Chapter 3: Euclidean Geometry

Ran Xie

February 20, 2022

1 Isometries of $\Re^3$

1.1

Consider

$$\begin{aligned}|C(p+a)-C(p)-C(a)|^2 &= C(p+a)\cdot C(p+a)+C(p)\cdot C(p)+C(a)\cdot C(a)\\ &\quad -2C(p+a)\cdot C(p)-2C(p+a)\cdot C(a)+2C(p)\cdot C(a)\\ &= (p+a)^2+p^2+a^2-2(p+a)p-2(p+a)a+2pa\\ &= p^2+2pa+a^2+p^2+a^2-2p^2-2pa-2pa-2a^2+2pa\\ &= 0\end{aligned}$$

Therefore $C(p+a)=C(p)+C(a)$. It follows that $CT_a(p)=C(p+a)=C(p)+C(a)=T_{C(a)}C(p)$ ■