Phys 512 Problem Set 3

Due on github Wednesday October 23 at 4 PM. You may discuss problems, but everyone must write their own code.

In this problem set, we will use the power spectrum of the Cosmic Microwave Background (CMB) to constrain the basic cosmological parameters of the universe. The parameters we will measure are the Hubble constant, the density of regular baryonic matter, the density of dark matter, the amplitude and tilt of the initial power spectrum of fluctuations set in the very early universe, and the Thomson scattering optical depth between us and the CMB. In this excercise, we will only use intensity data, which does a poor job constraining the optical depth.

For the data, we will use the WMAP satellite 9-year data release. (The Planck satellite has new and better data, but its greater sensitivity means it is more complicated to use). The data can be found at https://lambda.gsfc.nasa.gov/. Browse down to WMAP data products, and go to the TT power spectra link. We want the combined (not binned) version of the spectrum. This gives the measured variance of the sky as a function of multipole l. WMAP does not measure the monopole, and the dipole is set by the motion of the Earth/Milky Way relative to the CMB reference frame. So, the spectrum starts with the quadrupole. The first column is the multipole index, the second is the measured power spectrum, and the third is the error in that. For simplicity, we will treat the errors as Gaussian and uncorrelated, though that is not quite accurate. The final two columns break down the error into the instrument noise part and the "cosmic variance" part, due to the fact that we only have a finite number of modes in the sky to measure. These columns can safely be ignored. Further description, including plots, can be found in the WMAP 9-year result paper https://arxiv.org/pdf/1212.5226.pdf.

You'll also need to be able to calculate model power spectra as a function of input parameters. You can get the source code for CAMB from Antony Lewis's github page: https://github.com/cmbant. There's a short tutorial online at https://camb.readthedocs.io/en/latest/CAMBdemo.html as well. Note that CAMB returns the power spectrum starting with the monopole, so you may need to manually remove the first two entries.

To help you out, I have posted a sample script that calculates the power spectrum from CAMB, reads in the WMAP data, and plots them on top of each other for one guess for the cosmological parameters.

1) Using Gaussian, uncorrelated errors, what do you get for χ^2 for the model in my example script, where the Hubble constant $H_0 = 65\ km/s$, the physical baryon density $\omega_b h^2$=0.02, the cold dark matter density $\omega_c h^2 = 0.1$ the optical depth $\tau = 0.05$, the primordial amplitude of fluctuations is $A_s = 2 \times 10^{-9}$, and the slope of the primordial power law is 0.96 (where 1 would be scale-invariant). The baryon/dark matter densities are defined relative to the critical density

required to close the universe, scaled by h^2 where $h \equiv H_0/100 \sim 0.7$. Note that the universe is assumed to be spatially flat (for reasons too long to justify here), so the dark matter density relative to critical for these parameters would be $1 - (\omega_b h^2 + \omega_c h^2)/h^2$ =71.6% for the parameters assumed here. (You may want to play around plotting different models as you change parameters to get a sense for how the CMB depends on them.) If everything has gone well, you should get something around 1588 (please give a few extra digits) for χ^2 for this model.

2) Keeping the optical depth fixed at 0.05, write a Newton's method/Levenberg-Marquardt minimizer and use it to find the best-fit values for the other parameters, and their errors. What are they? If you were to keep the same set of parameter but now float τ, what would you expect the new errors to be? Note that CAMB does not provide derivatives with respect to parameters, so you'll have to come up with something for that. Pleae also provide a plot showing why we should believe your derivative estimates.

3) Now write a Markov-chain Monte Carlo where you fit the basic 6 parameters, including τ. However, note that we know the optical depth can't be negative, so you should reject any steps that try to sample a negative τ. What are your parameter limits now? Please also present an argument as to why you think your chains are converged. As a reminder, you can draw samples of correlated data from a covariance matrix with $r = np.linalg.cholesky(mat); d = np.dot(r, np.random.randn(r.shape[0]))$. You will want to use the covariance matrix from part 2) when drawing samples for the MCMC.

4) The Planck satellite has independently measured the CMB sky, and finds that the optical depth is 0.0544 ± 0.0073. Run a chain where you add this in as a prior on the value of τ. What are your new parameter values/constraints? You can also take your chain from part 3) and importance sample it (weighting by the prior) with the Planck τ prior. How to those results compare to the full chain results?