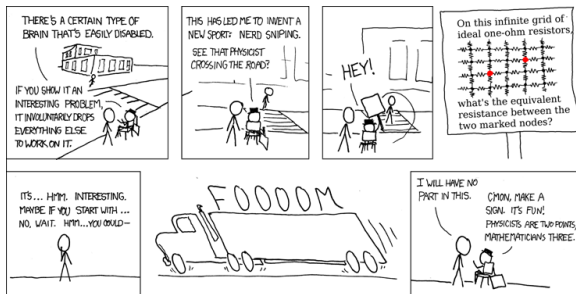


Solving the Nerd-Sniping Problem

Richard Xu

Harvard University

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Title text: I first saw this problem on the Google Labs Aptitude Test. A professor and I filled a blackboard without getting anywhere. Have fun.

What Makes a Nerd-Snipe?

“Easy to understand, hard to master”. As a result, it captivates our thoughts.

Other examples (promise me you won't think about them during the talk):

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- 1 How can you cut a circle into congruent pieces such that not every piece contains the origin?
- 2 Start with a positive integer, divide by 2 if it's even, and otherwise multiply by 3 and add 1. If you repeat this, will you always end up at 1?

Roadmap

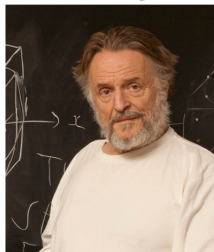
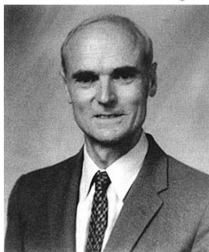
- 1 History of nerd-sniping.
- 2 Voltage, resistance.
- 3 Characteristic polynomials, and solving the resistance problem.
- 4 Final thoughts on nerd-sniping.

Nerd Sniping, First Mention

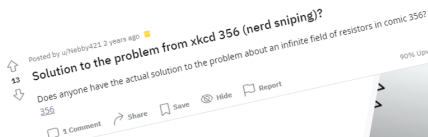


Nerd Sniping, Origin

Coxeter came to Cambridge and gave a lecture. Then he had this problem for which he gave proofs for selected examples, and he asked for a unified proof. I left the lecture room thinking. As I was walking through Cambridge, suddenly the idea hit me, but it hit me while I was in the middle of the road. When the idea hit me I stopped and a large truck ran into me and bruised me considerably and the man considerably swore at me. So I pretended that Coxeter had calculated the difficulty of this problem so precisely that he knew that I would get the solution just in the middle of the road. In fact I limped back after the accident to the meeting. Coxeter



Nerd Sniping, Attempts

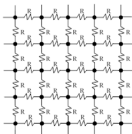


Infinite Grid of Resistors

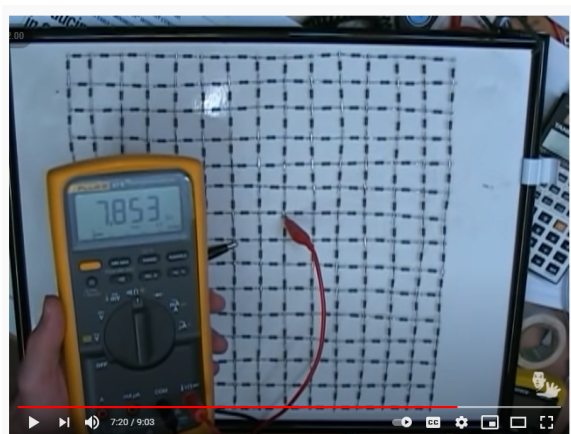
*Remain, remain thou here,
While sense can keep it on, and, sweetest, farewell,
As I my poor self did exchange for you,
To year so infinite loss, so in our trifles
I still win of you: for my sake wear this...*

Shakespeare

There is a well-known puzzle based on the premise of an "infinite" grid of resistors connecting adjacent nodes of a square lattice. A small portion of such a grid is illustrated below.



Nerd Sniping, Attempts



EEVblog #25 - The Infinite Resistor Puzzle

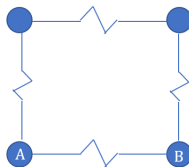
Setup: Electric Potential and Current

- 1 Start by defining a function V on all points in the space.
- 2 Ohm's Law: $V = IR$, $I_{u,v} = \frac{1}{R}(V_u - V_v)$.
- 3 Kirchoff's Law: sum of currents at a point is 0.
- 4 When all resistance are the same: if P is the averaging operator, then $PV = V$, $(P - id)V = 0$.

How to find Equivalent Resistance?

Suppose I add require 1 current to enter A and leave B . What is the potential difference between A and B ?

Exercises:



1-dimensional case

- ① $2V(0) - V(1) - V(-1) = 2$. Set $V(0) = 0$ and add symmetry to require that $V(1) = V(-1)$.
- ② We have $2V(n) = V(n+1) + V(n-1)$ for $n \neq 0$, and the characteristic polynomial is $2 = \mu + \frac{1}{\mu}$.

2-dimensional case

- 1 The characteristic polynomial becomes $4 = \mu + \frac{1}{\mu} + \nu + \frac{1}{\nu}$. Infinite solutions!
- 2 General solution given by linear combinations of the function $\mu^x \nu^y$.
- 3 Transform with $\alpha = \log(\mu)/i, \beta = \log(\nu)/i$. Then,
 $V(x, y) = \int_{-\pi}^{\pi} C(\alpha, \beta) e^{i(x\alpha + y\beta)} d\alpha$ and $2 - \cos(\alpha) - \cos(\beta) = 0$.
- 4 Use symmetry and conditions to get $C = \frac{1}{4\pi i \sin(\beta)}$.
- 5 Plug in to solve for $V(2, 1) = 1/4 - 2/\pi$.
- 6 Conclusion: $R = 4/\pi - 1/2$.

Concluding thoughts

Why try nerd-sniping problems?