

Multigrid as a preconditioner

$$\mathbf{u} = S_1^{v_1} \mathbf{u} + R_1 \mathbf{f}$$

$$\mathbf{d} = r(L_2 \mathbf{u} - \mathbf{f})$$

$$\mathbf{v} = L_1^{-1} \mathbf{d}$$

$$\mathbf{u} = \mathbf{u} - p\mathbf{v}$$

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$$= S_2^{v_2} \mathbf{u} - S_2^{v_2} p L_1^{-1} r L_2 \mathbf{u} + S_2^{v_2} p L_1^{-1} r \mathbf{f} + R_2 \mathbf{f}$$

$$= S_2^{v_2} (S_1^{v_1} \mathbf{u} + R_1 \mathbf{f}) - S_2^{v_2} p L_1^{-1} r L_2 (S_1^{v_1} \mathbf{u} + R_1 \mathbf{f})$$

$$+ S_2^{v_2} p L_1^{-1} r \mathbf{f} + R_2 \mathbf{f}$$

Multigrid as a preconditioner

$$\begin{aligned}u &= S_2^{\nu_2} (S_1^{\nu_1} u + R_1 f) - S_2^{\nu_2} p L_1^{-1} r L_2 (S_1^{\nu_1} u + R_1 f) + S_2^{\nu_2} p L_1^{-1} r f + R_2 f \\&= S_2^{\nu_2} S_1^{\nu_2} u + S_2^{\nu_2} R_1 f - S_2^{\nu_2} p L_1^{-1} r L_2 S_1^{\nu_1} u - S_2^{\nu_2} p L_1^{-1} r L_2 R_1 f + S_2^{\nu_2} p L_1^{-1} r f + R_2 f \\&= \left(S_2^{\nu_2} S_1^{\nu_2} - S_2^{\nu_2} p L_1^{-1} r L_2 S_1^{\nu_1} \right) u + \left(S_2^{\nu_2} R_1 - S_2^{\nu_2} p L_1^{-1} r L_2 R_1 + S_2^{\nu_2} p L_1^{-1} r + R_2 \right) f \\&= S_2^{\nu_2} \left(S_1^{\nu_2} - p L_1^{-1} r L_2 S_1^{\nu_1} \right) u + \left(S_2^{\nu_2} R_1 + R_2 + S_2^{\nu_2} p L_1^{-1} r - S_2^{\nu_2} p L_1^{-1} r L_2 R_1 \right) f \\&= S_2^{\nu_2} \left(I - p L_1^{-1} r L_2 \right) S_1^{\nu_1} u + \left\{ S_2^{\nu_2} R_1 + R_2 + S_2^{\nu_2} p L_1^{-1} r (I - L_2 R_1) \right\} f\end{aligned}$$

- $u = 0$ とすれば, \square は消え, 前処理行列は \square となる.