The Spread of COVID-19 in London: Network Effects and Optimal Lockdowns?

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Motivation

- The COVID-19 pandemic: 1.2M world-wide deaths, 47k in UK so far
- Unprecedented policy responses – lockdowns – to control the spread
- Significant social and economic cost
- Urgent need to evaluate effectiveness and inform optimal policy making

- So far, no spatial element in models of COVID-19 spread

- We use London as a salient case study and:
  - embed network elements in the workhorse SIR epidemiology model
  - map heterogeneous interactions among population using commuting networks
  - estimate the model and analyse effectiveness of policy response

- Given the estimated spatial dynamics, we study (counterfactual) optimal policy responses

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Main Findings

- Significant network externalities in COVID-19 epidemics
  - Decomposition of infections: network effects $\approx$ 42%; local/autoregressive $\approx$ 35%; endemic $\approx$ 23%.
  - Network Impulse-Response significantly different across geographic regions

  $\Rightarrow$ Most infections $\neq$ most externalities
- Lockdown policy has had major impact, with substantial network effects:
  - Total reduction of infected: 56.5%
  - Reduction due to network externalities: 36.1%
- Counterfactual analyses show that:
  - lockdown was somehow late, but further delay would have had more dramatic effects
  - the targeted lockdown of a small number of highly connected geographic regions is equally effective
  - targeted lockdowns based on a threshold of number of cases are not effective

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Literature I

- On-going discussions of trade-off of lockdowns: the planner's problem in balancing economic output and controlling fatality
- Review by Avery, Bossert, Clark, Ellison and Ellison, 2020
- Calibrations of SIR models, cost of life, productivity
  - Formulate and solve a planner's dynamic control problem, (Alvarez, Argente and Lippi 2020)
  - Externalities and history dependent equilibrium (Rowthorn and Toxvaerd, 2020)
  - Value of death is major policy determinant (Garriga, Manuelli, Sanghi 2020)
  - Equilibrium interactions between economic decision and epidemic dynamics: (Eichenbaum, Rebelo, and Trabandtz 2020)
  - Contagion externalities (Jones, Philippon, and Venkateswaran 2020)
  - Individual optimal reduction in social activities in SIR dynamics: (Farboodi, Jarosch and Shimer, 2020)
  - Heterogeneity in social interaction and productivity among social groups (Acemoglu, Chernozhukov, Werning, Whinston, 2020)
- Estimation of vanilla SIR(D): Fernandez-Villaverde and Jones (2020)
- We depart by estimating the network element and demonstrating its relevance and policy potential.

Literature II

- Network literature on epidemiology
- Jackson 2008, Chapter 7, Sections 7.1,7.2.
- Easley and Kleinberg, 2010, Chapter 21
- Key players in network (Ballester, Calvo-Armengol, and Zenou 2006; Denbee, Julliard, Li and Yuan 2020)

A Network SIR Model

SIR Models

The triplet: $\{S, I, R\}$ (Susceptible, Infectious, Removed) such that

$$\mathrm{d}S = -\beta_I \frac{S}{N} I \mathrm{d}t, \quad \mathrm{d}I = \underbrace{\left(\beta_I \frac{S}{N} - \beta_R\right)}_{\alpha_t} \times I \mathrm{d}t, \quad \mathrm{d}R = \beta_R I \mathrm{d}t$$

where: $\beta_I =$ contact rate, $\beta_R =$ removal rate; $N =$ population.

Probabilistic counterpart (a continuous-time Markov chain):

$$\mathbb{P}\left[y \text{ new infections in } (t, t+\mathrm{d}t) \mid I_t = x\right] = \begin{cases} \alpha_t x \mathrm{d}t & y = 1 \\ o(\mathrm{d}t) & y \geq 2 \end{cases}$$

Let $\mathbb{P}\left[y \text{ new infections in } (t, t+h) \mid I_t = x\right] = p_h(y \mid x)$
(holding α_t constant from t to $t+h$ and solving the Kolmogorov forward equation):

$$p_h(y \mid x) = \frac{\Gamma(y+x)}{\Gamma(x)\Gamma(y+1)} \left(e^{-\alpha_t h}\right)^x \left(1 - e^{-\alpha_t h}\right)^y$$

- probability mass function of the negative binomial distribution

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SIR Models

- Normalizing h to $h = 1$, we have

$$y_t \mid \mathcal{F}_{t-1} \sim \text{NegBinom}\left(p_t, x_{t-1}\right)$$

where

$$p_t = 1 - e^{-\alpha_t}$$

- If $S \approx N$, then $\alpha_t \equiv \alpha = \beta_I - \beta_R > 0$
- Infected remain infectious for L periods (weighted by decaying function $\nu(l)$)

$$x_t = \sum_{l=0}^{L-1} \nu(l) y_{t-l}$$

- Model moments

$$\mathbb{E}_{t-1}\left[y_t\right] = \mu_t = a x_{t-1}, \quad a = e^{\alpha} - 1 = \frac{p_t}{1-p_t} \in (0, \infty)$$

$$\text{var}_{t-1}\left[y_t\right] = (a+1) a x_{t-1}$$

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A Network SIR Model: Framework

- Multiple subpopulations (local authorities) $i = 1, \ldots, n$
  - Within subpopulation: autoregressive epidemic dynamics
  - Between subpopulations: network epidemic effects
- Distribution:

$$y_{i,t} \mid \mathcal{F}_{t-1} \sim \mathrm{NegBinom}(p_{i,t}, x_{i,t-1}), \text{ and } \mathbb{E}_{t-1}\left[y_{i,t}\right] = \mu_{i,t}$$

- Multivariate extensions of $\mu_t = a x_{t-1}$:

$$\underbrace{\boldsymbol{\mu}_t^{AR} + \boldsymbol{\mu}_t^{NE}}_{\text{epidemic effects}} = \boldsymbol{A}\boldsymbol{x}_{t-1} = \underbrace{\mathrm{diag}(\boldsymbol{A})\boldsymbol{x}_{t-1}}_{\boldsymbol{\mu}_t^{AR}\text{: autoregressive}} + \underbrace{(\boldsymbol{A} - \mathrm{diag}(\boldsymbol{A}))\boldsymbol{x}_{t-1}}_{\boldsymbol{\mu}_t^{NE}\text{: network}}$$

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A Network SIR Model: Spatial Form

- Autoregressive effects: $\mathrm{diag}(\boldsymbol{A}) = \gamma \boldsymbol{I}$

$$\boldsymbol{\mu}_t^{AR} = \gamma \boldsymbol{x}_{t-1}$$

- Network effects: driven by an adjacency matrix $\boldsymbol{W}$ ($\mathrm{diag}(\boldsymbol{W}) = \boldsymbol{0}$)

$$\mu_{i,t}^{NE} = \phi \left(\sum_{j \neq i} w_{ij} x_{j,t-1} \right) \quad \Longleftrightarrow \quad \boldsymbol{\mu}_t^{NE} = \phi \boldsymbol{W} \boldsymbol{x}_{t-1}$$

  - allowing for multiple graphs $\left\{ \boldsymbol{W}^{(g)} \right\}_{g=1}^{G}$:

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- Implication: parametrize $\boldsymbol{A}$ as

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A Network SIR Model: Policy Specification

- Policy regime $D_t \in \{0, 1\}$ may affect fundamental parameters $(\gamma, \phi^{(1)}, \dots, \phi^{(G)})$
- Autoregressive parameters:

$$\gamma(D_t) = \exp(D_t\gamma + \gamma_0)$$

  - before policy: $\exp(\gamma_0)$; after policy: $\exp(\gamma_0 + \gamma)$
  - policy impact: $\times \exp(\gamma)$
- Network parameters: for $g = 1, \dots, G$

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- Endemic effects:

$$\mu_{i,t}^{EN} = \exp\left(\boldsymbol{z}_t^\top \boldsymbol{\beta} + \eta_i\right) N_i$$

  - z_t: time trends interacted with D_t / fixed effects / detection ratio / testing
  - η_i: subpopulation fixed effects
  - N_i: subpopulation size

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Data and Estimation

Data and Estimation

- A case study of COVID-19 spread in London
  - Disease surveillance data of 32 local authorities
    `https://coronavirus.data.gov.uk/`
  - Sample: March 1st to June 4th
  - March 23rd (nationwide lockdown announcement) = policy change date ($D_t = 0$ before and $D_t = 1$ afterwards)

- Networks: residence/work links from 2011 UK census
  - A matrix $K = \{k_{ij}\}$: number of people who lives at i & goes to work at j
  - $W^1 = K$: work-to-home transmission
  - $W^2 = K^\top$: home-to-work transmission
  - $W^3 = KK^\top$: "home-to-home" transmission via common work places
  - Rescaled by the largest singular values

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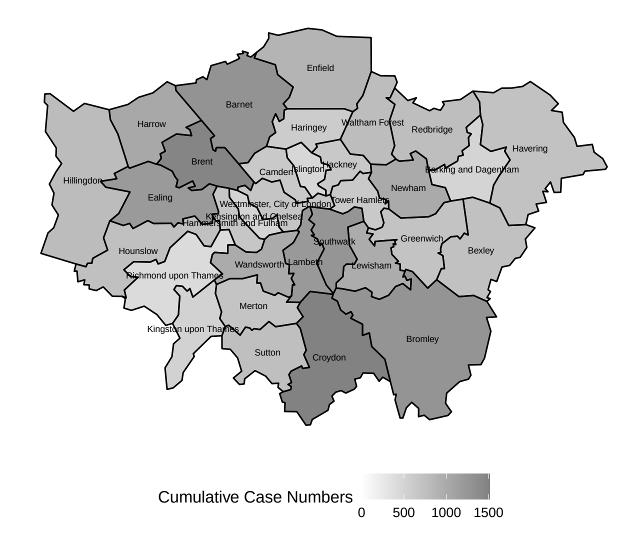
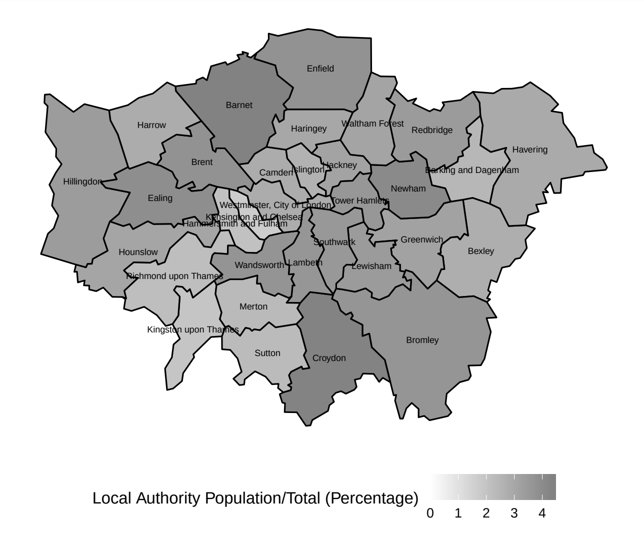
Data: Cumulative Cases Till June 4th



Total: 27,078 (9.3% of UK)

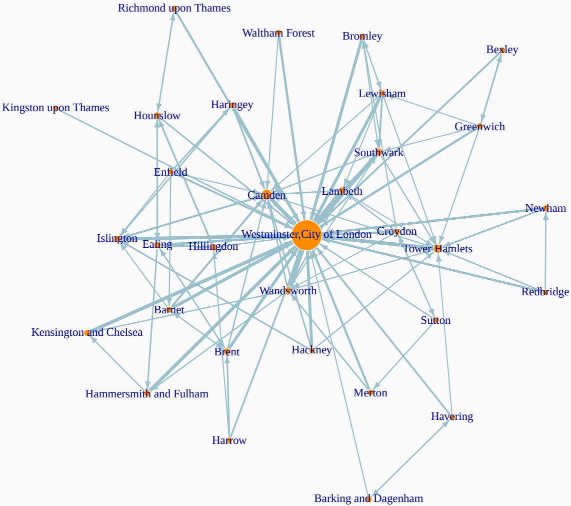
Data: Subpopulation Sizes



Corr(populaiton; cases) = 0.63/0.78 pre/post lockdown

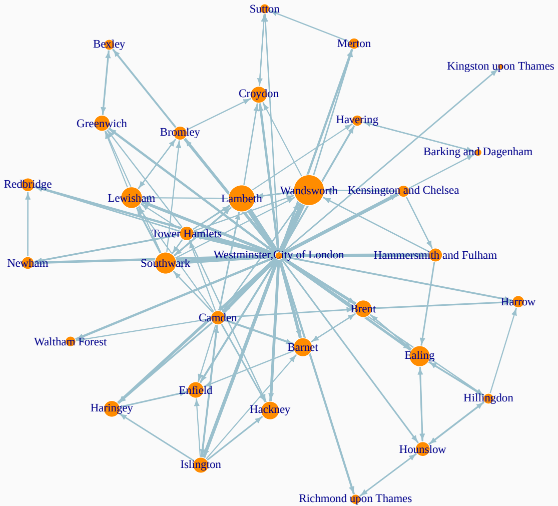
Data: Work-to-Home Transmission Network

Commuting from residential areas to work places: $W^{(1)} = K$



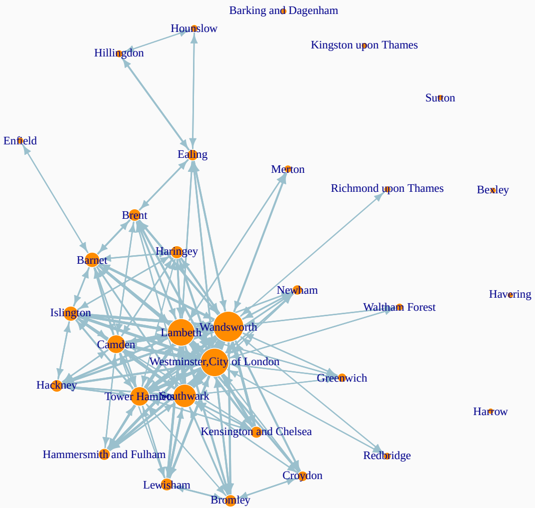
Data: Home-to-Work Transmission Network

Commuting from work places to residential areas: $W^{(2)} = K^{\top}$



Data: "Home-to-home" Transmission Network

Residential areas links via common work places: $W^{(3)} = KK^{\top}$



Model Estimation

Value	(K)		$(K^\top)$		$(KK^\top)$		all		$(K,K^\top)$	
	est.	*se.*	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*
Autoregressive effect										
$\exp(\gamma_0)$	0.311	0.035	0.286	0.040	0.286	0.037	0.252	0.041	0.253	0.041
$\exp(\gamma)$	0.617	0.077	0.958	0.136	0.750	0.102	0.760	0.129	0.754	0.127
Network effect										
K:										
$\exp(\phi_0)$	0.477	0.070					0.503	0.144	0.510	0.073
$\exp(\phi)$	0.885	0.136					0.821	0.169	0.827	0.127
$K^\top$:										
$\exp(\phi_0)$			0.115	0.022			0.062	0.036	0.064	0.023
$\exp(\phi)$			0.374	0.080			0.148	0.218	0.166	0.181
$KK^\top$:										
$\exp(\phi_0)$					0.261	0.035	0.000	0.085		
$\exp(\phi)$					0.570	0.082	0.284	0.000		
Testing-related endemic effect										
pos-to-test	−1.37	0.47	−1.09	0.28	−1.46	0.41	−1.46	0.48	−1.46	0.49
lag test	1.85	0.31	1.77	0.23	1.88	0.28	1.91	0.32	1.91	0.32
pseudo-R^2	82.09%		82.00%		82.02%		82.12%		82.12%	

est. w.o. control

Model Estimation

Value	(K) est.	(K) se.	$(K^\top)$ est.	$(K^\top)$ se.	$(KK^\top)$ est.	$(KK^\top)$ se.	all est.	all se.	$(K,K^\top)$ est.	$(K,K^\top)$ se.
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lag test	1.85	0.31	1.77	0.23	1.88	0.28	1.91	0.32	1.91	0.32
pseudo-R^2	82.09%		82.00%		82.02%		82.12%		82.12%	

est. w.o. control

Model Estimation

	(K)		$(K^\top)$		$(KK^\top)$		all		$(K,K^\top)$	
Value	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*
Autoregressive effect										
$\exp(\gamma_0)$	0.311	0.035	0.286	0.040	0.286	0.037	0.252	0.041	0.253	0.041
$\exp(\gamma)$	0.617	0.077	0.958	0.136	0.750	0.102	0.760	0.129	0.754	0.127
Network effect										
K:										
$\exp(\phi_0)$	0.477	0.070					0.503	0.144	0.510	0.073
$\exp(\phi)$	0.885	0.136					0.821	0.169	0.827	0.127
$K^\top$:										
$\exp(\phi_0)$			0.115	0.022			0.062	0.036	0.064	0.023
$\exp(\phi)$			0.374	0.080			0.148	0.218	0.166	0.181
$KK^\top$:										
$\exp(\phi_0)$					0.261	0.035	0.000	0.085		
$\exp(\phi)$					0.570	0.082	0.284	0.000		
Testing-related endemic effect										
pos-to-test	−1.37	0.47	−1.09	0.28	−1.46	0.41	−1.46	0.48	−1.46	0.49
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est. w.o. control

Model Estimation

	(K)		$(K^\top)$		$(KK^\top)$		all		$(K,K^\top)$	
Value	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*
Autoregressive effect										
$\exp(\gamma_0)$	0.311	0.035	0.286	0.040	0.286	0.037	0.252	0.041	0.253	0.041
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est. w.o. control

Model Estimation

Value	(K) est.	(K) se.	$(K^\top)$ est.	$(K^\top)$ se.	$(KK^\top)$ est.	$(KK^\top)$ se.	all est.	all se.	$(K,K^\top)$ est.	$(K,K^\top)$ se.
Autoregressive effect										
$\exp(\gamma_0)$	0.311	0.035	0.286	0.040	0.286	0.037	0.252	0.041	0.253	0.041
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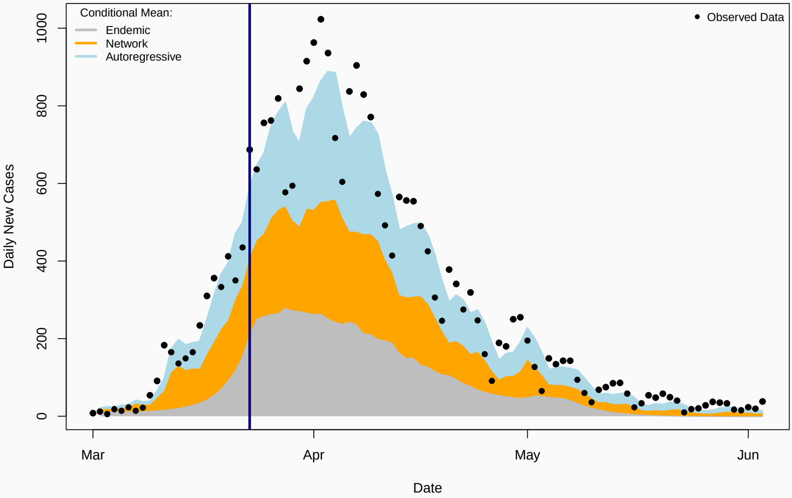
pseudo-R^2	82.09%		82.00%		82.02%		82.12%		82.12%	

est. w.o. control

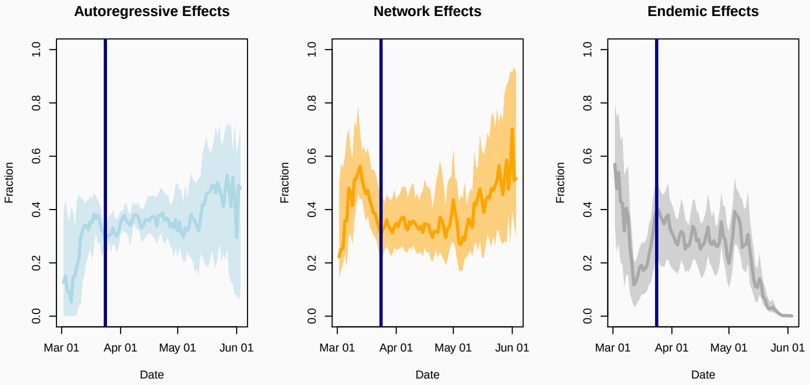
Decomposing the Conditional Mean

Conditional mean of disease cases: $\mu_t = \mu_t^{AR} + \mu_t^{NE} + \mu_t^{EN}$



Decomposing the Conditional Mean cont'ed

Fraction of conditional mean explained: $\frac{\mu_{i,t}^{AR}}{\mu_{i,t}}, \frac{\mu_{i,t}^{NE}}{\mu_{i,t}}. \frac{\mu_{i,t}^{EN}}{\mu_{i,t}}$

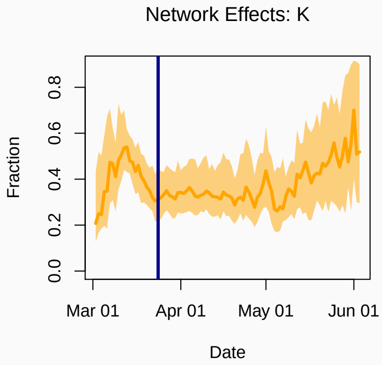


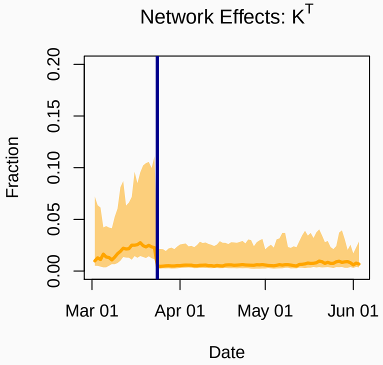
Bands represents 10- and 90-percent quantiles across 32 locations

Note: pre (post) lockdown average NE share: 45% (40%)

Decomposing the Conditional Mean

Fraction of conditional mean explained by each of the three networks: $\frac{\mu_{i,t}^{NE}}{\mu_{i,t}}(W^{(g)})$





Bands represents 10- and 90-percent quantiles across 32 locations

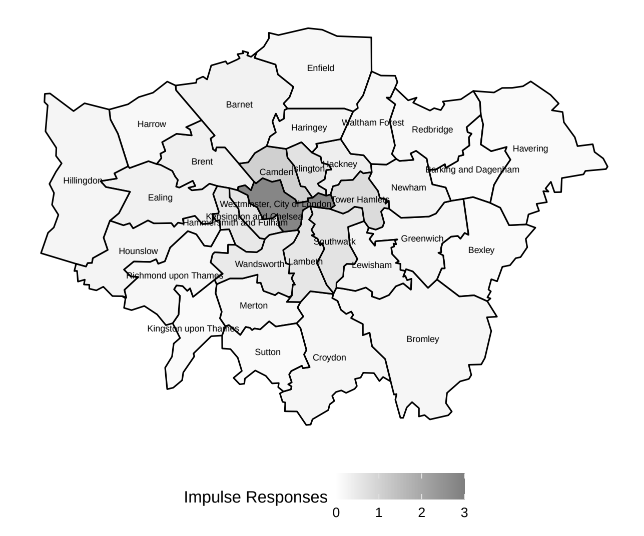
Network Impulse Responses

- "Shocks" to $y_{i,t}$ propagate through the network
- Let $\epsilon_{i,t} = y_{i,t} - \mu_{i,t}$
- We have the s-periods Network Impulse-Response Function:

$$NIRF_i(s) = \sum_{j=1}^{N} \frac{\partial \mathbb{E}_t[y_{j,t+s}]}{\partial \epsilon_{i,t}}$$

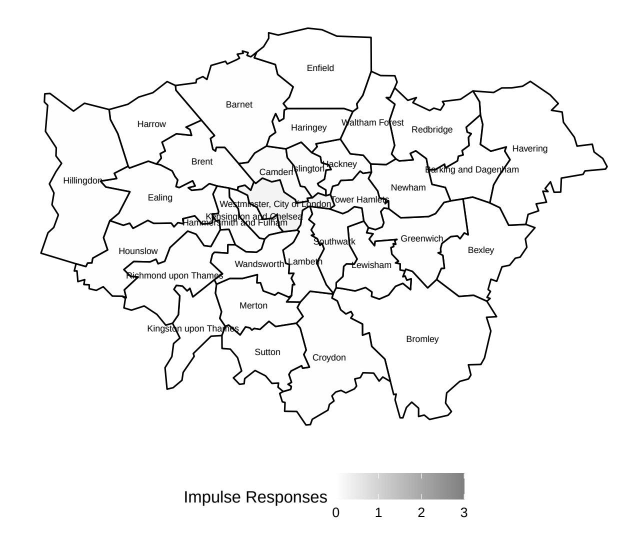
- Can be computed in closed form

Network Impulse Responses: Pre-lockdown, ($s = 7$ days)



Max N-IRF: 2.8

Network Impulse Responses: Post-lockdown, $(s = 7$ days)



Max N-IRF: 0.3

Epidemic Parameters

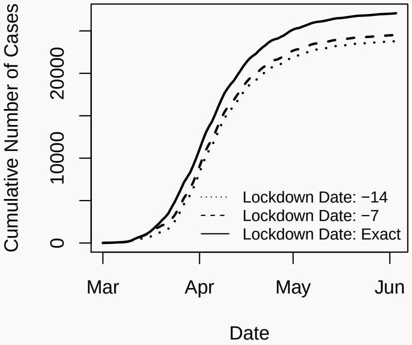
- The dominant (largest) eigenvalue $\sigma_{\max}$ of $\boldsymbol{A}(D_t) = \gamma(D_t)\boldsymbol{I} + \sum_{g=1}^{G} \phi^{(g)}(D_t)\boldsymbol{W}^{(g)}$ is key for epidemic dynamics since: $\boldsymbol{\mu}_t \propto \boldsymbol{A}(D_t)\boldsymbol{x}_t$
- The disease epidemic R_0 (basic reproduction number) can be approximated as

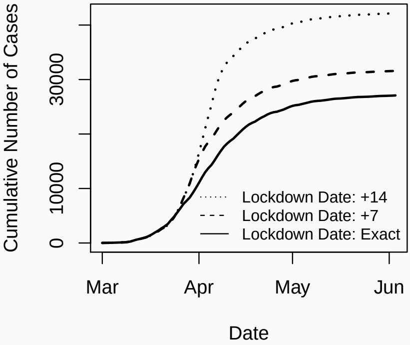
$$R_0 \leq \sum_{\ell=0}^{L} \widehat{v}(\ell)\, \sigma_{\max}\left(\widehat{\boldsymbol{A}}\right).$$

Model $\mathcal{G}$	(1)		(2)		(3)		(1,2,3)		(1,2)	
Value	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*
Before Lockdown Policy: $D_t = 0$										
$\sigma_{\max}$	0.840	0.590	0.511	0.106	0.639	0.287	0.853	0.533	0.766	0.537
R_0	1.467	1.030	1.577	0.327	1.531	0.688	1.391	0.876	1.369	0.953
After Lockdown Policy: $D_t = 1$										
$\sigma_{\max}$	0.510	0.369	0.270	0.038	0.301	0.123	0.501	0.398	0.422	0.392
R_0	0.891	0.644	0.832	0.118	0.722	0.296	0.818	0.649	0.754	0.701

Counterfactuals and Policy Experiments

Counterfactual: Different Lockdown Dates?





Each path is the median of 10,000 simulations

$-14 \Rightarrow -3{,}283\ (-12.1\%)$

$-7 \Rightarrow -2{,}553\ (-9.4\%)$

$+7 \Rightarrow +4{,}514\ (+16.7\%)$

$+14 \Rightarrow +15{,}085\ (+55.7\%)$

Optimal Lockdown Policies

Given the transition dynamics

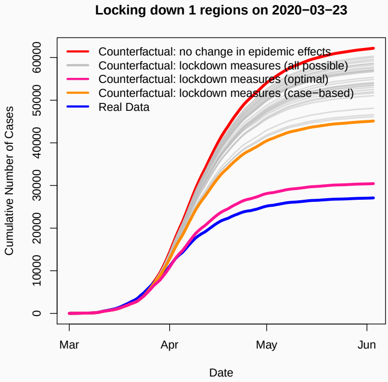
$$\mu_t \propto A x_{t-1}$$

there are two naive candidate optimal lockdown policies:

- Locking down areas with the largest number of cases ("case-based" policy):
  - targeting on x_{t-1}
- Locking down areas showing large externalities (larger NIRF):
  - targeting on A

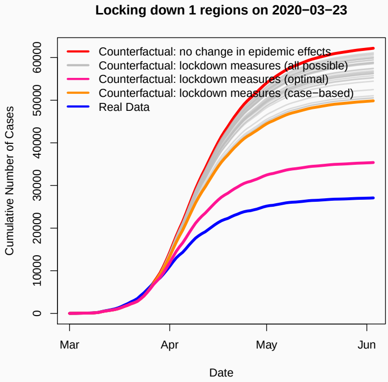
Partial Lockdowns: One Local Authority

Total lockdown



Optimal: Westminster, City of London

Partial lockdown

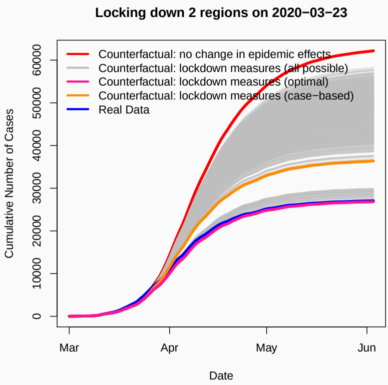


Optimal: Westminster, City of London

Note: Largest number of cases: Lambeth
Largest (integrated) NIRF: Westminster, City of London

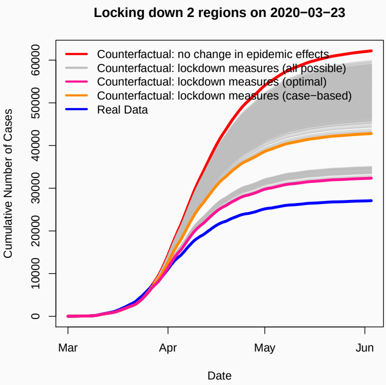
Partial Lockdowns: Two Local Authorities

Total lockdown



Optimal: Westminster, City of London/Southwark

Partial lockdown

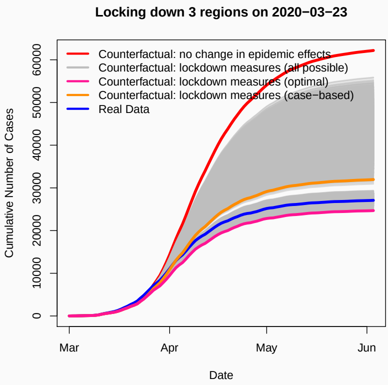


Optimal: Westminster, City of London/Southwark

Note: Largest number of cases: Lambeth/Southwark
Largest (integrated) NIRF: Westminster, City of London/Camden

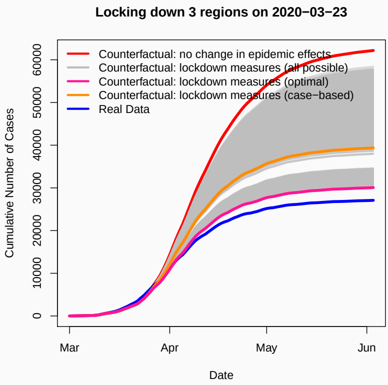
Partial Lockdowns: Three Local Authorities

Total lockdown



Optimal:
Westminster, City of London/Southwark/Lambeth

Partial lockdown



Optimal:
Westminster, City of London/Southwark/Lambeth

Note: Largest number of cases: Lambeth/Southwark/Brent
Largest (integrated) NIRF: Westminster, City of London/Camden/Tower Hamlets

Conclusion

We have embeded spatial dynamics in an empirical SIR model and found that:

1. Network externalities are first order
2. The national lockdown: $i)$ reduced the spread by more than half, $ii)$ was somehow late, and $iii)$ further delay would have had extreme conseqeunces.
3. "Case-based" lockdown policies appear suboptimal
4. Network externalities targeting, and partial lockdowns, seem very effective

Appendix

Estimation without controlling for test related variables

Model	(K)		$(K^\top)$		$(KK^\top)$		all		$(K,K^\top)$	
	est.	*se.*	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*	*est.*	*se.*
Autoregressive effect										
$\exp(\gamma_0)$	0.381	0.033	0.456	0.034	0.424	0.032	0.365	0.036	0.344	0.038
$\exp(\gamma)$	0.472	0.050	0.519	0.045	0.446	0.042	0.492	0.057	0.522	0.066
Network effect										
K:										
$\exp(\phi_0)$	0.563	0.066					0.496	0.091	0.592	0.071
$\exp(\phi)$	0.694	0.098					0.594	0.189	0.637	0.102
$K^\top$:										
$\exp(\phi_0)$			0.076	0.021			0.000	0.000	0.041	0.021
$\exp(\phi)$			0.598	0.175			0.000	0.000	0.203	0.424
$KK^\top$:										
$\exp(\phi_0)$					0.280	0.035	0.056	0.048		
$\exp(\phi)$					0.518	0.073	0.696	0.779		
pseudo-R^2	81.86%		80.90%		81.58%		81.89%		81.89%	