Problem 1

The probability that a married man watches a certain television show is 0.4. The probability that his wife watches the same show is 0.5. The probability that the man watches the show, given that his wife does, is 0.7. Find the probability that

(a) The couple watches the show together.
(b) The wife watches the show, given that her husband does.
(c) At least one member of the married couple will watch the show.

Answers: From the problem statement, we have the following:

H: husband watches show, $P[H] = 0.4$, W: wife watches show, $P[W] = 0.5$

$P[H \mid W] = 0.7$

(a) $P[H \cap W] = 0.35$

(b) $P[W \mid H] = 0.875$ or $P[W \mid H] = \dfrac{P[H \mid W]P[W]}{P[H]} = 0.875$

(c) $P[H \cup W] = 0.55$

Problem 2

A government agency employs three consulting companies (A, B, and C) with probabilities 0.4, 0.35, and 0.25, respectively. The agency checks for cost overruns. From past experience, it is known that the probability of cost overruns given that the work was done by A is 0.05, by B it is 0.03 and by C, it is 0.15. Suppose the agency experienced cost overrun. Define the event $O = \{\text{cost overrun}\}$.

(a) What are the values of the following probabilities $P[A]$, $P[B]$, $P[C]$, $P[O \mid A]$, $P[O \mid B]$ and $P[O \mid C]$?
(b) Show that if there is cost overrun, the probability that it is consulting company C is 0.5515.
(c) Show that if there is cost overrun, the probability that it is consulting company A is 0.2941

Answers: Define events

O: cost overrun, A: Consulting firm A, B: Consulting firm B, C: Consulting firm C

(a) From the statements, we are given:

$P[A] = 0.4,\ P[B] = 0.35,\ P[C] = 0.25,\ P[O \mid A] = 0.05,\ P[O \mid B] = 0.03,\ P[O \mid C] = 0.15$

(b) The probability we want is $P[C|O] = \dfrac{P[O|C]P[C]}{P[O]}$, that is, given there is cost overrun, what is the probability of C. The probability of cost overrun (using total probability) is

$P[O] = 0.068$

$P[C|O] = 0.5515$

(c) In this case, we want the probability
$P[A|O] = 0.2941$

Problem 3:
The probability that an automobile needs an oil change is 0.25; the probability that it needs a new oil filter is 0.4; and the probability that both oil and the filter need changing is 0.14.

(a) If the oil has to be changed, what is the probability that a new oil filter is needed?
(b) If a new oil filter is needed, what is the probability that the oil has to be changed?

Answers: From problem statement, we have the following:

C: an oil change is needed, $P[C] = 0.25$, F: an oil filter is needed, $P[F] = 0.4$

$P[F \cap C] = 0.14$

(a) $P[F|C] = 0.56$

(b) $P[C|F] = 0.35$