

Chapter 5: Properties of Random Samples

November 14, 2021

Exercise 5.1

The samples are drawn from Bernoulli trial with success rate 0.01. The probability of n samples not containing color-blind is 0.99^n . We want to find N such that for $n \geq N$, $0.99^n \leq 1 - 0.95$. $N \approx 299$.

Exercise 5.2

(a) Let T be the number of years until the first year's rainfall is exceeded. Then

$$\begin{aligned} P(T = k) &= P(X_2 \leq X_1, \dots, X_{k-1} \leq X_1, X_k > X_1) \\ &= \int_x P(X_2 \leq x, \dots, X_{k-1} \leq x, X_k > x | X_1 = x) f(x) dx \\ &= \int_x P(X_k > x) f(x) \prod_{i=2}^{k-1} P(X_i \leq x) dx \\ &= \int_x (1 - F(x)) f(x) F(x)^{k-1} dx \\ &= \int_x F(x)^{k-1} f(x) dx - \int_x F(x)^k f(x) dx \\ &= \frac{1}{k} F(x)^k \Big|_{-\infty}^{\infty} - \frac{1}{k+1} F(x)^{k+1} \Big|_{-\infty}^{\infty} \\ &= \frac{1}{k} - \frac{1}{k+1} \\ &= \frac{1}{k(k+1)} \end{aligned}$$

(b)

$$\mathbb{E}T = \sum_k k P(T = k) = \sum_k \frac{1}{k+1} = \infty$$