Question 1

Marks	(a)	In a fourth-year graduating class of 100 ECE students, 54 studied Digital Communication Systems, 69 studied Power Systems, while 35 studied both courses. If one of these students is selected at random, find the probability that
8 /8	(i)	The student studied Power Systems or Digital Communication Systems;
4 /4	(ii)	The student did not study Communication Systems and did not study Power Systems;
6 /6	(iii)	The student took Power Systems but not Digital Communications.
18 /18		

Event P — Power , Event D - digital

i) $P[P \cup D] = 0.54 + 0.69 - 0.35 = 0.88$

ii) $P[\bar{P} \cap \bar{D}] = 1 - P[P \cup D] = 0.12$

iii) $P[P \cap \bar{D}] = P[P \cap (S - D)]$

$P[P \cap S] - P[P \cap D]$

$= 0.69 - 0.35 = 0.34$

Question 1:

Marks		
	(a) b	A young boy has a collection of 2 music CDs and 3 sports games CDs in a box in his room. The boy asks his mother to bring 2 CDs. The room is dark so his mother picks 2 CDs in succession, without replacement. Using any method of your choice,
5 /5	(i)	Find the probability that the first CD his mother picks is a music CD and the second is a sports CD.
6 /10	(ii)	Find the probability that the two CDs his mother picks are both either music CDs or they are both sports games CDs .
11 /15		

2 music, 3 games, ⇒ 5 total CDs

i) $\text{Prob} = \dfrac{\binom{2}{1}\binom{3}{1}}{{}_5P_2} = \dfrac{6}{20} = \boxed{0.3}$ ✓

ii) $P = {}_2C_2 + {}_3C_2 = \dfrac{1+3}{{}_5P_2} = \dfrac{4}{20} = \dfrac{1}{5} = \boxed{0.2}$

-4

For question ii), the sample space must eliminate order as a factor, so $n_s = \binom{5}{2} = 10$

$P = \dfrac{{}_2C_2}{{}_5C_2} + \dfrac{{}_3C_2}{{}_5C_2} = \dfrac{1+3}{10} = 0.4$

Alternatively, you can keep the same sample space as i), and reflect that in the numerator ~~denominator~~

$P = \dfrac{{}_2P_2}{{}_5P_2} + \dfrac{{}_3P_2}{{}_5P_2} = \dfrac{2}{20} + \dfrac{6}{20} = \dfrac{8}{20} = \dfrac{4}{10} = 0.4$

Either way, you get the same answer

$\frac{50}{} \times \frac{49}{}$

Question 2:

Marks		
	(a)	A president and a treasurer are to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if
/3	(i)	There are no restrictions;
/4	(ii)	One person (say A) will only serve if he/she is president;
/6	(iii)	Two people (say B and C) will only serve together or not at all.
/13		

Note: **In parts (i), (ii) and (iii) of part (a) above, you need to state your reasonings and/or assumptions.**

i) 2 positions available $\quad 50 \times 49 = 2450$ possibilities $\left(\frac{3}{3}\right)$

order matters ✓ $\quad \frac{\text{President}}{50} \times \frac{\text{treasurer}}{49}$

$1 \times 49 + 49 \times 48 = 49 \times 49$

ii) $\underline{50} \times \underline{48} = 2400 \rightarrow$ order matters $\left(\frac{1}{4}\right)$

iii) $50 \times \underline{47} = 2350 \rightarrow$ order matters $\left(\frac{1}{6}\right)$

$2 + 48 \times 47$

Marks	(b)	Each time a modem transmits one bit, the receiving model analyzes the signal that arrives and decides whether the transmitted bit is a 0 or 1. It makes an error with probability p, independent of whether any other bit is received correctly. If the transmission continues until the receiving modem makes its first error, what is the PMF of X?
/3		

n times modem transmits signal

define error as success $\rightarrow p$

and working as fail $\rightarrow q$

$(1-p)^{x-1} p$

$\boxed{P_X(x) = (p)^{x-1}(1-p)}$

$\rightarrow$ first error

$\left(\frac{0}{3}\right) \rightarrow \frac{3}{3}$ ABS

geometric random variable

$\rightarrow$ many success

Marks		
	(c)	In the table below, you are given the PMF of a random variable X.
/4	(i)	Complete the entries for the CDF, $F_X(x)$ in the table;
/4	(ii)	Evaluate the probability $P[0.5 < X < 2.5]$;
/6	(iii)	Evaluate the probability $P[\{-3.0 \le X < 2.5\} \mid \{X \le 6.0\}]$
/4	(iv)	Evaluate the probability $P[\{-5.0 < X < 2.5\} \cap \{X \le 4\}]$
/18		

i)

$X=x$	−8	−3	−1	0	1	4	6	>6
$p_X(x)$	0.13	0.15	0.17	0.20	0.15	0.11	0.09	0
$F_X(x)$	0.13	0.28	0.45	0.65	0.8	0.91	1	1

$\left(\frac{4}{4}\right)$

ii) $P[0.5 < X < 2.5] = P[1] = \boxed{0.15}$ $\left(\frac{4}{4}\right)$

iii) $P[\underbrace{-3 \le x < 2.5}_{A} \mid \underbrace{X \le 6}_{B}]$

$P[-3] + P[-1] + P[0] + P[1] = 0.15 + 0.17 + 0.2 + 0.15 = \boxed{0.67}$ $\left(\frac{6}{6}\right)$

iii) $P[A \mid B] = \frac{P[A \cap B]}{P[B]} = \frac{0.67}{1} = 0.67$ — 0.67

iv) $P[(-5 < X < 2.5) \cap (X \le 4)] = P[-5 < X < 2.5]$

$= 0.15 + 0.17 + 0.2 + 0.15 = \boxed{0.67}$ ✓ $\left(\frac{4}{4}\right)$

$P[-3] + P[-1] + P[0] + P[1]$

$Q_2 - \left(\frac{23}{34}\right)$

Question 3:

Marks		
		Suppose we conduct an experiment and observe the number, X, of cell phones purchased by customers who randomly enter a phone store. We determine that the PMF of X is given by $\frac{20-5x}{10}+\frac{4-x}{10}$ $p_X(x)=\begin{cases}(4-x)/10, & x=0,1,2,3\\ 0, & \text{otherwise.}\end{cases}$ Find
/8	(i)	The expected value of X,
/6	(ii)	The second moment or mean-square value $E\left[X^2\right]$,
/5	(iii)	The variance of X,
/2	(iv)	The standard deviation of X.
/4	(v)	Suppose the customer service agent is paid a commission, denoted Y (in dollars), that is proportional to the number of phones purchased, that is, $Y=5X+1$. On average, what is the average dollar value of commission the agent would expect.
/3	(vi)	The second moment of mean-square value, that is, $E\left[Y^2\right]$,
/5	~~(vi)~~ (vii)	Find the variance of Y.
/33		

$E(x)=\mu(x) = 0 + (1)(3/10) + (2)\frac{(2)}{10} + \frac{(3)(1)}{10}$

i) $E(x) = \frac{10}{10} = \boxed{1}$ ✓

ii) $E[x^2] = \sum x^2 p(x) = (1)^2(3/10) + 4\left(\frac{2}{10}\right) + \frac{(9)(1)}{10}$

$= \frac{3+8+9}{10} = \frac{20}{10} = \boxed{2}$ ✓

iii) $\sigma_x^2 = E[x^2] - \mu_x^2 = 2 - 1^2 = \boxed{1}$ ✓

iv) $\sigma_X = \sqrt{1} = \boxed{1}$ ✓

v) $E(Y) = 5E(x) + 1 = 5(1)+1 = \boxed{6}$ ✓

$g(x) = 5x+1$

$= (5x+1)(P_X)(x)$

$E(x^2)$

$E(5x) + E(1)$

$E(x$

$\frac{0}{3}$ vi) $E(Y^2) = 5[E(x^2)] + 2 = \boxed{12}$ ✗ -3

$\frac{4}{5}$ Vii) $\sigma_Y^2 = 12 - 6^2 = \boxed{-24}$

ok given prev's values, but shouldn't be neg.

$Y = 5x+1$

$Y^2 = 25x^2 + 10x + 1$ $\rightarrow E[Y^2] = 25E[x^2] + 10E[x] + 1$

-4

$\frac{29}{33}$