Chapter 6: Principles of Data Reduction

November 29, 2021

Exercise 6.1

Yes

$$\frac{p(x|0,\sigma)}{q(|X||0,\sigma)} = \frac{p(x|0,\sigma)}{p(x|0,\sigma)+p(-x|0,\sigma)} = \frac{1}{2}$$

Does not depend on the paramters.

Exercise 6.2

The pdf for X_i is $f_{X_i}(x|\theta) = \exp(i\theta - x)\mathbb{1}_{x\geq i\theta}$. Then

$$\begin{aligned}
f_X(x_i|\theta) &= \prod_{i=1}^{n} f_{X_i}(x_i|\theta) \\
&= \exp\left(\sum_{i=1}^{n} i\theta - x_i\right)\prod_{i=1}^{n}\mathbb{1}_{x_i\geq i\theta} \\
&= \exp\left(\frac{n(n+1)\theta}{2}\right)\exp\left(\sum_i x_i\right)\prod_{i=1}^{n}\mathbb{1}_{\frac{x_i}{i}\geq\theta} \\
&= \exp\left(\frac{n(n+1)\theta}{2}\right)\mathbb{1}_{\min\frac{x_i}{i}\geq\theta}\exp\left(\sum_i x_i\right)
\end{aligned}$$

$g(T(x)|\theta) = g(\min\frac{x_i}{i}|\theta) = \exp\left(\frac{n(n+1)\theta}{2}\right)\mathbb{1}_{\min\frac{x_i}{i}\geq\theta}$ and $h(x) = \exp\left(\sum_i x_i\right)$. By factorization theorem, it is sufficient statistic.