Exercise 4

~~[illegible]~~) $f_n = \sum_{i=1}^{n} c_i b^{\lambda_i}$ (T.S → it has atmost $n-1$ zeroes)

Proof by induction where λ_i's are pairwise distinct

for $n = 1$ it holds true and not all c_i's can be zero.

By induction hypothesis assume for all functions f_N $(1 \leq N \leq n)$ ~~[illegible]~~

$n-1$ roots

Let $f_{n+1} = \sum_{i=1}^{n+1} c_i b^{\lambda_i}$ not all c_i's are zero

~~[illegible]~~ WLOG let c_1 ~~[illegible]~~ $\neq 0$

Then $f_{n+1} = \sum_{i=1}^{n+1} c_1 b^{\lambda_1} \left(1 + \sum_{i=2}^{n+1} \frac{c_i}{c_1} \left(\frac{b^{\lambda_i}}{b^{\lambda_1}}\right)\right) = c_1 b^{\lambda_1} (1 + f_N)$

for some $N \leq n$

Contrary to our hypothesis f_{n+1} has more than n roots. Then $(1 + f_N)$ has more than n roots. By Rolle's theorem $(1+f_N)' = f_N'$ has more than $n-1$ roots which contradicts our induction hypothesis.

∴ we can infer f_{n+1} has no more than n roots.

and f_n has atmost $n-1$ roots.