Model Uncertainty in the Cross Section

Jiantao Huang Ran Shi

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Motivation I

- Uncertainty in the macroeconomy and asset markets
  - Uncertainty shocks as jumps in the VXO/VIX index: Bloom (2009)
  - Macroeconomic and financial uncertainty: Jurado, Ludvigson, and Ng (2015); Ludvigson, Ma, and Ng (2015)
  - Policy uncertainty: Baker, Bloom, and Davis (2016)
- Observation: time-varying and countercyclical, related to real activities
- Uncertainty about variables v.s. Uncertainty about models?

Motivation II

- "Now we have a zoo of new factors." (Cochrane, AFA presidential address)
- New econometric approaches to uncovering the "true" linear factor model (e.g., Feng, Giglio and Xiu, 2019; Kozak, Nagel and Santosh, 2019)
- Or those models?
  - In the context of linear stochastic discount factor (SDF) models
  - What's the degree of model uncertainty?

Motivation III

- The appeal of smart beta (factor investing) is under attack

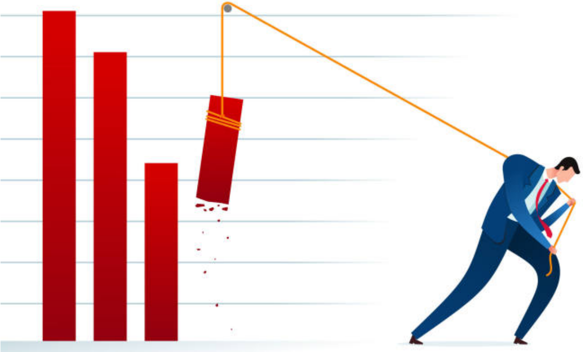


Figure 1: Declining returns on smart beta funds (source: **Financial Times**)

Motivation III

- The appeal of smart beta (factor investing) is under attack
- From **Financial Times**, *“nearly a third of investors surveyed by Invesco this year said their factor investments had underperformed, compared with just 10 per cent a year previously"*
- Linear SDF model uncertainty: a measure to quantify the uncertainty in factor investing

This Paper

- A framework to quantify uncertainty among the linear SDF models:
  - A transparent and easy-to-implement Bayesian econometric framework to compute posterior model probabilities
  - An entropy-based model uncertainty definition
  - Key observation:
    - ★ Countercyclical model uncertainty before 2008
    - ★ Persistently high model uncertainty in the past ten years
    - ★ Closely related to investors' sentiments
    - ★ Predict future mutual fund flows

Outline

Econometric Framework

The Linear SDF Model

- N test assets, $\boldsymbol{R}^\top = (R_1 \dots R_N)$, all excess returns
- p factors, $\boldsymbol{f}^\top = (f_1 \dots f_p)$, all tradable long-short portfolios
  - Hansen and Jagannathan (1991)
  - Factors possibly reflecting unobservable economic risks, e.g. long-run risk
- Factors are also included into test assets
- A linear SDF model

$$m = 1 - (\boldsymbol{f} - \mathbb{E}[\boldsymbol{f}])^\top \boldsymbol{b}, \tag{1}$$

  - Since $\boldsymbol{R}$ concatenates excess returns, $\mathbb{E}[\boldsymbol{R} \times m] = \boldsymbol{0}$, that is,

$$\mathbb{E}[\boldsymbol{R}] = \mathrm{Cov}[\boldsymbol{R}, \boldsymbol{f}]\boldsymbol{b} \tag{2}$$

Model Uncertainty

- p candidate factors $\Longrightarrow$ 2^p models for the linear SDF
- Introducing model $\mathcal{M}_\gamma$:

$$m_\gamma = 1 - \left(\boldsymbol{f}_\gamma - \mathbb{E}[\boldsymbol{f}_\gamma]\right)^\top \boldsymbol{b}_\gamma$$

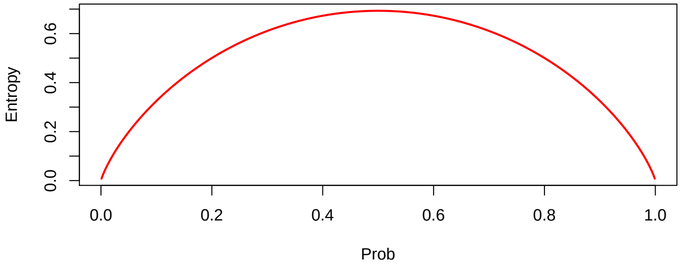
- $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_p]^\top$
  - $\gamma_j = 0$: the jth factor discarded
  - $\gamma_j = 1$: the jth factor included
  - $\boldsymbol{p}_\gamma = \sum_{j=1}^{p} I[\gamma_j = 1]$: the dimension of $\mathcal{M}_\gamma$
  - $\boldsymbol{f}_\gamma$: a p_γ-dimensional vector that contains all the included factors (the same for all γ-subscription of a vector)

Quantifying Model Uncertainty

- Measuring model uncertainty: the entropy of posterior model distribution

$$\mathcal{E}[\mathcal{M}_\gamma \mid \mathcal{D}] = -\sum_\gamma \log(p[\mathcal{M}_\gamma \mid \mathcal{D}])p[\mathcal{M}_\gamma \mid \mathcal{D}],$$

- $\mathcal{E}[\mathcal{M}_\gamma \mid \mathcal{D}] \in [0, \log(\#\,\text{models})]$



Bayesian Inference: Distributional Assumption

- Observed excess return data $\mathcal{D} = \{\boldsymbol{R}_t\}_{t=1}^{T}$ (including the factors)
- Distributional assumptions:

$$\boldsymbol{R}_1, \ldots, \boldsymbol{R}_T \overset{\text{iid}}{\sim} \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}). \tag{3}$$

- Model $\mathcal{M}_\gamma$: a restriction on this distribution through a moment condition:

$$\boldsymbol{\mu} = \mathbb{E}[\boldsymbol{R}] = \mathrm{Cov}[\boldsymbol{R}, \boldsymbol{f}_\gamma]\boldsymbol{b}_\gamma \triangleq \boldsymbol{C}_\gamma \boldsymbol{b}_\gamma, \tag{4}$$

- $\boldsymbol{C}_\gamma = \mathrm{Cov}[\boldsymbol{R}, \boldsymbol{f}_\gamma]$: a subset of column vectors in $\boldsymbol{\Sigma}$
- Posterior probability of a model from a Bayesian investors' perspective:

$$p\left[\mathcal{M}_\gamma \mid \mathcal{D}\right] \propto p\left[\mathcal{D} \mid \mathcal{M}_\gamma\right] \pi[\mathcal{M}_\gamma]$$

Bayesian Inference: Prior Specification

A fully Bayesian paradigm:

$$
\begin{aligned}
p\left[\mathcal{M}_\gamma \mid \mathcal{D}\right] &\propto p\left[\mathcal{D} \mid \mathcal{M}_\gamma\right] \pi[\mathcal{M}_\gamma] \\
&= \left(\int p\left[\mathcal{D} \mid \boldsymbol{b}_\gamma, \mathcal{M}_\gamma\right] p[\boldsymbol{b}_\gamma \mid \mathcal{M}_\gamma] \, \mathrm{d}\boldsymbol{b}_\gamma \right) \pi[\mathcal{M}_\gamma]
\end{aligned}
$$

- $p\left[\mathcal{D} \mid \boldsymbol{b}_\gamma, \mathcal{M}_\gamma\right]$ is the likelihood function
- $p[\boldsymbol{b}_\gamma \mid \mathcal{M}_\gamma]$:
  - A prior specification for $\boldsymbol{b}_\gamma$ under model $\mathcal{M}_\gamma$ is needed
  - Our approach: a generalized version of Arnold Zellner's g-prior
  - "Today's posterior is tomorrow's prior" (Lindley, 2000)

Bayesian Inference: Prior Specification (Cont.)

- A "conceptual" sample of size $T' = T/g$, denoted by $\mathcal{D}' = \{\boldsymbol{R}'_t\}_{t=1}^{T'}$
- Under model $\mathcal{M}_\gamma$: $\boldsymbol{R}'_1, \ldots, \boldsymbol{R}'_{T'} \overset{\text{iid}}{\sim} \mathcal{N}(\boldsymbol{C}_\gamma \boldsymbol{b}_\gamma, \boldsymbol{\Sigma})$
- A flat prior for $\boldsymbol{b}_\gamma$ that is fully uninformative
- Posterior of $\boldsymbol{b}_\gamma$ after "observing" $\mathcal{D}'$ under model $\mathcal{M}_\gamma$:

$$\left[\boldsymbol{b}_\gamma \mid \mathcal{M}_\gamma, \mathcal{D}'\right] \sim \mathcal{N}\left(\boldsymbol{b}_\gamma^a, \frac{g}{T}\left(\boldsymbol{C}_\gamma^\top \boldsymbol{\Sigma}^{-1} \boldsymbol{C}_\gamma\right)^{-1}\right)$$

- $\boldsymbol{b}_\gamma^a$ reflects our anticipatory value for $\boldsymbol{b}_\gamma$
- In the language of hypothesis testing: we must be interested in

$$H_0 : \boldsymbol{b}_\gamma = \boldsymbol{b}_\gamma^a \quad v.s. \quad H_1 : \boldsymbol{b}_\gamma \in \mathbb{R}^{p_\gamma}$$

- Clearly, we are mostly interested in whether $\boldsymbol{b}_\gamma = \boldsymbol{0}$ or not $\implies \boldsymbol{b}_\gamma^a = \boldsymbol{0}$

Bayesian Inference: Prior Specification (Cont.)

- "Today's posterior is tomorrow's prior"
- A (generalized) g-prior for $\boldsymbol{b}_\gamma$ under model $\mathcal{M}_\gamma$:

$$\boldsymbol{b}_\gamma \mid \mathcal{M}_\gamma \sim \mathcal{N}\left(\mathbf{0}, \frac{g}{T}\left(\boldsymbol{C}_\gamma^\top \boldsymbol{\Sigma}^{-1} \boldsymbol{C}_\gamma\right)^{-1}\right), \quad g > 0$$

- When $g \to \infty$, a flat prior
- Prior property: under the g-prior specification:

$$\mathrm{Var}[m_\gamma \mid g] = \frac{g p_\gamma}{T}$$

  - Higher model dimension: larger achievable Shape ratios in the economy
  - Models with too many factors are not likely to be realistic *a priori* (there may exist deals that are too good to be true, Cochrane and Saa-Requejo (2000))

Bayesian Inference: Posterior Calculation

Under the g-prior specification:

$$p[\mathcal{D} \mid \mathcal{M}_\gamma, g] = p[\mathcal{D} \mid \mathcal{M}_\mathbf{0}] \exp\left\{\frac{g}{2(1+g)} T\mathrm{SR}_\gamma^2 - \frac{\log(1+g)}{2} p_\gamma\right\}$$

- SR_γ: the maximal in-sample Sharpe ratio achievable from combining $\boldsymbol{f}_\gamma$
- $\mathcal{M}_\mathbf{0}$: the null model in which no factors are included, SDF being constant: a risk-neutral world
- $p[\mathcal{D} \mid \mathcal{M}_\mathbf{0}] \propto \exp\left\{-\frac{T}{2}\mathrm{SR}_{\max}^2\right\}$
    - $\mathrm{SR}_{\max}$: the maximal in-sample Sharpe ratio achievable using $\boldsymbol{R}$
    - Does not depend on g

Implications of the Posterior

The Bayes factor against the null model:

$$\mathrm{BF}_\gamma(g) = \frac{p[\mathcal{D} \mid \mathcal{M}_\gamma, g]}{p[\mathcal{D} \mid \mathcal{M}_\mathbf{0}]} = \exp\left\{\frac{g}{2(1+g)} TSR_\gamma^2 - \frac{\log(1+g)}{2} p_\gamma\right\}$$

- A tool for model comparison
- The Bayes factor for comparing any two models:

$$\mathrm{BF}_{\gamma,\gamma'}(g) = \frac{\mathrm{BF}_\gamma(g)}{\mathrm{BF}_{\gamma'}(g)}$$

  - Model comparison results do not depend on the choice of test assets
  - p_γ the same: models with larger SR_γ is more favorable (the GRS spanning tests)
  - The penalty on p_γ is necessary: adding assets to existing model $\implies$ $\mathrm{SR}_\gamma \uparrow$

Implications of the Posterior (Cont.)

To choose a model that includes one new factor, it must be

$$T(\Delta \mathrm{SR}^2) > \frac{1+g}{g} \log(1+g)$$

- The right-hand side is monotonically increasing w.r.t. g and unbounded
- Large g favors small models
- $g \to \infty$ (a flat prior for $\boldsymbol{b}_\gamma$), $\mathrm{BF}_\gamma(g) \to 0$
  - Flat priors always favor the null model (Bartlett's paradox)
  - Our g-prior might be "the subtle art of not giving a" flat prior
  - ...while maintaining 100% transparency
- But what about the choice of g?

A Prior for g

- Prior for g

$$\pi[g] = \frac{a-2}{2}(1+g)^{-\frac{a}{2}}, \quad g > 0.$$

- $\mathbb{E}[g] = \infty$ if $a \leq 4$, and that $\mathbb{E}[g] = 2/(a-4)$ if $a > 4$
- Unconditional volatility of the SDF under model $\mathcal{M}_\gamma$

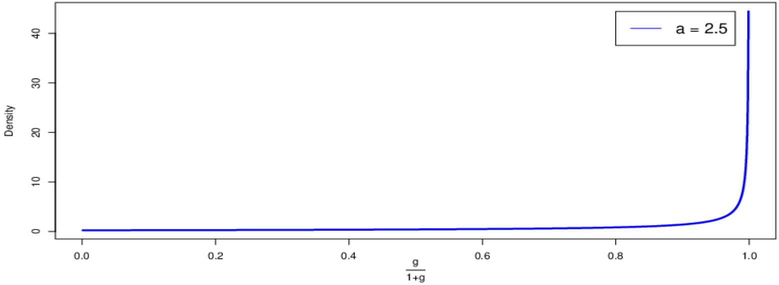
$$\mathrm{SR}^2_\infty = \mathrm{Var}[m_\gamma] \geq \mathbb{E}[\mathrm{Var}[m_\gamma \mid g]] = \frac{p_\gamma}{T}\mathbb{E}[g] = \frac{2p_\gamma}{(a-4)T}$$

- This results in

$$a \geq 4 + \frac{2}{T\mathrm{SR}^2_\infty}$$

A Prior for g

- $\mathbb{E}[\boldsymbol{\mu} \mid \mathcal{M}_\gamma, g, \mathcal{D}] = \frac{g}{1+g} \boldsymbol{C}_\gamma \left\{ \mathrm{Var}[\boldsymbol{f}_\gamma] \right\}^{-1} \left(\frac{1}{T} \sum_{t=1}^{T} \boldsymbol{f}_{\gamma,t} \right)$
- $\frac{g}{1+g}$ is the shrinkage operator, with $\frac{g}{1+g} \sim \mathrm{Beta}\left(1, \frac{a}{2} - 1\right)$



A Prior for g

- $\mathbb{E}[\boldsymbol{\mu} \mid \mathcal{M}_\gamma, g, \mathcal{D}] = \frac{g}{1+g} \boldsymbol{C}_\gamma \left\{ \mathrm{Var}[\boldsymbol{f}_\gamma] \right\}^{-1} \left(\frac{1}{T} \sum_{t=1}^{T} \boldsymbol{f}_{\gamma,t} \right)$
- $\frac{g}{1+g}$ is the shrinkage operator, and $\frac{g}{1+g} \sim \mathrm{Beta}\left(1, \frac{a}{2} - 1\right)$
- The ratio $g/(1+g)$ controls the relative importance of the information from the data v.s. the prior, the expectation of which is $2/a$; use the smallest possible a to lend more credits to the data
- Final choice of a:

$$a = 4 + \frac{2}{TSR_{\infty}^{2}} \simeq 4$$

Posterior Model Probabilities

- Integrating out g:

$$\mathrm{BF}_\gamma = \int_{g>0} \mathrm{BF}_\gamma(g)\,\mathrm{d}\pi[g]$$

$\implies$ Bayes factor is increasing in SR_γ but decreasing in p_γ

- Posterior probability of model $\mathcal{M}_\gamma$ is

$$p[\mathcal{M}_\gamma \mid \mathcal{D}] = \frac{\mathrm{BF}_\gamma \pi[\mathcal{M}_\gamma]}{\sum_\gamma \mathrm{BF}_\gamma \pi[\mathcal{M}_\gamma]}$$

- Prior model probability $\pi[\mathcal{M}_\gamma]$ can
  - Be the same across all 2^p models
  - Force some factors to coexist or be mutually exclusive

Time-Series of Model Uncertainty

US Evidence

Table 1: List of Factors

Source	Description	Reference
Ken French's Library	Fama-French 5 Factors	Fama and French (2016)
Macro Finance Society	new size, profitability & investment factors	Hou, Xue and Zhang (2015)
AQR Library	betting-against-beta (BAB)	Frazzini and Pedersen (2014)
AQR Library	Quality Minus Junk (QMJ)	Asness, Frazzini and Pedersen (2014)
AQR Library	HML Devil	Asness and Frazzini (2013)
Ken French's Library	Momentum	Carhart (1997)
Ken French's Library	Short-Term Reversal	Jegadeesh and Titman (1993)
Ken French's Library	Long-Term Reversal	Bondt and Thaler (1985)

US Evidence

- Consider models that contain at most one version of the factors in each of the following categories: (1) size (SMB or ME); (2) profitability (RMW or ROE); (3) value (HML or HML Devil); (4) investment (CMA or IA); $\Longrightarrow$ 5,184 models
- $\textit{Upper bound} \approx 8.55$
- Construct the time-series of model uncertainty using daily factor returns in 3-year rolling window
- Sample of daily factors starts from Jan 1967 $\Longrightarrow$ model uncertainty series begins from Dec 1969

Model Uncertainty in US Stock Markets before 2008

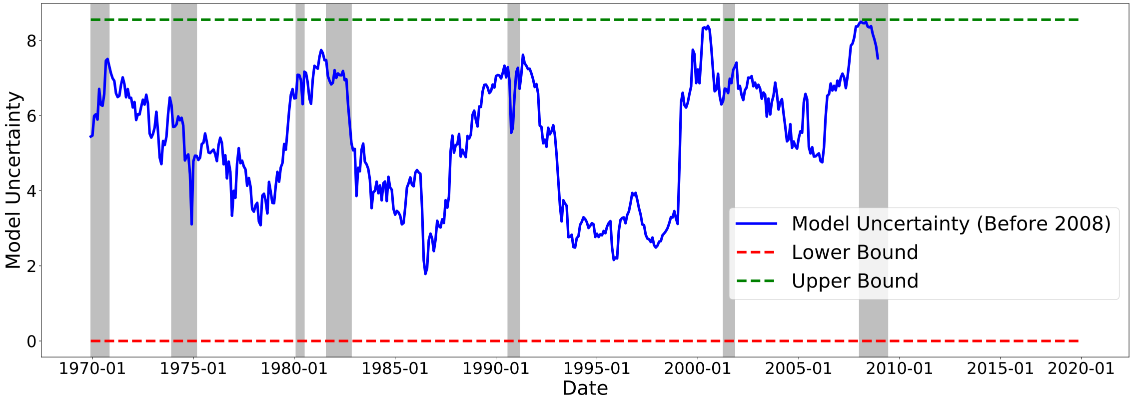


Figure 2: Model Uncertainty in US Markets (3-year rolling window)

Model Uncertainty in US Stock Markets after 2008

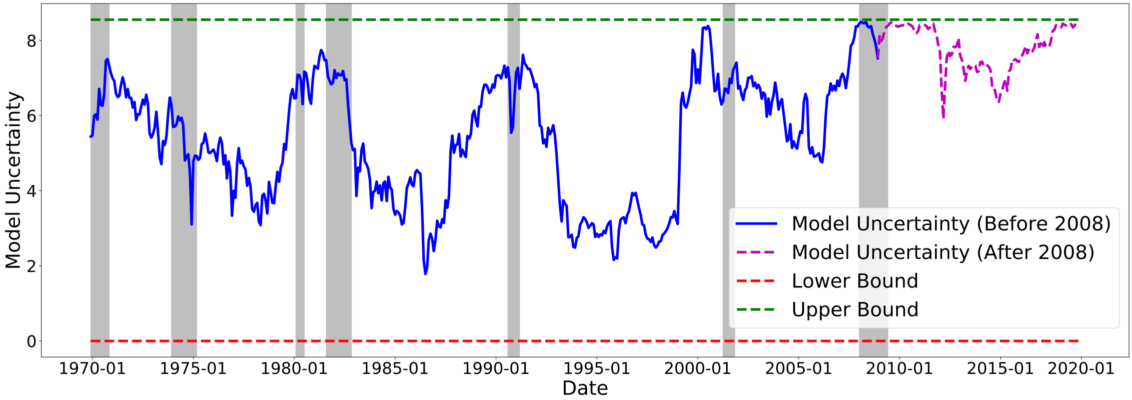


Figure 3: Model Uncertainty in US Markets (3-year rolling window)

International Evidence

- European and Asian Pacific stock markets;
- 9 factors: (1) Fama-French 5 Factors; (2) Momentumn; (3) BAB, QMJ and HML Devil from AQR (due to data availability)
- Only HML or HML devil can be selected
- 384 candidate models $\Longrightarrow$ $\textit{Upper bound} \approx 5.95$
- Sample of daily factors starts from July 1993 $\Longrightarrow$ model uncertainty series begins from June 1996

Model Uncertainty in European Stock Markets

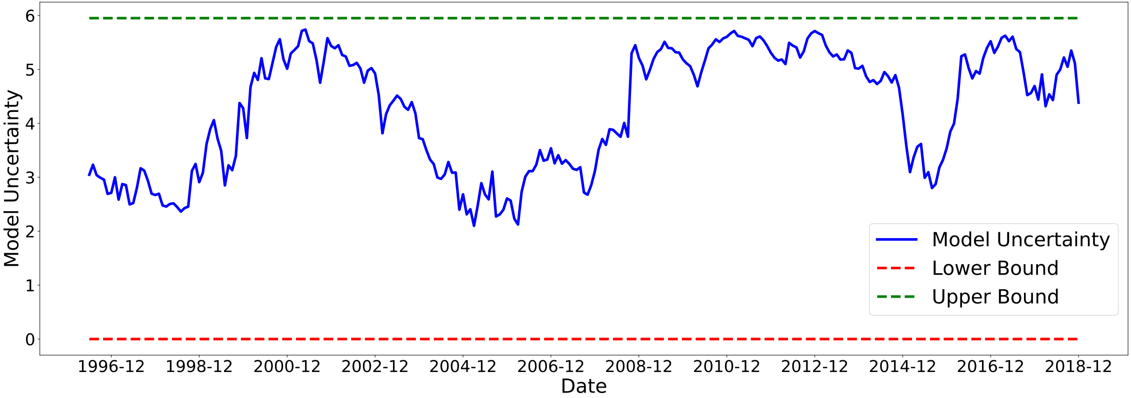


Figure 4: Model Uncertainty in European Markets (3-year rolling window)

Model Uncertainty in Asian Pacific Stock Markets

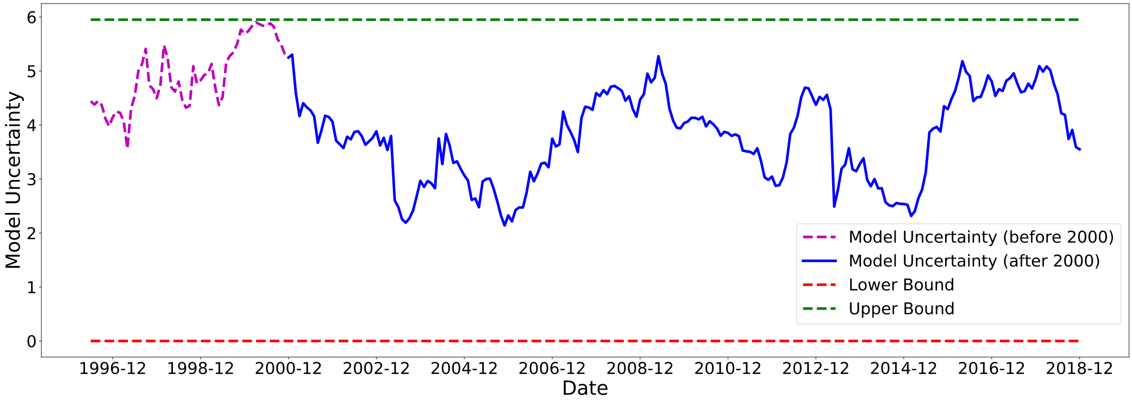


Figure 5: Model Uncertainty in Asian Pacific Markets (3-year rolling window)

Decomposing Model Uncertainty

- Model uncertainty quantifies dispersion in maximal in-sample Sharpe ratio, SR_γ, after imposing sensible penalty for dimensionality
- $SR_\gamma^2 = \mathbb{E}_T[\boldsymbol{f}_\gamma]^\top \boldsymbol{V}_\gamma^{-\mathbf{1}} \mathbb{E}_T[\boldsymbol{f}_\gamma] \implies$ three main determinants of SR_γ^2: (a) average daily returns of factors, (b) average daily factor volatility, (c) average pairwise correlation among daily factor returns

Average Daily Factor Return

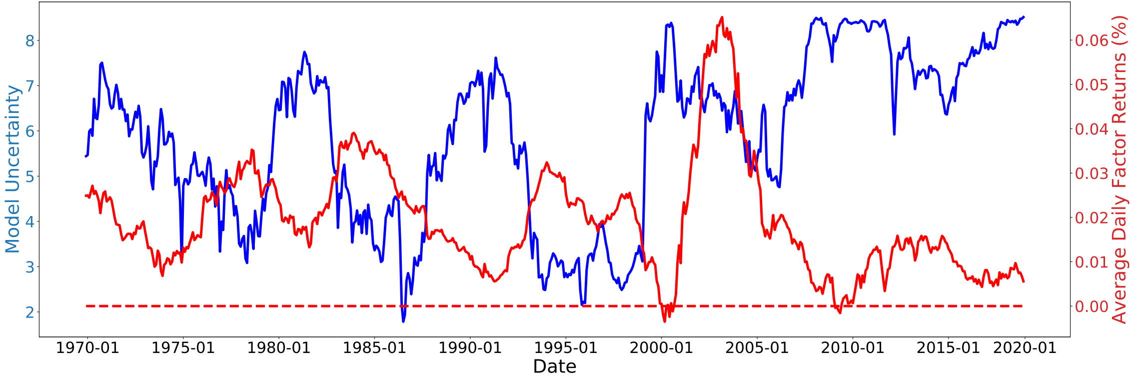


Figure 6: Average Daily Return of 14 Factors (3-year rolling window)

Average Daily Factor Volatility

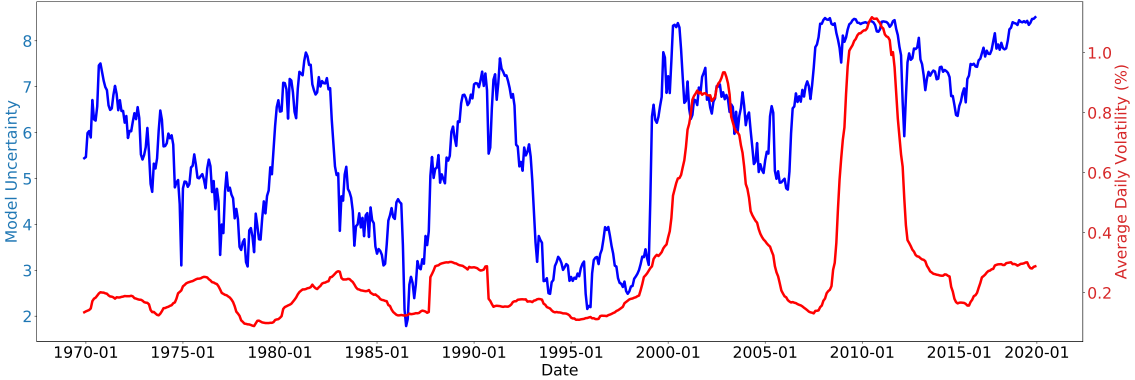


Figure 7: Average Daily Volatility of 14 Factors (3-year rolling window)

Average Pairwise (Absolute) Correlation

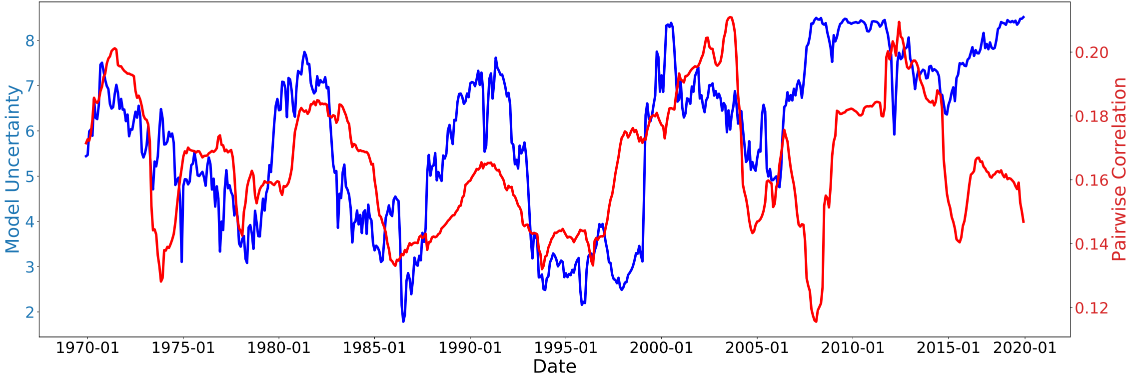


Figure 8: Average Pairwise Correlation of 14 Factors (3-year rolling window)

Investors' Sentiments

Model Uncertainty and Investors' Sentiments

- Survey conducted by American Association of Individual Investors (AAII)
  - completed weekly by registered members of AAII
  - % of bearish, neutral or bullish on the stock market in the next six months
- Shiller's stock market confidence indices
  - one-year confidence index: % of individual or institutional investors expecting an increase in the Dow in a year
  - crash confidence index: % of individual or institutional investors who think that the probability of a catastrophic stock market crash in the next six months is lower than 10%

Regression Analysis

- Time-series regression is

$$Sentiment_{t+1} = \beta_0 + \gamma Entropy_t + \psi X_t + \epsilon_{t+1}, \tag{5}$$

  where $Sentiment_{t+1}$ is the one-period ahead sentiment index, $Entropy_t$ is the model uncertainty measure in period t, and X_t includes other control variables up to time t, such as lagged sentiment indices, VIX and etc

- p-values: 36-lag Newey-West standard errors

Table 2: AAII Sentiment Index

	Bullish	Neutral		Bearish	
	(1)	(2)	(3)	(4)	(5)
$Entropy_t$	-0.0020	-0.0037**	-0.0034**	0.0058***	0.0056***
	(-0.57)	(-2.10)	(-2.05)	(2.73)	(2.65)
VIX_t		0.0004		-0.0001	
		(0.49)		(-0.13)	
$Sentiment_t$		0.4782***	0.4731***	0.3522***	0.3302***
		(7.84)	(8.05)	(8.51)	(6.82)
$Sentiment_{t-1}$		0.2144***	0.2154***	0.1859***	0.1973***
		(5.17)	(5.28)	(5.62)	(5.16)
Control for past 6-month returns	NO	NO	YES	NO	YES
N	379	377	377	377	377
R-Squared	0.0018	0.4402	0.4618	0.2749	0.2793

Table 3: Shiller's Confidence Index

	1-Year Confidence Index - Institution		1-Year Confidence Index - Individual		Crash Confidence Index - Institution		Crash Confidence Index - Individual	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Entropy_t$	-0.3836***	-0.2971***	-0.4522***	-0.3169***	-0.6762***	-0.7350***	-0.8117***	-0.8720***
	(-3.17)	(-2.61)	(-2.87)	(-2.76)	(-4.42)	(-5.20)	(-4.22)	(-4.94)
VIX_t	0.0313**		0.0291		-0.0978***		-0.0621***	
	(2.34)		(1.21)		(-3.30)		(-3.24)	
$Sentiment_t$	1.1528***	1.1636***	0.9788***	0.9891***	1.0744***	1.0846***	1.0178***	0.9409***
	(16.87)	(17.51)	(13.19)	(12.10)	(15.89)	(18.56)	(14.44)	(16.18)
$Sentiment_{t-1}$	-0.3004***	-0.2960***	-0.0530	-0.0480	-0.2567***	-0.2384***	-0.2006***	-0.1150***
	(-3.81)	(-3.62)	(-0.70)	(-0.60)	(-3.90)	(-3.73)	(-3.35)	(-2.94)
Control for past 6-month returns	NO	YES	NO	YES	NO	YES	NO	YES
N	208	208	208	208	208	208	208	208
R-Squared	0.8367	0.8389	0.9372	0.9383	0.8989	0.9017	0.9214	0.9278

Mutual Fund Flows

Model Uncertainty and Mutual Fund Flows

- CRSP survivor-bias-free US mutual fund database
- Investment Objective Codes: (a) all domestic equity mutual funds; (b) all domestic fixed income funds; (c) domestic “style", "sector", "large-cap" and “middle and small-cap" equity funds; (d) domestic government bond funds
- Calculate the net fund flows of each fund i in period t as

$$Flow_{i,t} = TNA_{i,t} - TNA_{i,t-1} \times (1 + RET_{i,t}) \tag{6}$$

- Aggregate the individual fund flows in each period across all funds in a group (e.g. all large-cap funds) and scale it by total market capitalization of all stocks in CRSP, or US GDP

Table 4: Regression of Mutual Fund Flows on Model Uncertainty

	Domestic Equity			Domestic Fixed-Income	
	(1)	(2)	(3)	(4)	(5)
$Entropy_t$	-0.0119***	-0.0121***	-0.0118***	0.0017	-0.0002
	(-5.39)	(-5.55)	(-5.48)	(0.29)	(-0.03)
VIX_t		-0.0006	-0.0002		0.0033**
		(-1.11)	(-0.37)		(2.19)
RET_t			0.2146		
			(1.33)		
$Flows_t$	0.2294***	0.2153***	0.1778***	0.1686***	0.1592***
	(3.85)	(3.38)	(3.27)	(4.20)	(4.01)
$Flows_{t-1}$	0.0325	0.0257	0.0544	0.1265**	0.1185**
	(0.35)	(0.27)	(0.62)	(2.36)	(2.21)
$Flows_{t-2}$	0.1306***	0.1260***	0.1404***	0.2164***	0.2104***
	(2.86)	(2.74)	(2.94)	(4.45)	(4.12)

Table 5: Regression of Mutual Fund Flows on Model Uncertainty: More Details

	(1) Style	(2) Sector	(3) Large-Cap	(4) Middle and Small Cap	(5) Government Bonds
$Entropy_t$	-0.0080***	-0.0001	0.0000	-0.0015*	0.0009*
	(-4.97)	(-0.59)	(0.00)	(-1.88)	(1.96)
VIX_t	-0.0001	0.0001	0.0000	0.0004***	0.0001
	(-0.27)	(0.87)	(0.01)	(2.63)	(0.79)
$Flows_t$	0.2253***	0.1815	0.1672*	0.3580***	0.3105***
	(4.28)	(1.50)	(1.68)	(5.11)	(4.84)
$Flows_{t-1}$	0.1069	0.2360***	-0.1248	0.1139**	0.1570**
	(1.59)	(3.17)	(-1.46).	(2.14)	(1.98)
$Flows_{t-2}$	0.1809***	0.0998**	0.0795	0.1077**	0.1826***
	(3.81)	(2.42).	(1.26)	(2.34)	(2.84)
RET_t	0.1106	0.0285**	-0.0874	0.0879***	
	(1.09)	(2.20)	(-1.21)	(2.81)	

Additional Related Literature

Additional Related Literature

- Bayesian inference for factor models / Bayesian portfolio choice:
  - Kandel and Stambaugh (1996), Avramov (2002): also use g-prior (but we propose a hyper-prior on g that overcomes Barlett's paradox and is new in finance literature)
  - Barillas and Shanken (2018a), Chib, Zeng, and Zhao (forthcoming): focus on time-series regression

Conclusion

Conclusion

- Develop a new measure of model uncertainty in the cross section
  - model uncertainty is countercyclical before 2008
  - reach to upper bounds in dot-com bubble and 2008 financial crisis
  - persistently high in recent 10 years
- Model uncertainty is closely related to investors' sentiments
  - investors are more pessimistic when model uncertainty becomes higher
- Model uncertainty is able to predict mutual fund flows
  - outflows in equity mutual funds when uncertainty is higher, particularly in “style" and “middle and small-cap" equity funds
  - inflows in US government bonds