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Abstract

This paper studies the predictability of cryptocurrencies time series. We compare several alternative univariate and multivariate models in point and density forecasting of four of the most capitalized series: Bitcoin, Litecoin, Ripple and Ethereum. We apply a set of crypto–predictors and rely on Dynamic Model Averaging to combine a large set of univariate Dynamic Linear Models and several multivariate Vector Autoregressive models with different forms of time variation. We find statistical significant improvements in point forecasting when using combinations of univariate models and in density forecasting when relying on selection of multivariate models.
Keywords: Cryptocurrency; Bitcoin; Forecasting; Density Forecasting; VAR; Dynamic Model Averaging

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1. Introduction

Bitcoin is the first decentralized cryptocurrency created in 2009 and documented in Nakamoto (2009). Since its introduction, it has gained a growing attention from the media, academics, and finance industry, and in recent months the global interest in Bitcoin and cryptocurrencies has spiked dramatically. The reasons for this are several, just to name few: Japan and South Korea have recognized Bitcoin as a legal method of payment (Bloomberg, 2017a; Cointelegraph, 2017); some central banks are exploring the use of the cryptocurrencies (Bloomberg, 2017c); a large number of companies and banks created the Enterprise Ethereum Alliance[1] to make use of the cryptocurrencies and the related technology called blockchain, (Forbes, 2017). Finally the Chicago Mercantile Exchange (CME) started the Bitcoin futures on 18th of December 2017, see Chicago Mercantile Exchange (2017), Nasdaq and Tokyo Financial Exchange will follow in 2018, see (Bloomberg, 2017b; Tokyo Financial Exchange, 2017).

This interest has been reflected on the cryptocurrencies market capitalization that exploded from around 19 billion in February 2017 to around 800 billion in December 2017 and more than 1000 cryptocurrencies. Although Bitcoin can be considered to be relatively new, there has already been

[1]Source: https://entethalliance.org/members/

some initial analysis into the cryptocurrency. Hencic and Gourieroux (2014) applied a non–causal autoregressive model to detect the presence of bubbles in the Bitcoin/USD exchange rate. Sapuric and Kokkinaki (2014) measure volatility of Bitcoin exchange rate against six major currencies. Chu et al. (2015) provide a statistical analysis of the log–returns of the exchange rate of Bitcoin versus the USD. Catania and Grassi (2018) provide a new model to analyze the main characteristics of the crypto-currency volatility. Hotz-Behofsits et al. (2018) apply a time-varying parameter VAR with t-distributed measurement errors and stochastic volatility.

Despide all this effort a detailed analysis of the forecasting performances of different models to this series has not been provided yet. This paper tries to fill this gap and compares a large set of different models for point and density forecasting of four of the most capatilized cryptocurrencies, precisely: Bitcoin, Litecoin, Ripple and Ethereum. We compare univariate autoregressive models to univariate linear regression models based on a large set of crypto–predictors. The predictors include commodity prices, other financial assets such as stock prices and bond prices, and volatility indices to proxy market sentiments, following evidence in Bianchi (2018) that returns on cryptocurrencies are mild correlated (in in-sample analysis) with commodities and few more financial assets. Moreover, we apply dynamic selection of the large set of models based on our predictor lists using dynamic model selection (DMS) and dynamic averaging of the same model set using dynamic model averaging (DMA) proposed by Raftery et al. (2010). DMS and DMA have been found to provide forecasting gains in macroeconomic applications, see for example Koop and Korobilis (2011) and Koop and Korobilis (2012), and have not yet applied to cryptocurrencies. Then, we generalize the exercise to multivariate models where we predict jointly the four series using Vector Autoregressive (VAR) models, Bayesian VAR, time–varying parameters and stochastic volatility VAR models as in Koop and Korobilis (2013), selection and averaging of these models with different degrees of smoothness and different set of predictors. See, among other, Stambaugh (1999), Pastor (2000), Pastor and Stambaugh (2000) and Barberis (2000) for the use of multivariate modelling and Bayesian inference in asset predictions and allocation; Dangl and Halling (2012) for application of model averaging to stock price prediction; and Johannes et al. (2014) for time-varying parameters and stochastic volatility VAR models for stock price prediction. We extend this methodology to cryptocurrencies and enlarge the model set by allowing for different sources of time variation and model uncertainty.

In total, we have 24 class of models and combine in the univariate up to 2'621'440 models and in the multivariate case up to 4 time-varying VAR models. We separate univariate analysis from multivariate analysis and in the former one we predict and report results separately for each cryptocurrencies; in the letter one we predict and evaluate forecast jointly for the four cryptocurrencies giving information for building portfolios of cryptocurrencies. We consider prediction from one day ahead to seven days (one week) ahead.

Our results show that DMA and DMS of a large set of models provide forecasting gains in terms of point forecasting relative to the autoregressive benchmark. Gains are economically and statistically significant for Bitcoin at short horizons, one and two days ahead, up to 4% and for Ethereum for several horizons up to 4%. Evidence is weaker for Litecoin and Ripple. When focusing on density forecasting, gains in predicting Bitcoin and Ethereum disappear, even if moderate improvements emerge for Litecoin and Ripple. Therefore, combinations of a large set of predictors increase point forecast accuracy for the major currencies but do not improve density forecasting.

The evidence is opposite when focusing on multivariate models. Very few models provide marginally more accurate point predictions and only for longer horizons, but generally predictability does not emerge. However, when the complete distribution is predicted, most of the multivariate schemes offer statically significant gains at all horizons. In particular, selection of time–varying VARs with different set of predictors and different level of smoothness provide the largest gains. Our finding corroborates and extends evidence in Hotz-Behofsits et al. (2018), which also find with their time-varying parameter VAR sizeable improvements in density forecasting but not on point forecasting, and Catania et al. (2018), which show that (more) sophisticated volatility models can improve volatility predictions at different forecast horizons, to a large set of multivariate models.

The remaining of the paper proceeds as follows. Section 2 provides details on cryptocurrencies and crypto–predictors. Section 3 presents our univariate and multivariate models. Section 4 presents the metrics used to assess our results and explains in details the major findings, and finally Section 5 concludes.

2. Dataset description

2.1. Cryptocurrency

The data used in this study are the cryptocurrencies closing log returns. The crypto–market is open 24 hours a day, seven days a week; hence for computing returns we use the closing price at midnight (UTC). Those data are freely available from CoinMarketcup.[2]

Since the introduction of Bitcoin in 2009, hundreds of other cryptocurrencies have been created and, as of January 2018, 1440 cryptocurrencies exist. The analysis and forecast of such a big dataset is outside the scope of this paper. Here we focus on four major cryptocurrencies: i) Bitcoin, ii) Ethereum, iii) Ripple, and iv) Litecoin.

Bitcoin is the most popular and prominent cryptocurrency based on the decentralization and cryptography. The decentralization means that the Bitcoin network is controlled and owned by all of its users, who must adhere to the same set of rules. The cryptography controls money creation (fixed

[2]https://coinmarketcap.com/

to a maximum of 21 million coins) and transactions, no central bank is needed, see Nakamoto (2009). This decentralized nature offers many advantages, such as being free from government control and regulation, but critics often argue that apart from its users, there is nobody overlooking the whole system and that the value of Bitcoin is unfounded. Despite that, since its creation from a starting price of few cents in 2010, it touched 12015 USD in December 2017 and at the time of writing is above 10000 USD.

Ethereum is a decentralized platform that runs smart contracts which facilitates online contractual agreements applications that run away any possibility of downtime, censorship, fraud, or third party interference. The Ethereum also provides a cryptocurrency token called Ether which can be transferred between accounts and used to compensate participant nodes for computations performed, see Ethereum (2014).

Ripple, developed by the banking industry in 2012, is a blockchain network which incorporates a payment system and a currency system known as XRP. It enables banks to send real–time international payments across them and for this reason is currently used by many banks such as UBS, Santander, and Standard Chartered among others, see Ripple (2012).

Litecoin has been created in 2011 and is based on the same protocol used by Bitcoin. For this reason it is often considered Bitcoin's leading rival. It has one main feature which distinguishes it from Bitcoin: it is significantly faster regarding transactions, and it is particularly attractive in time–critical situations, see Litecoin (2014).

We collect data in the sample between August 8, 2015 to December 28, 2017, for a total of 874 daily observations and compute percentage daily log returns. Table 1 reports some descriptive statistics for the cryptocurrencies and Table A.1 in Appendix A describes data transformation. Series are far from being normal distributed as documented in Chu et al. (2015) and display high volatility, non-zero skewness and very high kurtosis. Series present several spikes too, see Figure 1.

2.2. Crypto-predictors

Currently, cryptocurrencies are mainly considered as alternative investment since their use as payment is still limited. This can create correlations with other assets for at least two main reasons. The first reason is that investors usually allocate wealth in a global portfolio and hedge across investments; the second reason is that market sentiments spread fast among different assets. See Bianchi (2018) for similar arguments.

Our list of crypto–predictors includes international stock index prices, precisely S&P 500, Nikkei 225, Stoxx Europe 600; commodity prices, precisely gold and silver prices; interest rates and CDS, precisely 5-year Europe credit default swap and 1-Month US Treasury and 10-year US 10-Year Treasury rates; volatility index, VIX closing price. At midnight (UTC) when we compute daily

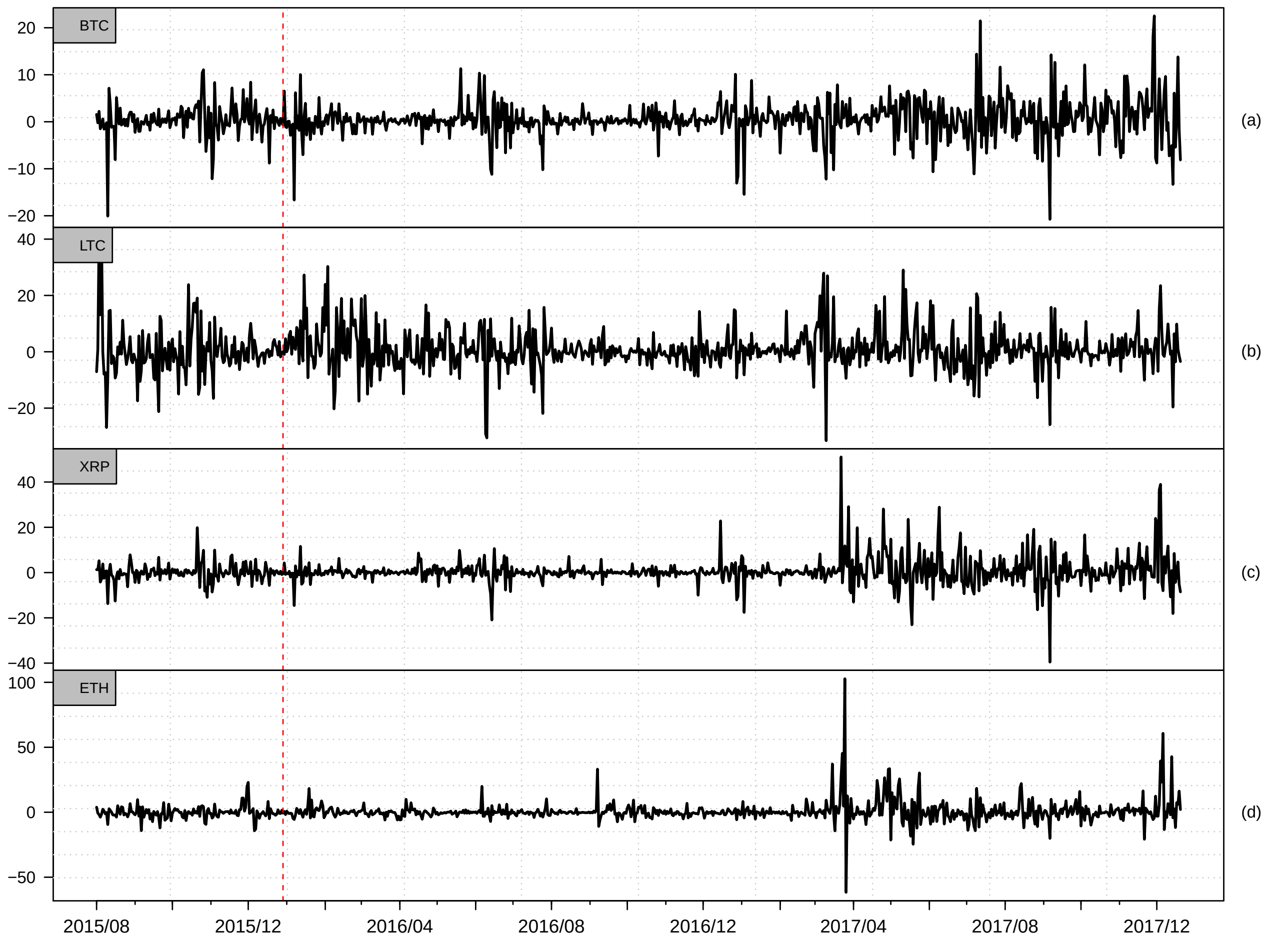


Figure 1: *The plots show the four crypto–currencies daily percentage log returns considered in this study: Bitcoin (BTC) in panel (a); Litecoin (LTC) in panel (b); Ripple (XRP) in panel (c); and Ethereum (ETH) in panel (d). The dashed horizontal red line indicates the beginning of the out-of-sample period on the 6th June, 2016. The full sample spans from 9th August, 2015 to 28th December, 2017 for a total of 873 observations.*

crypto–returns all series are available. See Table A.1 in Appendix A for data transformation.

We also apply lags of each and other cryptocurrencies, and a transformation of previous day cryptocurrencies, labelled in the paper crypto–explicative, to account for intra-day patterns by taking the difference between the highest and the lowest price, as a proxy of cryptocurrencies volatility.

3. Competing models

In this section, we introduce our different models to forecast daily cryptocurrencies. As anticipated, we consider univariate and multivariate models with and without exogenous variables and selection and combinations of such models. For a full list of univariate models see Table 2 and of multivariate models see Table 3. We compare both univariate and multivariate models to the autoregressive of order 1, AR(1), benchmark.[3]

[3]We also compute forecasts using a random walk model and economically and statistical evidence is almost identical to the AR(1). Results are available upon request.

Coin	Bitcoin	Ethereum	Ripple	Litecoin
Created	03-Jan-09	01-Aug-14	01-Jul-13	01-Nov-13
Supply	21 Millions Total	18 Millions Yearly	100 Billions Total	84 Millions Total
Market Cap	277 Billions	466 Billions	27 Billions	15.5 Billions
Maximum	22.512	41.234	51.035	102.736
Minimun	-20.753	-130.211	-39.515	-61.627
Mean	0.453	0.639	0.467	0.591
Median	0.318	-0.051	0.000	-0.338
Std Dev.	3.84	8.535	5.785	7.743
Skewness	-0.091	-3.721	1.637	3.767
Kurtosis	9.391	67.442	19.091	50.455

Table 1: *Descriptive statistics for the four larger cryptocurrencies by market capitalization calculated between 08/08/2015 to 28/12/2017, for a total of 874 observations. The table reports the name of the coins, the creation date, the maximum number of coins in Million (Mil.) and Billion (Bil.) and the market capitalization as in December 2017 reported in https://coinmarketcap.com/. The Ethereum has a total supply of 18 Millions coins per year, the other three have a total prefix amount.*

3.1. Univariate models

Linear regression models. Our data set includes 13 different crypto–predictors including macro and finance variables and crypto–explicative for each series. We apply a linear regression where we include lags of the dependent variable and all predictors, labelled KS, and a restricted version where we only include lags of the cryptocurrencies and crypto–explicative, labelled KS-noreg.

Model combinations. The previous linear equations can suffer of massive model uncertainty. Indeed, a model with 16 predictors and up to 3 lags of the dependent variable results in more than 524'288 possible combinations.[4] To mitigate it, we propose to apply model combination techniques.

In Raftery et al. (2010) introduce an estimation technique to predict the output strip thickness for a cold rolling mill, which they refer to as DMA. Recently, DMA has also shown to be useful in macroeconomic and financial applications see Koop and Korobilis (2011) and Koop and Korobilis (2012).

To provide more details on the underlying mechanism of DMA, we start by assuming that any combination of the elements on the right–hand–side of a linear regression can be expressed as a Dynamic Linear Models (DLM), see West and Harrison (1999) and Raftery et al. (2010). Particularly, let $\mathbf{F}_t^{(i)}$denote a $p \times 1$ vector based on a given combination of our total predictors, $\mathbf{F}_t = (1, y_{t-1}, y_{t-2}, y_{t-3}, x_{1,t-1}, \ldots, x_{16,t-1})'$. Then, we can express our i–th DLM as:

$$\begin{aligned} y_t &= \mathbf{F}_t^{(i)\prime} \boldsymbol{\beta}_t^{(i)} + \varepsilon_t^{(i)}, \quad \varepsilon_t^{(i)} \sim N\left(0, V_t^{(i)}\right) \\ \boldsymbol{\beta}_t^{(i)} &= \boldsymbol{\beta}_{t-1}^{(i)} + \boldsymbol{\eta}_t^{(i)}, \quad \boldsymbol{\eta}_t^{(i)} \sim N\left(0, \mathbf{Q}_t^{(i)}\right), \end{aligned} \tag{1}$$

[4]We also include an intercept term in all models, hence we have a total of 19 predictors resulting in $2^{19} = 524'288$ combinations.

where the $p \times 1$ vector of time–varying regression coefficients, $\boldsymbol{\beta}_t^{(i)} = \left(\beta_{1t}^{(i)}, \ldots, \beta_{pt}^{(i)}\right)'$, determines the impact of $\mathbf{F}_t^{(i)}$ on y_t. The Random Walk specification of $\boldsymbol{\beta}_t^{(i)}$ do not assume any systematic movements but consider changes in $\boldsymbol{\beta}_t^{(i)}$ as unpredictable.

The conditional variances, $V_t^{(i)}$ and $\mathbf{Q}_t^{(i)}$, are unknown quantities. We assume an Inverted–gamma prior for $V_0^{(i)}$ and update it's estimate at each point in time via its posterior distribution as in Prado and West (2010). Regarding $\mathbf{Q}_t^{(i)}$ we employ the forgetting factor approach detailed in Dangl and Halling (2012). We allow λ to take one of the $d = 5$ values in the grid $\{0.91, 0.93, 0.95, 0.97, 0.99\}$ and augment the number of possible models by to $k = 2^{19}d = 2'621'440$.[5] Notably, when $\mathbf{Q}_t^{(i)} = \mathbf{0}$ for $t = 1, \ldots, T$, then $\boldsymbol{\beta}_t^{(i)}$ is constant over time. Thus, (1) nests the specification of constant regression coefficients. For $\mathbf{Q}_t^{(i)} \neq \mathbf{0}$, $\boldsymbol{\beta}_t^{(i)}$ varies according to Equation 1. However, this does not mean that $\boldsymbol{\beta}_t^{(i)}$ needs to change at every time period, we can easily have periods where $\mathbf{Q}_t^{(i)} = \mathbf{0}$ and thus $\boldsymbol{\beta}_t^{(i)} = \boldsymbol{\beta}_{t-1}^{(i)}$. Ultimately, the nature of time variation in the regression coefficients is dependent on the data at hand.

DMA then averages forecasts across the k different combinations using a recursive updating scheme based on the predictive likelihood, that measures the ability of a model to predict y_t, thus making it the central quantity of interest for model evaluation. Besides averaging, we can also use the model receiving the highest probability among all model combinations to forecasts, this is the so called DMS, see Koop and Korobilis (2012). For both DMA and DMS, we also include restricted versions with only lags of the dependent variable, see Table 2 for more details. Estimation and prediction is made exploiting the **eDMA** package for R of Catania and Nonejad (2018).

Abbreviation	*Full Description*
AR(1)	Autoregressive model of order one, benchmark model.
KS	Kitchen Sink specification, *i.e.*, a linear multiple regression including all variables.
KS–noregr	Kitchen Sink specification with only the lagged values of the series as covariates.
DMA	Dynamic Model Averaging across all models and forgetting factor combinations. See Dangl and Halling (2012).
DMS	Dynamic Model Selection, selecting at each t the best model between: all models and forgetting factor combinations. See Dangl and Halling (2012).
DMA–noregr	Dynamic Model Averaging with only the lagged values of the series as covariates.
DMS–noregr	Dynamic Model Selection with only the lagged values of the series as covariates.

Table 2: *Univariate models considered in the forecasting exercise. The first column is the model's abbreviation. The second column provides a brief description of each individual model.*

3.2. Multivariate models

Constant parameter VARs. The first class of multivariate models we consider is the constant parameter Vector autogressive (VAR) specification. VARs are among the most common models

[5] Dangl and Halling (2012) refer to this parameter as δ.

applied in financial and macroeconomic forecasting, see among others Ltkepohl (2007) and Koop and Korobilis (2010). We have four-variate VARs with three lags selected using BIC. We apply two version of them: a frequentist VAR estimated using OLS and a Bayesian VAR (BVAR) as in Koop and Korobilis (2013).

Time–varying parameter specifications. Cryptocurrencies are subject to several instabilities, both in mean and at higher moments. Large parametrized constant parameter VAR models might fail to capture these instabilities and we extend the model set with 14 different time–varying specifications.

The starting point for the analysis is the time-varying parameter vector autoregression model (TVP-VAR) as described in Koop and Korobilis (2013) (henceforth KK). KK provide a new approach to estimate large–dimensional TVP-VAR. Focusing on the case where the VAR has one lag and intercepts are suppressed, the TVP-VAR(1) model can be written as:

$$\mathbf{y}_t = \mathbf{T}_t \mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{2}$$

with $\mathbf{y}_t$ an $(M \times 1)$ vector containing observations on M time series variables, T_t a $(M \times M)$ matrix containing M^2 parameters and $\boldsymbol{\varepsilon}_t \sim N(0, \Sigma_t)$. KK close the model by specifying dynamics for the time-varying VAR parameters and rewrite the model in state space form:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{F}_t \boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim \mathrm{N}(0, \boldsymbol{\Sigma}_t) \\ \boldsymbol{\beta}_{t+1} &= \boldsymbol{\beta}_t + \boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t \sim \mathrm{N}(\mathbf{0}, \mathbf{Q}_b) \end{aligned} \tag{3}$$

where the $(M \times M^2)$ matrix $\mathbf{F}_t$ collects lagged observations, and the time-varying vector $\boldsymbol{\beta}_t$ captures the time-varying VAR parameters (with $\boldsymbol{\beta}_t = vec(\mathbf{T}_t') = vecr(\mathbf{T}_t)$) which are taken random walks with innovations $\boldsymbol{\eta}_t \sim \mathrm{N}(\mathbf{0}, \mathbf{Q}_t)$. As for the constant parameter VARs, we consider three lags of the dependent variables.

As the model is in state space form, the usual estimation approaches based on the Kalman filter seems attractive. For example, one could use the frequentist approach of maximizing the likelihood, or the Bayesian approach of using Markov Chain Monte Carlo (MCMC) methods. Unfortunately, for large-dimensional VARs these standard approaches turn out to be unfeasible. For a VAR of dimension 7 and a model with 4 lags the amount of time-varying parameters would be $4 \times 7^2 = 196$, making either approach computationally very demanding.

To reduce the computational burden, KK propose to make two adjustments such that the usual Kalman filter can still be used. The parameters of the model are in the variance matrices $\mathbf{Q}_t$ and $\boldsymbol{\Sigma}_t$, the idea of KK is to take $\mathbf{Q}_t$ out of the model and replace it by an approximation. In this case $\boldsymbol{\beta}_t$ can be obtained using closed-form expressions without having to maximize a likelihood first in order to get parameter estimates (or do so using MCMC methods). Typically the latent state innovation variance $\mathbf{Q}_t$ enters the Kalman filter in the updating step, where the state variance matrix is updated through $\mathbf{P}_{t|t} = \mathbf{P}_{t-1|t-1} + \mathbf{Q}_t$ (see Durbin and Koopman, 2012). If one instead writes $\mathbf{P}_{t|t} = \frac{1}{\lambda}\mathbf{P}_{t-1|t-1}$ (for some λ), an estimate of $\mathbf{Q}_t$ is no longer necessary. This is often referred to as a forgetting factor set-up. The second adjustment is to replace the measurement error variance matrix $\mathbf{\Sigma}_t$ by an estimate using an exponentially weighted moving average (EWMA) filter. The EWMA filtering recursion $\hat{\mathbf{\Sigma}}_t = \kappa\hat{\mathbf{\Sigma}}_{t-1} + (1-\kappa)\bar{\varepsilon}_t\bar{\varepsilon}_t'$ (with $\bar{\varepsilon}_t = \mathbf{y}_t - \mathbf{F}_t\boldsymbol{\beta}_{t|t-1}$) gives the measurement error variance estimates, which we can plug into the filter. From the discussion above this methodology requires the specification of the hyperparameters λ and κ (and the specification of the initial condition of the states $\boldsymbol{\beta}_0$ and $\mathbf{\Sigma}_0$), we refer to KK for an extensive discussion of the problem.

As explained in KK we carry out model selection using a model space involving all the variable reported in Table A.1. We cluster them in four different database and have consequently four TVP-VARs of different size: a small TVP-VAR with only the four cryptocurrencies; a medium TVP-VAR with the four cryptocurrencies series plus the four crypto–explicative; a second medium TVP-VAR with the four cryptocurrencies series plus the seven financial and macro variables for a total of eleven dependent variables; and a larger TVP-VAR with all the seventeen variables. As described in KK the algorithm selects between the four TVP-VARs based on past predictive likelihoods for the set of variables the researcher is interested in forecasting, allowing for model switching accordingly to an ad hoc hyperparameter α. Moreover, as described in KK the forgetting factor λ can be dynamically selected together with the optimal value of the shrinkage parameter at different points in time. This results in 14 different models, labelled $\mathcal{M}_4$–$\mathcal{M}_{18}$ in Table 3.

4. Empirical Section

Our results are based on one, two, three, four, five, six and seven day-ahead forecasting process using an expanding window and an initial insample period of 146 days for Bitcoin, Litecoin, Ripple and Ethereum. Multistep ahead predictions are obtained through direct forecasting, see Marcellino et al. (2006). Hence, our forecast evaluation period is from January 1, 2016 to December 28, 2017. In all the analysis we consider an autoregressive model of order 1, AR(1), as benchmark and also compute forecasts using univariate and multivariate models described in the previous section. We discuss forecast metrics in 4.1 and present univariate results in 4.2 and multivariate results 4.3. For the univariate analysis, we present results for each cryptocurrency separately; for multivariate application we provide joint results in line with model predictions.[6]

[6] Separate statistics for each currency are reported in Appendix B.

Abbreviation	*Full Description*
$\mathcal{M}_1$	Autoregressive model of order one, benchmark model.
$\mathcal{M}_2$	Vector Autoregressive with 3 lags estimated using the OLS.
$\mathcal{M}_3$	Baysian Vector Autoregressive with 3 lags as described in Koop and Korobilis (2010).
$\mathcal{M}_4$	TVP-VAR(3) with 4 cryptocurrencies series and stochastic volatility. Optimal value of the shrinkage parameter is selected using DMS as outlined in Koop and Korobilis (2013). In this model λ is dynamically selected and $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_5$	TVP-VAR(3) with 4 cryptocurrencies series and stochastic volatility. Optimal value of the shrinkage parameter is selected using DMS as outlined in Koop and Korobilis (2013). In this model $\lambda = 0.99$, $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_6$	TVP-VAR(3) with 4 cryptocurrencies series and stochastic volatility. Optimal value of the shrinkage parameter is selected using DMS as outlined in Koop and Korobilis (2013). In this model λ is dynamically selected and $\kappa = 0.99$ and $\alpha = 0.001$.
$\mathcal{M}_7$	TVP-VAR(3) with two models: the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus crypto-explicative. Stochastic volatility is also present. Models and optimal value of the shrinkage parameter are selected using DMS as outlined in Koop and Korobilis (2013). In this model λ is dynamically selected and $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_8$	TVP-VAR(3) with two models: the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus crypto-explicative. Stochastic volatility is also present. Models and optimal value of the shrinkage parameter are selected using DMA as outlined in Koop and Korobilis (2013). In this model λ is dynamically selected and $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_9$	TVP-VAR(3) with two models: the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus crypto-explicative. Stochastic volatility is also present. Models and optimal value of the shrinkage parameter are selected using DMS as outlined in Koop and Korobilis (2013). In this model $\lambda = 0.99$, $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_{10}$	TVP-VAR(3) with two models: the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus crypto-explicative. Stochastic volatility is also present. Models and optimal value of the shrinkage parameter are selected using DMA as outlined in Koop and Korobilis (2013). In this model $\lambda = 0.99$, $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_{11}$	TVP-VAR(3) with two models: the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus macroeconomic variables. Stochastic volatility is also present. Models and optimal value of the shrinkage parameter are selected using DMS as outlined in Koop and Korobilis (2013). In this model λ is dynamically selected and $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_{12}$	TVP-VAR(3) with two models: the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus macroeconomic variables. Stochastic volatility is also present. Models and optimal value of the shrinkage parameter are selected using DMA as outlined in Koop and Korobilis (2013). In this model λ is dynamically selected and $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_{13}$	Time varying parameters VAR(3) with two models: the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus macroeconomic variables. Stochastic volatility is also present. Models and optimal value of the shrinkage parameter are selected using DMS as outlined in Koop and Korobilis (2013). In this model $\lambda = 0.99$, $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_{14}$	TVP-VAR(3) with two models: the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus macroeconomic variables. Stochastic volatility is also present. Models and optimal value of the shrinkage parameter are selected using DMA as outlined in Koop and Korobilis (2013). In this model $\lambda = 0.99$, $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_{15}$	TVP-VAR(3) with four models: the first uses the 4 cryptoseries, the second uses cryptocurrencies plus crypto-explicative, the third uses cryptocurrencies plus macroeconomic variables. The fourth uses all the series. Stochastic volatility is also present. Models and optimal value of the shrinkage parameter are selected using DMS as outlined in Koop and Korobilis (2013). Here λ is dynamically selected, $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_{16}$	TVP-VAR(3) with four models:the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus crypto-explicative, the third uses cryptocurrencies plus macroeconomic variables. The fourth uses all the series. Stochastic volatility is also present. Models and optimal value of the shrinkage parameter are selected using DMA as outlined in Koop and Korobilis (2013). Here λ is dynamically selected and $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_{17}$	TVP-VAR(3) with four models: the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus crypto-explicative, the third uses cryptocurrencies plus macroeconomic variables. The fourth uses all the series. Stochastic volatility is also present. . Models and optimal value of the shrinkage parameter are selected using DMS as outlined in Koop and Korobilis (2013). Here λ is dynamically selected, $\kappa = 0.99$ and $\alpha = 0.99$.
$\mathcal{M}_{18}$	TVP-VAR(3) with four models: the first uses the 4 cryptocurrencies series, the second uses cryptocurrencies plus crypto-explicative, the third uses cryptocurrencies plus macroeconomic variables. The fourth uses all the series. Stochastic volatility is also present. . Models and optimal value of the shrinkage parameter are selected using DMA as outlined in Koop and Korobilis (2013). Here $\lambda = 0.99$, $\kappa = 0.99$ and $\alpha = 0.99$.

Table 3: *The table reports all the multivariate models considered in the forecasting exercise. The first column is the model's abbreviation. The second column provides a brief description of each model.*

4.1. Forecast metrics

We assess the goodness of our forecasts using different point and density metrics. Considering the accuracy of point forecasts, we use the mean squared errors (MSEs) for each of the forecast horizon, $h = 1, \cdots, 7$ we consider.[7] For the univariate exercise, the metric is computed seperately for each cryptocurrency series, $i =$ Bitcoin, Litecoin, Ripple and Etherum:

$$\mathrm{MSE}_{i,h} = \frac{1}{T-R} \sum_{t=R}^{T-h} \left(\hat{y}_{i,t+h|t} - y_{i,t+h}\right)^2, \tag{4}$$

where T is the number of observations, R is the length of the rolling window, $\hat{y}_{i,t+h|t}$ is the ith-cryptocurrency forecasts made at time t for horizon h and $y_{i,t+h}$ is the realization. For the multivariate application, we compute a average squared errors for each forecast and an average MSE as:

$$\mathrm{MSE}_{h} = \frac{1}{(T-R)} \sum_{t=R}^{T-h} \frac{1}{4} \sum_{i=1}^{4} \left(\hat{y}_{i,t+h|t} - y_{i,t+h}\right)^2. \tag{5}$$

To evaluate density forecasts, we use predictive log score (LS). The LS is commonly viewed as the broadest measure of density accuracy, see Geweke and Amisano (2010). As for the MSE, we compute it for each horizon and series separately in the univariate application

$$s_{i,h}(y_i) = \sum_{t=R}^{T-h} \ln\left(f(y_{i,t+h}|I_{i,t})\right), \tag{6}$$

where $f(y_{i,t+h}|I_{i,t})$ is the predictive density for $y_{i,t+h}$ constructed using information up to time t. The multivariate version is

$$s_{h}(y) = \sum_{t=R}^{T-h} \ln\left(f(\mathbf{y}_{t+h}|I_{t})\right), \tag{7}$$

where $f(\mathbf{y}_{t+1}|I_t)$ is the joint predictive density for the 4-variate $\mathbf{y}_{t+h}$ constructed using information up to time t. For the AR, we assume a joint distribution composed by the four independent marginal predictions; therefore we assume a diagonal variance-covariance matrix.

More specifically, we report the MSEs and the LSs for the AR benchmark model. For the other models, we report the ratios of each model's MSE to the baseline AR model, such that entries less than 1 indicate that the given model yields forecasts more accurate than those from the baseline and differences in score relative to the AR baseline, such that a positive number indicates a model beats the baseline. In order to statistically assess the differences among alternative models, we apply Diebold and Mariano (1995) t-tests for equality of the average loss (with loss defined as squared error and negative log score) of each movel versus the AR benchmark and we also employ the Model

[7] In Appendix B we also report mean absolute deviations (MADs).

Confidence Set procedure of Hansen et al. (2011) using the R package MCS detailed in Bernardi and Catania (2016) to compare jointly all predictions. Differences are tested separately for each horizon h.

4.2. Univariate forecasting results

Table 4 reports mean squared errors for predicting the four cryptocurrencies using univariate models.[8] Largest gains are found when using DMA for predicting Bitcoin at shorter horizons and Ethereum at all horizons. For Bitcoin, DMA gives statistically significant reductions at one and two day-ahead of 4% and 2% respectively. Considering the large volatility of the series and that we focus on a daily forecast horizons, forecast gains are economically sizeable, above all when compared to other high volatile assets such as stock prices and exchange rates. At both horizons the AR benchmark is not included in the model set confidence. Precisely, only models based on dynamic averaging, DMA and DMA-noreg, are included in the model set.

Table 6 shows the inclusion probability of the most probable five crypto–predictors. For Bitcoin, one of the other cryptocurrencies or of the crypto–explicative is always included, but also other assets, such as VIX and silver at one day-ahead horizon, both bonds and SP_500 at two day-ahead horizon, have large positive probabilities. In general, correlation evidence in Bianchi (2018) are confirmed, but there is also large uncertainty on which predictors shall be included and several variables receive large probability, underlying the importance of combining them.

For the other models, straightforward linear regressions based on direct forecasting, labelled KS and KS-noreg in the Table, do not seem a credible strategy, with very different performances across horizons and possible enormous losses, see for example statistics at the second horizon.

When focusing on Ethereum, we find evidence of statistically superior predictability of alternative models to the benchmark at several horizons. DMA performs accurately at one day ahead with an 3% reduction in MSE; and DMA–noreg provides economically sizeable gains at five horizons over seven and statistically significant gains at three horizons over seven. Furthermore, it is always included in the model confidence set. Interestingly, a combination of its own lags and lags of other currencies is a more valuable strategy for Ethereum than Bitcoin, somewhat confirming its central role as larger exchanger of currencies in the crypto–system and therefore highly connected to several currencies, more than the leading role of the Bitcoin that often drives movements in the crypto–market. Indeed, inclusion probabilities in Table 6 show that probabilities for macro and finance crypto–predictors become more relevant for longer horizons when DMA deteriorates performance relative to DMA–noreg. For Litecoin and Ripple, predictability is weaker and model averaging seems to reduce accuracy we have found with Ethereum and Ripple even if Table 6 indicates still large uncertainty among

[8]See Table B.2 in Appendix B for MAD scores.

h	1	2	3	4	5	6	7
				Bitcoin			
AR1	42.49	42.28	41.54	41.62	41.55	41.42	41.12
KS	1.52	12.25	1.84	**0.96**	1.06	1.06	1.12
KS-noreg	2.07	7.03	1.61	1.02	1.01	1.03	1.01
DMA-noreg	**0.96**	**0.98**	0.99	1.00	1.00	0.99	1.00
DMS-noreg	**0.97**	1.00	0.99	1.03	1.02	1.01	1.02
DMA	**0.97**	**0.97**	1.01	1.04	1.02	1.02	1.13
DMS	1.01	1.02	1.06	1.06	1.02	1.05	1.15
				Litecoin			
AR1	134.27	132.88	133.05	133.43	133.60	133.25	131.71
KS	1.02	1.17	7.64	1.01	1.17	1.11	1.09
KS-noreg	**0.96**	1.03	1.88	1.00	1.01	1.02	1.00
DMA-noreg	0.99	1.03	1.04	1.02	1.02	1.05	1.05
DMS-noreg	1.01	1.04	1.06	1.04	1.02	1.04	1.04
DMA	0.98	1.03	1.09	1.11	1.03	1.06	1.15
DMS	1.00	1.07	1.11	1.11	1.04	1.09	1.22
				Ripple			
AR1	224.02	221.31	222.02	221.13	218.93	219.62	219.45
KS	1.11	1.24	1.27	1.10	1.76	1.21	2.01
KS-noreg	1.03	1.04	1.02	1.01	1.08	1.00	1.02
DMA-noreg	0.99	1.03	1.03	1.05	1.06	1.00	1.05
DMS-noreg	1.02	1.03	1.04	1.07	1.08	1.03	1.08
DMA	1.20	1.03	1.22	1.25	1.10	1.18	1.17
DMS	1.27	1.05	1.22	1.26	1.11	1.21	1.21
				Ethereum			
AR1	180.57	174.99	175.61	175.56	175.79	175.90	174.08
KS	1.05	12.72	1.09	1.01	1.02	1.67	1.09
KS-noreg	1.01	3.40	1.02	1.00	1.00	1.01	1.00
DMA-noreg	**0.96**	1.00	0.98	**0.97**	**0.98**	0.98	1.00
DMS-noreg	0.98	1.01	1.01	0.99	0.99	1.00	1.01
DMA	0.97	1.01	1.03	1.01	1.04	1.04	1.04
DMS	1.02	1.04	1.08	1.05	1.05	1.09	1.04

Table 4: *Mean squared error (MSE), computed over the forecast horizon. Results are reported relative to the benchmark specification (AR1) for which the absolute score is reported. Models' description is reported in Table 2. Values in* ***bold****, indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold–Mariano test at the* 5% *confidence level. Gray cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level* 10%.

predictors. Only KS models provide economical and statistical gains at some horizons, but a clear pattern does not exist. Their lower capitalization relative to Bitcoin and Ethereum results in lower correlations with other assets and less accurate predictions. New predictors based on crypto–market sentiments might be considered to investigate heterogeneity across cryptocurrencies.

Table 5 provides log score results for univariate models. Evidence is different than for point forecasting. Gains vanishes for prediction of Bitcoin and Ethereum with no models providing an higher score than the benchmark and in the case of Ethereum the AR model is the only specification

h	1	2	3	4	5	6	7
				Bitcoin			
AR1	−884.29	−885.08	−876.44	−874.76	−872.13	−870.71	−865.93
KS	−384.99	−1733.72	−383.63	−18.42	−16.18	−54.75	−29.44
KS-noreg	−579.10	−1371.64	−420.92	−9.53	6.52	−11.00	10.75
DMA-noreg	−20.42	−1.06	−15.81	−10.13	−11.05	−5.49	−5.43
DMS-noreg	−23.00	−13.25	−24.20	−39.24	−34.63	−13.71	−27.47
DMA	−24.94	−8.41	−28.80	−21.42	−7.05	−7.73	−9.29
DMS	−42.82	−40.67	−64.87	−100.95	−101.66	−67.71	−138.00
				Litecoin			
AR1	−909.55	−909.03	−906.77	−903.98	−901.54	−898.99	−894.19
KS	**105.90**	**58.05**	−1253.89	**135.78**	**58.31**	34.51	**125.68**
KS-noreg	**141.25**	**110.27**	−256.93	**150.40**	**133.68**	**123.37**	**156.28**
DMA-noreg	**120.22**	**120.52**	**81.40**	**114.41**	**140.68**	**140.48**	**125.26**
DMS-noreg	**108.91**	**103.53**	**73.45**	**79.45**	**119.50**	**126.34**	**89.72**
DMA	**132.07**	**122.20**	**102.71**	64.61	**101.14**	**134.11**	**84.64**
DMS	**91.53**	**105.62**	**69.44**	−24.63	45.05	**102.23**	−63.56
				Ripple			
AR1	−772.26	−766.65	−765.52	−763.71	−762.24	−759.32	−748.10
KS	**97.18**	−0.83	17.67	**107.70**	−260.45	−24.84	−488.58
KS-noreg	**141.42**	**115.97**	**145.36**	**125.82**	**86.93**	**152.24**	**145.89**
DMA-noreg	**142.93**	**132.62**	**141.18**	**108.81**	**111.68**	**141.85**	**97.90**
DMS-noreg	**114.99**	**130.49**	**116.91**	**94.92**	**95.26**	**130.15**	**75.24**
DMA	−5.56	**76.59**	−0.87	27.06	**80.77**	60.29	**69.71**
DMS	−68.25	**70.99**	−48.87	−19.64	**73.09**	25.38	**67.49**
				Ethereum			
AR1	−611.31	−604.34	−606.14	−604.58	−602.32	−601.87	−600.36
KS	−228.53	−1972.69	−243.49	−212.21	−215.65	−578.28	−222.97
KS-noreg	−206.94	−881.72	−198.98	−187.75	−192.16	−193.34	−187.38
DMA-noreg	−205.45	−232.79	−201.42	−198.84	−199.22	−204.81	−201.60
DMS-noreg	−212.74	−238.07	−210.52	−202.34	−200.69	−206.75	−202.01
DMA	−220.16	−244.29	−219.43	−218.75	−212.86	−220.10	−217.00
DMS	−224.69	−250.47	−237.06	−219.91	−228.38	−230.53	−224.63

Table 5: *Log Score (LS), computed over the forecast horizon. Results are reported relative to the benchmark specification (AR1) for which the absolute score is reported. Models' description is reported in Table 2. Values in* ***bold****, indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold–Mariano test at the* 5% *confidence level. Gray cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level* 10%*.*

included in the model confidence set at all horizons. However, evidence reverts for Litecoin and Ripple with several models outperforming the AR models at all horizons. In particular, DMA-noreg and DMS–noreg improve accuracy at all horizons for both series. Univariate DMA even if it allows for time–varying volatility seems to fail to capture dynamics of cryptocurrency higher moments. Catania et al. (2018) find that sophisticated univariate models are required to produce accurate forecasts of cryptocurrency volatility. The next subsection investigates (time–varying) multivariate models for prediction of cryptocurrency returns.

$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$	$h=7$
			bitcoin			
ETH(62)	ETH_HL(37)	Lag3(70)	LTC(44)	LTC(40)	VIX(42)	CDS_5y(45)
VIX(44)	ETH(13)	Lag4(69)	CDS_5y(40)	ETH_HL(39)	NK_225(38)	SV(41)
ETH_HL(41)	BD_10y(12)	SV(57)	Lag6(36)	NK_225(38)	BTC_HL(37)	BD_10y(40)
XRP_HL(31)	SP_500(12)	ETH(43)	SP_500(36)	Lag5(38)	GLD(37)	Lag8(37)
SV(25)	BD_1m(11)	ES_600(43)	BD_10y(35)	VIX(35)	XRP(34)	LTC_HL(37)
			litecoin			
ES_600(46)	ETH_HL(60)	Lag5(76)	SV(40)	BTC(50)	SP_500(60)	LTC_HL(46)
ETH_HL(45)	SP_500(57)	SV(47)	Lag4(39)	ETH(49)	BTC(59)	SP_500(40)
SP_500(44)	ES_600(52)	ETH(35)	GLD(37)	ES_600(47)	ES_600(58)	ES_600(39)
NK_225(42)	Lag3(52)	BTC(31)	ETH_HL(36)	SV(46)	GLD(58)	SV(39)
BD_1m(41)	NK_225(48)	XRP(27)	ES_600(33)	NK_225(45)	Lag8(56)	XRP_HL(39)
			ripple			
XRP_HL(64)	SP_500(59)	SV(49)	LTC_HL(45)	ES_600(50)	SP_500(59)	SV(55)
Lag2(50)	VIX(55)	CDS_5y(46)	BTC(32)	BTC_HL(47)	XRP_HL(58)	ES_600(54)
Lag1(48)	NK_225(52)	ETH_HL(44)	Lag6(30)	BTC(47)	BD_10y(55)	BTC(53)
NK_225(46)	GLD(52)	ES_600(43)	ETH(29)	LTC(46)	SV(50)	NK_225(53)
VIX(45)	Lag3(51)	LTC(42)	VIX(28)	ETH_HL(46)	BD_1m(46)	BD_1m(53)
			ethereum			
Lag3(50)	Lag2(52)	GLD(42)	BD_1m(37)	BTC_HL(42)	CDS_5y(37)	NK_225(38)
XRP_HL(43)	Lag3(47)	BD_10y(40)	ES_600(35)	BTC(35)	Lag8(35)	CDS_5y(36)
BD_1m(42)	Lag4(40)	SP_500(40)	NK_225(34)	NK_225(34)	BTC_HL(34)	Lag7(35)
LTC_HL(40)	BTC(24)	Lag3(39)	SP_500(33)	ETH_HL(34)	Lag7(33)	ES_600(32)
GLD(39)	ETH_HL(23)	ETH_HL(38)	LTC(33)	LTC(34)	BTC(32)	BTC(31)

Table 6: *Top 5 crypto predictors for different cryptocurrencies and forecast horizon. Number in brackets is the average (%) inclusion probability of the selected predictor over the forecasting period.*

4.3. Multivariate forecasting results

Tables 7 and 8 report MSE and predictive log score for the multivariate models. The evidence is striking and results are almost opposite to the univariate case in terms of forecast metrics: no model provides economic gains when point forecasting cryptocurrencies; several models provide large gains when density forecasting cryptocurrencies. Focusing on MSE results, simpler constant–parameter VAR and BVAR specifications, labelled $\mathcal{M}_2$ and $\mathcal{M}_3$ respectively, are very imprecise at short horizons with losses up to 20%, but they perform more similar to the AR(1) at longer horizons with mild improvements for $\mathcal{M}_2$. Time–varying specifications provide more similar performance across horizons but they are never superior to the benchmark, even if several of them are included in the 5% model confidence set.

Focusing on predictive log score, most of the models in Table 8 provide statistically superior forecasts relative to the benchmark at almost all horizons. Model $\mathcal{M}_9$, selection among a model with only cryptocurrencies and a model with also crypto–explicative; model $\mathcal{M}_{13}$, selection among a model

h	1	2	3	4	5	6	7
$\mathcal{M}_1$	21.66	21.68	21.72	21.74	21.92	22.07	22.22
$\mathcal{M}_2$	1.12	1.11	1.07	1.01	**0.99**	**0.99**	**0.99**
$\mathcal{M}_3$	1.22	1.08	1.02	1.02	1.00	1.00	1.00
$\mathcal{M}_4$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_5$	1.01	1.00	1.01	1.00	1.00	1.00	1.00
$\mathcal{M}_6$	1.02	1.00	1.01	**1.00**	1.00	1.00	1.00
$\mathcal{M}_7$	1.01	1.01	1.00	1.00	1.01	1.00	1.00
$\mathcal{M}_8$	1.00	1.01	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_9$	1.01	1.02	1.01	1.01	1.01	1.00	1.01
$\mathcal{M}_{10}$	1.01	1.01	1.01	1.00	1.00	1.00	1.01
$\mathcal{M}_{11}$	1.00	1.00	1.00	1.01	1.01	1.00	1.00
$\mathcal{M}_{12}$	1.00	1.01	1.01	1.00	1.00	1.00	1.00
$\mathcal{M}_{13}$	1.01	1.01	1.01	1.01	1.00	1.00	1.01
$\mathcal{M}_{14}$	1.01	1.01	1.01	1.01	1.00	1.00	1.00
$\mathcal{M}_{15}$	1.00	1.00	1.00	1.01	1.01	1.00	1.00
$\mathcal{M}_{16}$	1.00	1.00	1.00	1.00	1.00	0.99	1.00
$\mathcal{M}_{17}$	1.01	1.01	1.02	1.01	1.01	1.01	1.01
$\mathcal{M}_{18}$	1.01	1.01	1.01	1.01	1.01	1.01	1.01

Table 7: *(Multivariate) Mean Squared Error, computed over the forecast horizon. Results are reported relative to the benchmark specification (AR1) for which the absolute score is reported. Models' description is reported in Table 3. Values in* ***bold****, indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold–Mariano test at the* 5% *confidence level. Gray cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level* 10%.

with only cryptocurrencies and a model with also macroeconomic variables; model $\mathcal{M}_{17}$, selection among a model with only cryptocurrencies, a model with also crypto–explicative, a model with also macro predictors and a model with all variables give the higest gains. In particular $\mathcal{M}_{17}$ is included in the 5% model confidence set in six horizons over seven and at two day-ahead and four day-ahead horizons it is the only model in the confidence set. At three day-ahead horizon it is not included, but model $\mathcal{M}_9$ and $\mathcal{M}_{13}$ are the only two included. As in the univariate case, crypto–explicative and macro and financial predictors improve forecast accuracy. However, differently than the univariate case, a selection of models containing clusters of them, instead of averaging predictors, provides the largest gains. We speculate that the importance of crypto–predictors differs across currencies and a flexible multivariate combination scheme which allows for very different weights across series and clusters of predictors could improve accuracy, see for example Casarin et al. (2018).

When focusing on performance over time, Figure 2 reports the cumulative predictive log score over time relative to the benchmark for three different horizons, h=1, 4 and 7. At each point in time, a positive number indicates that the alternative model outperforms the benchmark. Plots show that DMS of TVP-VAR models provides constant gains relative to the AR benchmark over all the out-of-sample period and in some circumstances the increase is very large, such as at the end of March 2017 when all currencies experienced a break in volatility, see Figure 1. Models based on DMA also

h	1	2	3	4	5	6	7
$\mathcal{M}_1$	−3478.72	−3478.98	−3487.65	−3492.26	−3502.20	−3508.01	−3513.73
$\mathcal{M}_2$	**129.77**	**165.02**	**186.89**	**237.95**	**318.04**	**284.02**	**283.34**
$\mathcal{M}_3$	42.25	**155.45**	**278.58**	**283.21**	**301.68**	**303.23**	**301.74**
$\mathcal{M}_4$	**694.76**	**511.70**	**451.84**	**443.52**	**400.53**	**413.30**	**387.20**
$\mathcal{M}_5$	**684.82**	**506.99**	**456.74**	**449.26**	**418.59**	**424.72**	**385.97**
$\mathcal{M}_6$	**672.21**	**530.81**	**478.04**	**448.88**	**463.85**	**443.22**	**407.06**
$\mathcal{M}_7$	**706.90**	**532.33**	**474.76**	**459.00**	**452.44**	**423.65**	**388.44**
$\mathcal{M}_8$	**364.12**	**289.74**	**225.30**	202.69	173.61	160.91	109.54
$\mathcal{M}_9$	**723.90**	**556.84**	**530.91**	**503.06**	**514.39**	**451.97**	**426.99**
$\mathcal{M}_{10}$	**395.03**	**316.48**	**250.84**	**243.64**	**212.34**	197.99	153.79
$\mathcal{M}_{11}$	**717.53**	**529.22**	**479.58**	**457.88**	**448.87**	**444.57**	**367.63**
$\mathcal{M}_{12}$	12.82	−68.29	−138.84	−172.83	−197.46	−210.15	−254.22
$\mathcal{M}_{13}$	**726.15**	**550.67**	**529.80**	**514.83**	**509.57**	**473.75**	**407.17**
$\mathcal{M}_{14}$	48.83	−2.64	−75.41	−99.42	−124.50	−132.56	−177.41
$\mathcal{M}_{15}$	**697.97**	**549.74**	**493.45**	**487.39**	**440.75**	**443.40**	**397.99**
$\mathcal{M}_{16}$	−304.17	−390.70	−449.44	−498.01	−531.88	−551.27	−591.65
$\mathcal{M}_{17}$	**723.08**	**621.99**	**517.05**	**533.95**	**517.43**	**500.67**	**443.85**
$\mathcal{M}_{18}$	−296.44	−327.60	−411.81	−429.96	−456.00	−476.11	−516.87

Table 8: *(Multivariate) Log Score (LS), computed over the forecast horizon. Results are reported relative to the benchmark specification (AR1) for which the absolute score is reported. Models' description is reported in Table 3. Values in* ***bold****, indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold–Mariano test at the* 5% *confidence level. Gray cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level* 10%.

outperform the benchmark, but gains are lower. VAR ($\mathcal{M}_2$) and BVAR ($\mathcal{M}_3$) provide some gains in the initial part of the sample, but their performance deteriorates substantially with instability in March 2017. Only the BVAR at long horizons maintains part of its predictability power at long horizons, but scores are substantially lower than DMS alternatives.

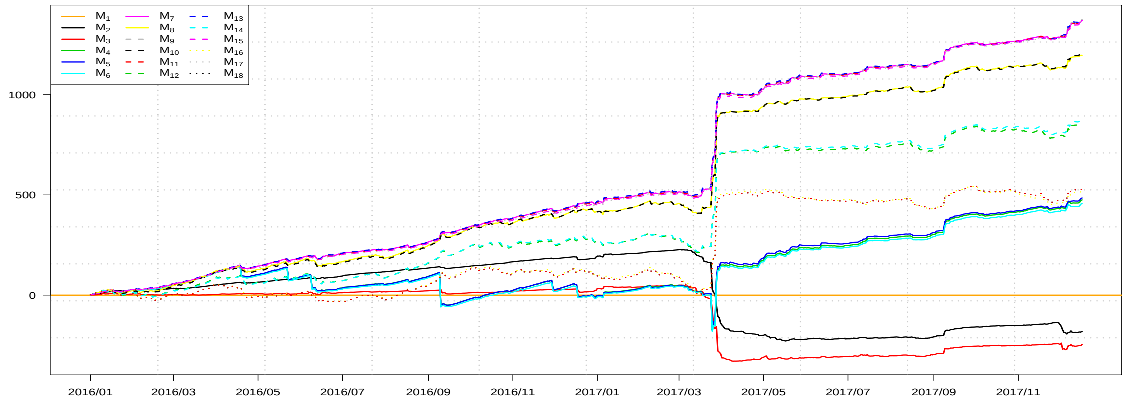
As final check, we report in Tables B.4-B.7 in Appendix B log score results for multivariate models in predicting each cryptocurrency separately. Previous evidence is confirmed for each currency and gains are not a result of the models performing well just for a subset of asset.

5. Conclusions

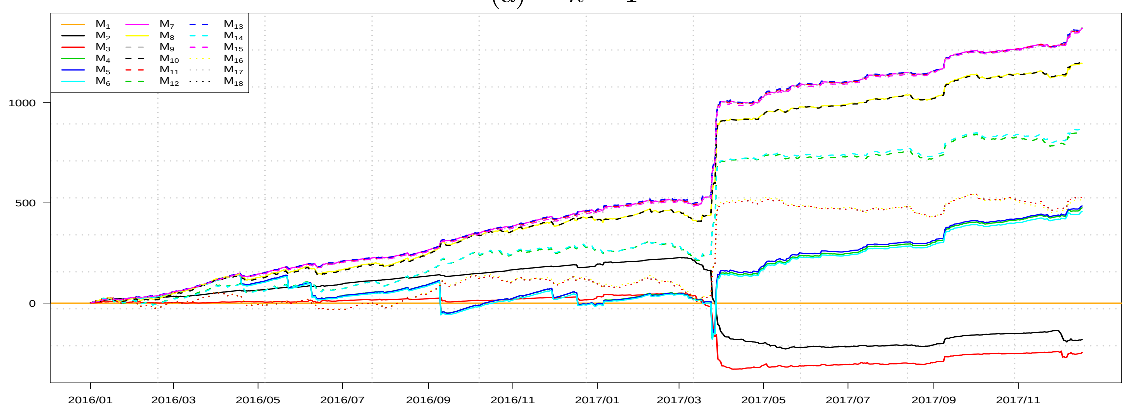
This paper compares several alternative univariate and multivariate models for predicting four of the most capitalized cryptocurrencies, Bitcoin, Litecoin, Ripple and Ethereum. A set of crypto–predictors is applied and univariate and multivariate model combinations are proposed to combine these predictors. The results show large statistical significant improvements in point forecasting of Bitcoin and Ethereum when using combinations of univariate models and in density forecasting for all the cryptocurrencies when relying on selection of time–varying multivariate models.

We believe that our analysis opens various research agenda in predicting cryptocurrencies. For example, flexible multivariate combination schemes which allow for different weights across series

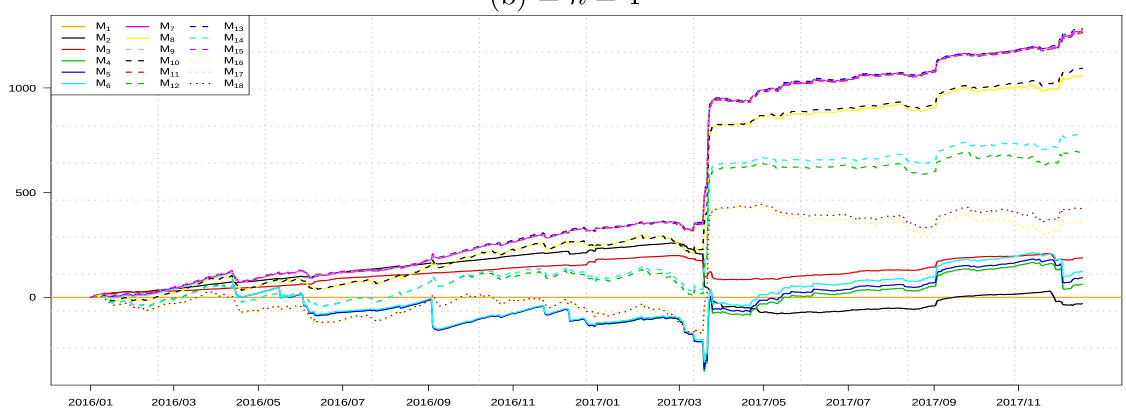
Figure 2: *Cumulative Log Score*



(a) = $h = 1$



(b) = $h = 4$



(c) = $h = 7$

The plots show cumulative log score relative to the AR(1) benchmark ($\mathcal{M}_1$), computed over one day forecast horizon (panel (a)), four day forecast horizon (panel (b)) and seven day forecast horizon (panel (c)).

could improve point and density forecast accuracy. Moreover, new predictors based on crypto–market sentiments might be considered to investigate heterogeneity across cryptocurrencies and could result in (point) forecast gains across all series.

References

Barberis, N. (2000). Investing for the Long Run when Returns Are Predictable. *The Journal of Finance*, 55:225 – 264.

Bernardi, M. and Catania, L. (2016). Portfolio Optimisation Under Flexible Dynamic Dependence Modelling. *ArXiv e-prints*.

Bianchi, D. (2018). Cryptocurrencies As an Asset Class? An Empirical Assessment. Technical report, SSRN Working Paper.

Bloomberg (2017a). Japan's BITPoint to Add Bitcoin Payments to Retail Outlets. https://www.bloomberg.com/news/articles/2017-05-29/japan-s-bitpoint-to-add-bitcoin-payments-to-100-000s-of-outlets.

Bloomberg (2017b). Nasdaq Plans to Introduce Bitcoin Futures. .

Bloomberg (2017c). Some Central Banks Are Exploring the Use of Cryptocurrencies. https://www.bloomberg.com/news/articles/2017-06-28/rise-of-digital-coins-has-central-banks-considering-e-versions.

Casarin, R., Grassi, S., Ravazzolo, F., and van Dijk, H. K. (2018). Predictive Density Combinations with Dynamic Learning for Large Data Sets in Economics and Finance. Technical report.

Catania, L. and Grassi, S. (2018). Modelling Crypto–Currencies Financial Time–Series. Technical report, SSRN Working paper.

Catania, L., Grassi, S., and Ravazzolo, F. (2018). Predicting the Volatility of Cryptocurrency Time-Series. Technical report, MIMEO.

Catania, L. and Nonejad, N. (2018). Dynamic Model Averaging for Practitioners in Economics and Finance: The eDMA Package. *Journal of Statistical Software, (forecaming).*

Chicago Mercantile Exchange (2017). CME Group Announces Launch of Bitcoin Futures. .

Chu, J., Nadarajah, S., and Chan, S. (2015). Statistical Analysis of the Exchange Rate of Bitcoin. *PloS one*, 10:1–27.

Cointelegraph (2017). South Korea Officially Legalizes Bitcoin, Huge Market For Traders. https://cointelegraph.com/news/south-korea-officially-legalizes-bitcoin-huge-market-for-traders.

Dangl, T. and Halling, M. (2012). Predictive Regressions with Time–Varying Coefficients. *Journal of Financial Economics*, 106:157–181.

Diebold, F. and Mariano, R. (1995). Comparing Predictive Accuracy. *Journal of Business and Economic Stastistics*, 13:253–263.

Durbin, J. and Koopman, S. (2012). *Time Series Analysis by State Space Methods*. Oxford University Press, Oxford, UK., 2nd edition.

Ethereum (2014). Ethereum Wiki. https://github.com/ethereum/wiki/wiki/White-Paper/.

Forbes (2017). Emerging Applications For Blockchain. https://www.forbes.com/sites/forbestechcouncil/2017/07/18/emerging-applications-for-blockchain.

Geweke, J. and Amisano, G. (2010). Comparing and evaluating Bayesian predictive distributions of asset returns. *International Journal of Forecasting*, 26:216–230.

Hansen, P. R., Lunde, A., and Nason, J. M. (2011). The Model Confidence Set. *Econometrica*, 79:453–497.

Hencic, A. and Gourieroux, C. (2014). Noncausal Autoregressive Model in Application to Bitcoin/USD Exchange Rate. *Proceedings of the 7th Financial Risks International Forum*, pages 1–25.

Hotz-Behofsits, C., Huber, F., and Zorner, T. O. (2018). Predicting crypto-currencies using sparse non–Gaussian state space models. Technical report, arXiv 1801.06373v1, forthcoming in the Journal of Forecasting.

Johannes, M., Korteweg, A., and Polson, N. (2014). Sequential Learning, Predictive Regressions, and Optimal Portfolio Returns. *Journal of Finance*, 69:611–644.

Koop, G. and Korobilis, D. (2010). Forecasting Inflation Using Dynamic Model Averaging. *Bayesian Multivariate Time Series Methods for Empirical Macroeconomics*, 3:267–358.

Koop, G. and Korobilis, D. (2011). UK Macroeconomic Forecasting with Many Predictors: Which Models Forecast Best and When Do They Do So? *Economic Modelling*, 28:2307–2318.

Koop, G. and Korobilis, D. (2012). Forecasting Inflation Using Dynamic Model Averaging. *International Economic Review*, 53:867–886.

Koop, G. and Korobilis, D. (2013). Large Time-Varying Parameter VARs. *Journal of Econometrics*, 177:185–198.

Litecoin (2014). Litecoin Wiki. https://litecoin.info/Litecoin/.

Ltkepohl, H. (2007). *New Introduction to Multiple Time Series Analysis.* Springer Publishing Company, Incorporated.

Marcellino, M., Stock, J. H., and Watson, M. W. (2006). A comparison of direct and iterated multistep ar methods for forecasting macroeconomic time series. *Journal of Econometrics*, 135(1-2):499 – 526.

Nakamoto, S. (2009). Bitcoin: A Peer-to-Peer Electronic Cash System. https://Bitcoin.org/Bitcoin.pdf.

Pastor, L. (2000). Portfolio Selection and Asset Pricing Models. *Journal of Finance*, 55:179–223.

Pastor, L. and Stambaugh, R. F. (2000). Comparing Asset Pricing Models: an Investment Perspective. *Journal of Financial Economics*, 56:335–381.

Prado, R. and West, M. (2010). *Time series: Modeling, Computation, and Inference.* CRC Press, Boca Raton.

Raftery, A. E., Kárnỳ, M., and Ettler, P. (2010). Online Prediction Under Model Uncertainty via Dynamic Model Averaging: Application to a Cold Rolling Mill. *Technometrics*, 52:52–66.

Ripple (2012). Welcome to Ripple. https://ripple.com/.

Sapuric, S. and Kokkinaki, A. (2014). Bitcoin is Volatile! Isn't That Right? *Business Information Systems Workshops*, pages 255–265.

Stambaugh, R. F. (1999). Predictive regressions. *Journal of Financial Economics*, 54:375 – 421.

Tokyo Financial Exchange (2017). Tokyo Financial Exchange Plans for Bitcoin Futures Launch. .

West, M. and Harrison, J. (1999). *Bayesian Forecasting & Dynamic Models.* Springer–Verlag, Berlin.

Appendix A. Data transformation

Data Overview		
Abbreviation	Full name	Transformation
Cryptocurrencies time series		
BTC	Bitcoin	First difference of Log
ETH	Ethereum	First difference of Log
XRP	Ripple	First difference of Log
LTC	Litecoin	First difference of Log
Additional crypto–explicative time series		
BTC_HL	Bitcoin High minus Bitcoin Low	Log
ETH_HL	Ethereum High minus Ethereum Low	Log
XRP_HL	Ripple High minus Ripple Low	Log
LTC_HL	Litecoin High minus Litecoin Low	Log
Additional financial and macro time series		
CDS_5y	Europe credit default swap index 5 years	First difference of Log
ES_600	Stoxx Europe 600 - Price Index	First difference of Log
GLD	Gold Bullion LBM	First difference of Log
NK_225	Nikkei 225 Stock Average - Price Index	First difference of Log
SP_500	S&P 500 Composite - Price Index	First difference of Log
SV	Silver Handy & Harman Base Price	First difference of Log
BD_1m	1-Month US Treasury Constant Maturity Rate	First difference
BD_10y	10-Year US Treasury Constant Maturity Rate	First difference
VIX	VIX closing price	Log

Table A.1: *This table provides an overview of the data we use in the paper. The table reports the main four cryptocurrencies used in the paper. The series are available over the period August 8, 2015 to December 28, 2017. The table also reports the additional cryptocurrency time series e.g. Bitcoin High minus Bitcoin Low together with the macro and financial time series used in the study. For each series the table reports the abbreviation we use in the paper, the full name of the series, and the transformation applied on the raw data series.*

Appendix B. Results for alternative forecasting metrics

h	1	2	3	4	5	6	7
				Bitcoin			
AR1	10.88	10.82	10.63	10.66	10.64	10.60	10.53
KS	2.13	115.05	2.51	**0.91**	1.06	1.03	1.15
KS-noreg	3.24	43.78	2.27	1.02	1.03	1.07	1.02
DMA-noreg	**0.95**	**0.97**	1.01	1.03	1.03	0.99	1.04
DMS-noreg	**0.95**	1.00	1.01	1.07	1.05	1.03	1.05
DMA	**0.90**	**0.95**	1.01	1.15	1.02	1.05	1.31
DMS	0.97	1.03	1.11	1.21	1.02	1.10	1.33
				Litecoin			
AR1	21.99	21.76	21.79	21.85	21.88	21.82	21.57
KS	**0.96**	1.24	44.67	**0.97**	1.22	1.10	1.12
KS-noreg	**0.97**	1.06	3.44	0.99	1.00	1.03	0.99
DMA-noreg	1.04	1.09	1.06	1.05	1.04	1.11	1.12
DMS-noreg	1.06	1.11	1.11	1.06	1.06	1.10	1.09
DMA	0.99	1.07	1.17	1.26	1.09	1.12	1.34
DMS	1.01	1.13	1.24	1.19	1.06	1.16	1.60
	0.88	**0.89**					
				Ripple			
AR1	27.14	26.81	26.89	26.79	26.52	26.60	26.58
KS	1.23	1.44	1.46	1.22	2.60	1.24	3.19
KS-noreg	1.07	1.07	1.04	0.99	1.14	**0.95**	1.02
DMA-noreg	0.99	1.09	1.07	1.14	1.19	0.97	1.16
DMS-noreg	1.06	1.07	1.06	1.20	1.21	1.05	1.22
DMA	1.61	1.11	1.70	1.75	1.32	1.46	1.51
DMS	1.77	1.12	1.65	1.81	1.36	1.56	1.74
				Ethereum			
AR1	25.93	25.13	25.22	25.21	25.25	25.26	25.00
KS	1.06	153.01	1.13	0.98	1.00	2.47	1.08
KS-noreg	0.99	6.92	1.02	**0.97**	**0.98**	1.00	**0.98**
DMA-noreg	**0.88**	1.00	**0.94**	**0.91**	**0.91**	**0.93**	0.97
DMS-noreg	**0.90**	0.99	0.98	**0.93**	**0.92**	0.96	0.97
DMA	**0.89**	1.03	1.00	0.96	1.06	1.06	1.05
DMS	0.97	1.09	1.10	1.02	1.07	1.16	1.01

Table B.2: *Mean absolute deviation (MAD), computed over the forecast horizon. Results are reported relative to the benchmark specification (AR1) for which the absolute score is reported. Models' description is reported in Table 2. Values in* ***bold****, indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold–Mariano test at the* 5% *confidence level. Gray cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level* 10%.

h	1	2	3	4	5	6	7
$\mathcal{M}_1$	3.08	3.08	3.08	3.09	3.10	3.11	3.12
$\mathcal{M}_2$	1.06	1.05	1.03	1.01	1.00	1.00	1.00
$\mathcal{M}_3$	1.12	1.04	1.01	1.01	1.00	1.00	1.00
$\mathcal{M}_4$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_5$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_6$	1.01	1.01	1.01	1.00	1.00	1.00	1.00
$\mathcal{M}_7$	1.00	1.01	1.00	1.00	1.01	1.00	1.00
$\mathcal{M}_8$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_9$	1.00	1.01	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_{10}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_{11}$	1.00	1.00	1.00	1.00	1.01	1.00	1.00
$\mathcal{M}_{12}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_{13}$	1.01	1.00	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_{14}$	1.01	1.00	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_{15}$	1.01	1.01	1.00	1.01	1.01	1.01	1.01
$\mathcal{M}_{16}$	1.00	1.01	1.00	1.00	1.00	1.00	1.00
$\mathcal{M}_{17}$	1.01	1.00	1.01	1.01	1.01	1.00	1.01
$\mathcal{M}_{18}$	1.01	1.01	1.01	1.01	1.01	1.00	1.01

Table B.3: *(Multivariate) Mean Absolute Error, computed over the forecast horizon. Results are reported relative to the benchmark specification (AR1) for which the absolute score is reported. Models' description is reported in Table 3. Values in* ***bold****, indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold–Mariano test at the* 5% *confidence level. Gray cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level* 10%.

h	1	2	3	4	5	6	7
$\mathcal{M}_1$	−876.09	−876.76	−877.37	−877.44	−884.73	−885.18	−888.04
$\mathcal{M}_2$	−7.18	3.80	−3.17	2.64	**5.84**	**2.55**	1.69
$\mathcal{M}_3$	−63.36	−24.54	−28.78	−16.28	−17.51	−12.89	−11.57
$\mathcal{M}_4$	**77.79**	**74.96**	**74.14**	**65.74**	**65.59**	**63.36**	**56.92**
$\mathcal{M}_5$	**74.87**	**76.18**	**70.55**	**60.88**	**63.09**	**59.46**	**58.48**
$\mathcal{M}_6$	**78.34**	**69.26**	**62.10**	**65.42**	**64.76**	**62.86**	**57.55**
$\mathcal{M}_7$	**76.46**	**69.07**	**72.04**	**59.34**	**66.49**	**58.07**	**54.18**
$\mathcal{M}_8$	36.46	20.11	18.73	8.41	11.60	−15.47	−19.80
$\mathcal{M}_9$	**70.56**	**72.01**	**65.85**	**62.17**	**69.27**	**58.49**	**57.06**
$\mathcal{M}_{10}$	30.69	34.24	32.60	15.56	10.15	−4.99	−8.38
$\mathcal{M}_{11}$	**79.82**	**71.53**	**68.84**	**60.63**	**66.56**	**63.26**	**55.26**
$\mathcal{M}_{12}$	−97.59	−119.69	−134.88	−145.26	−163.42	−182.32	−202.54
$\mathcal{M}_{13}$	**71.24**	**65.14**	**70.23**	**57.97**	**64.61**	**60.83**	**60.28**
$\mathcal{M}_{14}$	−82.33	−97.77	−106.70	−131.75	−137.77	−165.34	−178.96
$\mathcal{M}_{15}$	**70.73**	**71.43**	**68.34**	**60.72**	**66.61**	**63.79**	**55.62**
$\mathcal{M}_{16}$	−280.07	−282.17	−309.96	−331.16	−342.09	−360.59	−381.50
$\mathcal{M}_{17}$	**71.03**	**68.26**	**64.04**	**61.49**	**65.21**	**63.37**	**56.84**
$\mathcal{M}_{18}$	−257.19	−282.18	−299.56	−312.96	−355.97	−348.63	−353.54

Table B.4: *Bitcoin. Predictive Log Score (LS), computed over the forecast horizon. Results are reported relative to the benchmark specification (AR1). Models' description is reported in Table 2. Values in* ***bold****, indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold–Mariano test at the 5% confidence level. Gray cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level 10%.*

h	1	2	3	4	5	6	7
$\mathcal{M}_1$	−767.01	−764.97	−765.69	−765.77	−766.76	−767.21	−768.98
$\mathcal{M}_2$	−3.66	−1.87	−4.55	−2.83	−11.92	−4.52	−4.08
$\mathcal{M}_3$	−73.29	−23.70	−0.99	−0.70	6.26	2.66	0.85
$\mathcal{M}_4$	**113.42**	**95.24**	**90.64**	**82.94**	**77.47**	**80.49**	**79.38**
$\mathcal{M}_5$	**108.89**	**94.36**	**91.33**	**82.92**	**77.08**	**78.51**	**79.10**
$\mathcal{M}_6$	**110.94**	**92.16**	**90.69**	**82.69**	**75.63**	**76.57**	**77.28**
$\mathcal{M}_7$	**111.09**	**91.62**	**87.86**	**80.72**	**79.01**	**75.73**	**79.66**
$\mathcal{M}_8$	**120.02**	**92.77**	**70.55**	45.33	**49.40**	40.52	**54.89**
$\mathcal{M}_9$	**108.45**	**95.54**	**85.94**	**75.50**	**79.85**	**74.26**	**77.43**
$\mathcal{M}_{10}$	**116.89**	**97.96**	**84.52**	**52.72**	**63.88**	**60.01**	**57.59**
$\mathcal{M}_{11}$	**107.95**	**92.28**	**90.89**	**79.71**	**73.93**	**75.14**	**79.56**
$\mathcal{M}_{12}$	25.71	1.53	−29.88	−79.51	−110.80	−91.24	−83.66
$\mathcal{M}_{13}$	**106.47**	**95.31**	**88.36**	**75.24**	**75.99**	**77.56**	**75.53**
$\mathcal{M}_{14}$	30.84	18.07	−38.09	−58.62	−77.83	−56.30	−62.70
$\mathcal{M}_{15}$	**108.36**	**86.86**	**91.48**	**79.74**	**72.36**	**75.74**	**75.66**
$\mathcal{M}_{16}$	−72.90	−148.07	−176.85	−243.73	−255.74	−221.11	−249.88
$\mathcal{M}_{17}$	**107.25**	**87.21**	**84.83**	**77.66**	**76.04**	**76.45**	**76.63**
$\mathcal{M}_{18}$	−105.33	−117.89	−178.88	−209.26	−226.10	−252.94	−230.30

Table B.5: *Litecoin. Predictive Log Score (LS), computed over the forecast horizon. Results are reported relative to the benchmark specification (AR1). Models' description is reported in Table 2. Values in* ***bold****, indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold–Mariano test at the 5% confidence level. Gray cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level 10%.*

h	1	2	3	4	5	6	7
$\mathcal{M}_1$	−603.55	−600.59	−605.76	−606.77	−607.88	−611.34	−611.94
$\mathcal{M}_2$	−63.53	−65.14	−60.18	−8.37	**5.65**	4.08	2.39
$\mathcal{M}_3$	−71.22	−40.96	−19.64	−24.53	−13.99	−9.17	−9.28
$\mathcal{M}_4$	**69.15**	**48.74**	**42.50**	**40.79**	**43.47**	**46.14**	**45.46**
$\mathcal{M}_5$	**64.38**	**50.46**	**40.88**	**40.27**	**41.02**	**45.02**	**46.16**
$\mathcal{M}_6$	**55.39**	**50.72**	**38.69**	**43.20**	**41.58**	**45.09**	**48.91**
$\mathcal{M}_7$	**63.01**	**52.21**	**42.89**	**40.76**	**39.91**	**41.61**	**41.03**
$\mathcal{M}_8$	**98.08**	**73.92**	**51.54**	44.25	**48.03**	42.58	42.09
$\mathcal{M}_9$	**57.46**	**43.90**	**37.33**	**36.67**	**36.36**	**41.40**	**42.54**
$\mathcal{M}_{10}$	**103.79**	**71.86**	**53.67**	44.73	**56.69**	**54.15**	**57.61**
$\mathcal{M}_{11}$	**66.62**	**49.29**	**43.40**	**43.20**	**40.92**	**44.21**	**47.15**
$\mathcal{M}_{12}$	18.56	−6.65	−34.98	−43.00	−36.64	−40.04	−45.79
$\mathcal{M}_{13}$	**55.15**	**43.80**	**34.79**	**34.51**	**40.07**	**45.68**	**42.33**
$\mathcal{M}_{14}$	52.09	3.91	−30.94	−31.81	−12.71	−10.02	−23.29
$\mathcal{M}_{15}$	**62.15**	**43.78**	**46.13**	**42.04**	**39.58**	**45.13**	**39.53**
$\mathcal{M}_{16}$	−84.91	−104.22	−136.71	−158.44	−158.15	−141.25	−144.49
$\mathcal{M}_{17}$	**57.78**	**44.28**	**37.78**	**38.74**	**36.46**	**41.63**	**38.78**
$\mathcal{M}_{18}$	−63.05	−112.33	−138.49	−139.10	−154.96	−112.62	−122.29

Table B.6: *Ripple. Predictive Log Scores (LS), computed over the forecast horizon. Results are reported relative to the benchmark specification (AR1). Models' description is reported in Table 2. Values in* ***bold****, indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold–Mariano test at the 5% confidence level. Gray cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level 10%.*

h	1	2	3	4	5	6	7
$\mathcal{M}_1$	−906.59	−908.45	−908.15	−908.29	−907.92	−907.63	−907.24
$\mathcal{M}_2$	**69.96**	**79.13**	**87.61**	**98.82**	**99.23**	**99.55**	**101.28**
$\mathcal{M}_3$	**36.49**	**68.67**	**84.86**	**80.77**	**80.81**	**81.79**	**80.62**
$\mathcal{M}_4$	**179.47**	**175.62**	**168.81**	**162.80**	**158.85**	**157.37**	**157.34**
$\mathcal{M}_5$	**178.71**	**175.86**	**170.06**	**163.32**	**158.64**	**156.99**	**153.51**
$\mathcal{M}_6$	**169.02**	**168.41**	**167.10**	**160.62**	**158.95**	**156.60**	**155.77**
$\mathcal{M}_7$	**182.64**	**171.69**	**168.22**	**162.06**	**156.91**	**153.61**	**152.01**
$\mathcal{M}_8$	**136.15**	**117.50**	**111.20**	**92.17**	**92.63**	**80.97**	**81.78**
$\mathcal{M}_9$	**177.00**	**170.93**	**170.49**	**159.29**	**154.67**	**152.72**	**155.73**
$\mathcal{M}_{10}$	**136.75**	**131.59**	**124.51**	**118.97**	**100.11**	**91.11**	**94.30**
$\mathcal{M}_{11}$	**176.76**	**161.40**	**166.38**	**152.81**	**149.81**	**146.08**	**153.89**
$\mathcal{M}_{12}$	−15.11	−29.84	−37.84	−52.61	−71.38	−74.89	−87.89
$\mathcal{M}_{13}$	**170.48**	**170.89**	**169.05**	**156.51**	**156.92**	**152.68**	**151.31**
$\mathcal{M}_{14}$	0.52	−6.06	−23.49	−28.42	−45.68	−51.36	−51.01
$\mathcal{M}_{15}$	**169.74**	**160.08**	**157.84**	**146.28**	**149.88**	**145.13**	**147.50**
$\mathcal{M}_{16}$	−175.79	−192.04	−204.95	−251.08	−244.24	−263.94	−252.94
$\mathcal{M}_{17}$	**165.87**	**162.03**	**166.61**	**152.53**	**155.36**	**139.99**	**146.36**
$\mathcal{M}_{18}$	−149.92	−159.93	−205.19	−213.03	−212.76	−234.01	−227.41

Table B.7: *Ethereum. Predictive Log Score (LS), computed over the forecast horizon. Results are reported relative to the benchmark specification (AR1). Models' description is reported in Table 2. Values in* ***bold****, indicate rejection of the null hypothesis of Equal Predictive Ability between each model and the benchmark according to the Diebold–Mariano test at the 5% confidence level. Gray cells indicate those models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure at confidence level 10%.*

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