Problem 1:

Suppose the length X, in minutes, of a phone conversation is a random variable with PDF given by

$$f_X(x) = \begin{cases} \frac{1}{5}e^{-\frac{x}{5}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

(a) What is the mean length, $E[X]$, of this type of conversation? (Answer: 5)

(b) What is the variance and standard deviation of X? (Answers: 25 and 5)

(c) Find $E\left[(X+5)^2\right]$? (Answer: 125)

Problem 2:

The life, in years, of a certain type of electrical switch has an exponential distribution with an average life $\lambda = 2$. If 100 of these switches are installed in different systems, what is the probability that at most 30 fail during the first year?

Problem 3:

The average zinc concentration recovered from a sample of measurements taken in 36 different locations in a river is found to be 2.6 grams per milliliter. Find the 95% and 99% confidence intervals for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3 grams per milliliter.

Answers:
$$\begin{cases} \alpha = 0.05 \Rightarrow z_c \approx 1.96 \Rightarrow 2.50 \le \mu_X \le 2.70 \\ \alpha = 0.01 \Rightarrow z_c \approx 2.575 \Rightarrow 2.47 \le \mu_X \le 2.73 \end{cases}$$

Problem 4:

You are given the following data samples:

i	1	2	3	4	5	6	7	8	9	10
X_i	8.81	15.91	20.39	8.63	13.80	21.73	27.23	7.32	14.83	20.26

(a) Find the sample quantile,

(b) Find the theoretical quantile (for a standard Gaussian CDF),

(c) Find the theoretical quantile (for a regular Gaussian CDF),

(d) Sketch (i) the sample quantile versus the theoretical (standard Gaussian) quantile and (2) the sample quantile versus the theoretical (regular Gaussian) quantile on the same graph.

(e) Would you say the data approximately fits a Gaussian distribution?

Problem 5:

A machine produces metal pieces that are cylindrical in shape. A sample of pieces is taken, and the diameters are found are as follows:

1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99 1.01 1.03 centimeters

Find the 99% confidence interval for the mean diameter of pieces from the machine, assuming approximately Gaussian distribution.

(**Answer:** $0.978 \le \mu_X \le 1.033$)

Problem 6:

An electrical firm manufactures light bulbs that have a length of life that is approximately Gaussian distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm. (Answer: $765 \le \mu_X \le 795$ milliliter.

(**Answer:** $0.978 \le \mu_X \le 1.033$)

Problem 7:

An electrical firm manufactures light bulbs that have a length of life that is approximately Gaussian distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm. (Answer: $765 \le \mu_X \le 795$)

Problem 8:

Test the hypothesis that the average content of a container is 10 liters if the contents of a random sample of ten containers (measured in liters) are

10.2	9.7,	10.1,	10.3,	10.1,	9.8,	9.9,	10.4,	10.3,	9.8

Use a 0.01 significance level and assume the distribution of the contents is Gaussian.
Answer: Accept hypothesis

Problem 9:

An operation has six components. Each component has a failure probability q, independent of any other component. A successful operation requires both of the following conditions:

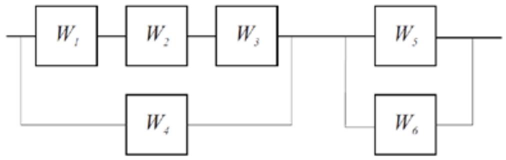
- Components 1, 2, and 3 all work, or component 4 works.
- Component 5 or component 6 works.

(a) Draw a block diagram for this operation.

(b) Derive a formula for the probability $P[W]$ that the operation is successful.

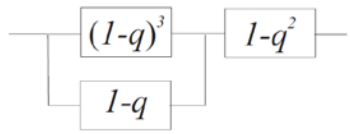
<u>Solutions:</u>

(a)



(b)

$$P[W_1 \cap W_2 \cap W_3] = (1-q)^3$$
$$P[W_5 \cup W_6] = 1 - \left[\overline{W5} \cap \overline{W_6}\right] = 1 - q^2$$



$$P[W] = \left[1 - q\left(1 - (1-q)^2\right)\right]\left[1 - q^2\right]$$

<u>Problem 10:</u>

A sample of 10 transistors is selected from a production line and the gains are measured. The measured gains are provided in the table below.

X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
112	77	113	83	95	105	102	120	73	95

(a) What is the sample mean $\bar{X}$?
(b) What is the sample standard deviation S_X ?
(c) Determine and sketch the empirical CDF for the measured transistor gain.
(d) Determine the sample quantiles and theoretical quantiles
(e) Sketch the Quantile-Quantile plots and conclude whether the data samples come from a Gaussian population.
(f) Examining the Quantile-Quantile plot, (1) is the intercept approximately equal to the sample mean? (2) is the slope approximately equal to the sample standard deviation?

<u>Problem 11:</u>

Consider the pair of measurements in the table below.

X_i	0.77	4.39	4.11	2.91	0.56	0.89	4.09	2.38	0.78	2.52
Y_i	14.62	22.21	24	19.42	14.69	15.23	24.48	16.88	8.56	16.24

Compute the following statistics:

(a) The sample means $\overline{X}$ and $\overline{Y}$

(b) The sample variances S_x^2 and S_y^2

(c) The covariance C_{xy}

(d) The correlation coefficient ρ_{xy}

Practice problems on Hypotheses Testing may be found in the supplementary lecture notes.