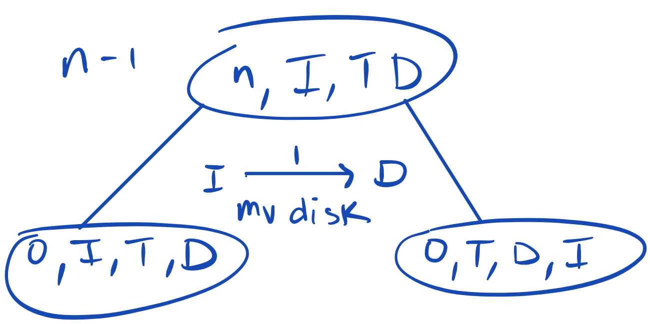
Recursion

	iterative	recursive
P1: Factorial	n	n
P2: Fibonacci	$n-1$	$\exp(n)$
P3: Towers of Hanoi	$2^n - 1$	$2^n - 1$
P4: Binomial Coefficient		

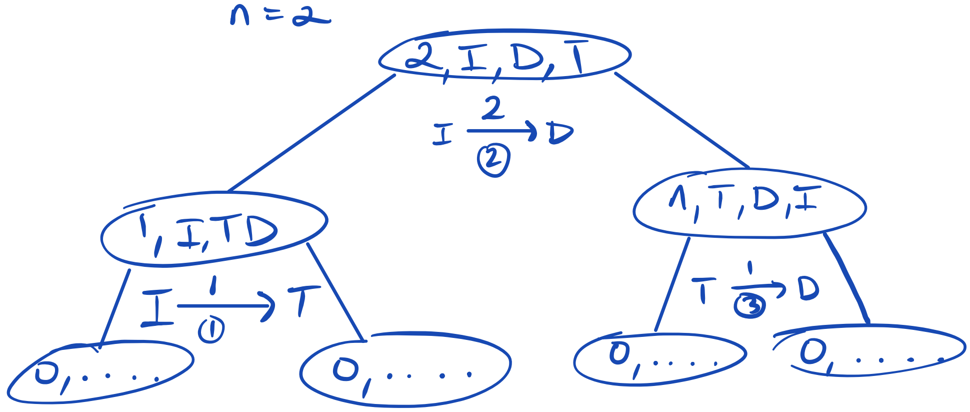
```
p.s.v. solve (int n, String I, String D, String T)
{ if (n > 0)
  {
    solve (n-1, I, T, D);
S.O.P.L ("remove disk" + n + "from" + I + "to" + D);
   solve (n-1, T, D, I);
  }
}
```

call from main: solve (n, "I", "D", "T");

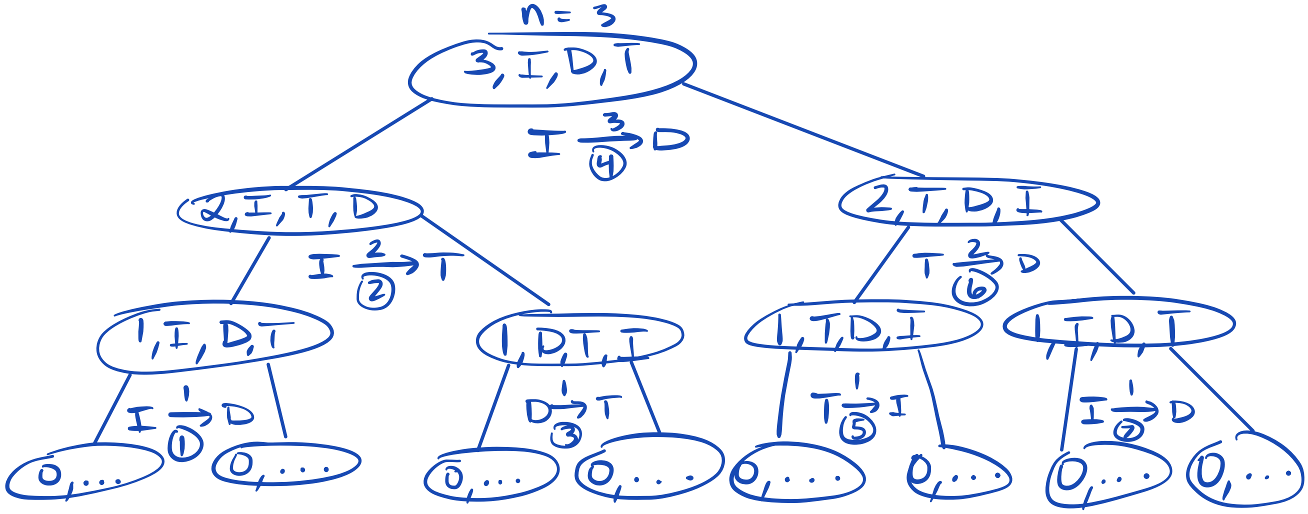
n-1



$n = 2$



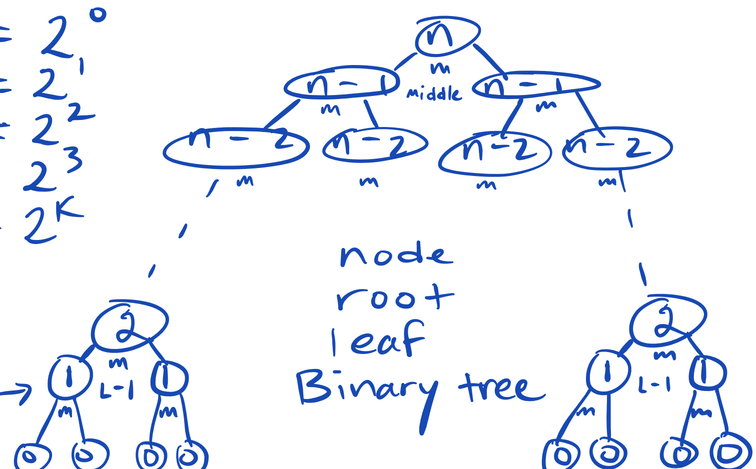
$n = 3$



1. $I \xrightarrow{1} D$
2. $I \xrightarrow{2} T$
3. $D \xrightarrow{1} T$
4. $I \xrightarrow{3} D$
5. $T \xrightarrow{1} I$
6. $T \xrightarrow{2} D$
7. $I \xrightarrow{1} D$

$1 = 2^0$
$2 = 2^1$
$4 = 2^2$
$= 2^3$
$= 2^K$

$2^{n-1} \rightarrow$
$2^n \rightarrow$



node
root
leaf
Binary tree

$1 = n - (n-1)$
$0 = n - n$
$\uparrow$
K

crit ops

calls $= 2^0 + 2^1 + 2^2 + \dots + 2^{n-1} + 2^n = \sum_{i=0}^{n} 2^i$

$$= \frac{2^{(n-1)+1} - 1}{2-1} = \boxed{2^n - 1}$$

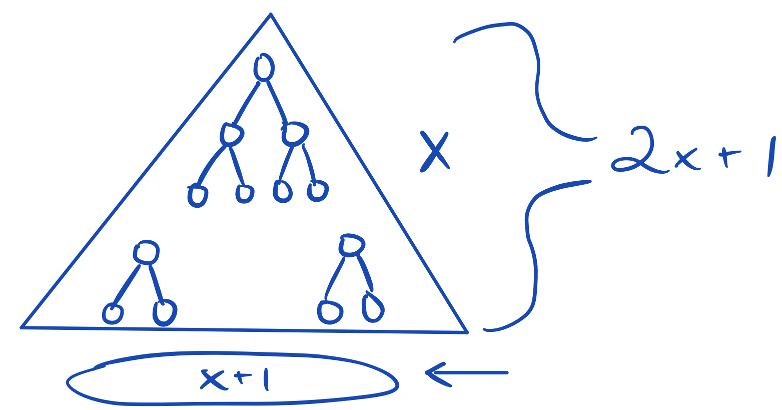
$$\sum_{i=0}^{n} a^i = 1 + a + a^2 + \dots + a^n = S$$

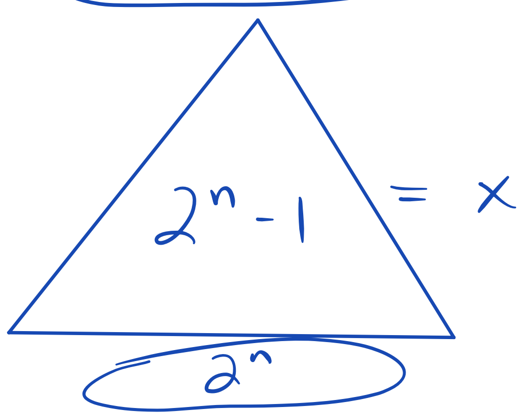
$$(1 + a + a^2 + \dots + a^n)(a-1) = S(a-1)$$

$$\cancel{a} + \cancel{a^2} + \cancel{a^3} \dots + \boxed{a^{n+1}} = S(a-1)$$

$$\boxed{-1} - \cancel{a} - \cancel{a^2} \dots - {}^{n}$$

$$S = \boxed{\frac{a^{n+1} - 1}{a - 1}} = \frac{1 - a^{n+1}}{1 - a}$$





Binomial Coefficient

$$C(n,k) = \sum_{k=0}^{n} C(n,k)\, x^{k} y^{n-k}$$

$(x+y)^{n}$

$(x+y)^{0} = 1$ $\qquad$ $I = 1$

$(x+y)^{1} = x + y$

$(x+y)^{2} = 1x^{2} + 2xy + 1y^{2}$

$(x+y)^{3} = 1x^{3} + 3x^{2}y + 3xy^{2} + 1y^{3}$

$(x+y)^{4} = 1x^{4} + 4x^{3}y + 6x^{2}y^{2} + 4xy^{3} + 1y^{4}$

exp	coefficient
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1