ABS
12/07/2021

ENEL 419: Probability and Random Variables

Final Exam for Fall 2021

Instructor: Dr. Abu Sesay

December 20, 2021

ID NUMBER	LAST NAME (PRINTED):	OTHER NAMES

Signature:

Note: Please read the entire instructions before you start the exam.

Exam Rules, Instructions and Information:

1. **You must sign and submit the attached Academic Integrity Statement with your completed exam.**
2. This exam is open book and open notes.
3. This will be made available starting from 2:30 pm December 20 and must be completed and submitted by 5:30 pm December 20, 2021, which is the Registrar's scheduled date and time.
4. This exam has 4 questions worth a total of 35 marks (or 100%).
5. If you write anything you do not want marked, put a large "X" through it and write "rough work" beside it.
6. Please print or write your answers legibly. What cannot be read cannot be marked
7. This exam is being administered entirely through D2L.
8. All final answers should be uploaded directly into D2L and **NOT in the QUIZ**.
9. You can write solutions by hand on loose-leaf, lined paper, tablet, etc. but uploaded as PDF
10. You will need access to a computer and internet, as well as an ability to scan and upload handwritten work. Microsoft Office Lens is recommended when using a smartphone or tablet to scan handwritten work.
11. You are not permitted to search the internet, collaborate, or consult with others when developing solutions and determining answers.
12. For questions and clarifications or technical issues, contact instructor by email (sesay@ucalgary.ca). Please note that my response may not be instantaneous.

Marks Summary

	Q1	Q2	Q3	Q4	Total
Marks obtained					
Maximum marks	38	17	37	8	100

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Question 1: Answer the following questions on the blank pages provided (or on loose sheets of paper, Tablets, etc.). If there are calculations involved, **you must show the steps** leading to your answer, otherwise, you will lose some points.

Marks		
		Two Internet providers (A and B) declared a data transfer rate of 5.5 MBps . However, the actual download speed is lower at almost every moment. The observed download speed losses for both internet providers are given (in MBps) in the table below.
/1 /4 /9 /4 /9	(a)	It is desired to develop a procedure for testing the mean difference of the download speed losses to establish whether the two providers have the same download speed loss. Assume that download speed losses are random. There is no assumption that the two population variances are equal. (i) What is the parameter of interest for such a test? (ii) What are the hypotheses for such a test? (iii) Determine the parameters required to set up the test statistic for such a test? (iv) What is the expression for the test statistic for such a test? (v) Determine the acceptance interval and conclude whether we can accept the hypothesis that the download speed losses are equal. Use a 95% confidence level.
/9 /2	(b)	For the same data and assumptions in Part (a) above, (i) Find a confidence interval for the mean difference of the download speed loss. (ii) Conclude whether there is a significant difference in the mean speed loss of the internet service providers and explain why.
/38		

Table of Internet speed Losses for Providers A and B

A	0.16	0.34	0.457	0.839	0.979	0.25	0.255	0.792	0.224	0.992	0.942	0.022	0.581	0.434	0.541
B	0.137	0.703	0.22	0.834	0.573	0.214	0.13	0.552	0.391	0.387	0.343	0.355	0.699		

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Question 2: Answer the following questions on the blank pages provided (or on loose sheets of paper, Tablets, etc.). If there are calculations involved, **you must show the steps** leading to your answer, otherwise, you will lose some points.

Marks		**Use the Q-function Table only for the following questions:**
/3.5	(a)	The temperature in a thermostatically controlled electronics laboratory T, (in degrees Fahrenheit), is a Gaussian random variable with a mean value of $68^{\circ}\mathrm{F}$. In addition, $P[T < 66] = 0.1587$. What is the value of the variance of T?
/1.5	(b)	Suppose X is a Gaussian random variable with a mean value of 50 and a standard deviation of 10. Find $P[X > 65]$
/2.5	(c)	Suppose X is a Gaussian random variable with a standard deviation of 10, and $P[X \le 10] = 0.933$. Find the mean of X.
/3	(d)	Suppose X is a Gaussian random variable with an unknown mean value μ_X, and a standard deviation of 2. Find $P[X \le \mu_X + 1]$
/4 /2.5	(e)	The peak temperature T, in degrees Fahrenheit, on a July day in Antarctica is a Gaussian random variable with a variance of $225^{\circ}\mathrm{F}$. Suppose $P[T > -75] = 0.5$. Find (i) $P[T > 0]$. (ii) $P[T < -100]$.
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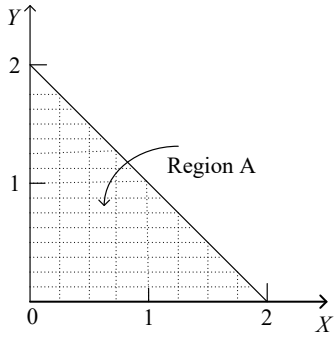
Question 3: Answer the following questions on the blank pages provided (or on loose sheets of paper, Tablets, etc.). If there are calculations involved, **you must show the steps** leading to your answer, otherwise, you will lose some points.

Marks		Two random voltages X and Y have a joint probability distribution $f_{XY}(x,y)=c$, inside the shaded triangular region A, shown in the diagram below. 
/4	(a)	Find the value of the constant c such that $f_{XY}(x,y)$ is a valid PDF in the region A.
/6	(b)	Show whether X and Y are independent.
/6	(c)	Show whether X and Y have equal mean values.
/6	(d)	Show whether X and Y have equal mean-squared values.
/6	(e)	Show whether X and Y have equal standard deviations.
/7	(f)	Find the correlation coefficient of X and Y.
/2	(g)	What is your conclusion about the correlation between X and Y.
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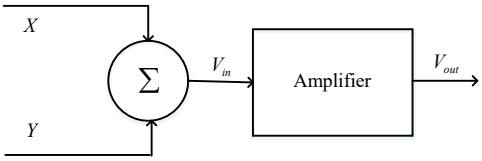
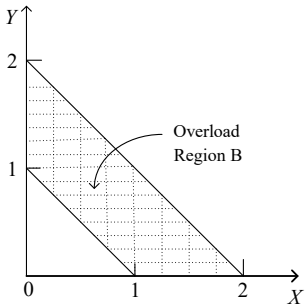
Question 4: Answer the following questions on the blank pages provided (or on loose sheets of paper, Tablets, etc.). If there are calculations involved, **you must show the steps** leading to your answer, otherwise, you will lose some points.

Marks		
/8		Suppose we want to use two voltages X and Y as inputs to investigate a summer amplifier. The amplifier is said to overload if the input values fall inside the overload Region B in the graph below.   What is the probability that the amplifier overloads? Assume the joint PDF is $f_{XY}(x,y)=\frac{1}{2}, \quad 0\le x\le 2,\ 0\le y\le 2,$

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