TS: A is well defined:

11 Ax II 2 = 
$$(\Xi 1 A x 1^2)^{1/2} = (\Xi 1 A x 1^2)^{1/2} = (\Xi 1 X 1^2)^{1/2} = (\Xi 1 X 1^2)^{1/2}$$

A is injective

consider any sequence (Andneire, lyndein s. E.

(b) TS there is a constant C>O s.t.

11 AN11 2 4 CHXII2 4 XET(IN)

FOT x = 0 Ax=0

for x to 11 An 11 5 clix11

=> IIANII & C

=> \[ \langle \frac{11 \text{A \text{A \text{II}}}{\text{I \text{A \text{II}}}} : \text{7 \text{E \text{2}}^2 (1N) - \langle 0} \] is bounded above by some CEIR ... supremum exists.

The supremum is  $\|A\| = \sup_{x \in \mathbb{R}} \sqrt{\frac{\|Ax\|}{\|x\|}} : x \in L^{2}(\mathbb{N}) - Logg$ 

CORR HAIL = 11KAIL

=> 11Ax11 = 11A1111x11 => 11A11 is the smallest possible value of c.

## To show: A is cond.

kut (xn) new ->x in & l2(1N)

⇒ 2n-x →0 ⇒1124-x1100

.. Xu'x E Y, (IN) => (Xu-x) E J, (IN)

bounded it A sounded

11 A (xn-xi) 11 5 C 11 xn-x 11 -> 0 as a as n > 0

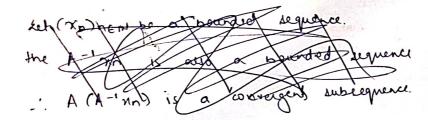
OF IIRA-MAII

=) Ann-An -o

=) Ann - An in l2(IN) so, h is ont.

## Exercise 4

At A-1 -> bounded toward operator



Red (Kn) nery be a bounded segmence

A 3 compad -> there is a subsequence S(xn), 3 such that A x mrs, converges.

Elia (m) Es bounded and

And since A-1 is bounded => continuous the sequence \$ A-1(A many)
converges

→ @ A-1 A is compact

=) I is compad

nowerer it is not possible eine & A: x - y where x descinfinite .: Our assumption is wrong and dimensional