} = \frac{950}{3} \boxed{\approx 316.67}$$

$$\boxed{E[X^2] = \frac{950}{3} \approx 316.67}$$

iii) $\sigma^2 = V(x) = E[X^2] - \mu^2 = \frac{950}{3} - \left(\frac{35}{2}\right)^2 = \frac{125}{12} = 10.42$

$$\boxed{\sigma^2 = V(x) = \frac{125}{12} \approx 10.42}$$

iv) $P[15 \le X \le 20] = \int_{15}^{20} f(x)\, dx = \int_{15}^{20} \frac{1}{10}\, dx = \left[\frac{x}{10}\right]_{15}^{20}$

$$= \frac{20-15}{10} = \frac{5}{10} = \frac{1}{2} = 0.5$$

$$\boxed{P[15 \le X \le 20] = \frac{1}{2} = 0.5}$$