Sample Estimation and Confidence Intervals:

Problem 1: An electrical firm manufactures light bulbs whose length of life is Gaussian distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm:

Solution: $n = 30,\ \bar{X} = 780,$ and $\sigma = 40.$ $Q(z_c) = \alpha/2 = 0.02 \Rightarrow z_c \approx 2.075$

$$780 - 2.075 \times \frac{40}{\sqrt{30}} < \mu < 780 + 2.075 \times \frac{40}{\sqrt{30}} \Rightarrow 764.85 < \mu < 795.15$$

Problem 2: The heights of a random sample of 50 ECE students showed a mean of 174.5 centimetres. The population has a standard deviation of 6.9 centimetres.

(a) Find a 98% confidence interval for the mean height of all students.
(b) What is the possible size of the error if we estimate the mean height of all students to be 174.5 centimetres?

Solution:

(a) $n = 50,\ \bar{X} = 174.5,$ and $\sigma = 6.9.$ $Q(z_c) = \alpha/2 = 0.01 \Rightarrow z_c \approx 2.325$

$$174.5 - 2.325 \times \frac{6.9}{\sqrt{50}} < \mu < 174.5 + 2.325 \times \frac{6.9}{\sqrt{50}} \Rightarrow 172.23 < \mu < 176.77$$

(b) $e = 2.325 \times \frac{6.9}{\sqrt{50}} = 2.2687$

Problem 3: What is the sample size needed in ***Problem 1*** if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?

Solution:

$$n = \left(\frac{\sigma z_c}{e}\right)^2 \left(40 \times \frac{2.075}{10}\right)^2 = 69 \text{ when rounded up}$$

Problem 4: A random sample of 100 automobile owners in Alberta shows that an automobile is driven on average 23,000 kilometers per year. The standard deviation is 3900 kilometers. Assume the distribution of measurements is approximately Gaussian.

(a) Find a 99% confidence interval for the average number of kilometers driven by car owners in Alberta..
(b) What is the possible size of the error if we estimate the number of kilometers driven by car owners in Alberta to be 23,500 kilometers per year?

Solution: $n = 100,\ \bar{X} = 23{,}500$ and $\sigma = 3900.$ $Q(z_c) = \alpha/2 = 0.005 \Rightarrow z_c \approx 2.575$

$$23{,}500 - 2.575 \times \frac{3900}{10} < \mu < 23{,}500 + 2.575 \times \frac{3900}{10} \Rightarrow 22{,}496 < \mu < 24504$$

(c) $e = 2.575 \times \dfrac{3900}{10} = 1004.25$

Problem 5: A machine produces metal pieces that are cylindrical in shape. A sample of pieces is taken and the measured diameters are given by the following table:

1.01	0.97	1.03	1.04	0.99	0.98	0.99	1.01	1.03

Find a 99% confidence interval for the mean diameter of all pieces from the machine, assuming Gaussian distribution.

Solution: The variance is not assumed known, therefore, it needs to be estimated along with the mean.

$n = 9,\ \upsilon = 8, \bar{X} = 1.0056$ and $S = 0.0245.\ t(z_c) = \alpha/2 = 0.005 \Rightarrow t_c \approx 3.355$

$$1.00056 - 3.355 \times \frac{0.0245}{3} < \mu < 1.00056 + 3.355 \times \frac{0.0245}{3} \Rightarrow 0.978 < \mu < 1.033$$

Problem 6: The recorded random measurements for drying time, in hours, of a brand of paint is given by the following table:

3.4	2.5	4.8	2.9	3.6	2.8	3.3	5.6	3.7	2.8	4.4	4.0	5.2	3.0	4.8

The measurements are assumed to be Gaussian distributed. Find the 95% prediction interval for the drying time for the next trial of the paint.

Solution:

$n = 15,\ \upsilon = 14, \bar{X} = 3.7867$ and $S = 0.9709.\ t(z_c) = \alpha/2 = 0.025 \Rightarrow t_c \approx 2.145$

$$3.7867 - 2.145 \times \frac{0.9709}{\sqrt{15}} < \mu < 3.7867 + 2.145 \times \frac{0.9709}{\sqrt{15}}$$

$$\Rightarrow 3.249 < \mu < 4.3244$$

Problem 7: The sample mean $\bar{X} = 128.5$, sample variance is $S_X^2 = 1230$, and sample size is $n = 32$. What is the 99% confidence interval of the mean?

The degrees of freedom is $\upsilon = n - 1 = 31$ and the value of t_c in the column for area 0.99 is 2.744. The 99% confidence interval is:

$$128.5 - 2.744 \times 35.07/\sqrt{32} < \mu < 128.5 + 2.744 \times 35.07/\sqrt{32}$$

$$128.5 - 2.744 \times 6.2 < \mu < 128.5 + 2.744 \times 6.2$$

$$\Rightarrow 111.5 < \mu < 145.5$$

Note: The t-Table provided to you does not have $v = 31$. In a test a value like $v = 31$ will not be asked for.