1.a

$$M = \begin{pmatrix} x_1^1 & x_2^1 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_1^1 & x_2^1 & 0 & 1 \\ x_1^2 & x_2^2 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_1^2 & x_2^2 & 0 & 1 \\ x_1^3 & x_2^3 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_1^3 & x_2^3 & 0 & 1 \end{pmatrix}$$

$$\det M = x_1^1 \cdot \det \begin{pmatrix} 0 & x_1^1 & x_2^1 & 0 & 1 \\ x_2^2 & 0 & 0 & 1 & 0 \\ 0 & x_1^2 & x_2^2 & 0 & 1 \\ x_2^3 & 0 & 0 & 1 & 0 \\ 0 & x_1^3 & x_2^3 & 0 & 1 \end{pmatrix} - x_2^1 \cdot \det \begin{pmatrix} 0 & x_1^1 & x_2^1 & 0 & 1 \\ x_1^2 & 0 & 0 & 1 & 0 \\ 0 & x_1^2 & x_2^2 & 0 & 1 \\ x_1^3 & 0 & 0 & 1 & 0 \\ 0 & x_1^3 & x_2^3 & 0 & 1 \end{pmatrix} +$$

$$+ \det \begin{pmatrix} 0 & 0 & x_1^1 & x_2^1 & 1 \\ x_1^2 & x_2^2 & 0 & 0 & 0 \\ 0 & 0 & x_1^2 & x_2^2 & 1 \\ x_1^3 & x_2^3 & 0 & 0 & 0 \\ 0 & 0 & x_1^3 & x_2^3 & 1 \end{pmatrix} = -x_1^1 x_2^2 \det \begin{pmatrix} x_1^1 & x_2^1 & 0 & 1 \\ x_1^2 & x_2^2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ x_1^3 & x_2^3 & 0 & 1 \end{pmatrix} +$$

$$+ x_1^1 \det \begin{pmatrix} 0 & x_1^1 & x_2^1 & 1 \\ 0 & x_1^2 & x_2^2 & 1 \\ x_2^3 & 0 & 0 & 0 \\ 0 & x_1^3 & x_2^3 & 1 \end{pmatrix} + x_2^1 x_1^2 \det \begin{pmatrix} x_1^1 & x_2^1 & 0 & 1 \\ x_1^2 & x_2^2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ x_1^3 & x_2^3 & 0 & 1 \end{pmatrix} - x_2^1 \det \begin{pmatrix} 0 & x_1^1 & x_2^1 & 1 \\ 0 & x_1^2 & x_2^2 & 1 \\ x_1^3 & 0 & 0 & 0 \\ 0 & x_1^3 & x_2^3 & 1 \end{pmatrix} -$$

$$- x_1^2 \det \begin{pmatrix} 0 & x_1^1 & x_2^1 & 1 \\ 0 & x_1^2 & x_2^2 & 1 \\ x_2^3 & 0 & 0 & 0 \\ 0 & x_1^3 & x_2^3 & 1 \end{pmatrix} + x_2^2 \det \begin{pmatrix} 0 & x_1^1 & x_2^1 & 1 \\ 0 & x_1^2 & x_2^2 & 1 \\ x_1^3 & 0 & 0 & 0 \\ 0 & x_1^3 & x_2^3 & 1 \end{pmatrix} =$$

$$= \left(x_1^2 x_2^1 - x_1^1 x_2^2\right)\begin{pmatrix} x_1^1 & x_2^1 & 1 \\ x_1^2 & x_2^2 & 1 \\ x_1^3 & x_2^3 & 1 \end{pmatrix} + \left(x_1^1 - x_1^2\right) x_2^3 \begin{pmatrix} x_1^1 & x_2^1 & 1 \\ x_1^2 & x_2^2 & 1 \\ x_1^3 & x_2^3 & 1 \end{pmatrix} +$$

$$+\left(x_2^2 - x_2^1\right) x_1^3 \begin{pmatrix} x_1^1 & x_2^1 & 1 \\ x_1^2 & x_2^2 & 1 \\ x_1^3 & x_2^3 & 1 \end{pmatrix} =$$

$$= \left(x_1^2 x_2^1 - x_1^1 x_2^2 + x_1^1 x_2^3 - x_1^2 x_2^3 + x_1^3 x_2^2 - x_1^3 x_2^1\right) \det \begin{pmatrix} x_1^1 & x_2^1 & 1 \\ x_1^2 & x_2^2 & 1 \\ x_1^3 & x_2^3 & 1 \end{pmatrix} =$$

$$= -\left(x_1^2 x_2^3 - x_2^3 x_1^1 - x_1^2 x_2^1 + x_1^3 x_2^2 + x_1^1 x_2^2 + x_1^3 x_1^2 + \underline{x_1^1 x_2^1} - \underline{x_1^1 x_2^1}\right) \cdot$$

$$\cdot \left(x_1^2 x_2^3 - x_1^3 x_2^2 - x_1^1 x_2^3 + x_1^3 x_2^1 + x_1^1 x_2^2 - x_1^2 x_2^1 + \underline{x_1^1 x_2^1} - \underline{x_1^1 x_2^1}\right) =$$

$$= -\left[\left(x_1^2 - x_1^1\right)\left(x_2^3 - x_2^1\right) - \left(x_1^3 - x_1^1\right)\left(x_2^2 - x_2^1\right)\right]^2 =$$

$$= -\det \begin{pmatrix} x_1^2 - x_1^1 & x_2^2 - x_2^1 \\ x_1^3 - x_1^1 & x_2^3 - x_2^1 \end{pmatrix}^2 .$$

1.b

$$N = \begin{pmatrix} x_1^1 & -x_2^1 & 1 & 0 \\ x_2^1 & x_1^1 & 0 & 1 \\ x_1^2 & -x_2^2 & 1 & 0 \\ x_2^2 & x_1^2 & 0 & 1 \end{pmatrix}$$

Prove:

$$\det N = \|x^1 - x^2\|^2$$

Proof:

$$\det(N) = \det\begin{pmatrix} x_1^1 & x_1^1 & 1 \\ x_1^2 & -x_2^2 & 0 \\ x_2^2 & x_1^2 & 1 \end{pmatrix} + \det\begin{pmatrix} x_1^1 & -x_2^1 & 0 \\ x_2^1 & x_1^1 & 1 \\ x_2^2 & x_1^2 & 1 \end{pmatrix} =$$

$$= \det\begin{pmatrix} x_1^2 & -x_2^2 \\ x_2^2 & x_1^2 \end{pmatrix} + \det\begin{pmatrix} x_2^1 & x_1^1 \\ x_1^2 & -x_2^2 \end{pmatrix} - \det\begin{pmatrix} x_1^1 & -x_2^1 \\ x_2^2 & x_1^2 \end{pmatrix} + \det\begin{pmatrix} x_1^1 & -x_2^1 \\ x_2^1 & x_1^1 \end{pmatrix}$$

$$= (x_1^2)^2 + (x_2^2)^2 - x_2^1 x_2^2 - x_1^2 x_1^1 - x_1^1 x_1^2 - x_2^1 x_2^2 + (x_1^1)^2 + (x_2^1)^2 =$$

$$= (x_1^1 - x_1^2)^2 + (x_2^1 - x_2^2)^2 = \|x^1 - x^2\|_2^2 .$$