Problem 1:

An integrated circuit manufacturer has three machines X, Y and Z. A circuit selected from each machine can be either defective (not acceptable) or non-defective (acceptable). Suppose we run an experiment by randomly picking three circuits in sequence from X, Y and Z, respectively. We test each one and record the observation as "a" if acceptable or "f" if it fails. For example, if the circuits from X and Y are acceptable (pass the test) but the circuit from Z is defective (fails the test), then the observation is aaf. Use set operations to answer the following questions:

    (a) What are the elements of the sample space of the experiment?

    (b) What are the elements of the set $A = \{\text{circuit from } Z \text{ fails}\}$

    (c) What are the elements of the set $B = \{\text{circuit from } X \text{ acceptable}\}$

    (d) Are A and B mutually exclusive? Why?

    (e) Are A and B collectively exhaustive? Why?

    (f) What are the elements of the set $D = \{\text{more than one circuit is acceptable}\}$?

    (g) What are the elements of the set $E = \{at\ least\ two\ circuits\ fail\}$?

    (h) Show whether D and E are mutually exclusive?

    (i) Show whether D and E are collectively exhaustive?

Problem 2:

Comparing all the events listed below, which events are equal?

    (a) $A = \{2,3\}$.

    (b) $B = \{0,1,2,3,4,5,6\}$.

    (c) $C = \{x \mid x^2 - 5x + 6 = 0\}$

    (d) $D = \{x \mid x \text{ is the number of heads when six coins are tossed}\}$

Problem 3:

Consider the following sets:

$S = \{0,1,2,3,4,5,6,7,8,9\}$	$A = \{0,2,4,6,8\}$	$B = \{1,3,5,7,9\}$
$C = \{2,3,4,5\}$	$D = \{1,6,7\}$	

List the elements of the sets corresponding to the following events:

(a) $A \cup C$	(b) $A \cap B$	(c) $\overline{C}$
(d) $\left(\overline{C} \cap B\right) \cup D$	(e) $\overline{(S \cap C)}$	(f) $\overline{A} \cap C \cap D$

Problem 4:
Consider two non-disjoint events A and B and sample space S. Sketch Venn diagrams and shade the regions for each of the events resulting from the following operations:

(a) $C = \overline{A \cap B}$	(b) $D = \overline{A \cup B}$	(c) $A \cap \overline{B}$

Problem 5:
From past experience, a stockbroker believes that under present economic conditions a customer will invest in bonds (event B) with a probability of 0.6, will invest in mutual funds (event M) with a probability of 0.3, and invest in both bonds (B) and mutual funds (M) with a probability of 0.15. Note that events are not disjoint or mutually exclusive. Find the probability that a customer will make
  (a) An investment in either bonds or mutual funds or both; (answer: 0.75)
  (b) No investment at all; (answer: 0.25).

Problem 6:
A cell phone manufacturer is concerned about a possible recall of its best-selling phone. If there were a recall, there is a probability 0.25 of a defect in the battery, 0.18 of a defect in the electronics, 0.17 of a defect in the operating system, and 0.40 of a defect in the display. Define events: $A =$ defect in the battery and $B =$ defect in the electronics, $C =$ defect in the operating system and $D =$ defect in display.
  (a) Find the probability that there is defect in the battery or the electronics, or both. The probability of defects in both battery and electronics, simultaneously, is $P[A \cap B] = 0.15$?
  (b) Find the probability that no defects will be involved in any of the two (battery or electronics).
  (c) Find the probability that a battery is defective, and the electronics are not defective? The probability of defects in both battery and electronics, simultaneously, is $P[A \cap B] = 0.15$? Note: $P[A \cap S] = P[A]$