1. With the straightforward averaging method, the results are as follows:

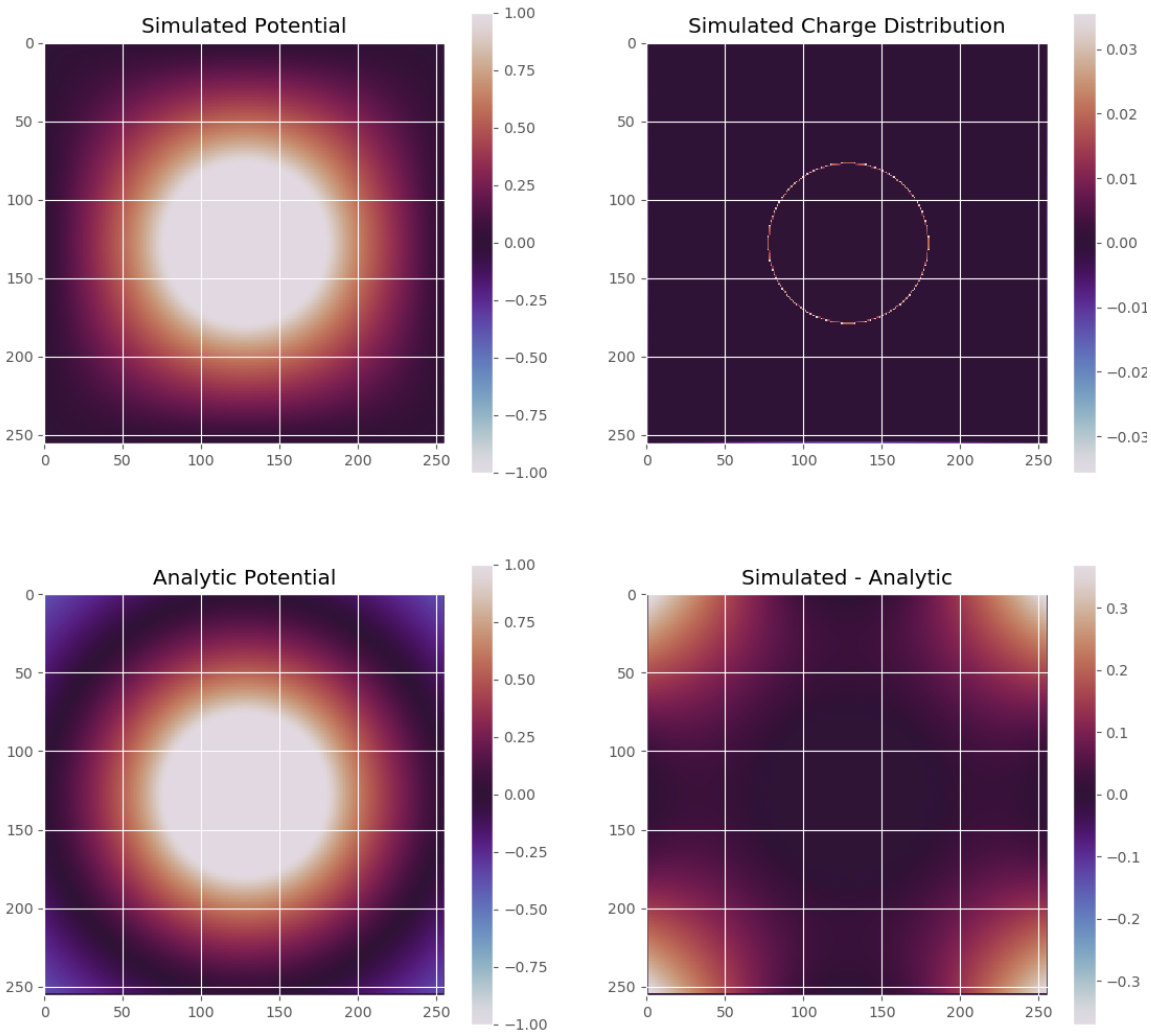
   Considering a gaussian surface outside of the disk of fixed potential. We find that the radial electric field around the disk is $E = 2kQ/r$. Taking a point of 0 potential at the wall (W), results in a potential at a point $R >$ radius of the disk, given by

$$V = -\int_W^R \frac{2kQ}{r}\mathrm{d}r = 2kQ\int_R^W \frac{1}{r}\mathrm{d}r = 2kQ\log\left(\frac{W}{R}\right).$$

   Knowing that at the radius of the disk R_d we want $V = V_d$ we can set Q as $\frac{Vc}{2k\log\left(\frac{W}{R_d}\right)}$ (k was simply set to one in the simulation).

   Simulations were performed on a 256x256 grid spanning a distance of 40 units with a disk of radius 8 units. The threshold was set to be a difference of the total summed potential less than 0.01. For the simple simulation the output was as follows:

```
{TRUNCATED}
Itartion 20460, delta = 0.010002
Itartion 20461, delta = 0.009999
Converged!
Charge density is: 0.125166
```

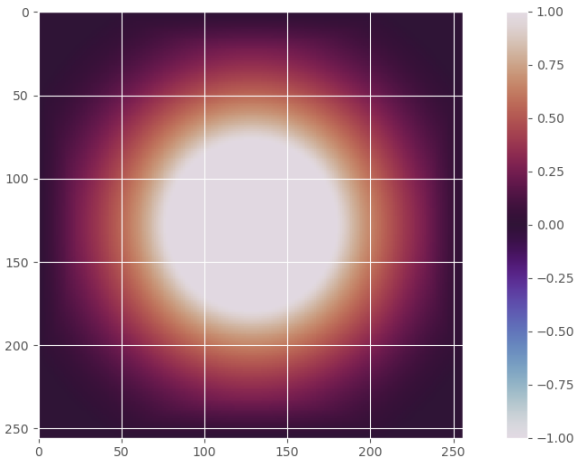


We see that the charge accumulates at the surface of the disk. Summing the entire charge and dividing by the circumference results in a value of 0.123 for the surface charge.

Comparing to the analytics, we see that in the central region this looks good, but at the edges of the square this is not the case. This is mostly an artefact from how I calculated the analytic solution, as it considered the potential of the wall to be zero a radius W from the center, which is not the case for the corners.

2. With conjugate gradient, the results are obtained much faster.

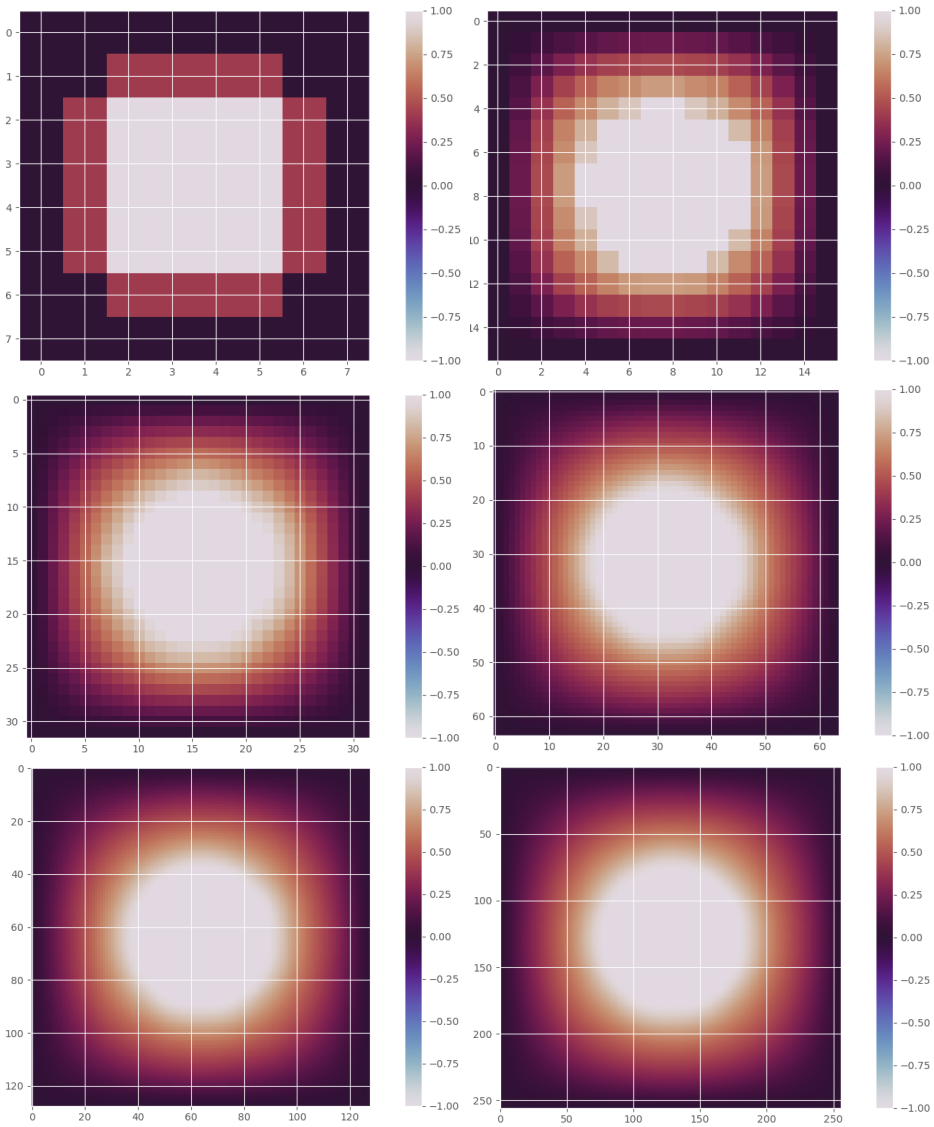
```
{TRUNCATED}
iteration 139, resid is 0.010155
iteration 140, resid is 0.009699
Converged!
```



This method took only 140 iterations, vs the 20461 from before!

3. With a scaling conjugate gradient method, the results are yet again faster to obtain.

```
Solving pot with downscale factor: 32
iteration 0, resid is 3.200000
iteration 1, resid is 0.105785
Converged!
Solving pot with downscale factor: 16
iteration 0, resid is 0.230784
iteration 1, resid is 0.117348
Converged!
Solving pot with downscale factor: 8
iteration 0, resid is 0.165132
iteration 1, resid is 0.124915
iteration 2, resid is 0.125942
iteration 3, resid is 0.093801
iteration 4, resid is 0.094508
iteration 5, resid is 0.076115
Converged!
Solving pot with downscale factor: 4
iteration 0, resid is 0.055974
iteration 1, resid is 0.038500
Converged!
Solving pot with downscale factor: 2
iteration 0, resid is 0.017265
Converged!
Solving pot with downscale factor: 1
iteration 0, resid is 0.009016
Converged!
Total iterations = 14
```



This method took only 14 iterations, vs the 140 from before!

4. Adding, a bump, we see that the electric field is roughly 2 times larger at its surface then at the surface of the rest of the initial disk.

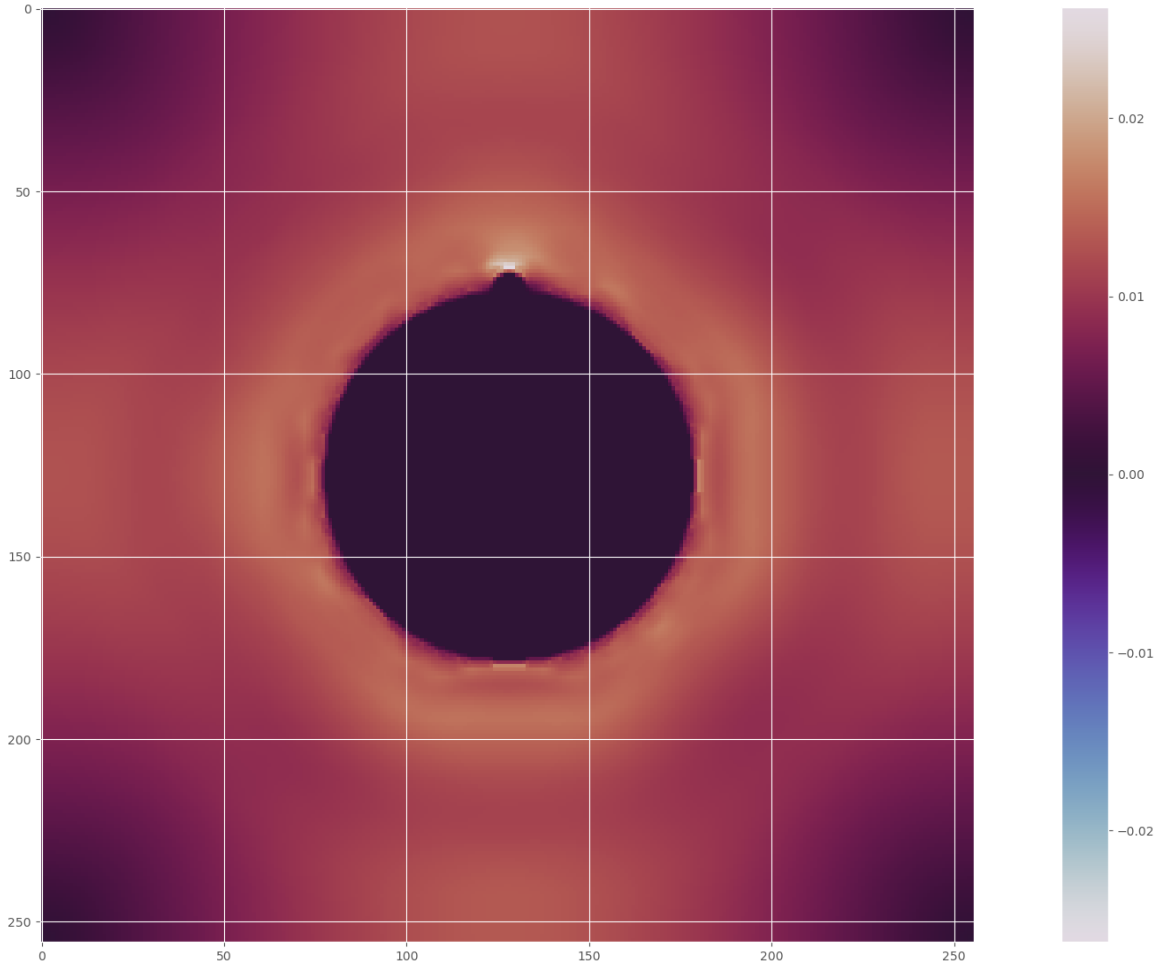


Figure 1: Electric field around disk with bump.

5. In the adiabatic limit, the box has time to reach equilibrium within a time less than that during which a substantial change is made to the boundary conditions. This was modelled by incrementing the boundary wall temperature and then running a full conjugate gradient, starting with the previous temperature distribution. This was done with 100 steps, incrementing the temperature by 0.1 degrees after each conjugate gradient run. The following plot shows a trace of temperature from the heated wall to the center of the box at each time step. I believe this simulation as is, corresponds to a box filled with a material of unit heat capacity.

   To more accurately model heat flow, the boundary temperature could be incremented while the conjugate gradient method is being run, but I ran out of time to program this.