## **MVA ASSIGNMENT 8**

## MEMBER INFORMATION

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## LOGISTIC REGRESSION ANALYSIS

First, we ran the overall logistic regression to all the variables with the outcome of purchasing the deposit product as our dependent binary variable. Here is the outcome.

```
glm(formula = y \sim ., family = "binomial", data = bank)
Deviance Residuals:
                      Median
    Min
                10
                                               Max
-4.0169
          -0.3814
                               -0.1579
                     -0.2567
                                           3.0346
Coefficients:
                        Estimate Std. Error z value Pr(>|z|).462e+00 6.038e-01 -4.077 4.55e-05
                                                                    * * *
(Intercept)
                      -2.462e+00
                                                         4.55e-05
                                    7.125e-03
 Bank$age
                      -4.232e-03
                                                 -0.594 0.552537
 jobblue-collar`
                                    2.420e-01
3.811e-01
                      -3.924e-01
                                                 -1.621 0.104937
                      -2.498e-01
                                                 -0.655 0.512199
 obentrepreneur
jobentreprene
jobhousemaid
                      -3.530e-01
                                    4.176e-01
                                                 -0.845 0.398000
jobmanagement
                                    2.407e-01
                      -7.302e-02
                                                 -0.303 0.761602
jobretired
                       6.315e-01
                                    3.112e-01
                                                  2.029 0.042454
 jobself-employed`
                                    3.533e-01
2.729e-01
                      -1.812e-01
                                                 -0.513 0.608167
                                                 -0.534 0.593542
 observices
                      -1.457e-01
                                    3.750e-01
2.301e-01
<u>iobstudent</u>
                                                  1.009 0.312958
                       3.784e-01
jobstadent
jobtechnician
jobunemployed
                      -1.926e-01
                                                 -0.837 0.402496
                                    4.214e-01
                                                 -1.518 0.129138
                      -6.395e-01
iobunknown
                       5.207e-01
                                    5.853e-01
                                                  0.890 0.373669
maritalmarried
maritalsingle
                                                                    **
                      -4.696e-01
                                    1.743e-01
                                                 -2.694 0.007058
                                    2.038e-01
                      -3.051e-01
                                                 -1.497 0.134354
                                    2.022e-01
2.337e-01
educationsecondary
                       8.011e-02
                                                  0.396 0.691924
educationtertiary
                       3.208e-01
                                                  1.373 0.169897
                                    3.572e-01
4.315e-01
                                                 -1.179 0.238561
educationunknown
                      -4.210e-01
                                                  1.262 0.206824
default
                       5.446e-01
 Bank$balance`
                      -3.911e-06
                                    1.749e-05
                                                 -0.224 0.823014
                      -2.600e-01
                                    1.381e-01
housing
                                                 -1.883 0.059676
                                                 -3.149 0.001640
-0.302 0.762900
loan
                      -6.296e-01
                                    2.000e-01
                                    2.327e-01
2.277e-01
contacttelephone
                      -7.020e-02
                      -1.416e+00
                                                 -6.219 4.99e-10
contactunknown
 Bank$day
                       1.641e-02
                                    8.161e-03
                                                  2.011 0.044362
                                    2.494e-01
                                                 -1.235 0.216655
                      -3.081e-01
monthaug
monthdec
                       1.144e-01
                                    6.573e-01
                                                  0.174 0.861784
                       2.022e-01
                                    2.937e-01
                                                  0.688 0.491290
monthfeb
monthjan
                      -1.123e+00
                                    3.816e-01
                                                 -2.944 0.003245
                      -7.515e-01
                                    2.498e-01
                                                 -3.008 0.002630 **
monthjul
```

```
monthjun
                      5.542e-01
                                  3.003e-01
                                               1.845 0.065009
                                                3.842 0.000122 ***
                                  3.901e-01
                      1.498e+00
monthmar
                                  2.340e-01
2.737e-01
                                              -2.094 0.036246
                     -4.900e-01
monthmay
                     -8.430e-01
                                              -3.080 0.002072
monthnov
                                                                ***
monthoct
                      1.361e+00
                                  3.300e-01
                                               4.124 3.72e-05
                                               1.597 0.110265
20.912 < 2e-16
monthsep
                      6.572e-01
                                  4.115e-01
                                  2.020e-04
2.821e-02
9.959e-04
3.818e-02
duration
                                                                * * *
                      4.225e-03
                                              20.912
                                               -2.496 0.012549 *
campaign
                     -7.042e-02
pdays
previous
                                              -0.098 0.921684
-0.144 0.885249
                     -9.791e-05
                     -5.511e-03
                                  2.692e-01
                                               1.825 0.068019
poutcomeother
                      4.912e-01
                                                       < 2e-16 ***
                      2.445e+00
                                  2.773e-01
                                               8.818
poutcomesuccess
                                  3.199e-01
                                              -0.380 0.703822
poutcomeunknown
                     -1.216e-01
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 3231.0 on 4520 degrees of freedom
Residual deviance: 2173.7
                                       degrees of freedom
                             on 4478
AIC: 2259.7
Number of Fisher Scoring iterations: 6
```

In order to simplify this, we employ the stepwise selection method with the BIC as the standard. And here is the result of selection.

```
Call:
glm(formula = y ~ jobretired + maritalmarried + loan + contactun
    monthjan + monthjul + monthmar + monthmay + monthnov + month
oct +
    duration + campaign + poutcomesuccess, family = "binomial",
    data = bank
Deviance Residuals:
              10
                    Median
    Min
                                          Max
-4.0803 -0.3972
                   -0.2651 -0.1672
                                       3.0209
Coefficients:
                   Estimate Std. Error z value Pr(>|z|)
                                                          * * *
                             0.1412887 -18.582
                                                 < 2e-16
(Intercept)
                 -2.6254372
                 0.7191126
jobretired
                                          3.523 0.000426 ***
                             0.2040986
                                         <u>-3.617</u> 0.000298
                -0.4137360
                             0.1143798
maritalmarried
                             0.1953026
                                         -3.229 0.001241 **
                 -0.6306864
loan
                                         -6.980 2.96e-12
contactunknown
                 -1.2605895
                             0.1806082
                                         -2.796 0.005167
monthjan
                                                          **
                 -0.9397973
                             0.3360726
                             0.1813340
                                         -4.444 8.82e-06 ***
monthjul
                 -0.8058806
                                          4.468 7.90e-06 ***
monthmar
                 1.5364708
                             0.3438877
                 -0.7157138
                             0.1565216
monthmay
                                         -4.573 4.82e-06 ***
                                         -3.763 0.000168 ***
                 -0.8209415
                             0.2181389
monthnov
                             0.2791453
                                          5.161 2.45e-07
                 1.4407692
monthoct
                                        20.929 < 2e-16 ***
-2.758 0.005819 **
                                                 < 2e-16 ***
duration
                 0.0041161
                             0.0001967
                -0.0741598
                             0.0268909
campaign
```

```
poutcomesuccess 2.5008916 0.2171477 11.517 < 2e-16 ***
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 3231.0 on 4520 degrees of freedom
Residual deviance: 2224.4 on 4507 degrees of freedom
AIC: 2252.4

Number of Fisher Scoring iterations: 6
```

Then we are going to check if there is any complete separation or quasi-complete separation here. After we check all these dummy variables, we do not find any complete separation or quasi-complete separation. Here is the proof.

```
xtabs(~y+monthjan,data=bank)
 monthjan
     0
        132
0 3868
  505
         16
   abs(~y+contactunknown,data=bank)
 contactunknown
     0
0 2737 1263
1 460
xtabs(~y+monthjul,data=bank)
monthjul
         61
0 3355
        645
1 460
         61
  tabs(~y+monthmar,data=bank)
monthmar
0 3972
         28
   500
         21
         /+monthmay,data=bank)
 monthmay
0 2695 1305
  428
     s(~y+monthoct,data=bank)
 monthoct
         43
0 3957
1 484
         37
        y+monthnov,data=bank)
 monthnov
0 3650
        350
         39
         /+maritalmarried,data=bank)
 maritalmarried
```

```
1480 2520
         y+jobretired,data=bank)
 jobretired
 3824
         54
   467
      (~y+loan,data=bank)
 loan
0 3352
        648
   478
         43
          +poutcomesuccess,data=bank)
 poutcomesuccess
0 3954
         46
         83
```

Then we compute the McFadden's pseudo R<sup>2</sup> using attributes of the logistic regression. Here is the outcome.

## [1] 0.3115455

After that, we compute the p values of overall fitting test. Here is the result.

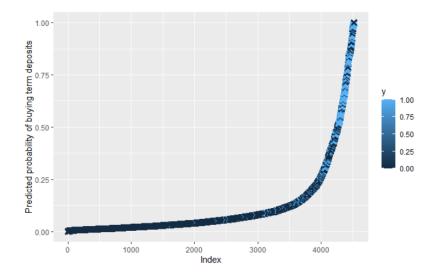
```
> 1 - pchisq(2*(ll.proposed - ll.null), df=(length(a$coefficien
ts)-1))
[1] 0
> 1 - pchisq((a$null.deviance - a$deviance), df=(length(a$coeff
icients)-1))
[1] 0
```

It appears that both p values indicate that the joint null hypothesis is rejected. That means the overall fitting is somewhat significant. And coefficients are significant.

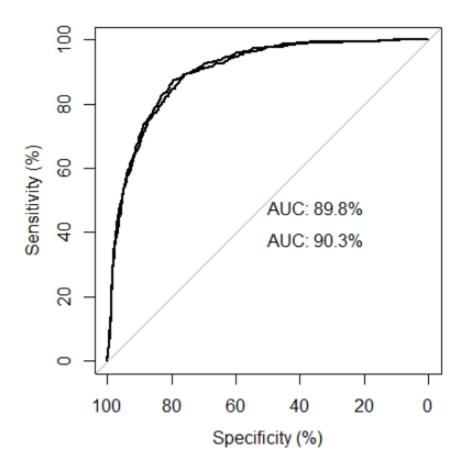
Then we make predictions based on the resulting model selected by the BIC. And we also got the confusion matrix here. Here is the result.

```
Predicted 0 Predicted 1 Total
Actual 0 3920 80 4000
Actual 1 341 180 521
Total 4261 260 4521
```

This matrix can describe the power of prediction to some extent. Then we tried to use graphs to show the prediction of probabilities about buying the term deposit product.



This plot shows the result of prediction and the truth. You can see that blue points mean that this client bought the product in truth while black dots means not buying in reality. While the x-axis represents the sample size, the y-axis means the probabilities of buying predicted from the model. As you can see, the model has some power in prediction. Later, we drew curves of precision and recall for both models above. Here is the outcome.



It turns out that both curves overlapped to some extent. That means their differences are minor while the former one has higher AUC OF 90.3%. But it also has higher BIC. On the other hand, the curve also indicates good performances of these models. Then we used another way to get the confusion matrix which is the same as that above. But this one has analysis with more details. And we set that probabilities over 0.5 indicate buying while probabilities below 0.5 mean not buying. Here it is.

```
Confusion Matrix and Statistics
            Reference
Prediction
             bought not bought
  bought
                180
  not bought
                341
                          3920
               Accuracy: 0.9069
                          (0.898, 0.9152)
                 95% CI :
    No Information Rate: 0.8848
    P-Value [Acc > NIR] : 9.283e-07
                  Kappa : 0.4161
 Mcnemar's Test P-Value : < 2.2e-16
            Sensitivity: 0.34549
                         0.98000
            Specificity
         Pos Pred Value:
                          0.69231
         Neg Pred Value: 0.91997
             Prevalence : 0.11524
                          0.03981
         Detection Rate:
   Detection Prevalence :
                          0.05751
      Balanced Accuracy: 0.66274
       'Positive' Class : bought
```