Exercise 1

(a) $\mathrm{supp}\left(e_{k}^{(j)}\right) = [2^{-j}.k, 2^{-j}.(k+1)] \quad \forall j,k \in \mathbb{Z}$

j: resolution, k: translation

where $e_k^{(j)} = 2^{j/2}.\, e(2^j.x-k) \quad \forall j,k \in \mathbb{Z}$

and $e(x) := \chi_{[0,1]}(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

$$e_k^{(j)} = 2^{j/2}\chi_{[0,1]}(2^j.x-k)$$

$$= 2^{j/2}\begin{cases} 1 & 0 \le 2^j x - k \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$= 2^{j/2}\begin{cases} 1 & k \le 2^j x \le 1+k \\ 0 & \text{otherwise} \end{cases}$$

$$= 2^{j/2}\begin{cases} 1 & \frac{k}{2^j} \le x \le \frac{(1+k)}{2^j} \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 2^{j/2} & 2^{-j}k \le x \le 2^{-j}(k+1) \\ 0 & \text{otherwise} \end{cases}$$

Thus $\mathrm{supp}\left(e_k^{(j)}\right) = [2^{-j}k, 2^{-j}(k+1)]$

Also, $\langle e_k^{(j)}, e_l^{(j)}\rangle_{L^2(\mathbb{R})} = \int e_k^{(j)} e_l^{(j)}\, dx = 0$

however if $k = l$ then $= 1$